Optimal Interest Rate Policy in a Small Open Economy

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Abstract

Using an optimizing model we derive the optimal monetary and exchange rate policy for a small stochastic open economy with imperfect competition and short run price rigidity. The optimal monetary policy has an exact closed-form solution and is obtained using as welfare criterion the utility function of the representative agent. The paper concludes that the optimal policy depends crucially on the source of stochastic disturbances affecting the economy, much as in the standard literature pioneered by Poole (1970). Under the optimal policy the exchange rate floats, but there is a positive correlation between domestic and rest-of-the-world interest rates. That means that the nominal exchange rate does not carry all the burden of adjustment, and may therefore move “relatively little” in equilibrium. This may provide a theoretical rationale for the “fear of floating” recently documented empirically for open emerging economies.

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1. Introduction

After the exchange rate crises of the last decade many small open economies, both rich and poor, have adopted flexible exchange rates in combination with some kind of monetary or interest rate rule. Exactly what such a rule should look like, however, remains very much an open question. In closed economies, inflation-targeting or Taylor-type rules are common, even though the optimality of such rules is yet to be established analytically. In the open economy, host of additional tricky issues turn up. Should monetary policy respond systematically to the nominal or real exchange rate? Equivalently, should the float be completely clean or not? If dirty, should monetary policy act through direct intervention in the exchange market or through changes in target interest rates? If the exchange rate is targeted, however loosely, how to avoid the credibility problems that are endemic to this kind of policy?

These crucial questions are only now beginning to receive systematic attention in the context of state-of-the-art models. There is work on optimal monetary policy that considers different model structures with price stickiness. However, previous research has been based on closed or two-country models, or has lacked of an adequate welfare criterion for open economies.

It seems important to have for the small open economy a simple representation of the optimal monetary policy, derived explicitly from the utility function of the representative agent. The aim of this paper is to develop such an optimizing model and its corresponding policy rule. We consider a small stochastic open economy with imperfect competition and short-run price rigidity (one-period nominal contracts for prices.) The utility function of the representative local agent has an exact closed-form solution, and both the first and the second moments of variables matter for utility. Using this welfare functional we can compute the optimal interest rate policy, which also as an exact, closed-form solution. In particular, we study how monetary policy should respond to shocks to productivity and international interest rates. Implicitly, this sheds light on the question of what the optimal exchange rate system should be. We also compare, in terms of welfare, this optimal monetary policy with alternative rule-of-thumb regimes.

Our results appear to be in accordance to conventional wisdom and empirical

\footnote{In the context of an earlier generation of models, some of these issues were discussed, for instance, by Fischer (1977) and Flood (1979).}

\footnote{We consider only interest rate rules, but given money demand one could easily determine the quantity movements that could implement or support the chosen interest rules.}
evidence. We find that the optimal monetary policy of this small open economy requires a positive correlation between domestic and the rest of the world’s interest rates. Indeed, the parameter associated to the foreign interest rate shock in the reaction function of the monetary authority is positive. It is also less than one, which implies that the optimal policy does not require a full offset by the domestic interest rate.

The intuition is as follows. An unanticipated shock of the foreign interest rate can cause movements in the level of domestic consumption and the real exchange rate. In particular, a positive foreign interest rate innovation, other things equal, would lead to a depreciation of the exchange rate, affecting domestic consumption via both the level and the volatility of the relative price of home consumption. Some smoothing of the movements in the real exchange rate is therefore called for. The monetary authority of our small open economy model, when faced with such an external shock (a foreign interest rate shock,) should respond increasing its interest rate until it compensates, to some extent, for this external rate hike.

Hence, under the optimal policy the exchange rate floats, but given the required co-movement between domestic and rest-of-the-world interest rates, the nominal exchange rate does not carry all the burden of adjustment, and may therefore move “relatively little” in equilibrium. This may provide a theoretical rationale for the “fear of floating” recently documented empirically for open emerging economies.3

We also study the optimal interest rate reaction to a temporary productivity shock, and show that interest rate policy should not react when faced with such an innovation. The intuition in this case is given by the fact that domestic output is demand-determined in the short-run; therefore, a temporary positive productivity shock does not produce any effect on consumption or the real exchange rate; the productivity increase is simply absorbed by a fall in domestic labor supply. Since price exceeds marginal cost (because of monopolistic competition), domestic output remains unchanged.

How is our contribution in this paper related to the earlier literature? Recently a wave of contributions has attempted to explain macroeconomic consequences of monetary policy using two different approaches. First, in dynamic neo-Keynesian models, which consider optimizing agents, nominal rigidities, and a key role for monetary policy, the solution is determined numerically. These models compare alternative monetary policies and exchange rate regimes.4 On the other hand,

4See Clarida, Galí and Gertler (1999) for a survey of this approach. For open economy appli-
there are also more tractable models, in which the solution is obtained analytically. These models are mostly not dynamic, but they permit closed-form solutions and exact utility calculations.5

Three features set our paper apart from the literature contained by these two approaches. First, in contrast to a large number of papers that have assumed a closed economy or two equal-size country models, we have assumed a small open economy. In particular, the effects of variability of the exchange rate play a key role in the reaction of the economy to specific shocks.

Second, our approach derives the optimal monetary policy from a welfare function based on the utility of the consumer, and not arbitrary loss functions present in most of the literature.

Finally, it is also different from previous analyses in that we consider the interest rate as the instrument of monetary policy rather than money supply, as it is found in many models based on Obstfeld and Rogoys (1995.) An exception is the paper by Galí and Monacelli (2000), which considers a small open economy version of a dynamic New Keynesian model with staggered price-setting à la Calvo.

The paper is organized as follows. Section 2 contains a description of the theoretical model. Section 3 presents the analysis of the optimal monetary policy. Section 4 presents a closed form solution of the welfare function based on the utility function of the representative agent. The final section summarizes the results and their implications for models of monetary policy.

2. A Sticky-Price Model

The model is a stochastic small open economy version of Obstfeld and Rogoys (1998) and Corsetti and Pesenti (2000) in which the current account is always zero in equilibrium.

In the small economy, Home agents consume a variety of Home and Foreign goods. Every agent is both consumer and producer who manufactures monopolistically a single tradable good. They can hold two types of assets, money and bonds, and supplies labor implicitly. As Obstfeld and Rogoys (1996) stress, monopoly plays a key role in the analysis because it helps to justify the Keynesian

assumption that output is demand determined in the short run when prices are fixed. The key assumption of the model is that agents set next period’s prices before production and consumption is realized.

Having described the general setup of the model, we proceed in three steps. First, we explain the country and economic size aspects of the model. Second, we outline the main building blocks of the model and its micro-foundations. Finally, in section 3 we embed these relationships in an otherwise conventional model.

2.1. Country Size

In this model, a small open economy (Home) and the rest of the world (Foreign) compose our economy. Home agents are indexed by the interval \([0; n]\), while Foreign agents reside on the interval \((n; 1)\). We assume that, as in Galí and Monacelli (1999), the rest of the world is a limiting case of an open economy, in which the consumption of the small open economy goods is insignificant. In other words, we consider the rest of the world as a closed economy where its aggregate consumer price index (CPI) is exactly equal to the price index of goods produced within this economy.

Similarly, every individual has a monopoly in producing a single tradable good, also indexed by \(n\) in the interval \([0; 1]\). Thus, \(n\) indicates both the population size and the economic size of the Home country. As we demonstrated, the model requires that the weight in the utility function (economic size) is the same as the country size.\(^6\)

2.2. Individual Preferences and Technology

The small open economy has a continuum of consumers-producers indexed by \(i \in [0; n]\), where a home representative individual maximizes the expected value of

\[
U^i_t = E_t \left( \frac{\mu}{1 + \pm \left( \frac{C^i_s}{1} \right)^{1/2}} \left( \frac{\kappa_s}{2} \right)^2 \right); \quad (2.1)
\]

where \(\pm\) is the rate of time preference and \(1=\frac{1}{2}\) is the elasticity of intertemporal substitution. The notation \(E_t[x_{t+j}]\) represents the expectation of variable \(x_{t+j}\) conditional on information available at \(t\).

The utility is separable in two arguments:

\(^6\)See appendix 1.
1. Consumption ($C_i$): Preferences over goods over time (utility is concave in consumption).

2. Individual Output ($Y_i$): Individuals produce output with labor input. This term captures the disutility the individual experiences from having to produce more output. The stochastic parameter $\kappa$ represents an inverse productivity shock.

The aggregate real consumption index $C_t$ for any person $i$ is given by

$$C_t = \frac{C_{H,t}^i n C_{F,t}^i n}{n^0(1 - n)^{\mu} n};$$

where $C_{H,t}^i$ and $C_{F,t}^i$ are the consumption quantities that Home agents consume of domestic and foreign goods, respectively. And $n$ is the share of the total consumption of the small open economy goods consumed by both regions.

The two consumption subindexes are symmetric and are defined, as in Dixit and Stiglitz (1977), by

$$C_{H,t}^i = \int_0^1 \frac{\mu n}{\mu} Z_n C(j) \frac{\mu - n}{\mu - n} dj; \quad C_{F,t}^i = \int_0^1 \frac{\mu n}{\mu} Z_n C(j) \frac{n - n}{n - n} dj.$$  

Analogously to Obstfeld and Rogoff (1998) and Corsetti and Pesenti (2000), the elasticity of substitution across goods produced within a country is $\mu > 1$, while the elasticity of substitution between indices of the small open economy and the rest of the world is 1.

Rest-of-the-world agents have identical preferences. Foreign values of the corresponding domestic variables will be denoted by an asterisk (*) and may differ from home variables. Preferences over consumption goods are symmetric across regions because it is assumed that the elasticity of substitution ($1 = \mu$) and the rate of time preference ($\pm$) are the same across countries.

With the specification allowing for a continuum of differentiated goods in both countries, we are explicitly assuming that goods markets are imperfectly competitive while labor markets are perfectly competitive. Implicitly, technology is linear in labor across countries, which is competitively supplied. Specifically, labor is homogeneous with respect to labor of other agents; hence, individuals do not have any degree of monopolistic power. Agents simply take the real wage as given and
then decide how much to work. On the other hand, rms hire labor and produce goods that are imperfect substitutes of each other. As a consequence, each rm enjoys some monopoly power in the goods market. Consequently, nominal rigidities are represented only by prices in the goods market. Therefore, the degree of monopolistic competition is measured by the elasticity of goods substitution \( (\mu = \mu^f) \).

2.3. Prices and Demand Curve Facing Each Monopolist

Home prices indexes for the two preceding consumption baskets, denoted by \( P_{H,t} \) and \( P_{F,t} \) are defined as

\[
P_{H,t} = \frac{1}{n} \sum_{j=0}^{n} P_T(j)^{1i} \mu_j \quad ; \quad P_{F,t} = \frac{1}{1/n} \sum_{j=0}^{1/n} P_T(j)^{1i} \mu_j
\]

where the domestic currency price index for overall real consumption \( C_t \) is given by

\[
P_t = P_{H,t}^n P_{F,t}^m.
\]

The Law of One Price (LOP) holds across all individual goods since agents of the small open economy and the rest of the world have identical preferences, so that \( P_t(j) = S_t P_t^n(j) \); \( 8 j \in [0;1] \), where \( P_t(j) \) and \( P_t^n(j) \) are the prices of good \( j \) in the small economy and the rest of the world, respectively, and \( S_t \) represents the nominal exchange rate. Equivalently, \( P_{F,t} = S_t P_{H,t}^n \) and \( P_{H,t} = S_t P_{F,t}^n \). Define the real exchange rate as \( Q_t = \frac{S_t P_t^n}{P_t} \). This relative price will play a key role in what follows.

\[
P_t = (P_{H,t})^n S_t P_{H,t}^n = (P_{H,t})^n (S_t P_t^n)^{1i} \quad \tag{2.6}
\]

The rest of the world behaves, in the limit, as a closed economy. This fact entails that foreigners expenditure share in home goods is negligible, i.e., \( P_t^n = P_{H,t}^n \). Hence, we can use \( P_{H,t} \) or \( P_t^n \) in the previous equations.

The small economy commodity demand functions (home good demand: \( j = h \) and foreign good demand: \( j = f \) ) resulting from cost minimization are:

\[
C_t(h) = \frac{1}{n} \frac{P_t(h)}{P_{H,t}} \mu_h C_{H,t}; \quad C_t(f) = \frac{1}{1/n} \frac{P_t(f)}{P_{F,t}} \mu_f C_{F,t} \quad \tag{2.7}
\]
Using the definition of total consumption (equation (2.2)), we can derive the demand functions for home and foreign goods

\[ C_{H:t} = n \frac{P_{H:t}}{P_t} C_t; \quad C_{F:t} = (1 - n) \frac{P_{F:t}}{P_t} C_t. \quad (2.8) \]

Thus, plugging equation (2.8) into equation (2.7), it follows that the Home demand for Home and Foreign goods is given by

\[ C_t(h) = \frac{P_t(h)}{P_{H:t}} \frac{P_{H:t}}{P_t} C_t; \quad C_t(f) = \frac{P_t(f)}{P_{F:t}} \frac{P_{F:t}}{P_t} C_t. \quad (2.9) \]

2.4. Asset Markets and Individual and Government Budget Constraints

Home Agents own an equal share of all domestic firms. They can also hold two assets: a domestic bond denominated in terms of home currency (\(B_t\)) and a foreign bond (\(B^\xi_t\)) denominated in terms of foreign currency. Thus, the individual household constraint, expressed in units of tradable goods, is given by

\[ P_t B_t + S_t B^\xi_t + P_t C_t = (1 + \xi_t^H) P_t B_{t-1} + (1 + \xi^\xi^F) S_t B^\xi_{t-1} + P^\xi_{H:t} Y^\xi_{H:t}; \quad (2.10) \]

where \(B_t\) and \(B^\xi_t\) denote end of period \(t\) bonds holdings. The variable \(Y^\xi_{H:t}\) is output produced by each agent, while \(P^\xi_{H:t}\) is its domestic currency price and \(\xi^F\) and \(\xi^\xi\) are nominal interest rates at home and abroad, respectively.

2.5. Market Clearing in the Small Open Economy

Regardless of whether prices are sticky or flexible, tradable goods market requires that output equals demand,

\[
\begin{align*}
nP_t C_t + (1 \cdot n) P_t C^\xi_t &= nP_{H:t} Y; \\
(1 \cdot n) [nP_t C_t + (1 \cdot n) P_t C^\xi_t] &= (1 \cdot n) P_{F:t} Y^\xi.
\end{align*}
\quad (2.11)
\]

From which we get

\[ Y_t^\xi = Y_t P_{F:t}; \quad (2.12) \]

This condition takes the same form as Corsetti and Pesenti (2000) and Obstfeld and Rogo\(x\) (1998). Expression (2.12), together with the assumption that the initial net international asset holdings, \(B\), is 0, implies that the current account is
always zero. Furthermore, this equation shows that countries has constant shares of world income at all times independently of any shock. Therefore, countries always consume all their real income, because of the assumption of isoelastic preferences over total consumption and the constant real income share, i.e., the current account is always 0 despite of the shocks. This implies that³

\[ Y_t = \frac{P_t}{P_{H:t}} C_t; \quad Y_t^u = \frac{P_t^u}{P_{H:t}} C_t^u. \]  

(2.13)

3. Equilibrium

When computing the equilibrium we assume that the Central Bank uses an interest rate rule, and as we stress before, prices are set one period in advance of the realization of the shocks.

3.1. First Order Conditions

The representative consumer chooses his optimal holdings of bonds, consumption and wage level to maximize his expected utility (equation (2.1)) subject to the usual budget constraint (equation (2.10)). We wage-setting problem in the next section, and focus on the other choices faced by the consumer here. If \( \lambda_t \) is the Lagrange multiplier associated with budget constraint 2.10, the FONCs with respect to consumption, domestic bonds, and foreign bonds respectively are

\[ \lambda_t = i C_t^{1/2} \]  

(3.1)

\[ \lambda_t = (1 + \xi_t) \frac{1}{1 + \xi_t} E_t \frac{1}{1 + \xi_t} \frac{p_t}{p_{t+1}} \]  

(3.2)

\[ \lambda_t = (1 + \xi_t) \frac{1}{1 + \xi_t} E_t \frac{1}{1 + \xi_t} \frac{p_t}{p_{t+1}} \frac{S_t}{S_{t+1}} \]  

(3.3)

Combining the three we have

\[ i C_t^{1/2} = -E_t i C_t^{1/2} (1 + \xi_{t+1}) \frac{p_t}{p_{t+1}} \]  

(3.4)

³It is important to note that the key assumption to get this result is to assume that economic size is the same as population size. See appendix 1.
which is the traditional intertemporal Euler equation for total real consumption. In turn, combining conditions (3.2) and (3.3) we have

\[ E_t (1 + \frac{\mathbb{E}_{t+1}}{P_{t+1}}) (1 + \frac{\mathbb{E}_{t+1}}{P_{t+1}}) \frac{S_{t+1}}{S_t} = 0; \tag{3.5} \]

which is the standard arbitrage condition between domestic and foreign assets.

Assuming that the natural logarithms of the exogenous variables are jointly normally distributed, we can express the equilibrium conditions in logs. For the sake of clarity, we define the natural logarithm of any variable \( X \) by \( x \), and the date \( t \) unconditional variance of \( x_t \), \( \text{Var}_{t} \{ x_t \} \), by \( \sigma^2_x \).

Taking logs of (3.4) we can express the consumption Euler equation as a function of endogenous variances:

\[
\frac{1}{2} (E_{t+1} C_{t+1} - C_t) = i \pm \frac{1}{2} \left[ \frac{1}{2} \sigma^2_p + \frac{1}{n} \sigma_{pq} \right] + \frac{1}{2} \left[ \frac{1}{2} \sigma^2_q + \frac{1}{n} \sigma_{cq} \right]; \tag{3.6}
\]

where \( i = \log(1 + \phi) \). We express the variance of the price in terms of the variance of the real exchange rate. Since \( p_t = n p_{H,t} + (1 - n) p_{F,t} \), \( p_{H,t} \) and \( p_{F,t} \) are predetermined, the variance of \( p_t \) depends only on the variance of the nominal exchange rate, i.e., \( \sigma^2_p = (1 - n)^2 \sigma^2_q \). Furthermore, the variance of the real exchange rate, \( \sigma^2_q \), is composed by the variance of the nominal exchange rate, the variance of the price level, and the covariance between them. However, after some simplifications, we get that \( \sigma^2_q = n^2 \sigma^2_q \). Thus, \( \sigma^2_p = i \frac{1}{n} \sigma^2_{pq} \).

Similarly, for the same reasons, instead of the covariance between consumption and price, we include an expression for the covariance between consumption and the real exchange rate, i.e., \( \sigma^2_{cp} = i \frac{1}{n} \sigma^2_{cq} \).

\[ A \text{ variable } X \text{ is log normally distributed if } x = \ln(X) \equiv N \left( 1, \frac{\sigma^2}{2} \right). \text{ Thus, if } \ln(X) = x \text{ then } X = e^x. \text{ In this case } E[X] = E[e^x] = m(x), \text{ where } m(x) \text{ is the moment generating function for } x \text{ and is given by}
\]

\[ M(x) = \int_{-\infty}^{\infty} e^{x} \frac{1}{2\pi \sigma_x^2} e^{-\frac{(x-x_0)^2}{2\sigma_x^2}} \, dx \]

Therefore,

\[ E[X] = e^{x} + \frac{1}{2} \sigma^2 \]
On the other hand, the rest of the world log Euler equation takes a closed-economy form because of our assumption of negligible consumption of small economy goods by foreigners. In that case, the log version of the foreign Euler equation becomes

$$\frac{1}{2} E_t c^v_{t+1} - c^p_t = i_t + \frac{1}{2} \gamma^2 \partial c_t + \frac{1}{2} \gamma^2 \partial p_t$$

where $i^v = (1 + \varphi)$. 

Next we log-linearize arbitrage condition 3.5:

$$i_t = i^v_t + E_t s_{t+1} + s_t + \frac{1}{2} \gamma^{3/2}$$

In real terms it becomes

$$i_t (E_t p_{t+1} + p_t) = i^v_t (E_t p^v_{t+1} + p^v_t + (E_t q_{t+1} + q_t) + \frac{1}{2} \gamma^{3/2} + \frac{1}{n^{3/2}})$$

The equation characterizes the real uncovered interest parity condition (UIPC), in which movements in the real exchange rate are given by ex-ante real interest rate differentials.

### 3.2. Aggregate Demand and Output Determination

Market clearing for our small open economy require that domestic output be equal to demand. Previous equations have seen that domestic expenditure in home goods is a fraction $n$ of total expenditures. Specifically, plugging equation (2.13) into equation (2.8), one obtains this obvious condition for home consumption of home goods in terms of domestic production

$$C_{H,t} = n Y_t$$

Similarly, using the same set of equations, we can get an expression for home consumption of foreign goods

$$C_{F,t} = (1 - n) \frac{p_{H,t}}{S_t} Y_t$$
Combining these two conditions, we can express overall consumption in terms of domestic output and terms of trade

\[
C_t = \frac{\bar{A}}{S_t} \frac{P_{H,t}}{P_{H,t}^{P_H}} Y_t: \quad (3.12)
\]

The log version of this equation in terms of domestic output and the real exchange rate will be

\[
c_t = y_t i \frac{\mu_{i_n} q_t}{n} \quad (3.13)
\]

Note that in deriving equation (3.13) we used the assumption that the foreign country behaves as a closed economy, i.e., \(P_t^{P} = P_{H,t}^{P_H}\) and that \(q_t = n(s_t + p_t^{P_H} p_{H,t}).\)

This equation establishes a simple relationship linking domestic consumption with output and a proportional factor of the real exchange rate.

### 3.3. Summary of Equilibrium

In previous subsections, nonlinear stochastic equations appear quite complex. However, with the assumption of lognormal disturbances, they lead to a rather simple closed form solution. Specifically, we assume that the natural logarithms of the exogenous variables—the productivity shock \(k_t\) and the foreign interest rate shock \(\bar{\rho}_t\)—are all jointly normally distributed. Thus, we can fully describe the equilibrium dynamics of the small open economy with eqs. (3.6), (3.9), (3.13), and a monetary rule for the domestic interest rate.

For the sake of clarity, we rewrite the system in this section, reorganizing some equations and replacing the expected value of all the variables with their steady state values. The reason is that we are dealing with transitory unanticipated shocks, and hence, after the period of the shock all variables adjust freely to the pre-shock steady state level. Therefore, in analyzing the effects of monetary policy in presence of price stickiness it is enough to focus what happens in the period of the shock.

\[
\frac{1}{2} \{ c_t \} = \pm i \frac{1}{2} \frac{1}{\eta_2} \frac{\mu_{i_n}}{n} q_t + 2 \frac{1}{2} \frac{1}{\eta_2} \frac{\mu_{i_n}}{n} q_t \quad (3.14)
\]

\(^9\text{See Appendix 2.}\)
\[(c_t, \ddot{c}) = (y_t, \ddot{y}) + \frac{\mu}{n} (q_t, \ddot{q}) ; \tag{3.15}\]

\[i_t i (E_t p_{t+1} i p_t) = i^n_t i E_t p^n_{t+1} i p^n_t (q_t \ddot{q}) + \frac{1}{2} \gamma^2 ; \tag{3.16}\]

We have three equations and four unknowns: consumption, \(c_t\); output, \(y_t\); the real exchange rate, \(q_t\); and the expected real interest rate. The fourth equation that will close the system is the domestic central bank’s policy rule.

The equilibrium dynamics for the rest of the world take the equivalent form, but for a closed economy where \(P^n_{t} = P^n_{H,t}\):

\[\frac{1}{2}(c^n_t, \ddot{c}^n) = \pm i (\ddot{c}^n_t, \ddot{c}^n) + \frac{1}{2} \gamma^2 + \frac{1}{2} \gamma^2 + 2\gamma^2 \frac{1}{2} \gamma^2 ; \tag{4.1}\]

\[(c^n_t, \ddot{c}^n) = (y^n_t, \ddot{y}^n) ; \tag{3.18}\]

Again in the rest of the world case, we have fewer equations than unknowns: two of the former and three of the latter: consumption, \(c^n_t\); output, \(y^n_t\); and the expected real interest rate. The last equation, as in the small open economy, is the policy function.

4. Price-Setting

Home agents set prices for period \(t\) based on period \(t-1\) information and must satisfy all the demand at the quoted prices. It follows that the problem of home agent \(i\) in period \(t-1\) is to choose its price, \(P^n_{i,H,t}\), to maximize its objective function (equation (2.1)), but with the expected value conditional on date \(t-1\) information, i.e.,

\[E_{t-1} \left( \frac{\mu}{1 + \frac{1}{1} \left( \frac{1}{2} \mu \right)^{\frac{1}{2}} \left( c^n_{i} \ddot{c}^n \right)^{\frac{1}{2}}} \right) ; \tag{4.1}\]

The maximization of equation (4.1) is subject to the demand for individual goods (equation (2.9)) and the individual’s intertemporal budget constraint (equation (2.10)). Thus, the FONC is
In order to simplify this expression we can use two facts. First, $P_{H;t}$ is predetermined so we can take out from the expected value and second, in symmetric equilibrium $P_{H;t} = P_{H;\overline{t}}$. Hence, it follows that

$$E_{t;1} \left( C_t \right) \frac{1}{\gamma} \left( \mu \right) \left( 1 \right) \frac{1}{P_{H;\overline{t}}} \frac{1}{(P_{H;\overline{t}})^{\frac{1}{\gamma}}} \mu \frac{1}{P_{t} \left( C_t \right)} \frac{1}{P_{t}}$$

or equivalently

$$P_{H;\overline{t}} = \frac{\mu \left( \frac{1}{P_{t} \left( C_t \right)} \frac{1}{P_{t}} \right)}{E_{t;1} \left( C_t \right) \frac{1}{\gamma} \left( \mu \right) \left( 1 \right)}.$$

Equation (4.2) represents the optimal preset home good price, $P_{H;\overline{t}}$, where $\mu$ is the elasticity of substitution across goods, and therefore, $\frac{1}{\mu}$ corresponds to a fixed markup. In a deterministic setup, equation (4.2) would imply that the marginal utility of the real price $(P_{H;\overline{t}} = P_{t})$ is equal to a markup times the marginal disutility of producing a unit of good.

We can also obtain a log version of equation (4.2), assuming that all exogenous shocks are lognormal

$$\log \left( 1 + \frac{1}{2} E_{t;1} C_t \right) = \frac{1}{2} \left( \frac{1}{\mu} \right) \left( \frac{1}{n} \right) \left( \frac{1}{\gamma} \right) \left( \frac{1}{1} \right) \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right)$$

In equation (4.3), as in Obstfeld and Rogoey (1998), the effects of consumption volatility, $\frac{1}{2} \gamma$, over expected consumption is unclear because it will depend on whether $\frac{1}{2} \gamma$ is less or higher than 3. If $\frac{1}{2} \gamma < 3$, then the relationship is negative. Later in the paper, when we calculate ex-ante utility, we simplify algebraic expressions by using log utility ($\frac{1}{2} = 1$) preference, and thus, we would be within
the range where consumption variability affect negatively expected consumption all the times.

Similarly, both the variance of the real exchange rate, $\frac{\sigma^2_q}{\mu}$, and the productivity volatility, $\frac{\sigma^2_k}{\mu}$, have a negative effect on expected consumption. The higher the volatility of the real exchange rate or productivity, the lower expected consumption. The reason is that in both cases, the demand for both regions' goods is modified, which implies a change in the ex-ante marginal disutility of work. On the other hand, the demand shock has a positive effect over expected consumption.

Analogously, the logarithmic version for the rest of the world is

\[
(1 + \frac{\sigma^2_q}{\mu} E_t c_{t+1}^{\alpha}) = i + \frac{1}{2} \mu \left[ \frac{1}{2} \left( \frac{\sigma^2_k}{\mu} \right) i E_t k_{t+1}^{\alpha} + \log \frac{\mu}{\mu - 1} i E_t p_{t+1}^{\alpha} + \frac{1}{2} \frac{\sigma^2_c}{\mu} \right].
\] (4.4)

Notice that in the case of the rest of the world, the price setting equation takes a closed-economy structure where expected consumption depends only on foreign variables. The reason, as we mentioned before, is that foreigners' consumption of Home goods is insignificant.

5. Specifying Monetary Policy

The astute reader will have noticed that so far money or monetary policy have not entered the model. We could avoid introducing money explicitly because we describe monetary policy entirely in terms of interest rules. This means that, whatever the shape or form of the money demand function, each central bank lets money supply adjust endogenously so that a) the nominal interest rate is equal to its chosen rate and b) money demand is satisfied. We now specify the monetary authority reaction functions, which specify the setting of such chosen nominal interest rates at home and abroad.

5.1. Rest-of-the-World Reaction Function

The rest of the world follows a mechanical monetary policy. It follows from equation (3.17) that in steady state (when $c_{t+1}^{\alpha} = 0$, where overbars denote steady state values) the following real (risk-premium exclusive) interest rate must obtain:

\[
i_{t+1}^{\alpha} = \frac{1}{2} \mu \left( \frac{\sigma^2_k}{\mu} k_{t+1}^{\alpha} + \frac{\sigma^2_c}{\mu} c_{t+1}^{\alpha} + 2 \frac{\sigma^2_{cp}}{\mu} p_{t+1}^{\alpha} \right).
\] (5.1)
We assume that the actual foreign expected real interest rate deviates stochastically from this steady state rate:

\[ i^\mu_{t+1} - i^E_{t+1} \cdot p^\mu_t - p^\mu_t = \bar{r}^\mu + \eta^\mu, \]  

(5.2)

where \( \eta^\mu_t = \log^\mu_t \) has mean 0 and variance \( \frac{\sigma^2}{2} \).

5.2. Small Open Economy Reaction Function

The monetary authority of the small open economy designs an optimal monetary policy. Again we can derive an expression for the steady state interest rate using the log version of the domestic Euler equation. It follows from equation (3.14) that the (risk-premium exclusive) steady state domestic real interest rate –that which is obtained when \( c_t \cdot \dot{c} = 0 \) – is given by

\[ i_t = \bar{r} + \alpha_{k} k_t + \alpha_{\mu} \eta^\mu_t. \]  

(5.3)

We now postulate a reaction function such that

\[ i_t = \bar{r} + \alpha_{k} k_t + \alpha_{\mu} \eta^\mu_t. \]  

(5.4)

where the \( \alpha_{k} \) is the coefficient associated to the productivity innovation, while \( \alpha_{\mu} \) is the coefficient associated with the foreign interest rate shock.

6. A Closed-Form Solution

In this section we are able to characterize the optimal monetary policy using welfare considerations based on the utility function of the representative agent. The solution requires three major steps. First, we express the price setting equation in terms of logs and variances of logs of endogenous variables. Second, we calculate how endogenous variables respond to exogenous disturbances; then we are able to express the variances in terms of the variances of exogenous shocks. Finally, we optimize the welfare function obtaining the parameters values of the optimal monetary policy.
6.1. Solving for the Real Exchange Rate and Consumption

In this section, we are interested, given the two optimal monetary policies, in getting the optimal solution for the remaining variables of the model. In particular, we need to compute the endogenous variances in terms of the underlying exogenous shocks.

Using the uncovered interest parity condition and the two policy rules (eqs. (5.4) and (5.2)), we can compute the real exchange rate, and consumption in the small open economy.

Plugging eq. (5.4) into equation (??) yield

\[
\frac{1}{2} (c_t - \bar{c}) = r_t = i \tilde{A}_k k_t + \tilde{A}_r^n n_t: \tag{6.1}
\]

Now, the real UIPC together with the foreign interest rate rule turns into

\[
r_t = r^n_t (q_t - \bar{q}) + \mu \frac{3}{2} i n \frac{1}{n^2} = \bar{r} + \tilde{A}_k k_t + \tilde{A}_r^n n_t:
\]

Combining these two equations, which after rearranging becomes

\[
(q_t - \bar{q}) = i \tilde{A}_k k_t + (\tilde{A}_r^n n) + \mu \frac{3}{2} i n \frac{1}{n^2} (\bar{r} + \bar{r}^n):
\tag{6.2}
\]

Equations (6.1) and (6.2) give \(c_t\) and \(q_t\) as a function of the shocks and the variance of the real exchange rate. Notice that \(\bar{r}\) also depends on endogenous variances - that is variances that depend, in equilibrium, on what the domestic monetary authority does - and that \(\bar{r}^n\) depends on variances that are exogenous, and which we can take as given.

The next step is to express utility as a function of the shocks and exogenous variances.

6.2. Variances

As we noted before, we need to express endogenous variances in terms of exogenous variances. With this purpose, we can use eqs. (6.1) and (6.2) to obtain the real exchange rate variance, the consumption variance, and the covariance between consumption and the real exchange rate, respectively

\[
\frac{3}{2} \xi = \mu \tilde{A}_k^2 \frac{3}{2} + \mu \tilde{A}_r^n \frac{3}{2} +: \tag{6.3}
\]
\[
\frac{3}{\theta_i^2} = \tilde{A}_k^{3/2} + (\tilde{A}_{\mu, i} 1)^2 \frac{3}{\theta_i^2};
\]
\[
\frac{3}{\theta_{eq}^2} = \frac{A_k^2}{\frac{1}{2} \theta_i^2} + (\tilde{A}_{\mu, i} 1)^2 \frac{3}{\theta_i^2};
\]

(6.4)

(6.5)

6.3. Calculating Ex-Ante Utility

In this section, we derive the ex-ante utility to get a welfare measure in a closed-form.

Max \( E_{t_1} [U_t] \) = Max \( E_{t_1} \sum_{t=0}^{\infty} k_t (Y_t)^2 \)

where \( E_{t_1} [U_t^i] = E_{t_1} \sum_{i=1}^{n} \frac{(C_i^i)^{1/2}}{1_i^{1/2}} k_t (Y_t)^2 \).

Using the condition of optimal price setting (equation (4.2)) and the fact that \( P_t i_t = Y_t \) we get a relation between the two components of the utility function

\[
\frac{E_{t_1} [U_t^i]}{\mu} = \frac{E_{t_1} [C_i^i]^{1/2}}{(1_i^{1/2})} = \frac{E_{t_1} [k_t (Y_t)^2]}{\mu}.
\]

(6.6)

Therefore, the expected utility in term of consumption will be

\[
E_{t_1} [U_t^i] = \frac{2\mu_i (1_i^{1/2})(\mu_i 1)}{2\mu} E_{t_1} \frac{(C_i^i)^{1/2}}{1_i^{1/2}}.
\]

It is worth noting that expected utility depends on the degree of monopolistic power, \( \mu \), even in the logarithmic case (\( 1/2 = 1 \)). Therefore, without loss of generality, we can continue with the case of log utility of consumption.

In that case, expected utility will be equivalent to

\[
E_{t} [U_t^i] = E_{t_1} [C_t^i] \frac{(\mu_i 1)}{2\mu};
\]

(6.7)
where from equation (4.3) we have that

\[ E_{t_1} [c_t] = \frac{1}{2} \left( \frac{1}{n} \mu \frac{1}{2} \left( \frac{1}{2} \frac{n}{2} \right)^{\frac{3}{4}} \frac{1}{2} \frac{n}{2} \right) \]

\[ + \frac{1}{2} \log \frac{\mu}{\mu} \left( \frac{1}{2} \frac{n}{2} \right) \]

However, in this equation we have that expected consumption not only depends on variances and constants, but also on the expected real exchange rate.

Combining eqs. (2.12) and (3.13) to express the expected real exchange rate in terms of expected consumption

\[ E_{t_1} [c_t] = E_{t_1} [q_t] + \frac{1}{2} \frac{n}{2} \left( \frac{1}{2} \frac{n}{2} \right)^{\frac{3}{4}} \]

Thus, using the previous equation to eliminate \( E_{t_1} [q_t] \) from equation (6.8), one obtains

\[ E_{t_1} [c_t] = E_{t_1} [\tilde{c}_t] + \frac{1}{2} \frac{n}{2} \left( \frac{1}{2} \frac{n}{2} \right)^{\frac{3}{4}} \]

where \( \tilde{c}_t = E_{t_1} [\tilde{c}_t] \).

This would mean that expected consumption is unambiguously decreasing in both the variability of consumption and the variability of the exchange rate.

The next step is to express expected consumption in terms of exogenous variances using eqs. (6.3), (6.4), and (6.5). It follows that

\[ E_{t_1} [c_t] = \frac{1}{2} \frac{n}{2} \left( \frac{1}{2} \frac{n}{2} \right)^{\frac{3}{4}} \]

\[ + \frac{1}{2} \frac{n}{2} \left( \frac{1}{2} \frac{n}{2} \right)^{\frac{3}{4}} \]

\[ E_{t_1} [k_t] \]

\[ E_{t_1} [\tilde{c}_t] \]

\[ E_{t_1} [\tilde{c}_t] \]

\[ E_{t_1} [\tilde{c}_t] \]
6.4. Optimal Monetary Policy

In this section, we calculate the optimal values of the parameters of the monetary authority reaction function.

Plugging equation (6.9) into equation (6.7), the expected utility in terms of exogenous variances and constants would be

\[
E_t \mathbb{U}_t = \mu \frac{1}{n} \sum_{i} \frac{1}{n} \bar{\eta}^2 \frac{\bar{A}^{23/\hat{F}}}{k} + (\bar{A}_{\epsilon} i 1)^{23/\hat{F}}
\]

(6.10)

From the previous equation we can conclude unambiguously that the variance of the productivity and the foreign interest rate shocks affect expected welfare negatively, since \(0 < n < 1\).

Finally, we can choose the optimal values of the parameters associated to each shock in the central bank’s policy function to maximize the welfare function. We are looking for the policy who maximizes the expected utility. In order to do so, it is necessary to choose the parameters of the reaction function - \(\bar{A}_k\) and \(\bar{A}_{\epsilon}\) - to maximize equation (6.10).

Therefore,

\[\bar{A}_k = 0;\]

\[\bar{A}_{\epsilon} = 1 + n^2;\]

Therefore, the optimal monetary policy rule for the small open economy would take the following form:

\[r_t = \hat{r} + \frac{1}{n} \frac{1}{n^2} \bar{\eta} \bar{y} \frac{1}{2};\]

where \(\hat{r}\) is the steady state level of the ex-post real interest rate.

These results appear to be in accordance to conventional wisdom and empirical evidence. We find that the parameter associated to the temporary productivity...
shock is 0. In other words, the optimal monetary policy of this small open economy does not require any type of reaction when faced with temporary productivity shock.

The intuition of this result is given by the fact that domestic output is demand-determined in the short-run; therefore, a temporary rise in productivity shock does not produce any effect over consumption and the real exchange rate. Thus, the monetary policy does not need to respond to this specific shock, because the productivity increase is absorbed by a fall in domestic labor. A temporary change in productivity does not modify the fact that price exceeds marginal cost (because of monopolistic competition), and hence domestic output will remain unchanged. In sum, in presence of a transitory domestic productivity shock, domestic production is still demand-determined, and thus it is not necessary a monetary response by a monetary authority who tries to maximize agents' welfare.

On the other hand, we also study the optimal interest rate reaction to a temporary foreign interest rate innovation. We find that the optimal monetary policy of this small open economy requires a positive correlation between domestic and the rest of the world's interest rates. Specifically, the parameter that corresponds to the foreign interest rate innovation in the reaction is positive but less than one, \(0 < \alpha_n < 1\), which implies that the central bank does not require a complete reaction of domestic interest rate, because this would provoke high consumption variability. In the limit, if \(n \to 1\) \(\alpha_n \to 0\), which seems plausible, would mean that the more closed the economy is, the less sensitive to changes in the foreign interest rate (or backwards, as in Galí and Monacelli (2000), \((1 - n)\) could be interpreted as an “openness index.”)

In this case the intuition is as follows. An unanticipated shock of the foreign interest rate would create movements in the level of domestic consumption and exchange rate. In particular, a positive foreign interest rate innovation, other things equal, would lead to a depreciation of the exchange rate, affecting negatively domestic consumption via both the level and the volatility of the real exchange rate (prices are preset.) The impact of the depreciation would be too costly in terms of sustaining smooth consumption over time, which would require to partially offsetting the former with some instrument. Therefore, the monetary authority of our small open economy model, when faced with a positive external shock (a foreign interest rate shock,) should respond increasing its policy rate until they compensate, to some extent, this external hike. Beyond a particular point, a higher domestic interest rate would lead to a non-optimal Pareto equilibrium, which means that there is a precise domestic interest rate response where welfare
is maximized. In other words, this result suggests that an active role by Central Banks in small open economies is necessary, but up to a certain point; otherwise, monetary policy would begin to reduce welfare.

This result would also suggest that this optimal monetary policy requires an active monetary policy and some degree of flexibility in exchange rates. The reaction of the domestic interest rate would be less than proportional to the foreign interest rate increase, while the rest would be absorbed by the exchange rate. Therefore, a flexible exchange rate or at least, a managed exchange rate regime would be required to stabilize this small open economy. Of course, this will depend on the degree of openness of the economy. As the country tends to close up, the optimal regime will approach to a fixed exchange rate. It is also worth noting that since prices are preset, movements in the nominal exchange rate are fully translated into movements in the real exchange rate, and hence, depreciations are given only by changes in the nominal exchange rate.
7. Conclusions

Since the second half of the 1990s, a substantial number of central banks have moved towards monetary policies that combine flexible exchange rates with some kind of monetary or interest rate rule. In this context, it is essential to discuss what type of monetary policy rule is more convenient in terms of economic stability and countries’ welfare. Research on optimal monetary policies has received limited consideration, especially for small open economies, and only recently, there has been an interest to study the optimality of such policies under normative frameworks.

Some literature on optimal monetary policy considers different model structures with price stickiness. However, previous research has been based on closed or two-country equally size models, or has lacked an adequate welfare criterion for open economies. In view of that, our paper extends Obstfeld and Rogoff (1998, 2000) framework to a small open economy context in which foreign variables are not affected by any small economy action. In doing so, we developed an optimizing model, in which we derived the optimal monetary and exchange rate policy for a small stochastic open economy with imperfect competition and short run price rigidity. The optimal interest rate policy has an exact closed-form solution and is obtained using a welfare criterion derived from the utility function of the representative agent. This welfare criterion suggests that the monetary authority has to avoid large changes in both consumption and the real exchange rate. In particular, we require a policy that minimizes the effects of higher volatility in exogenous variables over consumption and the real exchange rate. Hence, the welfare analysis allows us to consider the optimal monetary policy for a country, which is subject to productivity and foreign interest rate shocks. With such a tool, it is possible to answer questions like: given unexpected changes in productivity and in external financial conditions, what is the optimal monetary policy reaction? What determines the response of the optimal monetary policy instrument? What is the exchange rate regime associated with this optimal monetary policy?

Our results appear to be in accordance to conventional wisdom and empirical evidence. We find that the optimal monetary policy of this small open economy requires a positive correlation between domestic and the rest of the world’s interest rates. Indeed, the parameter associated to the foreign interest rate shock in the reaction function of the monetary authority is positive. Furthermore, it should be noted that it is also less than one, which implies that we do not require a complete reaction of domestic interest rate, because this would cause high consumption...
variability.

The reaction of the domestic interest rate would be less than proportional to the foreign interest rate increase, while the rest would be absorbed by the exchange rate. Thus, a flexible exchange rate or at least, a managed exchange rate regime would be required to stabilize the small open economy.

The analysis also finds out the optimal degree of intervention for a small open economy that undergoes a temporary productivity shock. We conclude that, the optimal monetary policy in this framework should not react when faced with a productivity innovation. The reason is that domestic output is demand-determined in the short-run; consequently, a temporary increase in productivity shock would not produce any effect over consumption and the real exchange rate. Therefore, monetary policy would not need to respond to this specific shock, because the productivity increase is absorbed by a fall in domestic labor.

A natural next step for further research would be to extend the analysis with other type of shocks, such as cost push shocks. It would be also attractive to answer the same questions but in a model with more complex financial structure, as in Bernanke and Gertler (1989) and more recently Céspedes, Chang, and Velasco (2000), among others.
8. Appendices


This appendix has two purposes. First, together with the assumption that initial net international asset holdings is zero, we show the necessary condition of having constant shares of per capita world real income. Second, for the general case in which both countries consume both home and foreign good, we demonstrate that the population size, denoted by \( n \), has to be equal to the economic size, represented by \( \textcircled{e} \).

The market clearing conditions require that

\[
\textcircled{e} [nP_t C_t + (1 \lessdot n)P_t C_t^u] = n P_H Y \quad (8.1)
\]
\[
(1 \lessdot n) [nP_t C_t + (1 \lessdot n)P_t C_t^u] = (1 \lessdot n) P_F Y^u
\]

\[
n Y = n C_H + (1 \lessdot n) C_F^u \quad (8.2)
\]
\[
(1 \lessdot n) Y^u = (1 \lessdot n) C_H^u + n C_F
\]

Set (8.1) characterizes the equilibrium condition in which output supplies equal demands, while set (8.2) represents the equilibrium of quantities produced and consumed.

Dividing the two equations of set (8.1) we can get the following expression

\[
\frac{P_H}{P_F} = \frac{Y}{Y} \frac{1}{n} \frac{\textcircled{e}}{\textcircled{e}} \frac{n}{(1 \lessdot n) Y} \quad (8.3)
\]

Now, plugging equation (8.3) into the ratio of demands functions for the composite home and foreign goods (eqs. (2.8)), we can obtain an expression for relative consumption of home and foreign goods in terms of relative prices

\[
\frac{C_H}{C_F} = \frac{\textcircled{e}}{\textcircled{e}} \frac{P_F}{P_H} \quad (8.4)
\]

The parallel foreign relation is

\[
\frac{C_H^u}{C_F^u} = \frac{\textcircled{e}}{\textcircled{e}} \frac{P_F^u}{P_H^u} \quad (8.5)
\]
Thus, including eqs. (8.4) and (8.5) into equation (8.3) we can get the first and second equality, respectively, of

\[
\frac{\mu}{Y} = \frac{1_i n}{C_H} = \frac{1_i n}{C_F} \quad \text{(8.6)}
\]

which implies that

\[
C_H = C_F \quad \text{(8.7)}
\]

Next use set (8.2). Dividing the two equations of this set, we obtain the following expression

\[
\frac{n}{1_i n} \frac{Y}{Y^n} = \frac{nC_H + (1_i n)C_F}{(1_i n)C_H + nC_F} \quad \text{(8.8)}
\]

Plugging eqs. (8.6) and (8.7) into equation (8.8) and multiplying by \(\frac{CF}{CF}\)

\[
\frac{C_H}{C_F} = \frac{nC_H + (1_i n)C_F}{(1_i n)C_H + nC_F}
\]

Let \(x = \frac{C_H}{C_F}\), then

\[
x[1_i n]x + n = nx + (1_i n) \quad x = 1
\]

\[
\quad C_H = C_F \quad \text{(8.9)}
\]

We also know that \(C = \frac{C_H}{n^n(1_i n)^{1_i n}}\); \(C^n = \frac{(C_H^n)^{n}(C_F^n)^{1_i n}}{n^n(1_i n)^{1_i n}}\). Therefore, if we divide these expressions and consider equation (8.7) and equation (8.9) we get

\[
\frac{C}{C^n} = \frac{C_H}{C_H^n} = \frac{C_F}{C_F^n} = 1
\]

\[
\quad C = C^n \quad \text{(8.10)}
\]

Finally, we require a balanced current account, i.e., \(Y \cdot \frac{P}{P_H} C = 0\). Factorizing the first equation of set (8.1)
\[
Y \frac{P}{P_H} C = i \left( 1 + \frac{\mu}{n} \right) \frac{P}{P_H} C + \frac{\mu}{n} \frac{P}{P_H} C^* \tag{8.11}
\]

Therefore,
\[
Y \frac{P}{P_H} C = 0 \text{ iff } \frac{C}{C^*} = \frac{\mu}{1 + \frac{\mu}{n}} \tag{8.12}
\]

But we already know that \( \frac{C}{C^*} = 1 \), then we need that \( \frac{\mu}{n} = 1 \).

### 8.2. Real Exchange Rate and Terms of Trade: Some Identities

This appendix presents the derivation of some important identities, such as the real exchange rate and the terms of trade.

Define the real exchange rate as
\[
Q_t = \frac{E_t}{P_{t}^H} \frac{P_{t}}{P_{t}^F} = \frac{E_t}{P_{t}^H} \frac{P_{t}}{P_{t}^F}
\]

where \( E_t \) is the nominal exchange rate, while \( P_t \) and \( P_t^H \) are the price level of the Home and Foreign countries, respectively.

Similarly, we can define the terms of trade as
\[
X_t = \frac{P_{t}^F}{P_{t}^H} = \frac{E_t P_{t}^F}{P_{t}^H} = \frac{P_{t}^F}{P_{t}^H}
\]

where \( P_{t}^F \) and \( P_{t}^H \) are the price level of the goods produced by Foreign and Home countries, respectively.

The general price level of Home and Foreign are defined as
\[
P_t = P_{t}^n \frac{P_{t}^H}{P_{t}^F} \quad \text{and} \quad P_t^H = P_{t}^H \frac{P_{t}^F}{P_{t}^F} \theta
\]

Applying natural logs to these four equations and introducing one of the key assumptions of the model, that is assuming that the share of the Home goods price is negligible in the Foreign country as in a closed economy (i.e. set \( n = 0 \)), we get the following set of equations

\[
q_t = s_t + P_t^H \ i \ p_t \tag{8.13}
\]

\[
x_t = s_t + P_t^H \ i \ p_{tH}^H \tag{8.13}
\]

\[
p_t = nP_{tH} + (1 - n)P_{tF} \tag{8.13}
\]

\[
p_t^H = P_{t}^H \tag{8.13}
\]

Combining all the four equations we can obtain an expression of the real exchange rate as a function of the terms of trade

\[
q_t = nx_t \quad \text{as } \theta = 1 \tag{8.14}
\]
8.3. Price Rigidity

Home agents set prices for period $t$ based on period $t-1$ information and must satisfy all demands at the quoted prices.

It follows that the problem of home agent $i$ in period $t-1$ is to choose its price, $P_{H,t}$, to maximize its objective function. It follows that the first order condition is

$$E[C^{1i} (\mu_{i-1})] = E[k\mu_1 \frac{P_C}{P_H}]$$

Since home prices are predetermined $E[P_H] = P_H$, we can get an expression for home prices

$$P_H = \frac{s}{\mu_i 1 E[C^{1i} \frac{1}{2}]}$$ (8.15)

We also know that

$$P_t = P_{H,t} P_{F,t}$$ (8.16)

Using eqs. (8.15) and (8.16)

$$\mu P_H \frac{1}{P_{F,t}} = \frac{s}{\mu_i 1 E[C^{1i} \frac{1}{2}]}$$ (8.17)

Solving each component (with expected value),

$$E[kE^{2(1i-n)}C^2] = E[f \exp[\log k + 2(1i-n)e + 2c]g$$

$$= \exp[f \log k] + 2(1i-n)E[e] + 2E[c] + \frac{1}{2}ke$$

$$+ 2(1i-n)^2k2 + 2\frac{1}{2}ke + 2(1i-n)^3ke + 2\frac{1}{2}ke + 4(1i-n)^3ke$$

Similarly,

$$E[C^{1i} \frac{1}{2}] = \exp[(1i \frac{1}{2})E[c] + \frac{1}{2}(1i \frac{1}{2}k2)$$

Therefore, equation (8.17) turns to
\[ \frac{\mu P_H}{P_H} = \mu \frac{\mu}{\mu i} \exp \left[ \frac{1}{2} E [\log k] + (1 i n) E [e] + \frac{1}{2} (1 + \gamma) E [\xi] \right] + \frac{1}{4} \gamma^2 + (1 i n)^2 \gamma^2 + [1 i \frac{1}{4} (1 i \gamma^2)^2 \gamma^2 + (1 i n) \gamma \xi] \]

Applying logs (and assuming that \( \gamma k = \gamma c \), and knowing that \( \gamma^2 = \gamma^2 \) and \( \gamma e = \gamma e \) because prices are predetermined) we get:

\[ (1 i n) (p_H i p_H i E [e]) = \frac{1}{2} \log \frac{\mu}{\mu i} \frac{\mu}{\mu i} + \frac{1}{2} E [\log k] + \frac{1}{2} (1 + \gamma) E [c] \]
\[ + \frac{1}{4} \gamma^2 + (1 i n)^2 \gamma^2 + [1 i \frac{1}{4} (1 i \gamma^2)^2 \gamma^2 + 2(1 i n) \gamma e] \]

But we know that the real exchange rate is proportional to the terms of trade in the following way:

\[ E [q] = n E [s] = n (E [e] + p_H i p_H) \]
\[ (p_H i p_H i E [e]) = i \frac{1}{n} q \]

(remember that prices are preset in which case \( E [p_H i] = p_H \) and \( E [p_H i] = p_H \))

Thus,

\[ i \frac{1}{2} \frac{n}{n} E [q] = \frac{1}{2} \log \frac{\mu}{\mu i} \frac{\mu}{\mu i} + \frac{1}{2} E [\log k] + \frac{1}{2} (1 + \gamma) E [c] \]
\[ + \frac{1}{4} \gamma^2 + (1 i n)^2 \gamma^2 + [1 i \frac{1}{4} (1 i \gamma^2)^2 \gamma^2 + 2(1 i n) \gamma e] \]

Finally, we can obtain an expression for expected consumption (equation (4.3))

\[ E_{ti} [c] = \frac{1}{2} + \frac{1}{2} i \frac{1}{2} (1 i n)^2 \gamma^2 + \frac{1}{2} i \frac{1}{2} (1 i \gamma^2)^2 \gamma^2 + \frac{1}{4} (1 i n) \gamma e \]
\[ + \frac{1}{4} \log \frac{\mu i}{\mu} i E_{ti} [k] i \frac{1}{2} \gamma^2 + \frac{1}{4} i \frac{1}{n} E_{ti} [q] \]
References


