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**Are there Market Fluctuations that Increase  
Trade and Welfare?**



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increase trade and welfare?*

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## 1. INTRODUCTION

For the original version of the Kiyotaki-Wright model [1989] with three goods, I construct in this paper new equilibria that converge to stationary cycles. Even assuming small transaction costs, trade is higher and, consequently, social welfare is greater than in the standard equilibria exhibited in the literature. The nontechnical intuition is that when market conditions change, economic agents may find profitable to restructure their portfolios. So, when market conditions are restored, agents may recompose their portfolios by reselling or buying back. Consequently, this “trading” may lead to greater market “liquidity” whose benefits overcome the costs of trade. In the model, the market conditions change cyclically because of self-fulfilling prophecies believed by rational agents. In particular, the cyclic fluctuations are not caused by optimal technological relations among the productive sectors, neither driven by shocks to the fundamentals of the model.

The Kiyotaki-Wright model is a discrete time model in which infinitely-lived agents are randomly matched pairwise in each period. The agents maximize expected discounted utility by choosing trading strategies for indivisible objects that are perfectly durable **but costly to store**. When there are more than two goods, there must be other than double-coincidence trade because of the assumed pattern of specialization in production and consumption. According to Aiyagari-Wallace [1997], the Kiyotaki-Wright model “is the only attractive model with an endogenous transaction pattern” (p. 2-3; see also Wallace [1996, p. 251]). In turn, Wallace [1997] indicates that this model is the first one “in which several objects are potential media of exchange and in which the relationship between the physical properties of those objects and their role as media of exchange can be studied” (p. 3).

To show the existence of dynamic equilibria in the model, Kehoe-Kiyotaki-Wright [1993] exhibit also stationary cycles driven by self-fulfilling prophecies. However, the endogenous transaction pattern of their cycles does not allow a great deal of trading. In particular, the least costly to-store good is universally accepted and, consequently, agents exchange it only for their respective consumption good regardless of market conditions. In fact, only on third of agents do “trading.” Of course, one could expect intuitively that if intrinsically

attractive objects have great acceptance, agents would be very reluctant to trade them away even if market conditions change.

In contrast, the endogenous transaction pattern along my cycles is at least roughly analog to what we would expect in financial markets; namely, that the acceptability of an asset varies inversely with the rate of return if other conditions are equal.<sup>1</sup> In fact, all agents in my examples do “trading.” The intuition for having more trade is that if intrinsically unattractive objects are those that are widely accepted, people would be less reluctant to trade them away and more trade may occur when market conditions change. Therefore, my equilibria may meet the Hicks’s challenge about explaining the rate-of-return dominance of money (famous in monetary economics) in the most unexpected way.

To have this “trading-enhancing” transaction pattern in the model, I make agents to play mixed trading strategies at least in one point of the cycle. This seems to be the simplest and surest way because agents have to be indifferent to trading to randomize trading strategies. Consequently, exogenous differences in storage costs have to be compensated by endogenous differences in acceptability; in particular, acceptability or “liquidity” of an object has to vary directly with its storage cost. Hence, I show by example that if transaction costs (or exchange costs) are small enough, there may exist a class of mixed-strategy equilibria that have good welfare properties because more trade occurs than in other equilibria. Therefore, this result may contradict possible cautiously skeptical views about this point; for instance, Wallace [1996, p. 252, fn. 6] states that transaction costs “can play a role by preventing the occurrence of trades in which one or more of the participants are indifferent to trading.”

Notice that my results may give a basis to question the popular analysis that arbitrarily focuses on the simplest steady states. Moreover, this basis may be particularly appealing because I show that the trade volume and welfare may vary directly with the length of the cycle.

My results may also have far reaching implications from a practical point of view. They may rationalize buy-back programs of shares and they may shed some light on benefic market

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<sup>1</sup> See for instance Wallace [2000] for the so-called liquidity structure.

fluctuations. In fact, one may observe that some fluctuations in financial markets are highly predictable and often regarded as benefic. For instance, rallies followed by “profit taking” periods --not rarely along with repurchase programs of shares.

Even though we seem to observe at least some times benefic fluctuations, economists traditionally might have insisted too much in macroeconomic stability. For instance, Modigliani [1977] resuming the monetarist controversy does not leave room for benefic macroeconomic fluctuations: They are neutral or bad and in the latter case the discussion is about if they are curable. Another example may be Friedman [1948] who proposes a monetary and fiscal framework to achieve long-run growth with the bonus of achieving also short-run stability. Perhaps, another more recent example is the advocacy of (low and) **stable** inflation as the central banks' ultimate goal.

This paper is organized as follows. In section 2, I give the description of the environment, the notation, and the equilibrium definition. In section 3, I establish the existence of equilibria converging to stationary cycles. In section 4, I make welfare analysis of alternative equilibria.

## 2. PHYSICAL ENVIRONMENT AND EQUILIBRIUM DEFINITION

I will use the physical environment, notation, and essentially the equilibrium concept of the Aiyagari-Wallace [1992] exposition of the Kiyotaki-Wright model [1989]. I give next a brief description of these items. For reader's convenience, I abstract of fiat objects.

### 2.1 The Physical Environment

Time is discrete and represented by the positive integers. There exist  $N \geq 3$  goods that are assumed to be perfectly durable and indivisible goods. The goods are indexed by the set  $\{1, \dots, N\}$ . There are  $N$  types of infinitely-lived agents indexed also by the set  $\{1, \dots, N\}$ . Type  $i$  consumes good  $i$  and produces good  $i+1$ . There is a  $[0, 1/N]$  continuum of agents for each type.

Agents maximize expected discounted utility with discount factor of  $\rho \in (0, 1)$ . In any period, a type- $i$  agent's utility of neither consuming nor storing anything is zero, the utility of consuming one unit of good  $i$  without storing anything is  $u_i > 0$ , and the utility of not consuming and storing one unit of good  $j$  from the given period until the next period is  $-c_{ij} \leq 0$ . After consuming one unit of good  $i$ , agent of type  $i$  produces one unit of good  $i+1$  which appears at the beginning of next period. At the beginning of the initial period  $t = 1$ , each agent is endowed with one unit a good. Finally, each period each agent is paired randomly with one other agent. It is assumed that paired agents know each other's type and current inventory but nothing else about trading histories.

In this paper, I assume additionally that **all** agents have small exchange costs in terms of utility; that is, that agents when exchange one good for another they have utility  $-\epsilon \leq 0$  which adds to the other utilities from consuming or storing.

## 2.2 Definition of Equilibrium

Next, I define a class of Nash equilibria with rational expectations. In particular, we assume that the strategies are *symmetric*, i.e., all agents of the same type in the same situation use the same strategy, and that trading strategies are *nondiscriminatory*, i.e., willingness to trade does not depend on the type of agents one meets. Moreover, we assume that agents do not dispose of goods and do not postpone consumption. We require that in equilibrium the actions of each agent are individually optimizing given the actions of the other agents and the path of inventory distribution of stocks. The notation presumes that each agent start each period with one unit of a good. That is, we assume additionally that agents never give a good away to another agent for nothing.<sup>2</sup>

Therefore, the timing of an agent's activities in period  $t$  is that he or she starts with one good, meets another agent, ends up with some good after meeting, and then stores or consumes and produces to start at period  $t+1$  with some good.

I give next notation for the trading strategies of the agents and other items. The probability of choosing to trade good  $j$  for good  $k$  by those who are type  $i$ , hold good  $j$ , and meet another agent with good  $k$  at period  $t$ , is denoted by  $s_{ij}^k(t)$ . The vector of these over  $(j,k)$  is denoted by  $s_i(t)$  and the vector of these over  $(i,j,k)$  is denoted by  $s(t)$ .

The symbol  $p_{ij}(t)$  denotes the proportion of agents who are type  $i$  and hold good  $j$  at the start of period  $t$ . The symbol  $p(t)$  denotes the vector of these elements or the distribution of inventories; that is,  $p(t)$  is the state of system. The law of motion of the sequence  $\{p(t)\}$  is denoted by

$$p(t+1) = h[p(t), s(t)],$$

where  $h$  is defined by the following two equations

$$(1.1) \quad a_{ij}(t) = p_{ij}(t) - p_{ij}(t) \sum_k \sum_{l \neq j} p_{kl}(t) s_{ij}^l(t) s_{kl}^j(t) + \sum_k \sum_{l \neq j} p_{il}(t) p_{kj}(t) s_{il}^j(t) s_{kj}^l(t)$$

$$(1.2) \quad p_{ij}(t+1) = (1 - \delta_{ij}) a_{ij}(t) + \delta_{i+1,j} a_{ii}(t)$$

where  $\delta_{kn} = 1$  if  $k = n$  and 0 otherwise.

In (1.1), the symbol  $a_{ij}(t)$  represents the proportion of agents who are type  $i$  and end up with good  $j$  after the random meetings at period  $t$ . On the RHS of (1.1), the first term is the proportion of agents who are type  $i$  and hold good  $j$  at the start of period  $t$  and, consequently, before the random meetings at period  $t$ ; the second term is the proportion of agents who are type  $i$  and held good  $j$  before the random meetings and traded for a different good; and the third term is the proportion of agents who are type  $i$  and who did not hold good  $j$  before the random meetings and who traded for good  $j$  at period  $t$ . In (1.2), if  $j = i$ , then  $p_{ij}(t+1) = 0$  because agents do not postpone consumption; if  $j = i+1$ , then  $p_{ij}(t+1) = a_{ij}(t) + a_{ii}(t)$  because  $a_{ii}(t)$  is the fraction of agents who produce good  $i+1$ ; lastly, if  $j \neq i$  and  $j \neq i+1$ , then  $p_{ij}(t+1) = a_{ij}(t)$ .

Let  $v_{ij}(t)$  denote the expected discounted utility of agent of type  $i$  ending with good  $j$  after trade at period  $t$ , but before consuming, storing, or disposing, and let  $v(t) \in \mathfrak{R}^{N(N)}$  denote the vector of expected utilities. Given the sequence  $\{p(t+1), s(t+1)\}$ , there exists a unique bounded sequence of vectors  $\{v(t)\}$  that satisfy the dynamic-programming equations

$$(2.1) \quad v_{ij}(t) = -c_{ij} + \rho \sum_k \sum_l p_{kl}(t+1) \left[ s_{ij}^l(t+1) s_{kl}^j(t+1) (v_{il}(t+1) - \varepsilon) + (1 - s_{ij}^l(t+1) s_{kl}^j(t+1)) v_{ij}(t+1) \right]$$

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<sup>2</sup> In fact, we can ignore gift giving because, as noticed by Aiyagari-Wallace [1991], it is never an equilibrium strategy given the following small and otherwise innocuous change to the model: let agents derive some small

if  $i \neq j$  and

$$(2.2) \quad v_{ii}(t) = u_i + \rho \sum_k \sum_l p_{kl}(t+1) \left[ s_{i,i+1}^l(t+1) s_{kl}^{i+1}(t+1) (v_{il}(t+1) - \varepsilon) \right. \\ \left. + (1 - s_{i,i+1}^l(t+1) s_{kl}^{i+1}(t+1)) v_{i,i+1}(t+1) \right]$$

The existence and uniqueness of the bounded sequence  $\{v(t)\}$  follow from the fact that the RHS of equations (2.1) and (2.2) are contractions as functions of the components of the vector  $v(t)$ .

The individual optimizing conditions for trading strategies are

$$(3.1) \quad v_{ij}(t) \geq v_{ik}(t) \quad \text{if } s_{ij}^k(t) = 0$$

$$(3.2) \quad v_{ij}(t) \leq v_{ik}(t) \quad \text{if } s_{ij}^k(t) = 1$$

$$(3.3) \quad v_{ij}(t) = v_{ik}(t) \quad \text{if } 0 < s_{ij}^k(t) < 1.$$

The condition about optimality of consumption after acquiring the consumption good is

$$(3.4) \quad u_i + c_{ij} + v_{i,i+1}(t) \geq -c_{ii} + \rho [u_i + c_{ij} + v_{i,i+1}(t)]$$

The condition about optimality of nondisposal is

$$(3.5) \quad v_{ij}(t) \geq 0$$

Therefore, we have the next definitions.

**DEFINITION 1.** A *symmetric equilibrium* from  $p(1)$ , in which agents (i) do not play discriminatory strategies, (ii) do not dispose of goods, and (iii) do not postpone consumption, is a path  $\{p(t+1), s(t)\}$  such that

- (1)  $p(t+1) = h[p(t), s(t)]$ , and
- (2) there exists a bounded sequence  $\{v(t)\}$  such that equations (2.1)-(2.2) and (3.1)-(3.5) hold.

Notice that the equilibrium condition (2) requires that the sequence  $\{s_i(t)\}$  be the best responses of any agent of type  $i$  taken as given the path  $\{p(t+1), s(t)\}$ .

**DEFINITION 2.** A *symmetric steady state*, in which agents (i) do not play discriminatory strategies, (ii) do not dispose of goods, and (iii) do not postpone consumption, is a constant  $(p, s)$  such that  $[p(t+1), s(t)] = (p, s)$  for all  $t \geq 1$  satisfies Definition 1 when  $p(1) = p$ .

### 3. EQUILIBRIA CONVERGING TO STATIONARY CYCLES

I describe next the simplest examples of equilibria with stationary cycles with good welfare properties that I have been able to construct so far. No surprisingly, I assume  $N = 3$ , and symmetry in parameters (i.e., equal storage costs) and in inventory distributions. Moreover, I assume exchange costs equal to zero (i.e.,  $\varepsilon = 0$ ) and discount factor,  $\rho$ , close enough to 1. Indeed, by continuity (see, for instance, Renero 1999), there exist similar equilibria with stationary cycles for storage costs nearly equal and exchange costs small enough.

#### 3.1 An equilibrium converging to a two-period stationary cycle

Given the assumptions on parameters and assuming that  $t$  is odd, I describe next the shortest cyclic stationary equilibrium possible.

$$s_{i,i+1}^{i+2}(t) = 0 \text{ and } s_{i,i+2}^{i+1}(t) = \frac{9 p_{13}(t+1) + \rho [-1 + 6 p_{13}(t)][1 + 6 p_{13}(t+1)]}{3 \rho p_{13}(t)[1 + 6 p_{13}(t+1)]},$$

$$s_{i,i+1}^{i+2}(t+1) = 1 \text{ and } s_{i,i+2}^{i+1}(t+1) = 0,$$

and

$$p_{i,i+2}(t) = 1/9 + 2(p_{i,i+2}(t+1))^2, \quad p_{i,i+2}(t+1) = \lambda,$$

where  $\lambda$  is the only real root of the polynomial

$$-\rho + (6 + 3\rho)x + 42\rho x^2 + (108 - 72\rho)\rho x^3 + 108\rho x^4 + 648\rho x^5.$$

Notice that for storage costs small enough, this is a stationary equilibrium because

$$v_{i,i+1}(t) - v_{i,i+2}(t) = 0 \text{ and } v_{i,i+1}(t+1) - v_{i,i+2}(t+1) = -\frac{3(u_i + c_{i,i+1})p_{i,i+2}(t+1)}{1 + 6 p_{i,i+2}(t+1)}.$$

For reader's convenience, I give next approximations of the limits (as the discount factor approaches to one) of equilibrium variables that depend on  $\rho$ .

$$\lim_{\rho \rightarrow 1} s_{i,i+2}^{i+1}(t) \approx 0.607$$

$$\lim_{\rho \rightarrow 1} p_{i,i+2}(t) \approx 0.124$$

$$\lim_{\rho \rightarrow 1} p_{i,i+2}(t+1) \approx 0.079$$

Finally, to make welfare evaluations, I give next the fraction of agents who consume each period and their average along the cycle.

$$\lim_{\rho \rightarrow 1} \sum a_{ii}(t) \approx 0.183$$

$$\lim_{\rho \rightarrow 1} \sum a_{ii}(t+1) \approx 0.315$$

$$\lim_{\rho \rightarrow 1} \frac{1}{2} \left[ \sum a_{ii}(t) + \sum a_{ii}(t+1) \right] \approx 0.249$$

I establish next that for an open set of parameters and initial conditions there exists an equilibrium converging to the stationary equilibrium described above. Notice that if agents are playing mixed strategies at period 1, the vector of trading strategies  $s(1)$  is not determined. Hence, we take advantage of this indeterminacy to select an  $s(1)$  such that the equilibrium converges in two periods. So if the initial condition  $p(1)$  is given by  $p_{i,i+2}(1) = 1/9$ ,  $s(1)$  given by

$$s_{i,i+1}^{i+2}(1) = 0 \text{ and } s_{i,i+1}^{i+2}(1) = \frac{3 p_{i,i+2}(t+1) - 2 p_{i,i+2}(1) - 3 (p_{i,i+2}(1))^2}{6 (p_{i,i+2}(1))^2}$$

do the job and the claim follows by continuity.

### 3.2 An equilibrium converging to a three-period stationary cycle

Given the assumptions on parameters and assuming that  $t$  is odd, I describe next a stationary equilibrium with a three-period cycle.

$$s_{i,i+1}^{i+2}(t) = \frac{5 + p_{13}(t+1) [3 - 72 p_{13}(t+2)] - 12 p_{13}(t+2) + 6 p_{13}(t) [1 + 6 p_{13}(t+1)] [-2 + 3 p_{13}(t+2)]}{[-1 + 3 p_{13}(t)] [1 + 6 p_{13}(t+1)] [-2 + 3 p_{13}(t+2)]},$$

and  $s_{i,i+2}^{i+1}(t) = 0$ ,

$$s_{i,i+1}^{i+2}(t+1) = 1 \text{ and } s_{i,i+2}^{i+1}(t+1) = 0,$$

$$s_{i,i+1}^{i+2}(t+2) = 0 \text{ and } s_{i,i+2}^{i+1}(t+2) = 1.$$

For reader's convenience, I give next approximations of the limits (as the discount factor approaches to one) of equilibrium variables that depend on  $\rho$ .

$$\lim_{\rho \rightarrow 1} p_{i,i+2}(t) \approx 0.071,$$

$$\lim_{\rho \rightarrow 1} p_{i,i+2}(t+1) \approx 0.106,$$

$$\lim_{\rho \rightarrow 1} p_{i,i+2}(t+2) \approx 0.133.$$

Notice that for storage costs small enough, this is a stationary equilibrium because

$$v_{i,i+1}(t) - v_{i,i+2}(t) = 0,$$

$$\lim_{\rho \rightarrow 1} [v_{i,i+1}(t+1) - v_{i,i+2}(t+1)] \approx -0.194 (u_i + c_{i,i+1}), \text{ and}$$

$$\lim_{\rho \rightarrow 1} [v_{i,i+1}(t+2) - v_{i,i+2}(t+2)] \approx 0.11 (u_i + c_{i,i+1})$$

Finally, to make welfare evaluations, I give next the fraction of agents who consume each period and their average along the cycle.

$$\lim_{\rho \rightarrow 1} \sum a_{ii}(t) \approx 0.271,$$

$$\lim_{\rho \rightarrow 1} \sum a_{ii}(t+1) \approx 0.300,$$

$$\lim_{\rho \rightarrow 1} \sum a_{ii}(t+2) \approx 0.213,$$

$$\lim_{\rho \rightarrow 1} \frac{1}{3} \left[ \sum a_{ii}(t) + \sum a_{ii}(t+1) + \sum a_{ii}(t+2) \right] \approx 0.261$$

I establish next that for an open set of parameters and initial conditions there exists an equilibrium converging to the stationary equilibrium described above. Notice that if agents are playing mixed strategies at period 1, the vector of trading strategies  $s(1)$  is not determined. Hence, we take advantage of this indeterminacy to select an  $s(1)$  such that the equilibrium converges in two periods. So if the initial condition  $p(1)$  is given by  $p_{i,i+2}(1) = 1/9$ ,  $s(1)$  given by

$$s_{i,i+1}^{i+2}(1) = \frac{3 \left[ -3 p_{i,i+2}(t+1) + 2 p_{i,i+2}(1) + 3 (p_{i,i+2}(1))^2 \right]}{(1 - 3 p_{i,i+2}(1))^2} \text{ and } s_{i,i+1}^{i+2}(1) = 0$$

do the job and the claim follows by continuity.

#### 4. WELFARE FOR NEARLY-EQUAL STORAGE COSTS

In the last section I showed that there exist two equilibria converging to stationary cycles in which the acceptance of a good varies directly with its storage costs. In this section, I will show that these equilibria Pareto superior to other equilibria with a different transaction pattern; in particular, equilibria with universal acceptance of less costly to-store goods. (Notice that the cycles of Kiyotaki-Wright [1993] do not exist in a small enough neighborhood of equal storage costs.)

There is more than one way to prove this claim. The one that we will follow here, which seems to be the easiest one, is to make use of the results in Renero [1999, Props. 3.2 and 3.3]. There, I prove (1) that there exists one and only one equilibrium in which the least costly to-store good is universally accepted, (2) that there exists an equilibrium in which the second least costly to store good is universally accepted.

The proof of the welfare ranking relies on the fact that all multiple equilibria converge and consequently the welfare ranking is determined by the ranking of the steady-state expected utilities for a discount factor  $\rho$  close enough to 1. For reader's convenience and to establish some notation, I will review next the associated steady states.

##### 4.2 Standard equilibria

For an open set of parameters, Kiyotaki-Wright [1989] exhibit a pure-strategy steady state  $(p^f, s^f)$  in which the least costly to-store good has universal acceptance if the storage costs

of goods,  $c_{ij}$ , satisfy either the inequalities  $c_{i1} < c_{i2} < c_{i3}$  or the inequalities  $c_{i1} < c_{i3} < c_{i2}$ . The vector  $s^f$  is given by  $s^f = (s_{12}^3, s_{13}^2, s_{23}^1, s_{21}^3, s_{31}^2, s_{32}^1) = (1, 0, 1, 0, 0, 1)$  and  $p^f$  is given by

$$p^f = \begin{bmatrix} p_{11}^f & p_{12}^f & p_{13}^f \\ p_{21}^f & p_{22}^f & p_{23}^f \\ p_{31}^f & p_{32}^f & p_{33}^f \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 & 2^{-1/2} & 1 - 2^{-1/2} \\ 2 - 2^{1/2} & 0 & 2^{1/2} - 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

The trading strategies of agents of type 1 are optimal if  $\frac{c_{13} - c_{12}}{\rho [u_1 + c_{12}]} < (p_{31}^f - p_{21}^f) = \frac{1}{3}(2^{1/2} - 1)$ .

Actually, the pair  $(p^f, s^f)$  is the only steady state with universal acceptance of the east costly to-store good if the inequality above holds.

For an open set of parameters, Kiyotaki-Wright [1989] also exhibit a pure-strategy steady state  $(p^s, s^s)$  in which the second least costly to-store good has universal acceptance if the storage costs of goods,  $c_{ij}$ , satisfy either the inequalities  $c_{i1} < c_{i3} < c_{i2}$ . (Remember that type- $i$  agents produce good  $i+1$  with modulus 3.) The vector of trading strategies  $s^s$  is given by  $s^s = (s_{12}^3, s_{13}^2, s_{23}^1, s_{21}^3, s_{31}^2, s_{32}^1) = (1, 0, 0, 1, 1, 0)$  and  $p^s$  is given by

$$p^s = \begin{bmatrix} p_{11}^s & p_{12}^s & p_{13}^s \\ p_{21}^s & p_{22}^s & p_{23}^s \\ p_{31}^s & p_{32}^s & p_{33}^s \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 & 2^{1/2} - 1 & 2 - 2^{1/2} \\ 0 & 0 & 1 \\ 2^{-1/2} & 1 - 2^{-1/2} & 0 \end{bmatrix}.$$

### 4.3 Welfare analysis

As in other welfare analysis of alternative equilibria in the Kiyotaki-Wright model, we also consider initial conditions in the neighborhood of the distribution implied by always trade,  $p^{AT}$ , where

$$p^{AT} = p^c = \begin{bmatrix} p_{11}^c & p_{12}^c & p_{13}^c \\ p_{21}^c & p_{22}^c & p_{23}^c \\ p_{31}^c & p_{32}^c & p_{33}^c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & 0 & \frac{2}{9} \\ \frac{2}{9} & \frac{1}{9} & 0 \end{bmatrix}.$$

The next proposition establishes that for an open set of parameters and initial conditions, there exist multiple equilibria that are Pareto ranked. In particular, that there exists an equilibrium converging to a stationary two-period cycle. Moreover, this equilibrium Pareto superior to other equilibria with universal acceptance of less costly to-store goods. Furthermore, there exists equilibrium converging to a stationary three-period cycle that Pareto superior to the equilibrium converging to a stationary two-period cycle.

**PROPOSITION 1:** Assume  $N = 3$  and either  $c_{i1} < c_{i2} < c_{i3}$  or  $c_{i1} < c_{i3} < c_{i2}$ . There exists a neighborhood  $P$  of  $p^{AT} = p^c$  and an open set  $Y$  of parameters  $(\rho, u_i, c_{ij}, \varepsilon)$  (a set that includes equal storage costs), such that for any  $p(1) \in P$  and any parameters in  $Y$ ,

- (i) the path  $\{p(t+1), s^f\}$  from  $p(1)$  converges and it is an equilibrium and the only equilibrium in which  $s_{ij}^1(t) = 1, j \neq i$ ;
- (ii) for  $c_{i1} < c_{i3} < c_{i2}$ , the path  $\{p(t+1), s^s\}$  from  $p(1)$  converges;
- (iii) there exists an equilibria from  $p(1)$  converging in two periods to the two-period stationary cycle described in Sec. 3.1; moreover, this equilibrium Pareto superior to the equilibria in (i), and (ii).
- (iv) there exists an equilibria from  $p(1)$  converging in two periods to the three-period stationary cycle described in Sec. 3.2; moreover, this equilibrium Pareto superior to the equilibria in (i)-(iii).

*Proof:* The existence and convergence follow by using Propositions 3.2-3.3 in Renero [1999] and by the stated in Sections 3.1-3.2. The claim of the Pareto ranking follows by continuity for  $\rho$  close enough to 1, since (1) all equilibria converge for  $p(1)$  in a neighborhood of  $p^c$  and (2) if for every type of agent all goods are equally costly to store the components of the expected utility vectors  $v(t)$  associated with the corresponding steady states are ranked according to the claimed Pareto ranking.

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