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Statistical Nonlinearities in the Business Cycle: A Challenge for the Canonical RBC Model*

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May 29, 2001

Abstract

Significant nonlinearities (skewness, kurtosis, conditional heteroskedasticity) are found in several cyclical components macroeconomic time series across countries. Standard equilibrium models of business cycles are successful at explaining most first and second moments of these time series. However, this paper shows that a model of this class cannot replicate the nonlinear features of the data. This finding is established applying Gallant and Tauchen's Efficient Method of Moments methodology (1996; 2000) to build an algorithm that searches over the model's parameter space to find the parameterization that allows the model to do its best at replicating all the statistical properties of the data. The results suggest that this parameterization captures the nonlinearities in investment but at the expense of failing to account for the observed properties of consumption.

JEL Codes: C13, E21, E32

Key Words: Nonlinear, Real Business Cycles, Efficient Method of Moments

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*I wish to thank Enrique Mendoza for guidance and support. I also would like to thank Larry Christiano, Michelle Connolly, Jonathan Heathcote, and the participants of the Duke Macro Lunchgroup and Workshop for many helpful comments. In particular, I would like to thank Katherine Smith and Kirk White. All errors that remain are my own. I also acknowledge support from the North Carolina Supercomputing Center for use of a Cray T90 Supercomputer to solve and estimate the model. A previous version of this paper circulated with the title "Can a Standard Nonlinear RBC Model Explain Nonlinearities in the U.S. National Accounts Data?"

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1 Introduction

The conditional distributions of cyclical components of many macroeconomic time series significantly deviate from a Gaussian distribution and exhibit conditional heteroskedasticity (ARCH, GARCH). This paper refers to these characteristics as nonlinearities in a statistical sense.[1] As Table 1 and Table 2 show, these nonlinearities are common for most National Accounts aggregates for the United States and for a sample of OECD countries including France, Italy, Japan, Mexico and the United Kingdom. Rothman (1996) and Razzak (2001) also document the existence of asymmetries in business cycles for many industrialized countries. Standard general-equilibrium models of business cycles are successful at explaining most first and second moments (mean, variance, covariance, persistence) of these time series. However, this paper shows that the canonical Real Business Cycle (RBC) model cannot replicate the nonlinearities reflected in the skewness, kurtosis, and conditional volatility characteristic of United States national accounts. This is the case even if the model's parameters are allowed to adjust to values that yield the best possible fit to the standard properties of the data. Moreover, the parameters that lead to simulations that most closely resemble the nonlinear features of the data differ sharply from those that would result if the nonlinearities are ignored, suggesting sensitivity of model parameters to nonlinear features.

Interest in the study of nonlinearities in macroeconomic time series surged during the 1980's. Neftci (1984) presented evidence that unemployment fluctuations were asymmetrical along the business cycle. He developed a statistical test of time series asymmetry and found significant evidence that increases in unemployment were of larger magnitude and lasted fewer periods than reductions in unemployment. In the financial time series literature, Hinich and Patterson (1985) found that daily movements in fifteen commonly traded stocks were generated by a nonlinear process.

In the second half of the 1980's there was great interest in the statistical modelling of macroeconomic and financial time series by using nonlinear deterministic chaos models. Some empirical studies in this vein posed the question of whether macroeconomic data were charac-

[1] See Gallant et al. (1993) This notion is different from the concept of nonlinearity used in the deterministic chaos literature(e.g. Potter (1999) or Brock (2000))

terized by nonlinear chaos, nonlinear stochastic processes or linear stochastic processes. Brock and Sayers (1988) tested whether real macroeconomic variables could be described by deterministic chaos. They found that while it was hard to find evidence of chaos, Post War employment, unemployment and industrial production could be well approximated by nonlinear stochastic models. Ashley and Patterson (1989) developed a test that detects deviations from linear stochastic processes, either in the form of nonlinear stochastic process or in the form of deterministic chaos. Their test of industrial production strongly "suggests that nonlinear dynamics (deterministic or stochastic) should be an important feature of any empirically plausible macroeconomic model" (Ashley and Patterson, 1989, p. 685).

Recent research has made further progress in the study of nonlinearities in macroeconomic time series. Extending the work by Gallant et al. (1993), Potter (2000) generalized the use of linear impulse response functions from the Vector Autoregression (VAR) literature to the case of nonlinear stochastic processes. Altug et al. (1999) tested whether technology shocks are the cause of the nonlinear structure observed in production. Several alternative specification of Solow residuals were derived and the hypothesis of linearity was tested. The authors found that there is little evidence of nonlinearity in technology shocks, indicating that it is a nonlinear transmission mechanism (economic process) that generates nonlinearities in the aggregate series. Evidently, linear models with Gaussian shocks cannot produce asymmetry and nonlinearity in the simulated time series.

The goal of this paper is to document the evidence of nonlinearities for macroeconomic time series and to explore whether a workhorse RBC model can explain those nonlinearities. Section 2 documents evidence of nonlinearities in quarterly National Account aggregates for five OECD countries. The evidence presented are for very mild forms of nonlinearity (skewness, kurtosis) and not for more general forms of nonlinearity (e.g., conditional heteroskedasticity). Nonlinearities are present across the sample of countries. However, nonlinearites are different for the same series across countries. For example, while Private Consumption in Canada is negatively skewed and has high kurtosis, in Great Britain it is positively skewed and has low kurtosis. Moreover, the null hypothesis for Investment is rejected in three of the five countries studied. Having obtained evidence of nonlinearities in an international setting, the paper documents that nonlinearities are also present in US quarterly time series. For the United States the series that exhibit the greatest deviations from the hypothesis of a Normal process are investment and exports.

The next step in the study is to evaluate whether a workhorse RBC model can capture the nonlinearities present in the time series. Section 3 presents a model similar to that studied by Brock and Mirman (1972). The model includes a labor/leisure choice and a investment adjustment cost and is solved using a nonlinear, exact solution method: value-function iteration. The reason for using this computationally intensive method is that it allows exploration of the full nonlinear properties of the underlying economic model. Linearization solution methods force simulated series that are linear and gaussian and hence cannot be used to explore whether the model can explain the observed nonlinearities. The model's ability to match the statistical properties of the data is evaluated by performing an experiment that links estimation and simulation techniques. This algorithm allows the model's parameters to change in order to do their best to match the data's statistical process.[2]

Sections 4 and 5 present the results of the analysis of the RBC model using the Efficient Method of Moments Approach (Gallant and Tauchen, 1996, 2000). The economic model is estimated to the series of consumption and investment. The estimation is based on these two series for two reasons. First, focusing on two series allows for easy comparison to previous studies because the study of these two time series has been the focus of protracted analysis in macroeconomics. The second reason is a technical one. The estimation technique depends on a complete statistical description of the observed time series. As the number of series under study increases, the number of parameters of the statistical model increases rapidly, making it harder to estimate it accurately. In order to focus on a complete set of statistical properties, estimation needs to focus on a few series of interest.

The econometric procedure is a two-step process. The first step is to fully characterize the statistical properties of investment and consumption for the United States. The statistical tool used is a Seminonparametric (SNP) estimator (Gallant and Tauchen, 1989). The SNP estimator estimates a family of statistical models of the time series and chooses the *most* parsimonious

[2]See Gallant (1995), Hansen and Heckman (1996) and Browning et al. (1999).

model that *best* fits the data. The first important result of the analysis is that conditional heteroskedasticity and non-Normality are important features of both the consumption and the investment series. This confirms the preliminary evidence documented in Section 2. A simple VAR structure is not sufficient to characterize the statistical properties of macroeconomic time series. The second step is to estimate the canonical RBC model using the Efficient Method of Moments (EMM) approach (Gallant and Tauchen, 1996, 2000). EMM uses the information from the statistical model of the data and chooses parameters that make the economic model most closely resemble the statistical model under a minimum chi-squared (GMM) criterion. A second result is that the canonical RBC model cannot replicate either the conditional heteroskedasticity or right conditional distribution (non-Normal) of consumption and investment. A third result is that the parameter estimates change after taking into account these nonlinearities.

2 Empirical Evidence

This section presents preliminary evidence of statistical nonlinearities in OECD and US data. The series of interest are: Private Consumption, Private Investment, Government Expenditures and Public Investment, Exports, Imports and Aggregate Output. Inventory Investment and Fixed Gross Capital Formation series are added together to get a measure of Private Investment.

The RBC literature primarily focuses on the business cycle frequencies of the macroeconomic time series, and it is common to filter the data to remove trending and seasonal effects. This is typically done using the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1980). The problem of using the HP filter is that it may distort sample second moments of the data in small samples (King and Rebelo, 1993). That is, the HP filter may distort the true volatility, co-movement and persistence of the observed data at business cycle frequencies. Hence, rather than using the HP filter, an optimal band pass filter (BP) (Christiano and Fitzgerald, 1999; Baxter and King, 1999) is used to filter the data [3] The BP filter used in this application

[3] The qualitative results of this paper would not change if an HP filter had been used instead. As noted in Section 2, significant nonlinearities are present regardless of what filter is used. The top of Table 2 gives US summary statistics for BP filtered quarterly time series data. The bottom of the table shows the same statistics for the HP filtered data for comparison purposes. The features of non-Normality for skewness and kurtosis are present regardless of the filter used. If anything, the HP filter accentuates the negative skewness and excess kurtosis of investment. In this application we use the BP filter by Christiano and Fitzgerald (1999).

minimizes the Mean Squared Error between the estimated spectral decomposition and the true spectral decomposition of a particular process and works well for standard macroeconomic time series. The filter used in practice is symmetric (stationary), nonlinear and isolates the business cycle properties of the data between 6 and 32 quarters. This filter assumes that the raw data is close to having a unit root. Twelve observations at the beginning and at the end of the sample are lost since the filter is a double-sided moving average filter.

Table 1 gives summary statistics of the BP filtered data for OECD National Accounts aggregates.[4] Three columns are of particular importance. The skewness and kurtosis estimates show important deviations from normality (the normal distribution has a skewness of 0 and a kurtosis of 3) for most macroeconomic time series. The Jarque-Berra (1987) statistic (J-B column) is a statistical test of the null assumption of normality based on the estimated kurtosis of the series of interest. This statistic only tests for a very mild form of nonlinearity, that of non-Normal probability processes. Macroeconomic time series could be Normal even if the economic model were linear, as long as the shocks were not Normally distributed. However, as pointed out above, Solow residual analysis has shown that the productivity shocks are Normally distributed. Thus, it must be the case that the economic transmission mechanism is nonlinear. This study will take into account additional nonlinearities (like conditional heteroskedasticity) when the RBC model is evaluated against US Data.

There is significant evidence of nonlinearity for most countries and macroeconomic time series. Four of the six reported values for the Australian National Accounts significantly deviate from Normality. For Canada, one can reject the null hypothesis of Normality for five of the six series at the 90% critical level. For Great Britain, the number of rejections is four. These deviations capture the significant asymmetry (skewness) and excess volatility (kurtosis) of many National Account statistics. Over the 5 countries, 19 of 30 data series have significant deviations form Normality.

It is important to note that there is not much robustness in the form of nonlinearities across countries. While in Australia and Great Britain private consumption shows too little kurtosis

[4]OECD data comes from the OECD Statistical Compendium. Updates are from the OECD Source Web Site. The data is quarterly and, in most cases, spans the time period from 1960Q1 to 1998Q4.

relative to the normal distribution, the same series exhibits excess kurtosis in Canada and Japan. Also, private consumption is negatively skewed in Canada, while it is positively skewed in Australia, Great Britain and Japan, and symmetric in Mexico. Similar lack of pattern is present for other National Account time series as well.

[Table 1 about here.]

The data for the United States comes from the National Income and Product Accounts (NIPA) collected from the Department of Commerce State of the Nation webpage. The data is quarterly and covers the period from 1959QI to 1998QIV, including the latest revision of the series released in November 1999. Two data series, private consumption and the sum of gross capital accumulation and changes in inventories, called investment, are later used in the estimation, but general statistical properties of all the main aggregates are reported.

Table 2 gives summary statistics for the band pass filtered data. For the United States only two series, Investment and Imports, show significant deviation from normality. Particularly investment shows a significant asymmetry (negative skewness) and excess volatility (kurtosis in excess of 3), features that have been emphasized in many microeconomic studies.

Graph 1 gives a graphical representation of the non-Normality of investment and the relative normality of consumption. The top row shows the two series as percentage deviations from the mean trend. The bottom row shows histograms for the two series. For consumption, the observed distribution appears to be symmetric. For investment, the observed distribution is nonsymmetric. There appear to be more frequent small positive improvements in investment, while there are not as many negative changes, but they can be larger in magnitude.

Graph 2 captures some of the conditional heteroskedasticity of the two series. The top row shows the squared consumptions residuals obtained from a VAR(3) regression. The squared residuals are a proxy for volatility of consumption. The bottom row shows the squared investment residuals for the same VAR(3) regression. From the two graphs, one can see periods of low volatility and periods of high volatility in both time series, suggesting the presence of conditional heteroskedasticity.

[Table 2 about here.]

3 Model

The RBC model is a simple variation of a Brock-Mirman stochastic growth model. The agent maximizes lifetime utility by choosing savings, consumption and labor decisions. There are no distortions in the RBC model so the equilibrium can be represented by a central planner problem.

The problem faced by the planner is to maximize her lifetime utility:

$$\max_{\{c_t, k_{t+1}, l_t\}} E_0 \sum_{t=0}^{t=\infty} \beta^t U(c_t, l_t) \tag{3.1}$$

subject to the budget constraint:

$$c_t \leq F(k_t, l_t) - k_{t+1} + k_t(1-\delta) - \frac{\phi}{2} k_t \left(\Psi - \frac{k_{t+1} - k_t(1-\delta)}{k_t} \right)^2 . \tag{3.2}$$

E_0 is the expectation given period zero information, the initial capital stock and the current state of the technology shock. c_t is consumption, k_t is the capital stock, l_t is the labor supply, all dated at period t. β is the subjective time discount rate. $F(k_t, l_t)$ is domestic product. The capital stock inherited from the previous period depreciates at a constant geometric rate, δ, and is augmented this period by gross investment, $k_{t+1} - k_t(1-\delta)$. There is a isoplastic adjustment cost to investment captured by the last term of (3.2). $\Psi \equiv \frac{i}{k}$ is the ratio of steady state investment to steady state capital. Adjustment costs are zero at the certainty equivalence steady state. Adjustment costs are introduced here not only because they have been shown to work reasonably well to explain aggregate investment behavior at the aggregate level (Cooper and Haltiwanger, 2000), but also because they may capture other distortions present in the real world that this simplified model does not consider and will give flexibility in the estimation of the model.

The utility function is defined as in Greenwood et al. (1988). The utility is defined in terms of a composite good made up of consumption, c_t an labor, l_t:

$$U(c_t, l_t) = \frac{\left(c_t - \frac{l_t^\omega}{\omega} \right)^{1-\theta} - 1}{1-\theta} . \tag{3.3}$$

$1-\omega$ is the inverse of labor elasticity with respect to consumption. The marginal rate of substitution between consumption and leisure is given in terms of consumption only. This allows us to solve for the optimal labor supply period-by-period, $\hat{l}_t$:

$$\hat{l}_t \equiv \arg\max_{l_t} E_0 \sum_{t=0}^{t=\infty} U(c_t, l_t)$$

The domestic product is given by a Cobb-Douglas production function:

$$F(k_t, l_t) = A k_t^{\alpha} \hat{l}_t^{1-\alpha} \exp z_t$$

Production is perfectly competitive, so $F(\cdot)$ exhausts all factor payments. α is the income share paid to capital, and $1-\alpha$ is the share paid to labor. z_t is a technological (productivity) shock with zero mean and variance σ. The stochastic shock, z_t, is assumed to follow an $AR(1)$ process. That is:

$$z_t = \rho z_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_\epsilon). \tag{3.4}$$

The economic model's parameters are collected into the parameter vector $\Theta = (\sigma_\epsilon, \rho, \alpha, A, \omega, \delta, \beta, \phi, \theta)$.

Given the model's parameters, Θ, the equilibrium is defined as an infinite sequence of variables $\{k_t, l_t, c_t\}_{t=0}^{\infty}$ that solve the problem in (3.1), and that satisfy the aggregate resource constraint (3.2), subject to initial conditions for the capital stock, and the shock, k_0, z_0. The model can also be casted as a solution to a stochastic dynamic programming problem in the usual way, and the equilibrium concept would be a recursive competitive equilibrium.

The stochastic growth model is solved using the Value Function iteration approach.[5] Since there is strong evidence of nonlinearity in the macroeconomic time series, using local methods that approximate the agent's problem using linear decision rules is not an option if the objective is for the model to replicate those nonlinearities. Even if the underlying problem is nonlinear, the solution technique will give no opportunity to the economic model to capture nonlinearities

[5] The grid for capital has 701 points for the capital stock and covers $K \pm 20\%$. A 9-point quadrature grid is used to approximate the stochastic process (Tauchen and Hussey, 1991). The simulations have a length of 20,000 periods, after dropping the first 10,000 periods to eliminate the impact of starting values on the simulation.

in the data. At most, a linearized economic model could capture non-Normality if the shocks were non-Normal. However, as pointed out in the introduction, there is little evidence that real shocks are nonlinear. Furthermore, the estimation technique studies the simulations' ergodic distribution and so requires the use of a global solution method. The approximation error of local solution methods grows as simulated series move "far away" from the deterministic steady state of the model, even if they are still part of the ergodic distribution. [6]

4 Statistical Properties of the Data: Seminonparametric Estimation

The approach followed here to assess the ability of the stochastic growth model proposed in Section 3 to account for the observed business cycle regularities is based on Efficient Method of Moments (EMM) methodology developed by Gallant and Tauchen (1996, 2000). This method consists of two steps. In the first step, the statistical properties of the data of interest are characterized using a Seminonparametric (SNP) estimator. Successively more complex statistical models are estimated with the data until a *full* statistical characterization is achieved (full in the sense that the quasi-Maximum Likelihood probability of the statistical model is maximized but also takes into account the number of statistical parameters being estimated). The statistical models are flexible enough so as to account for increasing time dependence in the mean of the process (VAR) for conditional heteroskedasticity (ARCH, GARCH) and non-Normal innovations. The key to this first step is to take a flexible approach to estimating the statistical model. As more data are available, increasingly rich statistical models are used. The estimates of the statistical model then serve as moments for the second step of the EMM procedure, the simulation stage, which is described in the next section. The resulting statistical model is therefore referred to as the score generator (it is also referred to as the auxiliary model in EMM terminology).

[6] An appendix, available from the author upon request, discusses the use of alternative solution techniques for solving stochastic general equilibrium (SGE) models of real business cycles. The appendix illustrates the failure of the Linear Quadratic solution method to solve models when the variance of the stochastic process is large or when the economic model is very nonlinear, while the Value Function solution method does not fail to satisfy the first order conditions (FOC) of the model.

The SNP estimator used here is adopted from Gallant and Tauchen (1998). Define y_t as the "true" stochastic process of a particular time series to be estimated (consumption and investment in this case). $\tilde{y}_t$ is the observed stochastic process, that is, the quarterly observations from the United States NIPA accounts. $f(y_t \mid y_{t-L}, \ldots, y_{t-1}, \Omega)$ is the distribution of the statistical model that describes the observed data, with Ω representing the parameter vector of this statistical model.

The goal of the SNP procedure is to estimate the transition density function of the observed data as a full characterization of all statistical properties of the time series. The SNP estimator requires parametric assumptions regarding the statistical properties of the data. In particular, SNP assumes that the data has the following structure:

$$f(y_t \mid y_{t-L}, \ldots, y_{t-1}, \Omega) \propto R_{x_{t-1}}^{-1} h \left[R_{x_{t-1}}^{-1} (y_t - \mu_{t-1}) \right],$$

where $x_{t-1} = (y'_{t-L}, \ldots, y'_{t-1})'$ is the vector of lagged data $R_{x_{t-1}}$ is an upper triangular matrix, $R_{x_{t-1}} R'_{x_{t-1}} = \Sigma_{x_{t-1}} = VAR_{t-1}(e_t)$; that is, the matrix square root of the conditional variance-covariance matrix. μ_{t-1} is the conditional mean of the process. The linear error is defined as $e_t \equiv y_t - \mu_{t-1}$.

The conditional mean is assumed to be given by:

$$\begin{aligned} \mu_{t-1} &= b_0 + B_1 y_{t-L_\mu} + B_2 y_{t-L_\mu+1} + \cdots + B_{L_\mu} y_{t-1} \\ \mu_{t-1} &= b_0 + B x_{t-1} \end{aligned}$$

That is, the conditional mean can be described by a vector autoregressive process. $\Psi = \text{vec}[b_0 \mid B_1 \cdots B_{L_\mu}]$ groups the parameters of the VAR structure in a parameter vector.

The conditional variance (scale) is assumed to be given by:

$$\text{vech}(R_{t-1}) = \rho_0 + \sum_{i=1}^{L_r} P_i |e_{t-1-L_r+i}| + \sum_{i=1}^{L_g} \text{diag}(G_i) R_{t-2-L_g+j}$$

The scale captures an ARCH structure through the P matrices and a GARCH structure through the G matrices. In general, SNP nests a GARCH(L_g, L_r) structure. The parameters that describe the scale are arranged into the following vector $T = \text{vec}[\text{vec}\,\rho \mid \text{vech}\,P_1 \cdots P_{L_r} \mid \text{vech}\,G_1 \cdots G_{L_g}]$. The variance matrix Σ_x is assumed to be diagonal meaning that the variances and covariances of consumption and investment only depend on their own lagged values and not on the other series' lagged values.

The SNP estimator nests a non-Gaussian transition density (Heterogeneity) because it assumes that the density of the error is a transformation of the normal distribution. A hermite polynomial is used for this purpose:

$$h\left[R_{x_{t-1}}^{-1}(y_t - \mu_{t-1})\right] \propto [P(z,x)]^2\,\phi(t \mid \mu, R)$$

$$P_K(z,x) = \sum_{\alpha=0}^{K_z}\sum_{\beta=0}^{K_x}(a_{\beta\alpha}x^{\beta})z^{\alpha}$$

where $z = R^{-1}(y_t - \mu_{t-1})$. By increasing the degrees of the polynomial $P(z,x)$, SNP attains increasingly rich statistical structures. If $K_z = 0$ and $K_x = 0$ then the statistical model has a Gaussian error structure. If $K_z > 0$, $K_x = 0$ then the statistical model has a semiparametric error structure. If $K_z > 0$, $K_x > 0$ then conditional distribution is fully non-parametric and depends on lags of the data. The parameters of the hermite polynomial, $a_{\beta\alpha}$, are collected in the matrix $A = [a_{\beta\alpha}]$. The number of parameters grows very rapidly as one expands through the hermite polynomial. Thus, SNP allows suppression of interactions between series through the control parameters I_z and I_x so that only the terms with interaction between different series of degree greater than $K_z - I_z$ and $K_x - I_x$ are estimated.

The entire SNP parameter vector Ω is given by $\Omega = [A \mid \Psi \mid T]$. [7] This structure is flexible enough to capture a very rich family of statistical processes, yet it is tractable enough to easily obtain scores to be later used in the the second step of the estimation. The empirical RBC literature has focused on a vector autoregressive structures to summarize the statistical properties of data. The SNP hierarchy nests VAR. It also allows for richer statistical features, such as periods of low volatility followed by periods of high volatility (i.e., conditional heteroskedasticity), asymmetric business cycles (i.e., skewness) and "excess volatility" (i.e., excess kurtosis).

[7] It is the flexibility of the SNP score generator to nest a variety of statistical models that leads EMM to be efficient. Gallant and Nychka (1987) show that as the number of parameters increases with the sample size, SNP is a consistent estimator of the transition density.

These are features of interest and are characteristic of many macroeconomic time series.

Results

Statistical model selection within the SNP procedure is done by expanding through the SNP hierarchy of candidate statistical models. Gallant and Tauchen (1998) recommend the use of the Schwarz Bayesian Information Criterion (BIC) to help choose amongst different statistical models. Table 3 summarizes the results of the statistical model selection.

Table 3 shows the expansion through the SNP family of models. The arrows on Table 3 show the BIC-preferred models for expansion through the SNP hierarchy. [8] In particular, increasing the degree of the hermite polynomial, K_z increases the BIC scores. While the lowest score was a statistical model with a VAR(3) structure for the conditional mean with GARCH(1,1) for the conditional variance, there is strong evidence that non-Normalities are important. Therefore, a VAR(3) structure with ARCH(1) and a polynomial of degree 4 will be used as the benchmark model in estimation, hereafter referred to as the Benchmark Model.

Two additional oversimplified statistical models are used in the estimation step to test for the robustness of the estimates to the particular statistical specification and to evaluate the importance of nonlinearities. The first is a linear VAR(3) score generator, hereafter referred to as the VAR Model.[9] This statistical model resembles the RBC procedure of choosing parameters to match the impulse responses (IR's) generated by an economic model to the impulse responses obtained from the macroeconomic time series captured by a VAR. With this specification, the EMM procedure picks the economic model's parameters so that the economic model's simulations match a VAR(3) of the data.

The second oversimplified statistical model is a VAR(3), ARCH(1) conditional variance

[8]One problem encountered in the SNP estimation is that for some specifications (those marked with an asterisk (*) in Table 3) it was not possible to obtain standard errors for the estimate. A second problem is that the estimator reached a limit on the number of iterations to find the optimum, probably because the objective function was too flat for the particular specification regardless of the starting value. In either case, the SNP scores cannot be used in the EMM estimation because both the estimates and the standard errors are used to build the objective function.

[9]While the VAR Model is the BIC-preferred model, the Benchmark Model is the overall preferred model because of the statistically significant parameters capturing the nonlinear structure and the ARCH structure of the data. There is evidence that BIC is too conservative of a criterion and chooses overly-simplistic statistical models of the data.

score generator, hereafter referred to as VAR-ARCH Model. This statistical model extends the VAR Model by adding conditional heteroskedasticity terms to the VAR(3) statistical model of the data. The VAR-ARCH model includes some form of nonlinearity (conditional heteroskedasticity) but the shocks are still Gaussian.

Table 4 shows the parameter estimates, standard errors and corresponding t-ratios for the Benchmark Model.[10] The Table shows that the parameter that captures the conditional volatility for investment ($P(c) = 0.321$) is significantly different from zero (t-ratio = 5.712). The parameter that captures the kurtosis for investment ($A(i^4) = 0.069$) is positive and significant (t=4.108), indicating excess kurtosis for investment. This confirms the observation from Table 2 that investment is not Normally distributed. Meanwhile, the behavior of consumption appears to be more Normally distributed. The parameter that captures the conditional volatility for consumption ($P(i) = 0.168$, t=0.740) is not significant nor are the parameters that capture the higher order terms of the hermite polynomial ($A(c^3) = -0.164$, t=-0.275,$A(c^4) = -0.032$, t=1.225).

[Table 3 about here.]

[Table 4 about here.]

5 Simulation and Estimation: The Efficient Method of Moments

In the second step of the EMM method, the economic model is simulated for a candidate set of parameters (the economic model is referred to as the structural model in the EMM terminology). A comparison is made between the statistical parameter estimates obtained in the SNP step and the statistical parameters obtained using the simulated data and the same statistical model. Then, the candidate parameters of the economic model are adjusted until the economic model's simulations have statistical properties similar to those of the data. The objective function is a Chi-squared (i.e., Generalized Method of Moments (GMM)) statistic.

[10]Similar tables are available for the VAR and the VAR-ARCH statistical models from the author

This econometric technique belongs to the class of Simulated Method of Moments (SMM) estimators. The estimation technique is made efficient by taking a flexible approach in the first step of estimation. An appendix, available from the author upon request, describes the approach in detail and gives some references.

Following Gallant and Tauchen (1999), the second step of the EMM methodology starts by using the score generator from SNP to define the GMM Objective Function. Under the null that the economic model is the "true" generator of the realized time series, the gradient of the SNP likelihood function should be approximately equal to zero at the estimated statistical parameters, $\hat{\Omega}$:

$$(\partial/\partial\Omega)log[f(y_t \mid y_{t-L}, \ldots, y_{t-1}, \hat{\Omega})] \approx 0 \tag{5.1}$$

This is a vector of moments of the same length as Ω. Define $\hat{y_t}(\Theta)$ as the simulated data process from the parameters Θ of the stochastic growth model. The EMM estimator "punches in" $\hat{y_t}(\Theta)$ into the SNP score generator and changes the parameters of the economic model until the moments for a particular parameterization Θ are approximately equal to zero.

The GMM criterion is used to evaluate the simulated time series to get $\hat{\Theta}$, the estimated structural parameters. The EMM objective function is distributed χ^2 with degrees of freedom equal to the difference between the number of parameters in the score generator and the number of parameters being estimated.

Table 5 gives the point estimates for the parameters of the stochastic growth model using the three alternative statistical models of consumption and investment (the SNP score generators). Perhaps the most striking difference amongst the three models is in the size of the investment adjustment cost coefficient, ϕ. For the VAR Model, the coefficient is very large (10.21), while for the VAR-ARCH Model it is smaller (3.01) and the coefficient is even smaller for the Benchmark model (2.93).

Intuition for these results can be gained by comparing the properties of the simulated series of consumption and investment reported in Table 6. In the VAR Model, the high estimate for the adjustment cost coefficient leads to an investment series that has much less kurtosis (2.359) than in the data (3.559) or in either of the other statistical models (2.824 for the Benchmark Model and 2.827 for the VAR-ARCH Model). This high ϕ works to smooth changes in investment and this is appropriate for the VAR Model because, by construction, it neglects the importance of the kurtosis of investment present in the data. Thus, the adjustment cost parameter, ϕ, can increase to help the economic model with other statistical features without punishment for producing low kurtosis. Additionally, in smoothing the data, the big adjustment cost works against producing any conditional volatility that could be generated by the economic model. In comparison, the Benchmark and the VAR-ARCH models take into account the conditional volatility and capture it, thus requiring much smaller parameter estimates for ϕ. The higher adjustment cost coefficient also results in less volatile series. The VAR Model yields a much lower level of volatility for consumption (0.16% Standard Deviation) and investment (0.25% S.D.) than either the Benchmark Model (0.39% for consumption and 0.72% for investment) or the VAR-ARCH Model (0.31% for consumption and 0.70% for investment).

The parameter estimates for the parameter that determines the labor elasticity of substitution, ω, are also significantly different across models. For this parameter, the main differences occur between the Benchmark Model (1.39), and the VAR Model and the VAR-ARCH Model (1.47 range), implying a labor elasticity of substitution of 2.13% for the Benchmark Model and of 2.56% for the VAR and VAR-ARCH models.

The parameter estimates for the volatility and the persistence of the shock process (3.4) are different for the three alternative models. For the VAR Model, the estimate for the standard deviation of the shock, σ_{ϵ_z}, is .29% while the estimates for the Benchmark and VAR-ARCH models are in the .9% range. The estimate for the autocorrelation parameter, ρ is 94% for the simple VAR Model (VAR(3) structure), but taking into account conditional heteroskedasticity and non-Normality lowered the point estimate for ρ to the 83–84% range. The low level of correlation of the errors and the difference in the standard deviation result in similar unconditional means for the shock. The two parameters are changing from one statistical model to another to capture the ARCH features of the investment. However, as explained below in the sections detailing the results for each of the statistical models, the economic model does not have a good way to capture the conditional heteroskedasticity.

A troubling result from the estimation is that overall the stochastic growth model of Section 3 fails to pass the null hypothesis that the economic model is the "true" model of the data. This is reflected in the high levels of the χ^2 statistic presented in the next to last row of Table 5. This is the "omnibus" specification test given by the value of the GMM objective function. It is distributed χ^2 with degrees of freedom equal to the number of SNP parameters minus the number of parameters being estimated (reported in the last row of the same table). The *better* the statistical model is in the SNP-sense (The Benchmark Model being the best, and the VAR Model being the worse) the worse the rejection of the hypothesis (233.820 for the VAR Model, 362 for the VAR-ARCH Model and 2211 for the Benchmark Model). This pattern does not indicate that the VAR statistical model is better at describing the data. Rather, it indicates that the economic model is closer to replicating the features in a purely linear statistical model than the features in nonlinear statistical models. It fails to replicate the conditional heteroskedasticity (Benchmark and VAR-ARCH models) and nonlinearity (Benchmark models) that are present in the data. The next three subsections study in more detail the estimation results for each alternative statistical model, as well as the failures of each as indicated by the sample scores obtained in each corresponding estimation.

[Table 5 about here.]

[Table 6 about here.]

5.1 Benchmark Model

The Benchmark Model has a VAR(3) structure for the conditional mean, an ARCH(1) structure for the conditional variance and a non-Normal innovation structure with a hermite expansion of the fourth degree with all interactions suppressed. Table 8 gives the stochastic growth model's parameter estimates, the criterion difference confidence intervals, the Wald standard errors and t-ratios obtained from the Wald standard errors. The criterion difference confidence intervals are obtained by inverting the objective function and allows for non-symmetric confidence intervals.

An initial grid search was done over the parameter space of the stochastic growth model to get good starting values for the estimation, and further testing is done to mitigate problems with local minima. For the Benchmark model, the objective function is distributed $\chi^2(21)$

and is equal to 2280.92, well above the 95% significance level of 32.7. The economic model is rejected as the "true" data generating mechanism, as mentioned before.

Columns (1)-(3) of Table 7 give the mean scores, standard errors and t-statistics for the estimated parameters. The sample score for the ARCH term of investment, $P(c)$, is significantly above zero. This indicates that the economic model cannot replicate investment's ARCH structure of conditional variance. This should not come as a surprise because the economic model does not have a good way to replicate the conditional heteroskedasticity observed in the data. An example of an economic feature that might help in this respect is the presence of financial friction that is occasionally binding and generates excess volatility when it binds. In this case, the simulated time series would have periods of low volatility when the constraint is not binding and periods of high volatility when it binds, generating the desired conditional heteroskedasticity. However, at least for the U.S., the consumption series do not seem to exhibit conditional heteroskedasticity. Therefore, the mechanisms that introduce conditional heteroskedasticity for investment must not do the same for consumption. A possible solution to this consumption/investment dichotomy would be to use small open economy models (SOE). In deterministic (SOE) models the savings decision and the investment are uncorrelated. While in stochastic models this no longer is the case (Mendoza, 1991), the correlation is still weakened creating the possibility that introducing financial frictions may lead to conditional heteroskedasticity in investment but not in consumption (this avenue is studied in Valderrama, 2001).

More importantly, the structural model fails to capture the non-Normality of the data. The mean score for $A(c^3)$, which captures the skewness in consumption, is negative and significant indicating that the consumption simulation is skewed (negatively) relative to the data. The mean score for $A(c^4)$, which captures the excess volatility in consumption, is negative and significantly different from zero. The negative mean score indicates that the economic model produces a simulation for consumption with overly thin tails (too little kurtosis) with respect to the data. Meanwhile, the terms of the hermite polynomial for investment are, all but one, insignificant. In summary, the stochastic growth model can replicate some of the non-normal features of investment but not the statistical features of consumption. As mentioned before, the stochastic also fails to replicate the conditional heteroskedasticity observed in the investment series.

[Table 7 about here.]

[Table 8 about here.]

5.2 VAR-ARCH Model

Table 9 gives the estimates for the stochastic growth model parameters for the first oversimplified statistical model, the VAR-ARCH Model. The first important observation is that the relative performance of the economic model in matching the statistical properties of the data improves relative to the Benchmark Model. The $\chi^2(12)$ statistic is 335.706. The 95% critical value is 22.4. Thus, the economic model is rejected as the "true" data generating mechanism. The economic model has a very simple AR(1) stochastic process and so the heteroskedasticity has to be captured by the preference and technology parameters.

Column (4) of Table 7 give the mean scores for the statistical parameters. The economic model fails to match the stochastic structure for consumption and investment (captured by the mean scores for the $T(\cdot)$ and $P(\cdot)$ parameters). All t-statistics but one are over 2, meaning that the stochastic growth model can not explain the ARCH nature of the observed data. The economic model does capture some of the features of the VAR structure of the data, but it has problems matching the evolution of the conditional mean for investment as a function of lagged consumption, ($B(i_{t-2}, c_{t-2})$ and $B(i_{t-1}, c_{t-1})$ are both significantly different from zero, and of lagged investment ($B(i_{t-3}, i_{t-3})$, $B(i_{t-2}, i_{t-2})$ and $B(i_{t-1}, i_{t-1})$). Additionally, the stochastic model cannot replicate effects on the conditional mean of consumption of lagged consumption of the statistical model of the data ($B(c_{t-3}, c_{t-3})$ and $B(c_{t-2}, c_{t-2})$).

[Table 9 about here.]

5.3 VAR Model

Table 10 gives the estimates for the VAR Model (VAR(3) with no conditional heteroskedasticity and Normal shocks). In this case, the objective function is distributed $\chi^2(9)$ and the objective function is equal to 233.820, well rejected at any significance level. The 99% Critical value is 21.7. As was mentioned before, for the stochastic growth model to match the data, estimation yields a large adjustment cost coefficient, ϕ at the quarterly frequency. More importantly, the estimates for two parameters that control the stochastic structure (σ_{ϵ_z} and ρ) are significantly different from the estimates in the Benchmark and VAR-ARCH models. The parameter estimate for the correlation of the error is much lower than in the other two models and the estimate for the standard error of the innovation is higher.

Columns (7)-(9) of Table 7 give the mean scores, standard errors and t-statistics for the estimated parameter values. The model does well in terms of matching the simple structure for the stochastic shock distribution (the $T(\cdot)$'s). All t-statistics are well under 2. The structural model has problems matching the evolution of the conditional mean for investment at all lag lengths, both as a function of lagged consumption, ($B(i_{t-3}, c_{t-3})$, $B(i_{t-2}, c_{t-2})$ and $B(i_{t-1}, c_{t-1})$) and as a function of lagged investment ($B(i_{t-3}, i_{t-3})$, $B(i_{t-2}, i_{t-2})$, $B(i_{t-1}, i_{t-1})$).

[Table 10 about here.]

6 Conclusions and Extensions

EMM analysis of the canonical RBC model indicates that the model cannot explain important nonlinearities present in the data that had been previously overlooked in the form of non-Normality and conditional heteroskedasticity. This suggests that while many RBC models are nonlinear they might not be capturing the "right" nonlinear features of the data. Additionally, conditional heterogeneity (ARCH/GARCH) is another feature of the data that the canonical RBC model cannot explain.

The evidence presented here points to the importance of looking past first and second moments of the data, especially for investment data. Macroeconomic time series of interest, especially in time series of emerging markets business cycles, will have similar nonlinear features and understanding these is critical. As a recent example, the East Asian crisis of 1997 and the Russian crisis of 1998 prompted many researches to study "excess volatility" and "sudden stops" that were observed in the data. Many of the theories that were developed to study these phenomena departed from traditional perfect credit market models and introduced frictions such as "borrowing constraints" and "margin requirements" that potentially lead to nonlinear transmission mechanisms. In models like these, it is nonlinearities that are really the objects of interest.

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Table 1: **Sample Statistics:** OECD National Account Data for 1960Q1-1998Q4

	Mean	Std. Dev.	Skewness	Kurtosis	J-B	P-value
Australia						
Private Consumption	8.037	0.009	0.193	2.054	6.092	0.048
Investment	7.053	0.047	0.276	4.080	8.590	0.014
Gov't Consumption	6.854	0.014	0.766	3.768	17.112	0.000
Exports	6.397	0.034	-0.257	2.723	1.984	0.371
Imports	6.520	0.064	0.152	3.285	1.015	0.602
GDP	8.541	0.013	-0.569	3.682	10.273	0.006
Canada (61Q1-98Q4)						
Private Consumption	9.366	0.010	-0.403	3.716	6.591	0.037
Investment	8.117	0.057	-0.284	4.771	19.607	0.000
Gov't Consumption	8.449	0.010	-0.055	2.521	1.370	0.504
Exports	8.304	0.031	-0.332	3.620	4.679	0.096
Imports	8.196	0.045	-0.617	5.548	45.423	0.000
GDP	9.912	0.013	-0.495	3.290	6.038	0.049
Great Britain						
Private Consumption	7.218	0.013	0.558	2.877	7.346	0.025
Investment	5.931	0.055	-0.165	2.980	0.637	0.727
Gov't Consumption	6.223	0.010	0.423	3.041	4.178	0.124
Exports	6.111	0.023	-0.165	3.363	1.400	0.497
Imports	6.136	0.033	0.466	3.764	8.482	0.014
GDP	7.708	0.013	0.574	3.446	8.849	0.012
Japan						
Private Consumption	7.185	0.011	0.863	3.757	20.705	0.000
Investment	6.452	0.036	0.486	2.907	5.560	0.062
Gov't Consumption	5.462	0.011	-1.306	9.250	267.680	0.000
Exports	5.148	0.040	-0.102	2.330	2.860	0.239
Imports	5.165	0.053	0.252	2.299	4.344	0.114
GDP	7.698	0.013	0.853	3.903	21.738	0.000
Mexico (82Q1-96Q3)						
Private Consumption	9.192	0.028	0.015	3.015	0.003	0.999
Investment	8.432	0.042	-0.227	3.442	0.989	0.610
Gov't Consumption	7.347	0.031	0.066	2.081	2.121	0.346
Exports	7.629	0.046	0.195	2.122	2.271	0.321
Imports	7.578	0.138	-0.953	4.572	15.015	0.001
GDP	9.556	0.023	-0.171	3.406	0.692	0.707

Observations from 1960Q1-1998Q4, except where noted. First and last 8 observations lost due to filtering. Christiano and Fitzgerald (1999) filter removes the trend, assumes a random walk and is symmetric and stationary. Skewness is 0 for a Normal distribution and Kurtosis is 3 for the normal distribution. J-B is the Jarque-Bera (1987) statistic, a Wald Test of Normality with distribution $\chi^2(2)$. The 90% $\chi^2(2)$ critical value is 4.61 and the 95% critical value is 5.99.

Table 2: **Sample Statistics:** US NIPA Data for 1959Q1-1998Q4

	Std. Dev.	Skewness	Kurtosis	J-B
I: Band Pass Filter				
Private Consumption	0.011	0.048	3.227	0.344
Investment	0.063	-0.588	3.559	9.617
Exports	0.034	-0.054	3.080	0.102
Imports	0.054	-1.069	5.039	49.474
Government Consumption and Investment	0.013	-0.186	3.366	1.546
Gross Domestic Product	0.014	-0.295	3.176	2.149
II: HP Filter				
Private Consumption	0.013	-0.127	2.961	0.373
Investment	0.074	-0.812	4.065	21.353
Exports	0.044	-0.314	3.325	2.835
Imports	0.052	-1.039	4.923	45.471
Government Consumption and Investment	0.017	-0.011	3.492	1.374
Gross Domestic Product	0.016	-0.312	3.188	2.408

The observations from 1959Q1-1998Q4 were used but the first and last 12 observations were lost due to filtering. Band Pass filter used is the Christiano and Fitzgerald (1999) filter that removes the trend, assumes a random walk and is symmetric and stationary. Skewness is 0 for a Normal distribution and Kurtosis is 3 for the normal distribution. J-B is the Jarque-Bera (1987) statistic, a Wald Test of Normality with distribution $\chi^2(2)$. The 90% $\chi^2(2)$ critical value is 4.61 and the 95% critical value is 5.99. Data is in deviations from the mean.

Table 3: **SNP Estimation:** Band Pass Filtered Data

L_μ	L_G	L_R	L_P	K_z	I_z	K_x	I_x	#P	BIC
1	0	0	1	0	0	0	0	9	0.918
2	0	0	1	0	0	0	0	13	-0.464
⇒ 3	0	0	1	0	0	0	0	17	-0.505
4	0	0	1	0	0	0	0	21	-0.449
⇒ 3	0	1	1	0	0	0	0	15	-0.494
3	0	2	1	0	0	0	0	17	-0.467
3	0	1	1	2	2	0	0	23	-0.441
3	0	1	1	3	3	0	0	25	-0.445
3	0	1	1	4	4	0	0	21	-0.444
3	0	1	1	5	5	0	0	23	-0.449
⇒ 3	0	1	1	6	6	0	0	25	-0.470
3	0	1	1	7	7	0	0	28	-0.435
3	0	1	1	6	4	0	0	32	-0.456
3	0	1	1	6	6	1	1	57	-0.069
3	0	1	1	2	2	0	0	21	-0.461
3	0	1	1	3	3	0	0	23	-0.435
3	0	1	1	4	4	0	0	25	-0.405
⇒ 3	1	1	1	0	0	0	0	21	-0.511
3	2	1	1	0	0	0	0	23	-0.715 (*)
3	3	1	1	0	0	0	0	25	-0.703 (*)
3	1	1	1	2	2	0	0	25	-0.439 (*)
3	1	1	1	3	3	0	0	27	-0.415 (*)

Results of SNP estimation of NIPA Series for Consumption and Investment. Band Pass filter used is the Christiano and Fitzgerald (1999) filter that removes the trend, assumes a random walk and is symmetric and stationary. 4 lags of filtered data were reserved by the SNP estimator. SNP assumes a diagonal ARCH/GARCH structure where appropriate. BIC is the Schwarz Bayesian Information Criterion. (*) Represents values for which standard errors are not available.

Table 4: **SNP Estimation:** Benchmark Model

	Estimate	Standard Error	t-statistic
VAR			
Intercept			
$b(c)$	0.044	0.030	1.439
$b(i)$	0.058	0.010	5.706
$L_\mu = 3$			
$B(c_{t-3}, c_{t-3})$	0.305	0.091	3.346
$B(c_{t-3}, i_{t-3})$	0.348	0.070	4.974
$B(i_{t-3}, c_{t-3})$	0.116	0.070	1.657
$B(i_{t-3}, i_{t-3})$	0.220	0.052	4.206
$L_\mu = 2$			
$B(c_{t-2}, c_{t-2})$	-1.483	0.159	-9.300
$B(c_{t-2}, i_{t-2})$	-0.675	0.114	-5.911
$B(i_{t-2}, c_{t-2})$	-0.184	0.118	-1.556
$B(i_{t-2}, i_{t-2})$	-1.112	0.085	-13.045
$L_\mu = 1$			
$B(c_{t-1}, c_{t-1})$	2.047	0.088	23.352
$B(c_{t-1}, i_{t-1})$	0.516	0.064	8.044
$B(i_{t-1}, c_{t-1})$	0.116	0.070	1.658
$B(i_{t-1}, i_{t-1})$	1.686	0.055	30.812
Variance			
$T(c)$	0.153	0.028	5.535
$T(ci)$	0.008	0.008	0.943
$T(i)$	0.046	0.006	7.112
ARCH			
$P(c)$	0.168	0.226	0.740
$P(i)$	0.321	0.056	5.712
Hermite			
$A(00)$	1.000	0.000	0.000
$A(i)$	-0.299	0.118	-2.530
$A(c)$	-0.259	0.258	-1.001
$A(i^2)$	-0.060	0.104	-0.574
$A(c^2)$	-0.368	0.139	-2.647
$A(i^3)$	0.011	0.027	0.426
$A(c^3)$	-0.016	0.059	-0.275
$A(i^4)$	0.069	0.017	4.108
$A(c^4)$	0.032	0.027	1.225

Results of SNP estimation of NIPA Series for Consumption and Investment. Filtered using Christiano and Fitzgerald (1999) band-pass filter. 4 lags of filtered data were reserved by the SNP estimator. SNP assumes a diagonal ARCH/GARCH structure where appropriate.

Table 5: **EMM Parameter Estimates:** Summary Results

	SNP Score Generator		
	Benchmark	VAR-ARCH	VAR
σ_{ϵ_z}	0.009	0.009	0.003
ρ	0.839	0.838	0.940
α	0.368	0.366	0.366
A	4.107	4.179	4.188
ω	1.390	1.479	1.479
δ	0.024	0.025	0.025
β	0.980	0.980	0.980
ϕ	2.931	3.011	10.213
θ	1.000	1.000	1.001
χ^2	2211.241	362.651	233.820
DOF	19	11	9

Table 6: **Simulation Statistics:**

	Std. Dev.	Skewness	Kurtosis	Jarque-Berra	p-value	obs.
Consumption						
Data	1.06E-02	0.048	3.227	0.344	8.420E-01	136
Calibrated	4.51E-02	-0.046	2.442	267.002	1.050E-58	20000
Benchmark	3.39E-02	-0.053	2.805	41.076	1.204E-09	20000
VAR-ARCH	3.14E-02	-0.058	2.803	43.451	3.670E-10	20000
VAR	1.65E-02	-0.040	2.495	217.758	5.181E-48	20000
Investment						
Data	6.35E-02	-0.588	3.559	9.617	8.160E-03	136
Calibrated	8.47E-02	-0.059	2.190	558.362	5.666E-122	20000
Benchmark	7.25E-02	-0.106	2.824	62.876	2.222E-14	20000
VAR-ARCH	6.96E-02	-0.102	2.827	59.545	1.175E-13	20000
VAR	2.63E-02	-0.036	2.359	347.331	3.784E-76	20000

Table 7: **EMM Mean Scores:**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Benchmark			VAR			VAR-ARCH		
	Mean Score	S.E.	t-stat	Mean Score	S.E.	t-stat	Mean Score	S.E.	t-stat
VAR									
Intercept									
$b(c)$	-0.14	77.80	0.00	69.47	93.69	0.74	41.87	80.77	0.52
$b(i)$	0.55	446.30	0.00	-232.61	33.71	-6.90	-109.74	146.32	-0.75
$L_\mu = 3$									
$B(c_{t-3}, c_{t-3})$	-17.76	40.36	-0.44	-109.92	100.14	-1.10	-436.59	78.77	-5.54
$B(c_{t-3}, i_{t-3})$	76.39	441.54	0.17	-5.81	24.44	-0.24	21.12	124.16	0.17
$B(i_{t-3}, c_{t-3})$	-2.99	9.51	-0.31	172.93	11.45	15.11	-7.06	9.99	-0.71
$B(i_{t-3}, i_{t-3})$	14.27	54.53	0.26	-667.58	4.12	-162.12	-327.09	17.88	-18.30
$L_\mu = 2$									
$B(c_{t-2}, c_{t-2})$	-23.70	58.67	-0.40	-137.02	108.27	-1.27	-538.06	79.49	-6.77
$B(c_{t-2}, i_{t-2})$	76.10	162.89	0.47	-26.70	39.39	-0.68	16.13	43.81	0.37
$B(i_{t-2}, c_{t-2})$	-5.94	9.50	-0.62	173.45	11.45	15.15	-54.50	9.87	-5.52
$B(i_{t-2}, i_{t-2})$	14.06	54.52	0.26	-676.86	4.12	-164.38	-325.21	17.88	-18.19
$L_\mu = 1$									
$B(c_{t-1}, c_{t-1})$	-12.30	60.63	-0.20	-40.94	120.20	-0.34	-116.16	87.10	-1.33
$B(c_{t-1}, i_{t-1})$	76.29	464.63	0.16	1.11	45.19	0.03	73.18	159.98	0.46
$B(i_{t-1}, c_{t-1})$	-0.12	9.50	-0.01	181.01	11.45	15.81	173.25	9.87	17.56
$B(i_{t-1}, i_{t-1})$	14.27	54.52	0.26	-655.68	4.12	-159.24	-286.73	17.88	-16.04
Variance									
$T(c)$	-106.51	301.19	-0.35	-1286.62	2223.73	-0.58	-5471.94	1390.09	-3.94
$T(ci)$	54.09	89.20	0.61	-80.90	696.61	-0.12	-1571.25	328.69	-4.78
$T(i)$	-10.76	1426.40	-0.01	-1072.52	1331.36	-0.81	-881.95	367.81	-2.40
ARCH									
$P(c)$	-1699.47	185.79	-9.15				-8297.11	4044.41	-2.05
$P(i)$	-223.43	405.99	-0.55				-165.45	131.83	-1.26
Hermite									
$A(00)$									
$A(i)$	-1.31	24.87	-0.05						
$A(c)$	-10.03	30.10	-0.33						
$A(i^2)$	16.26	24.17	0.67						
$A(c^2)$	-18.17	35.62	-0.51						
$A(i^3)$	21.15	26.94	0.79						
$A(c^3)$	-167.35	50.50	-3.31						
$A(i^4)$	30.91	40.61	0.76						
$A(c^4)$	-842.99	114.11	-7.39						

VAR Model (VAR(3)), VAR-ARCH (VAR(3),ARCH(1)), Benchmark Model (VAR(3),ARCH(1), $K_x = 4$). Quasi t-ratios.

Table 8: **EMM Parameter Estimates:** Benchmark Model

	Estimate	Wald S.E.	t-ratio	95% Confidence Interval	
σ_{ϵ_z}	9.009E-03	0.001	6.658	0.009	0.009
ρ	0.839	0.196	4.280	0.839	0.839
α	0.368	0.085	4.310	0.304	0.442
A	4.107	1.595	2.575	4.106	4.107
ω	1.390	0.291	4.783	1.390	1.611
δ	0.024	0.017	1.409	0.024	0.041
β	0.980				
ϕ	2.931	1.531	1.915	2.930	2.931
θ	1.000	3.513	0.258	1.000	1.091

Parameter estimates based on 30114400 SNP scores. EMM objective function: 2211.241. EMM Objective function distributed χ^2 with 19 degrees of freedom. The 99% critical value is 36.2.

Table 9: **EMM Parameter Estimates:** VAR-ARCH Model

	Estimate	Wald S.E.	t-ratio	95% Confidence Interval	
σ_{ϵ_z}	9.018E-03	0.000	84.843	0.009	0.009
ρ	0.838	0.006	144.698	0.838	0.838
α	0.366	0.003	140.978	0.366	0.366
A	4.179	0.093	44.809	4.179	4.180
ω	1.479	0.001	1434.581	1.479	1.479
δ	0.025	0.000	76.756	0.025	0.025
β	0.980				
ϕ	3.011	0.053	56.974	3.011	3.011
θ	1.000	0.095	10.576	1.000	1.000

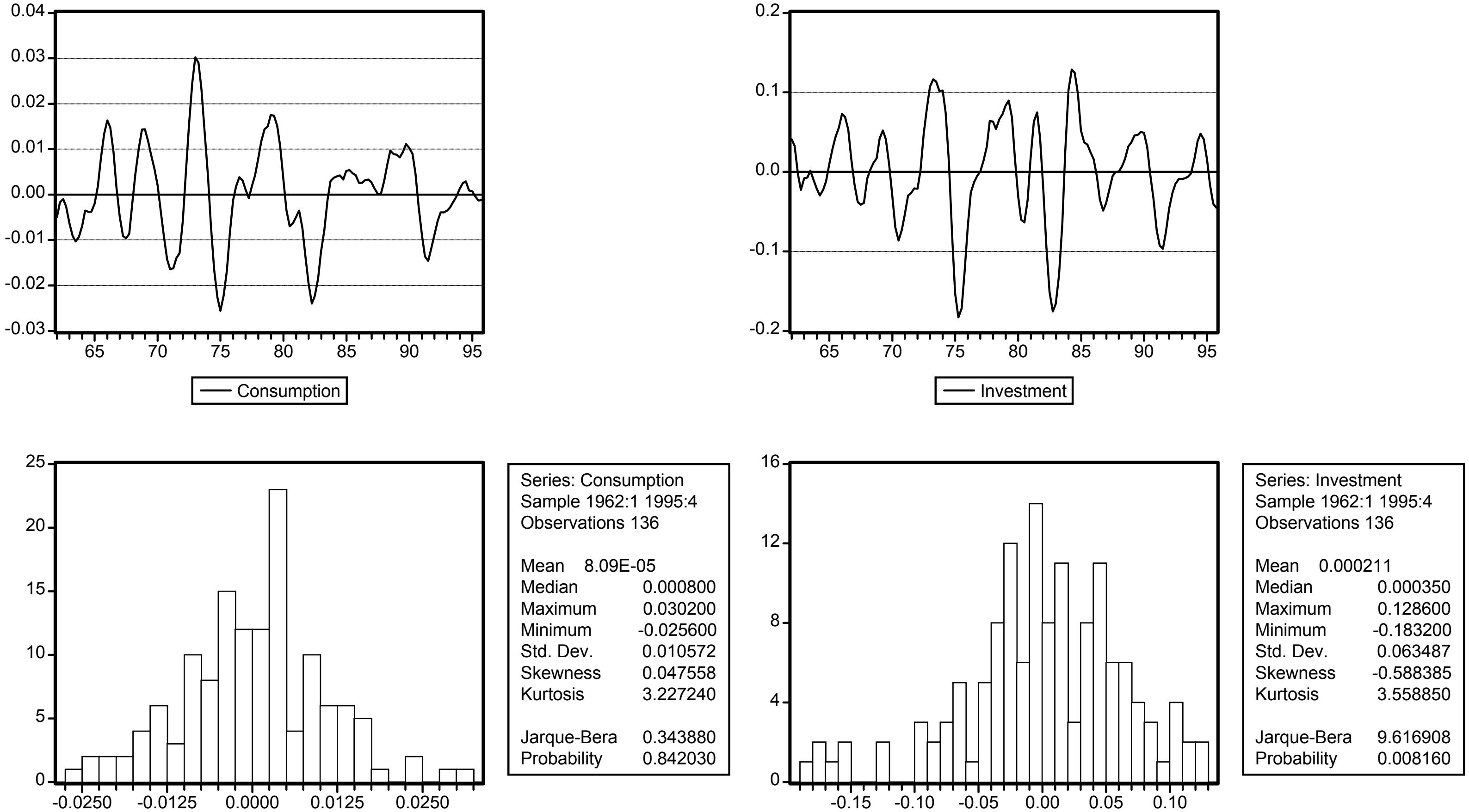
Parameter estimates based on 30110000 SNP scores. EMM objective function: 362.651. EMM Objective function distributed χ^2 with 11 degrees of freedom. The 99% critical value is 24.7.

Table 10: **EMM Parameter Estimates:** VAR Model

	Estimate	Wald S.E.	t-ratio	95% Confidence Interval	
σ_{ϵ_z}	2.941E-03	0.002	1.487	0.098	0.452
ρ	0.940	0.291	3.237	0.652	1.160
α	0.366	0.026	13.893	0.366	0.392
A	4.188	1.437	2.915	0.275	0.494
ω	1.479	0.011	138.761	0.147	0.149
δ	0.025	0.019	1.288	0.006	0.038
β	0.980				
ϕ	10.213	0.402	25.383	9.965	10.614
θ	1.001	3.244	0.309	1.000	4.227

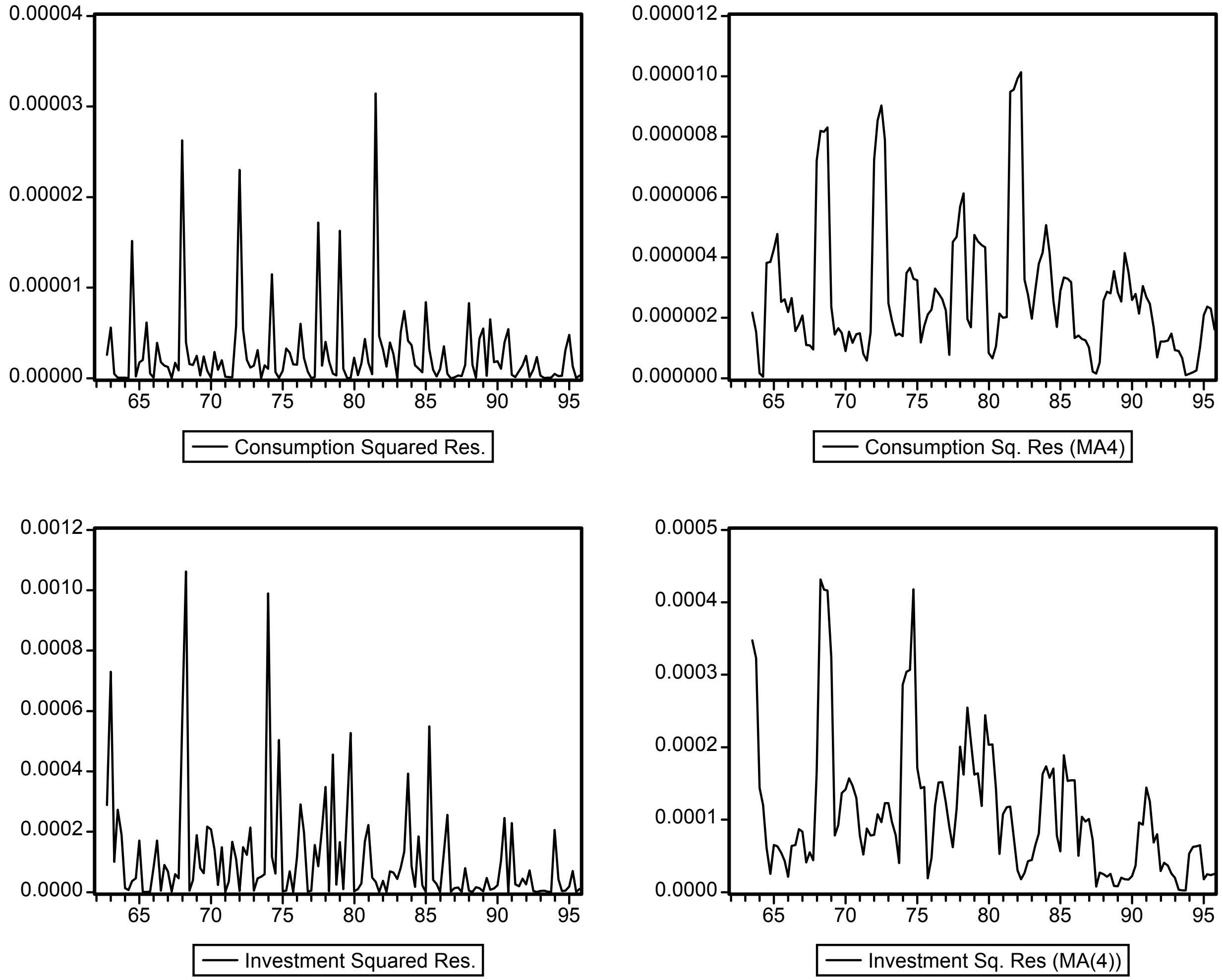
Parameter estimates based on 30010000 SNP scores. EMM objective function: 233.820. EMM Objective function distributed χ^2 with 9 degrees of freedom. The 99% critical value is 21.7.

Graph 1



Consumption
Investment
Series: Consumption
Sample 1962:1 1995:4
Observations 136
Mean 8.09E-05
Median 0.000800
Maximum 0.030200
Minimum -0.025600
Std. Dev. 0.010572
Skewness 0.047558
Kurtosis 3.227240
Jarque-Bera 0.343880
Probability 0.842030
Series: Investment
Sample 1962:1 1995:4
Observations 136
Mean 0.000211
Median 0.000350
Maximum 0.128600
Minimum -0.183200
Std. Dev. 0.063487
Skewness -0.588385
Kurtosis 3.558850
Jarque-Bera 9.616908
Probability 0.008160

Graph 2. Conditional Heteroskedasticity in
Consumption and Investment



Squared from a VAR (3) regression of consumption
and investment.