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Inflation and Log Run Output: The Role of Banks

Inflation and long-run output: the role of banks

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Abstract

This paper studies the role of banks as magnifiers of the inflation-tax distortion. It develops a general equilibrium model for a small open economy, with perfect foresight and has minimal departures from a neoclassical framework. The goal of this paper is to show that even abstracting from information issues commercial banks may still have a non-trivial role in amplifying disturbances. In order to stress this idea the paper considers a deterministic setup and assumes a zero-cost banking technology. The model shows that, in an inflationary environment, if banks are required to hold reserve deposits, then the steady-state equilibrium level of employment is lower. Moreover, the model finds a wide range of parameters for which the reserve requirement also affects the sensitivity of employment to inflation.These results depend on three key assumptions: banks are subject to a reserve requirement, the firms borrow from banks to cover their financial needs, and households must hold demand deposits to acquire some of the consumption goods.

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1 Introduction

Conventional macroeconomic models mostly abstract from financial intermediation considerations. The assumption of a financial system that works smoothly implies that real and financial variables do not interact with each other in interesting ways. Under this premise financial intermediaries simply do not play any role in determination of real variables. Interest on the role of financial intermediaries, however, has been growing steadily since the middle 1980s.[1] Contributions to this literature address financial issues in models of output determination that depart from the Real Business Cycle literature.[2]

More recently, the financial system, in particular the banking sector, has been the subject of close attention following the currency crises in Mexico, Russia, and East Asia. The recent literature emphasizes the role of the banking industry as either the trigger of these crises, or the key in explaining the output collapses in the aftermath of the crises.[3] This literature makes the distinction between fundamentals and multiple equilibria as explanations for the crises, but in either case, financial issues are at the core of the analysis.

All these different theoretical contributions, addressing issues as diverse as the behavior of the economic cycles and currency crises, share a common characteristic in the way the role of financial intermediation is modeled. In all these models, the presence of uncertainty is crucial for the financial intermediaries to play a non-trivial role. Decisions in these models are taken in an incomplete information environment, where acquiring information is costly.

The present paper examines the role of financial intermediaries in a model with perfect foresight and minimal departures from a neoclassical framework. My intention, of course, is not to question the importance of economies of information (of which financial issues constitute a clear example). Rather, the point of this paper is that even abstracting from information issues, both those arising directly (through uncertainty about the realization of shocks) and those arising indirectly (through, for example, information gains embedded in a particular bank's cost structure), financial intermediaries may still have a non-trivial role in amplifying disturbances. In order to stress this idea I consider a deterministic setup and assume a zero-cost banking technology.

The paper presents a general equilibrium model where commercial banks magnify the supply side's reaction to changes in monetary policy. It looks, in particular, at the role of commercial banks amplifying the distortionary effects of inflation. These results depend on three key assumptions: banks are subject to a reserve requirement, the firms borrow from banks to cover their financial needs, and households must hold demand deposits to acquire some of the consumption goods.[4]

The distortionary effects of inflation have been extensively addressed in the literature,

[1] See M. Gertler (1988) for an early and exhaustive overview.

[2] Some examples are Farmer (1984), Scheinkman and Weiss (1986), Williamson (1987), Bernanke and Gertler (1988), and most recently Edwards and Végh (1997).

[3] For empirical reserch in this issue see Kaminsky and Reinhart (1999). For theoretical models linking financial issues with currency crises see, for example Corsetti, Pesenti, and Roubini (1997), Chang and Velasco (1997), Burnside, Eichenbaun, and Rebelo (1998), Obsteld (1998), and Dooley (2000).

[4] We abstract from the benefits of having a reserve requirement (i.e. soundness of the financial system or fiscal considerations). In the model, instead, the government exogenously sets the reserve requirement.

particularly by Lucas' contributions.[5] To the extent that the inflation tax itself represents an important distortion, the role of banks in amplifying the distortion is a relevant problem to be considered in reality.

The paper extends the Lucas and Stokey (1987) model by introducing, in an open economy framework, a deposit-good in addition to their cash-good and leisure (credit-good). The general setup is related to Edwards and Végh's (1997) model for the study of the role of banks on the transmission of macroeconomic disturbances. The present model, however, provides a general equilibrium view of the way the demand and supply sides of the economy are related through the financial intermediary.[6]

The framework of this paper is closely related to the types of macro models on the credit channel and liquidity effects of monetary policy.[7] Models in these lines of work assume that firms have to cover their financial needs by borrowing from banks. This issue is of crucial importance in most developing economies where financial markets are not fully developed, and banking credit is by far the most important source of funds for firms. In the present model, however, the amount firms borrow from banks is determined endogenously.

The paper proceeds as follows. Section 2 develops the theoretical model, states the timing of the different markets, and solves for the problems of the different agents. Section 3 solves the model. It derives the aggregate resource constraint, and shows the existence of a steady-state equilibrium. Section 4 investigates the effects of monetary policy. It considers, as a benchmark case, an economy without reserve requirements on bank deposits, and compares it with the case where the reserve requirement is positive. Then, it discusses the role of banks in amplifying the inflation tax distortion and its effect on real variables at the steady-state equilibrium. Finally, Section 5 concludes.

2 The model

This section presents the basic structure of the model, which extends the Lucas and Stokey (1987) framework by introducing a deposit-good. In addition to the cash-good and leisure, I consider also a deposit-constraint to household's purchases of a consumption good paid for by check.

Consider a small open economy perfectly integrated with the rest of the world in the goods market. Assume, for simplicity, that only the representative commercial bank can borrow and lend in international financial markets.

There are three optimizing representative agents: the firm, the household, and the bank. The government exogenously sets the level of lump-sum taxes and transfers to household, T_t, the money supply, M_t, and the total domestic-currency-denominated debt held by households, B_t, bearing an interest rate i_t^b. The government also sets a reserve requirement, ϕ, on the bank's demand deposits. All agents have perfect foresight.

[5] See Lucas (1987) and (2000)

[6] By contrast, while in Edwards and Végh (henceforth E&V) both the household's deposit-in-advance constraint and bank's credit to the firm are fixed proportions of consumption and production respectively, in this model these restrictions are endogenous. E&V's results don't necessarily hold once these parameters are endogenized.

[7] See for example Bernanke and Blinder (1988) and Fuerst (1992).

The household owns the bank and the firm, and receives profits if there are any. There are two non-storable consumption goods, x_t is a non-tradable cash good and y_t is a tradable deposit-good. The household decides how much to purchase of the deposit-good y_t (purchased with checks backed up with demand deposits in the bank, D_t), and of the cash-good x_t (purchased with cash, C_t). The household also decides how much of its endowment of leisure (normalized to 1) to sell to the firm for a nominal wage w_t.

The model allows for the possibility that households hold domestic-currency-denominated saving deposits in the domestic bank, S_t, which pay the interest rate i_t^s, and domestic-currency-denominated bank loans, Z_t^h, that charge the interest rate i_t^z.

The firm produces both goods with the same constant-returns-to-scale (C.R.T.S.) technology, and hires labor as the only input. It directly exports and imports the deposit-good y_t, clearing accounts with the foreign sector at the end of the period. The firm has to borrow from the bank Z_t^f, equal to the amount by which cash good revenues fall short of the gross wage bill.

The bank carries out all its activities at zero cost. It accepts the household's saving and demand deposits, issues loans to the firm and to the household, and holds reserves at the central bank. It can also issue and buy in the foreign financial market the foreign-currency-denominated bond B_t^*, that yields the interest rate r_t.[8] None of the other domestic agents can either lend to or borrow from the foreign sector.

All markets are perfectly competitive.

The following subsections describe the timing in the different markets and present in detail the problems of the different agents .

2.1 Timing of different markets

As in the Lucas and Stokey model, different markets are open at distinct sub-periods.

1. At the beginning of each period there is a financial exchange. The household acquires cash for purchases of x_t and makes deposits D_t to back his payments by checks to buy y_t. The household makes his portfolio decisions Z_t^h, S_t, and B_t. The firm borrows from the bank Z_t^f (see below). No further portfolio reshu- ing is allowed after this sub-period.

2. In the goods exchange sub-period the household buys goods from the firm paying by cash or by checks backed up by demand-deposits.

3. The firm pays wages net of taxes to the household and the withheld taxes to the government. The firm can use the cash from sales of x_t, but needs to use bank loans, Z_t^f, for the rest of its financial needs.

4. Checks clear in next period financial exchange.

[8] $B^* > 0$ represents a net foreign asset for the bank and $B^* < 0$ net foreign liabilieties.

2.2 Government: treasury and central bank

The model consolidates the treasury and the central bank into one agent: the government. The government exogenously sets the level of lump-sum taxes net of transfers to the household, T_t, the money supply, M_t, the level of domestic-currency-denominated bonds, B_t, and the reserve requirement on bank's demand deposits, ϕ.

The treasury issues domestic-currency-denominated bonds, the central bank monetizes part of that debt, and the rest is held by households. The central bank transfers its profits to the treasury. The budget constraint of the consolidated government is:

$$M_t + B_t = M_{t-1} + (1 + i^b_{t-1})B_{t-1} - T_{t-1}. \tag{1}$$

2.3 Household's utility maximization problem

The representative household maximizes utility with respect to the two types of consumption, leisure, and the financial portfolio, over an infinite horizon. With β representing the standard household's rate of time preference ($0 < \beta < 1$), lifetime utility is given by [9]:

$$\max_{x_t, y_t, l_t, Z^h_t, S_t} \mathcal{C} = \sum_{t=0}^{+\infty} \beta^t \left[\log x_t + \gamma \log y_t + \theta \log(1 - l_t)\right] \text{ subject to}$$

$$p_t x_t \leqq C_t \tag{2}$$

$$\frac{q_t y_t}{1 + i^d_t} \leqq D_t \tag{3}$$

$$\begin{aligned} B_t + S_t + D_t + C_t + Z^h_{t-1}(1 + i^z_{t-1}) \quad = \quad & B_{t-1}(1 + i^b_{t-1}) + (1 + i^s_{t-1})S_{t-1} \\ & + w_{t-1} l_{t-1} - T_{t-1} \\ & + Z^h_t + (C_{t-1} - p_{t-1}x_{t-1}) \\ & + \left(D_{t-1} - \frac{q_{t-1}y_{t-1}}{1 + i^d_{t-1}}\right)\left(1 + i^d_{t-1}\right) \\ & + \Phi^f_{t-1} + \Phi^b_{t-1} \end{aligned} \tag{4}$$

$$S_t \geq 0 \tag{5}$$

$$Z^h_t \geq 0 \tag{6}$$

$$B_t \geq 0 \tag{7}$$

[9] We chose logarithmic utility for simplicity. It does not seem to play a key role in generating the results.

Equation (2) is the cash-in-advance constraint. Equation (3) is the deposit-in-advance constraint; it states that the household can write checks against the interests earned and the principal of its demand deposits.

From the bank's problem below (see section 2.4) we know that $i_t^s = i_t^z > i_t^d > 0 \, \forall t$, and therefore Equations (2) and (3) hold with equality.

The flow budget constraint can, therefore, be rewritten as:

$$\begin{aligned} B_t + S_t + \frac{q_t y_t}{1+i_t^d} + p_t x_t + Z_{t-1}^h(1+i_{t-1}^z) &= B_{t-1}(1+i_{t-1}^b) + (1+i_{t-1}^s)S_{t-1} \\ &\quad + w_{t-1}l_{t-1} - T_{t-1} + Z_t^h \\ &\quad + \Phi_{t-1}^f + \Phi_{t-1}^b \end{aligned} \tag{8}$$

Letting λ_t, υ_t, μ_t, and ν_t denote the shadow price of nominal wealth, and the multipliers associated with the non-negative constraints on Z_t^h, B_t, and S_t respectively, the first-order conditions (F.O.C.s) are given by the following expressions:

with respect to (w.r.t.) y_t :

$$\frac{\gamma}{y_t} = \lambda_t \frac{q_t}{1+i_t^d} \tag{9}$$

w.r.t. x_t :

$$\frac{1}{x_t} = \lambda_t p_t \tag{10}$$

w.r.t. l_t :

$$\frac{\theta}{1-l_t} = \beta \lambda_{t+1} w_t \tag{11}$$

w.r.t. B_t and the non-negativity and complementary slackness conditions:

$$\begin{gathered} [\mu_t - \lambda_t] + \beta \lambda_{t+1} \left(1+i_t^b\right) \leqq 0 \\ \left[(\mu_t - \lambda_t) + \beta \lambda_{t+1} \left(1+i_t^b\right)\right] B_t = 0 \\ B_t \geq 0 \end{gathered}$$

w.r.t. Z_t^h and the non-negativity and complementary slackness conditions:

$$\begin{gathered} \beta^t \left[\upsilon_t + \lambda_t\right] - \beta^{t+1} \left[\lambda_{t+1} \left(1+i_t^z\right)\right] \leqq 0 \\ \left[\beta^t \left[\upsilon_t + \lambda_t\right] - \beta^{t+1} \left[\lambda_{t+1} \left(1+i_t^z\right)\right]\right] Z_t^h = 0 \\ Z_t^h \geq 0 \end{gathered}$$

w.r.t. S_t and the non-negative and complementary slackness conditions:

$$\begin{gathered} \beta^t \left[\nu_t - \lambda_t\right] - \beta^{t+1} \left[\lambda_{t+1} \left(-1-i_t^s\right)\right] \leqq 0 \\ \left[\beta^t \left[\nu_t - \lambda_t\right] - \beta^{t+1} \left[\lambda_{t+1} \left(-1-i_t^s\right)\right]\right] S_t = 0 \\ S_t \geq 0 \end{gathered}$$

w.r.t. υ_t and the non-negativity and complementary slackness conditions:

$$\beta^t Z_t \geq 0$$
$$\upsilon_t \geq 0$$
$$\beta^t Z_t \upsilon_t = 0$$

w.r.t. ν_t and the non-negativity and complementary slackness conditions:

$$\beta^t S_t \geq 0$$
$$\nu_t \geq 0$$
$$\beta^t S_t \nu_t = 0$$

and w.r.t. μ_t and the non-negativity and complementary slackness conditions:

$$\beta^t B_t \geq 0$$
$$\mu_t \geq 0$$
$$\beta^t B_t \mu_t = 0$$

Equations (9) and (10) show the standard result that, at an optimum, the household equates the marginal utility of consumption to the shadow value of wealth times the effective price of the good. Equation (11) states that the household equates the marginal utility of leisure to its opportunity cost, given by the nominal wage times next period's shadow value of wealth, discounted one period at the rate of time preference. The nominal wage is discounted one period because the household can not spend its labor income until the next goods exchange.

The remaining inequalities and equations are complementary slackness conditions for the inequality constraints.

The stock of government debt is assumed to be always strictly positive, $B_t > 0$. In equilibrium, therefore, the household will always hold all the bonds that the government issues at a competitive and endogenously determined interest rate i_t^b. All the possible solutions, therefore, involve only the cases with $B_t > 0$.

The non negative restrictions on saving deposits and loans imply that four different cases should be considered: the household holds saving deposits (in addition to government bonds), or takes loans, or both, or does not hold neither deposits nor take loans. This latter alternative, where $Z_t^h = 0$, $S_t = 0$, and $B_t > 0$, constitutes the natural benchmark case of the model, and it will be the one discussed in the paper.

2.3.1 $Z_t^h = 0$, $S_t = 0$, and $B_t > 0$

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = 1 + i_t^b \tag{12}$$

$$\theta \left(\frac{x_t}{1 - l_t} \right) = \frac{w_t}{p_t} \frac{1}{1 + i_t^b} \tag{13}$$

$$\frac{\theta}{\gamma}\left(\frac{y_t}{1-l_t}\right)=\frac{w_t}{q_t}\frac{1+i_t^d}{1+i_t^b} \tag{14}$$

In this case, the household is holding only government bonds. Therefore, i_t^b is the relevant interest rate and this is shown in the expression for the interest rate given by Equation (12). Equations (13) and (14) show that the optimal choice between either of the consumption goods and leisure is distorted by the interest rate. This distortion is equivalent to the cash-good credit-good distortion in the Lucas and Stokey setup. With endogenous labor, the household has the ability to substitute labor away to reduce the payment of the inflation-tax.[10]

2.4 The bank's profit maximization problem

The bank has a zero cost technology: it issues loans and receives deposits without using any resources.

The bank's profits are given by:

$$\Phi_t^b = Z_{t-1}i_{t-1}^z - D_{t-1}i_{t-1}^d - S_{t-1}i_{t-1}^s + B_{t-1}^*\left(e_t\left(1+\rho_{t-1}\right)-e_{t-1}\right).$$

where in addition to interest payments on deposits and interest earnings on loans and foreign assets, profits include gains and loses due to changes in the exchange rate.[11]

The bank chooses its portfolio optimally, maximizing the present value of profits over an infinite horizon. Because the household owns the bank, the future flow of profits is discounted at a rate $\beta^t\lambda_t$, where λ_t is the shadow value of wealth derived in the household's utility maximization problem, and β is the household's subjective rate of time preference.

The bank solves the following maximization problem:

$$\max_{D_t,S_t,Z_t,B_t^*} \mathcal{L}=\sum_{t=0}^{\infty}\beta^t\lambda_t\Phi_t^b \text{ subject to}$$

$$\Delta\left(e_tB_t^*+Z_t+\phi D_t-S_t-D_t\right)=\Phi_t^b \tag{15}$$

$$D_t+S_t-\left(e_tB_t^*+Z_t+\phi D_t\right)\geq 0 \tag{16}$$

Equation (15) is the bank's flow budget constraint where Δ is the difference operator. Inequality (16) requires that the bank's lending activities and reserve deposits have to be completely financed through liabilities[12]. As shown below, the interest rate on any of the

[10] Notice that in the case of y, the interest rate on demand-deposits is also included: the effective price of y_t is $\frac{q_t}{1+i_t^d}$ because checks can be written against interest earned from t to $t+1$.

[11] Profits defininition includes capital gains (losses) that arise from depreciation (appreciation) of the domestic currency. ρ_t is real and nominal for dollar denominated assets (because $\frac{P_{t+1}^*}{P_t^*}=1$). When buying the foreign bond in period $t-1$ the bank pays $e_{t-1}B_{t-1}^*$, and at the end of the period, when selling it back earns: $e_tB_{t-1}^*\left(1+\rho_{t-1}\right)$.

[12] In other words, equity is zero.

bank's liabilities is strictly positive in equilibrium. Condition (16), therefore, will hold as a strict equality. Letting η_t and ψ_t, denote the Lagrange multipliers associated with conditions (15), and (16), the solution to the system of F.O.C.s is:

$$i_t^s = i_t^z \tag{17}$$

$$i_t^d = i_t^z(1-\phi) \tag{18}$$

$$1 + i_t^s = \frac{e_{t+1}}{e_t}(1+\rho_t) \tag{19}$$

Equation (17) is a direct result of the assumption of a zero cost technology in a perfectly competitive market: there is no spread between saving-deposit and loan interest rates (and the bank charges borrowers the opportunity cost of funds). There is, however, as shown in Equation (18), a spread between demand-deposit and loan interest rates because the former is subject to a non-interest-bearing reserve requirement.

2.5 Firm's profit maximization problem

The representative firm has the following C.R.T.S. technology for the production of both the deposit and the cash-good[13]

$$x_t + \overline{y}_t = l_t. \tag{20}$$

In the financial exchange of period $(t-1)$ the firm has to borrow from the bank Z_{t-1}^f, equal to the amount in which cash-good revenues of $(t-1)$ fall short of the gross wage bill of $(t-1)$. That is:

$$Z_{t-1}^f = w_{t-1}l_{t-1} - p_{t-1}x_{t-1}.$$

In the following financial exchange, in period t, when all checks are cleared, the firm pays principal and interest on what it borrowed, $Z_{t-1}^f\left(1+i_{t-1}^z\right)$, settles the import-export account with the foreign sector, and is paid for the deposit-good sales, $q_{t-1}y_{t-1}$.

The firm's period t profit is:[14]

$$\Phi_t^f = q_{t-1}y_{t-1} - e_{t-1}(y_{t-1} - \overline{y}_{t-1}) - Z_{t-1}^f(1+i_{t-1}^z).$$

Therefore, profits can be rewritten as:

$$\Phi_t^f = q_{t-1}y_{t-1} - e_{t-1}(y_{t-1} - \overline{y}_{t-1}) - (w_{t-1}l_{t-1} - p_{t-1}x_{t-1})(1+i_{t-1}^z). \tag{21}$$

The firm pays the same wage in both sectors. Foreign trade does not involve any transaction costs.

[13] Labor productivity is exogenous and normalized to 1.

[14] The price of y_t in the foreign market, P_t^*, is nomalized to 1.

The firm solves the following maximization problem:

$$\max_{x_t, \overline{y}_t, l_t} \mathcal{F} = \sum_{t=0}^{\infty} \beta^t \lambda_t \Phi_t^f \text{ subject to } x_t + \overline{y}_t = l_t.$$

As in the bank's problem above, the future flow of profit is discounted at a rate $\beta^t \lambda_t$, where λ_t is the shadow value of wealth derived in the household's utility maximization problem, and β is the household's subjective rate of time preference. The solution to the system of F.O.C.s is:

$$q_t = e_t \tag{22}$$

$$\frac{w_t}{p_t} = 1 \tag{23}$$

$$q_t = (1 + i_t^z)\, p_t \tag{24}$$

Equation (22) is the result of assuming no imperfections in the deposit-good market: its domestic price is equal to the world's price (normalized to 1), expressed in the domestic currency. Equation (23) gives the real wage in a perfectly competitive market. Finally, Equation (24) states that prices p_t and q_t must be such that the firm is indifferent between sales for cash versus checks: because the firm has to borrow from the bank when selling y_t, q_t includes interest paid on loans.

The assumption of perfect competition together with a C.R.T.S. technology, implies that the zero-profit condition, $\Phi_t^f = 0$, holds. This result will be used later in the paper (see section 3.1 below).

3 Solution to the model

This section computes the solution to the model and characterizes the steady state equilibrium. First, it derives the economy's aggregate resource constraint. Second, it shows that the model has a steady state equilibrium. Then, it shows under which conditions the bank, and therefore the aggregate economy, is a net borrower in the foreign market. Finally, it presents the steady state solutions for the endogenous variables.

3.1 The aggregate resource constraint

Subtracting Equation (1), the government's flow budget constraint, from Equation (8), the household's flow budget constraint, yields:

$$M_{t-1} - M_t + S_t + \frac{q_t y_t}{1 + i_t^d} + p_t x_t + Z_{t-1}^h (1 + i_{t-1}^z) = (1 + i_{t-1}^s) S_{t-1} + w_{t-1} l_{t-1} + Z_t^h. \tag{25}$$

Money demand is given by:

$$M_t^d = p_t x_t + [w_t l_t - p_t x_t] + \phi \frac{q_t y_t}{1+i_t^d},$$

where the first term is the household's demand for money to acquire the cash-good, the second term is the firm's money demand equal to the amount in which cash-good sales fall short of the gross wage bill, and the last term is the bank's reserves held at the central bank.

In equilibrium $M_t^s = M_t^d = M_t$, and after simplifying we get:

$$M_t = w_t l_t + \phi \frac{q_t y_t}{1+i_t^d}. \tag{26}$$

Imposing zero profit for the firm, and using the firm's F.O.C.s, Equation (26) can be rewritten as:

$$M_t = \frac{1}{1+i_t^z} q_t \overline{y}_t + p_t x_t + \phi \frac{1}{1+i_t^d} q_t y_t. \tag{27}$$

Plugging Equation (27) into Equation (25), and using the findings from the bank's problem that $i_t^z = i_t^s = i_t$, yields:

$$Z_t^h - S_t = \phi \frac{q_{t-1}}{1+i_{t-1}^d} y_{t-1} - \frac{q_t}{1+i_t} \overline{y}_t + (1-\phi) \frac{q_t}{1+i_t^d} y_t + \left(Z_{t-1}^h - S_{t-1} \right) (1+i_{t-1}). \tag{28}$$

The following derivations use the balance-sheet identity for the bank, some results derived from the firm's zero profit condition, and the finding that $e_t = q_t$ obtained from the firm's problem. The balance-sheet identity for the bank can be expressed as:

$$Z_t^h - S_t + q_t B_t^* = \frac{q_t}{1+i_t^d} y_t (1-\phi) - \frac{q_t}{1+i_t} \overline{y}_t, \tag{29}$$

and plugging Equation (29) into Equation (28), the flow aggregate budget constraint is:

$$B_t^* = \frac{q_{t+1}}{q_t} \frac{1}{(1+i_t)} B_{t+1}^* - \frac{\overline{y}_t - y_t}{(1+i_t)} \tag{30}$$

Defining $\alpha_T = \prod_{j=0}^{j=T} \frac{1}{1+i_{t+j}}$ and imposing the solvency/transversality condition that $\lim_{T \to +\infty} \alpha_{T-1} \frac{q_{t+T}}{q_t} B_{t+T}^* = 0$, the corresponding present-value aggregated budget constraint is:

$$q_t B_t^* = -\sum_{j=0}^{+\infty} \alpha_j \; q_{t+j} \left(\overline{y}_{t+j} - y_{t+j} \right). \tag{31}$$

Equation (31) states that the nominal value of net foreign assets is equal to the (negative) present value of all nominal trade balance results. When $B_t^* > 0$ the economy is a net lender in the foreign financial market and the present value of the trade balance is negative. When $B_t^* < 0$, the economy is a net borrower and the present value of the trade balance is positive.

3.2 The existence of a steady state equilibrium

This section shows that there exists a constant level of net foreign assets, B^*, that satisfies the flow aggregate budget constraint given by Equation (30).

To allow for a steady-state equilibrium, assume that the gross inflation rate on tradable goods, defined as $\pi_t = \frac{q_{t+1}}{q_t}$, and the interest rate on foreign bonds are time invariant, that is $\pi_t = \pi$ and $\rho_t = \rho\ \forall t$. All nominal interest rates are, therefore, also constant. I further assume that $\beta = \frac{1}{1+\rho}$.[15]

This set of assumptions yields constant levels of y, x, l,and $\overline{y}$.

Using these findings in Equation (31) yields:

$$B^* = -\frac{1}{\rho\pi}\left(\overline{y} - y\right), \tag{32}$$

confirming that this economy has a steady state equilibrium where the level of net foreign bonds is constant.

3.3 The bank as a net borrower in the foreign market: $B^* < 0$

This section solves the model for the bank's holdings of foreign bonds. It shows that under certain assumptions, the bank, and therefore, the economy, is a net borrower in the foreign market, $B^* < 0$.

Using the bank's balance-sheet identity $eB^* = S - Z^h - Z^f - D(\phi - 1)$, the results from the household's and the firm's problems, and assuming that at the steady state equilibrium the household's saving-deposits net of loans are zero, $Z^h - S = 0$[16], bank's net foreign assets, expressed in term of parameters, are:

$$B^* = -\phi\frac{\gamma}{\left(\gamma\pi(1-\phi) + \phi\gamma + \theta\pi^2 + \theta\pi^2\rho + \pi\right)(1+\rho)\,\pi} < 0. \tag{33}$$

A reserve requirement that is strictly positive, $\phi > 0$, which I assume to be the case in this economy, is a sufficient condition for $B^* < 0$. Equation (33) states, therefore, that as long as there is a reserve requirement on bank deposits, the bank is a net debtor in the foreign financial market. This result is a direct consequence of the equilibrium in the market of the deposit-good, y. The firm borrows from the bank to finance the value of the production of y, and the household, in turn, holds deposits for the same amount. The bank, however, must hold a portion ϕ of the deposits at the central bank. Assuming that the household neither borrows nor holds saving-deposits, it follows that, for the equilibrium in the domestic credit market to hold, the bank must borrow abroad, $B^* < 0$.

[15] This latter assumption about the subjective discount rate is common in the literature because it eliminates explosive dynamics.

[16] This is the obvious benchmark case for the starting conditions. With positive (negative) $Z^h - S$, the bank's foreign debt would be larger (smaller or even lending instead of borrowing). With constant labor productivity, no uncertainty, and $\beta = \frac{1}{1+\rho}$, the household will not change its initial endowment of Z^h and S. In particular, we assume that $Z^h = S = 0$.

3.4 The steady state solution

In addition to the value of the foreign debt, B^*, computed already in the previous section, this section presents the steady state equilibrium values of the endogenous variables as a function of the parameters of the model.

$$B^* = -\phi \frac{\gamma}{\left(\gamma\pi(1-\phi) + \phi\gamma + \theta\pi^2 + \theta\pi^2\rho + \pi\right)(1+\rho)\pi}$$

$$x = \frac{\pi}{\theta(1+\rho)\pi^2 + \left(\gamma(1-\phi)+1\right)\pi + \phi\gamma}$$

$$1 - l = \frac{(1+\rho)\,\theta\pi^2}{\theta(1+\rho)\pi^2 + \left(\gamma(1-\phi)+1\right)\pi + \phi\gamma}$$

$$\overline{y} = \frac{\left(\pi(1-\phi)+\phi\right)\gamma}{\theta(1+\rho)\pi^2 + \left(\gamma(1-\phi)+1\right)\pi + \phi\gamma}$$

$$y = \frac{\left(\pi\,(1+\rho)\,(1-\phi)+\phi\right)\gamma}{(1+\rho)\left(\theta(1+\rho)\pi^2 + \left(\gamma(1-\phi)+1\right)\pi + \phi\gamma\right)}$$

$$p = \frac{M}{\left(\theta\pi(1+\rho)+\phi\gamma\right)\pi}\left(\theta\pi^2\,(1+\rho) + \left(\gamma\,(1-\phi)+1\right)\pi + \phi\gamma\right)$$

All the solutions are rather cryptic, but the comparative statics analysis included in the next section provides more clear insights about the steady state equilibrium characteristics.

4 The effects of monetary policy

This section investigates the effects of monetary policy on the steady state equilibrium. It looks at the impact of changes of the inflation rate (due to money growth changes) on employment. In order to give a more complete description of the macroeconomic interactions of the model, this section also explores the effect of inflation on other variables like foreign debt, consumption, and the bank's balance sheet.

The central bank chooses the rate of inflation and the size of the reserve requirement. The money supply is endogenous.[17]

I consider first, as a benchmark case, an inflationary economy without a reserve requirements. Then, I compare these results with those obtained when the reserve requirement is strictly positive, and highlight the existence of an amplification role for banks.

[17] In this model there is price indeterminacy. Fixing the level of the initial money supply, however, is enough to get rid of the indeterminacy problem.

4.1 Inflation without reserve requirement: $\pi > 1$ and $\phi = 0$

Since a positive nominal interest rate distorts the consumption-leisure decision (as shown in section 2.3.1), the supply side of the economy is not isolated from changes in monetary policy. When the government increases the growth rate of the money supply, inflation rises and the level of employment falls, or equivalently, as shown below, leisure rises:

$$\frac{d}{d\pi}(1-l) = \theta(1+\rho)\frac{\gamma+1}{(\gamma+\pi\theta(1+\rho)+1)^2} > 0. \tag{34}$$

The decline in the equilibrium level of employment is, therefore, a result of the household's maximizing behavior: the household cuts down its supply of labor to reduce the inflation-tax payment.

Consumption of both the deposit-good, y, and the cash-good, x, fall when inflation rises:

$$\frac{d}{d\pi}x = -\frac{\theta(1+\rho)}{(\gamma+\pi\theta(1+\rho)+1)^2} < 0, \tag{35}$$

$$\frac{d}{d\pi}y = -\gamma\frac{\theta(1+\rho)}{(\gamma+\theta\pi(1+\rho)+1)^2} < 0. \tag{36}$$

When the bank does not have to hold reserves, $\phi = 0$, the optimal choice between y and x is not distorted. In this case, as explained below, there is no difference between the price of x and the effective price of y. On one hand, q rises because the firm's financial cost of selling y rises with inflation, and therefore, the price of y rises faster than the price of x. On the other hand, the effective price of y includes the interest earned on the demand-deposits required to acquire the credit-good. Because demand-deposits are fully indexed when $\phi = 0$, the interest earned rises and the financial cost rises in exactly in the same proportion. It follows, then, that the relative consumption between x and y does not change with inflation.[18]

Consider next the changes in the bank's balance sheet. As already shown in section 3.3, when $\phi = 0$, net foreign bonds are also zero, $B^* = 0$. Given that there are no saving-deposits, neither loans to the household, nor bank reserves, it follows from the balance-sheet identity that real demand-deposits and real loans have to be exactly the same. Their derivatives with respect to inflation are negative:

$$\frac{d}{d\pi}\frac{D}{q} = \frac{d}{d\pi}\frac{Z^f}{q} = -\gamma\frac{2\theta\pi(1+\rho)+\gamma+1}{\pi^2(\theta\pi(1+\rho)+\gamma+1)^2(1+\rho)} < 0$$

Finally, and again because $B^* = 0$, production and consumption of the deposit-good, $\overline{y}$ and y are equal to each other. This is so because the deposit-good is the tradable one and in the steady-state equilibrium the trade surplus is equal to interest payment on B^*: zero in this case. It follows, then, that $\overline{y}$ falls by the same magnitude that y does (as shown in Equation (36)).

[18] Recall $i^d = i^z(1-\phi)$ and $q = p(1+i^z)$, so $MRSyx = \gamma$ when $\phi = 0$

4.2 Inflation with positive reserve requirement: $\pi > 1$ and $\phi > 0$:

The first thing to notice is that, when the bank faces a non interest-bearing reserve requirement, the steady-state equilibrium involves $B^* < 0$.[19] A higher rate of inflation reduces the total amount of debt the economy can sustain in the steady-state equilibrium:

$$\frac{d}{d\pi}B^* = \phi\gamma\frac{2\gamma\pi(1-\phi)+3\theta\pi^2(1+\rho)+2\pi+\phi\gamma}{(\gamma\pi(1-\phi)+\phi\gamma+\theta\pi^2(1+\rho)+\pi)^2(1+\rho)\pi^2} > 0.$$

Inflation has a contractionary effect on the level of employment:

$$\frac{d}{d\pi}(1-l) = (1+\rho)\theta\pi\frac{\gamma\pi(1-\phi)+2\phi\gamma+\pi}{(\gamma\pi(1-\phi)+\phi\gamma+\theta\pi^2(1+\rho)+\pi)^2} > 0, \tag{37}$$

Equation (37) states that the higher the inflation, the higher the leisure. This result is a direct consequence of the nominal interest distortion in the household's labor-leisure decision, already discussed in the previous section for the case where $\phi = 0$.

The production of the deposit-good decreases when inflation rises:

$$\frac{d}{d\pi}\overline{y} = \gamma\frac{\pi(1+\rho)(-\pi(1-\phi)) - 2\phi)\theta - \phi}{(\gamma\pi(1-\phi)+\phi\gamma+\theta\pi^2(1+\rho)+\pi)^2} < 0. \tag{38}$$

The effect of inflation on consumption of both y and x is ambiguous: the sign of the derivatives depends on the values of the parameters:

$$\frac{d}{d\pi}y < 0 \Leftrightarrow \gamma < \frac{\theta\pi^2(1+\rho)^2}{\phi\rho} + \frac{1+2\theta\pi(1+\rho)}{\rho(1-\phi)} \tag{39}$$

$$\frac{d}{d\pi}x < 0 \Leftrightarrow 1 < \frac{\theta}{\phi\gamma}\pi^2(1+\rho). \tag{40}$$

Recall that leisure unambiguously increases when inflation rises. Therefore, a simultaneous increase in both y and x is only possible because of the interest payment reduction associated with the increase in B^*. From the analysis of (39) it follows that y would increase if, and only if, household's preference for the good, given by γ, were sufficiently large. Similarly, from (40), an increase in x would require a preference for x, which is normalized to 1, to be large enough compared to a function of the other parameters.[20] Nevertheless, a wide range of sensible values for the parameters guarantees that consumption of both x and y will actually decline with inflation.[21][22]

[19] Again, assuming that saving-deposits and loans to households are both zero.

[20] $\frac{d}{d\pi}y > 0$ only if $\frac{d}{d\pi}x > 0$: the price of y includes firm's financial cost which increases with inflation.With rising inflation, therefore, the price of y rises faster than the price of x and consumption of y unambiguously falls respect to x, $\frac{d}{d\pi}\frac{y}{x} < 0$.

[21] With log-utility, the coefficients in the utility function for x, y, and $(1-l)$, are also the expenditure shares. An increase in y would require a relatively large γ. The share of x is normalized to 1 for a total expenditure given by $1+\gamma+\theta$. Considering that x is the non-tradable good and y the tradable, it seems realistic to assume $\gamma < 1$. For example, with $\pi = 1.5$, $\rho = \phi = 0.1$, and $\theta = 1$, $\frac{d}{d\pi}y > 0$ holds only for $\gamma > 320$.

[22] At this point it may be interesting to compare our results to those in E&V. In their model the bank's

4.3 The bank's amplification role

Models that give a non-neutral role to financial intermediation in the transmission and amplifications of monetary policy rely heavily, directly or indirectly, on information issues. Information issues are modeled directly, for example, through uncertainty in liquidity models, or indirectly, like in E&V, through the bank's cost function. Here, instead, I want to stress the fact that, in a simpler setup, it is still possible to have a non-neutral financial intermediation system. In this model, with a deterministic setup and with banks that produce at zero cost, banks still amplify the monetary policy distortions on the level of output.

To prove this claim I examine how the size of the reserve requirement, ϕ, affects the equilibrium level of leisure, $(1-l)$.

From sections (4.1)and (4.2) , which set $\phi = 0$ and $\phi > 0$ respectively, we know that, in both cases, the level of leisure is affected by the inflation rate, π. Leisure is higher the higher π is, regardless of the value of the reserve requirement, ϕ. The existence of a reserve requirement, however, makes inflation even more distortionary:

$$(1-l)\,|_{\phi>0} - (1-l)\,|_{\phi=0} =$$

$$\frac{(\pi-1)\,\theta\,(1+\rho)\,\pi\phi\gamma}{(\theta\pi^2(1+\rho)+\pi\gamma(1-\phi)+\pi+\phi\gamma)\,(\theta\pi(1+\rho)+\gamma+1)} > 0.$$

As shown in the equation above, leisure is higher when the reserve requirement is positive. Moreover, the bigger the reserve requirement, the higher the level of leisure:

$$\frac{d}{d\phi}(1-l) = \frac{(1+\rho)\,\theta\pi^2\gamma\,(\pi-1)}{(-\theta\pi^2(1+\rho)-\pi\gamma(1-\phi)-\pi-\phi\gamma)^2} > 0.$$

This distortionary effect of ϕ on the level of leisure, however, is only present in an inflationary environment, or in other words, when the gross rate of inflation is greater than 1, $\pi > 1$. That is:

$$\frac{d}{d\phi}(1-l)\,|_{\pi=1} = 0.$$

I now provide the intuition for this result and explain why it is a consequence of the way the bank behaves.

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cost function is the key for inflation to have a negative effect on output. E&V assume that the marginal cost of loans is an increasing function of the loans-to-deposits ratio. Inflation causes deposits to fall but it does not affect the demand for loans. As a result, the cost of borrowing from the bank rises and the firm reduces its demand for labor. Their results, however, do not necessarily hold once the proportions of deposit to consumption and borrowing to production are endogenous. In our model, a rate of inflation that is small enough is a sufficient condition for the loans-to-deposits ratio to actually fall.

Analyze first the effect of inflation in isolation. This is a monetary economy, where the household uses money both to buy x and to make the deposits to buy y. The household, therefore, gives up some of its endowment of leisure for a monetary wage. According to the timing assumed, the household has to hold money until the next period. Simply for this reason, whenever $\pi > 1$, and even with $\phi = 0$, the household supplies less labor to reduce its inflation tax payment. It follows then, that with rising inflation, leisure unambiguously increases relative to consumption of both x and y :

$$\frac{d}{d\pi}\frac{1-l}{x} = (1+\rho)\,\theta > 0,$$

and

$$\frac{d}{d\pi}\frac{1-l}{y} = (1+\rho)^2\,\theta\pi\frac{\pi(1+\rho)(1-\phi)+2\phi}{(-\pi(1+\rho)(1-\phi)-\phi)^2\,\gamma} > 0.$$

So far, these results are equivalent to those in Lucas and Stokey (1987), in the sense that with inflation, the household substitutes the credit-good (leisure), for the cash-goods (x and y). Why is this increase in leisure magnified by the reserve requirement?. The key is the bank's role in the transmission of the monetary changes.

The bank holds non-interest-bearing reserves as long as $\phi > 0$. If there is inflation, $\pi > 1$, the bank passes to the firm the higher cost of holding reserves, by charging a higher interest rate on loans. The firm, in turn, passes to the household this increase in the financial cost of producing y, by selling the good at a higher price. It then follows that with $\pi > 1$, and only if $\phi > 0$, the household faces a higher relative price of y with respect to x. We know that the household always consumes some positive amount of good y, because the marginal utility derived from y is infinite when $y = 0$. Only when $\phi > 0$, therefore, an increase in the inflation rate yields a decline in the consumption of y relative to x :

$$\frac{d}{d\pi}\frac{y}{x} = \begin{cases} -\frac{\phi\gamma}{\pi^2(1+\rho)} < 0 \text{ when } \phi > 0 \\ 0 \text{ when } \phi = 0 \end{cases}. \tag{41}$$

Summarizing from the previous discussion, there are two different forces that determine a higher equilibrium level of leisure (lower level of employment). The first one is inflation. Inflation is a tax on labor income, and, as a direct consequence, the household chooses a higher level of leisure. The second one is the existence of a reserve requirement. With a reserve requirement, the increase in leisure goes even further: the inflation tax is relatively higher in y, and the household substitutes x *and* leisure for y. As a result, the reserve requirement amplifies the distortionary effect of inflation on employment.

Finally, this section looks at the effect of ϕ on the *sensitivity* of leisure with respect to changes in inflation. The previous discussion emphasizes the magnifying role of the reserve requirement on the *level* of leisure. Changes in ϕ, however, affect also the magnitude of the *change* on leisure due to changes on the inflation rate. It was determined already that leisure is an increasing function of both π and $\dot{\phi}$. Now, I investigate the effect of the interaction between the two of them.

The cross-partial derivative is:

$$\frac{d}{d\pi d\phi}(1-l) =$$

$$-\pi(1+\rho)\theta\gamma\frac{\theta\pi^3(1+\rho)-\pi^2\gamma(1-\phi)-\pi^2-3\pi\phi\gamma-2\theta\pi^2(1+\rho)+2\phi\gamma}{(\theta\pi^2(1+\rho)+\pi\gamma(1-\phi)+\pi+\phi\gamma)^3},$$

with the following conditions for the derivative of the interaction term to have a positive sign:

$$\frac{d}{d\pi d\phi}(1-l) > 0 \Leftrightarrow \left(\pi^2(1-\phi)+(3\pi-2)\phi\right)\gamma+\pi^2(1+\rho)(2-\pi)\theta+\pi^2>0,$$
$$\text{where } \pi \leqq 2 \text{ is a sufficient condition.}$$

There exists, therefore, a wide range of parameters for which a change in the reserve requirement exacerbates also the sensitivity of leisure to changes in the rate of inflation.

The intuition behind these conditions is related to the concavity of the utility function. The level of leisure always increases when inflation rises. The level of leisure is higher, and, therefore, its marginal utility is lower, the higher is the rate of inflation and/or the higher is the preference for leisure, θ. The increase in leisure due to inflation, $\frac{d}{d\pi}(1-l)$, is increasing in ϕ, $\frac{d}{d\pi d\phi}(1-l)>0$, whenever the level of leisure is not already "too" high, or equivalently, its marginal utility is not already "too" low.

5 Conclusions

This paper has constructed a general equilibrium model that, even abstracting from information issues, gives financial intermediaries a non-trivial role in magnifying disturbances. Commercial banks are typically required to hold deposits at the central bank at an interest rate lower than the market rate. The presence of this reserve requirement on bank demand-deposits magnifies the distortionary effects of inflation. For any given rate of inflation, the larger the reserve requirement, the larger the contraction of the labor supply.

Central bankers typically set the size of the reserve requirement based on considerations about the soundness of the banking industry. The policy implications of this model, however, show that, in an inflationary environment, the long-run level of economic activity may also be affected. Furthermore, the model shows that, for a wide range of parameters, a change in the reserve requirement exacerbates not only effects of inflation on the level of employment but also its sensitivity with respect to the rate of inflation.

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