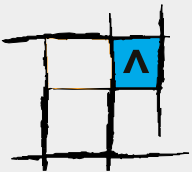




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and Minseong Kim
Decisiveness and Viability of Anarchy



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Decisiveness and The Viability of Anarchy

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Abstract

In Jack Hirshleifer's one-factor model of anarchy agents use in appropriative competition the same resource that they are competing to appropriate. Analyzing this model we find that equilibrium consumption is adequate for the viability of anarchy only if the decisiveness parameter of the technology of appropriation is small. For this reason Hirshleifer concludes that anarchy is fragile. This paper shows that Hirshleifer's one-factor assumption is not an innocent simplification. Specifically, we find that, if the resources that agents compete to appropriate and the resources that agents use in the appropriative competition are distinct, then anarchy can be viable even for large values of the decisiveness parameter. In other words, with more than one factor of production, anarchy does not seem to be fragile, at least not for the reason that Hirshleifer's analysis suggests.

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In the abstract of his paper, “Anarchy and Its Breakdown”, Jack Hirshleifer wrote,

Anarchy...is not chaos, but rather a spontaneous order. However, anarchy is fragile... Anarchy is sustainable only when there are strongly diminishing returns to fighting effort (the “decisiveness parameter” is sufficiently low)... (Hirshleifer, 1995, page 26)

To be viable, which is probably a more apt term than sustainable, a social system must generate “adequate” per capita consumption. Hirshleifer defines adequate consumption to be the “minimum...required to sustain life for an individual actor or for a group to preserve its institutional integrity”. (Hirshleifer, 1995, page 33)¹

Hirshleifer defines anarchy to be an unregulated social system in which autonomous agents compete to appropriate resources. Hirshleifer contrasts anarchy with “amorphous”, a situation in which, as in the state of nature imagined by Rousseau, people exist in isolation from each other. Hirshleifer also contrasts anarchy with organized social systems, in which category he includes both “hierarchy”, a system in which a higher authority, such as a state or a super state, regulates the appropriation of resources, as well as systems of informal social controls over the appropriation of resources. Examples of anarchy, as defined by Hirshleifer, range from instances of external relations among sovereign states to the California Gold Rush of 1849.²

¹There is some ambiguity in Hirshleifer’s terminology. In places he uses the terms “sustainable”, “viable”, and “stable” interchangeably. In other places, he refers to viability and stability as separate components of sustainability. Also, Hirshleifer does not formulate an explicitly dynamic analysis. Similarly, the present paper uses a static framework to replicate and to extend Hirshleifer’s analysis of viability, defined as generating adequate consumption.

²Helen Milner (1991) provides a survey and critique of the concept of anarchy as applied to the relations among sovereign states. Milner emphasizes that, in agreement with Hirshleifer, scholars who view the relations among sovereign states to be anarchic find “regularized, predictable patterns of behavior among states [and] order lurking in the seeming chaos of international relations” (page 70). Milner also stresses

In “Anarchy and Its Breakdown” Hirshleifer analyzes a model of anarchy in which there is only one factor of production. This one-factor model assumes that agents use in the appropriative competition the same resources that they are competing to appropriate. As this assumption presents a chicken-and-egg conundrum, presumably Hirshleifer views it to be a convenient simplification, which is not meant to be taken literally.

This model has two critical implications for the equilibrium level of consumption: First, equilibrium consumption depends negatively on the decisiveness parameter that determines the marginal effect of allocating resources to the appropriative competition. Second, this effect is so strong that equilibrium consumption could be adequate for the viability of anarchy only if the decisiveness parameter were small.

To be more precise, define potential consumption to be the level that per capita consumption would approach if the amount of resources allocated to appropriative competition were to go to zero. In general, if adequate consumption is large relative to potential consumption, then anarchy is viable only if the decisiveness parameter is small. More problematically, in the model analyzed in “Anarchy and Its Breakdown”, even if adequate consumption is small relative to potential consumption, the maximum decisiveness parameter that is consistent with the viability of anarchy is still small. For example, with a large number of agents competing to appropriate a large amount of resources, even if adequate consumption were close to zero, anarchy would be viable only if the decisiveness parameter was smaller than one.

that anarchy differs from an organized social system like a state only in “the manner in which order is provided” (page 74). Hirshleifer’s specific definition of anarchy as excluding informal social controls accords with the definition typically used by economists. See, for example, Dan Usher (1992) and Herschel Grossman (2000). In contrast, the literature that analyzes informal social controls as an alternative to a state or a super state often uses a broader definition of anarchy, according to which anarchy precludes specialized authorities who enforce collective choices, but anarchy does not preclude socially constructed punishments that support cooperative equilibria. Such equilibria can exist only if agents interact repeatedly and, among other things, are sufficiently foresighted. For extended discussions see, for example, Michael Taylor (1982), Robert Axelrod (1984), or Emerson Niou and Peter Ordeshook (1990).

We have no reason to presume that the decisiveness parameter is as small as one, let alone as much smaller than one as Hirshleifer’s model requires for anarchy to be viable. Thus, Hirshleifer’s model suggests that anarchy is fragile. What happens if the decisiveness parameter is too large for anarchy to be a viable social system? According to Hirshleifer, “Anarchy is always liable to ‘break down’ into amorphous or ‘break up’ into organization.” (Hirshleifer, 1995, page 48)

This paper questions the conclusion that a large decisiveness parameter would threaten the viability of anarchy. We show that this conclusion depends on the one-factor assumption in the model in “Anarchy and Its Breakdown”. This assumption turns out not to be an innocent simplification. Specifically, we find that, if the resources that agents compete to appropriate and the resources that agents use in the appropriative competition are distinct, then, although equilibrium consumption depends negatively on the decisiveness parameter, anarchy can be viable even for large values of the decisiveness parameter. In other words, with more than one factor of production, anarchy does not seem to be fragile, at least not for the reason that Hirshleifer’s analysis suggests.³

1. A Generic Model of Anarchy

Consider an anarchic situation in which a group of $n + 1$ identical autonomous agents, $n \in \{1, 2, 3, \dots\}$, have the opportunity to appropriate among themselves a common pool of $(n + 1)E$ units of heretofore unappropriated resources.⁴ We take these resources to be

³Of course, anarchy can fail to survive for other reasons. Most obviously, because a state or a super state can enforce collective choices, a state or a super state can be an attractive alternative, even a Pareto improvement over anarchy. For example, the ability to enforce collective choices can enable states or super states to subjugate anarchic groups, making the formation of a state or a super state a dominant strategy for the members of a group. For more on these issues, see Grossman (2000).

⁴Following Hirshleifer, we consider only the appropriation of resources from a common pool. In contrast, in Grossman and Minseong Kim (1995) and in Grossman (2001) we analyze appropriative interactions in which agents have initial claims over resources. With initial claims we have to model the distinction between

inputs into the production of consumables, but the analysis would be isomorphic if we were to take the resources to be consumables themselves. The agents can be individuals, or they can be groups, such as families or tribes, who act as unitary agents. The agents can even be sovereign states, again as long as we can assume that they act as unitary agents.

To model the technology of appropriation, assume that, as in a standard “contest-success function”, an agent appropriates e units from the common pool of resources, where

$$(1) \quad e = \frac{r^m}{r^m + nR^m}(n + 1)E.$$

In equation (1), r is the amount of resources that an agent allocates to the appropriative competition, and R is the amount of resources that other agents on average allocate to the appropriative competition. The parameter m is the “decisiveness parameter.”⁵

We assume that the resources allocated to appropriative competition have an alternative use as inputs into the production of consumables. But, for now we do not specify whether and, if so, how the resources that agents appropriate from the common pool differ from the resources that agents allocate to the appropriative competition.

Equation (1) exhibits several important properties that we can plausibly consider to be generic to the appropriation of heretofore unappropriated resources.

the challenging of initial claims and the defending of initial claims. In the context of appropriation from a common pool this distinction does not arise.

⁵Equation (1) is a black box that does not explicitly specify the process of appropriation, just as the standard generic production function does not explicitly specify the process of production. In other words, equation (1) does not tell us how agents appropriate resources any more than a production function tells us how to make cars. The relation between the resources allocated to appropriative competition and the appropriative outcome described by equation (1) could involve such disparate processes as a nonviolent scramble, a division under the threat of force, or a violent struggle. We could easily extend the analysis to allow the appropriative process to despoil some of the resources by assuming that E is a decreasing function of R . Another interesting extension would be to allow for differences among agents in their abilities to appropriate resources.

First, e is bounded between zero and $(n + 1)E$, the entire pool of resources.

Second, $\partial e/\partial r$ is positive. In other words, for any given value of R , e is an increasing function of r .

Third, if r equals R , then e equals E . In other words, an agent who allocates the same amount of resources to appropriation as the average agent appropriates the average amount of resources from the common pool.

We can see from equation (1) why the decisiveness parameter is important. A straightforward calculation reveals that $\partial^2 e/\partial r \partial m$, evaluated at r equal to R , is positive.⁶ This property means that in a symmetrical equilibrium, in which every person makes the same choice of r , the larger is the decisiveness parameter the larger is the marginal effect of allocating more resources to appropriative competition on the amount of resources that an agent appropriates from the common pool. Hirshleifer provides some illustrative examples of the decisiveness parameter being high or low.

An agent's consumption, denoted by c , depends positively on the amount of resources that the agent appropriates from the common pool and depends negatively on the amount of resources that the agent allocates to the appropriative competition rather than to production. Specifically, we assume a production technology such that

$$(2) \quad c = F(e, r),$$

where $\partial F(e, r)/\partial e > 0$, $\partial^2 F(e, r)/\partial e^2 \leq 0$, $\partial F(e, r)/\partial r < 0$, and $\partial^2 F(e, r)/\partial r^2 \leq 0$. For now we do not specify any other properties of the function $F(e, r)$. Let \underline{c} denote the minimum level of per capita consumption that is adequate for the viability of anarchy.⁷

⁶The sign of $\partial^2 e/\partial r \partial m$ is the same as the sign of $[1 + m \ln(r) + m \ln(R)] (r^m + nR^m) - 2m [r^m \ln(r) + nR^m \ln(R)]$. With r equal to R this expression becomes $(n + 1) R^m$.

⁷If the resources that the agent appropriates from the common pool and the resources that the agent allocates to appropriative competition were themselves consumables, then we would interpret $F(e, r)$ to be a utility function, and we would interpret \underline{c} to be the minimum level of utility that is adequate for the

An agent chooses r to maximize its consumption, taking R as given.⁸ Using equations (1) and (2) the first order condition for the maximization of an agent's consumption implies

$$(3) \quad \frac{dc}{dr} = \frac{\partial F(e, r)}{\partial e} \frac{\partial e}{\partial r} + \frac{\partial F(e, r)}{\partial r} = 0,$$

$$\text{where } \frac{\partial e}{\partial r} = \frac{mr^{m-1}nR^m}{(r^m + nR^m)^2} (n+1)E.$$

We assume that the second order condition, $d^2c/dr^2 < 0$, is satisfied.

In a symmetrical equilibrium in which every agent makes the same choice of r , we have for every agent $r = R$ and, hence, $e = E$. Accordingly, in a symmetrical equilibrium equation (3) becomes

$$(4) \quad \frac{dc}{dr} = \frac{\partial F(E, R)}{\partial e} m \frac{n}{n+1} \frac{E}{R} + \frac{\partial F(E, R)}{\partial r} = 0.$$

Let R^* denote the value of R that solves equation (4). Because $\partial F(e, r)/\partial e$ is positive, given that the second-order condition, $d^2c/dr^2 < 0$, is satisfied, equation (4) implies that dR^*/dm is positive. In other words, with each agent maximizing its consumption, the larger is the decisiveness parameter, m , the larger is the equilibrium allocation of resources to appropriative competition, R^* . More precisely, equation (4) implies that R^* satisfies

$$(5) \quad m \frac{n}{n+1} \frac{E}{R^*} = - \frac{\partial F(E, R^*)}{\partial r} / \frac{\partial F(E, R^*)}{\partial e}.$$

The left side of equation (5) is the marginal effect of allocating resources to the appropriative competition on amount of resources appropriated from the common pool. The right side of

viability of anarchy. In Grossman and Juan Mendoza (2000) we analyze a model in which appropriative competition uses time and effort, the alternative use of which is in leisure activities. In that model appropriative competition decreases the utility obtained from leisure activities rather than from consumption.

⁸The analysis in Hirshleifer (1995) assumes that an agent chooses the ratio, r/e , rather than r . This assumption implies that an agent faces a constraint on its action variable, a constraint for which Hirshleifer provides no rationale. Although the assumption that an agent chooses r/e alters the anarchic equilibrium, this assumption does not affect the answer to the question of whether a large decisiveness parameter would threaten the viability of anarchy.

equation (5) is the ratio of the marginal productivity of resources from the common pool to the marginal productivity of resources that agents could allocate to the appropriative competition.

Let c^* denote the equilibrium value of per capita consumption. In a symmetrical equilibrium c^* equals $F(E, R^*)$. Because $\partial F(e, r)/\partial r$ is negative and dR^*/dm is positive, dc^*/dm is negative. In other words, the larger is the decisiveness parameter, m , the smaller is the equilibrium value of per capita consumption. This result, derived in a generic model of anarchy, generalizes an essential property of Hirshleifer's one-factor model.

The viability of anarchy requires that c^* be at least as large as \underline{c} . The result that dc^*/dm is negative suggests that for large enough values of m this condition might not be satisfied. Let \bar{m} denote the maximum value of m that is consistent with c^* being as large as \underline{c} . The viability of anarchy requires that m be not larger than \bar{m} .

Unlike in Hirshleifer's one-factor model, in a generic model of anarchy \bar{m} can be a large number. We will see that \bar{m} depends on the nature of the resources that agents compete to appropriate and the nature of the resources that agents use in the appropriative competition, as well as on adequate consumption, \underline{c} , and potential consumption, \hat{c} .

It is useful to observe that in the limit, if R were to approach zero, then per capita consumption would approach $F(E, 0)$. Let \hat{c} denote $F(E, 0)$. We call \hat{c} potential consumption. We now turn to specific examples of the production technology, $F(e, r)$.

2. A One-Factor Model

In the one-factor model in "Anarchy and Its Breakdown" agents use both in the appropriative competition and in the production of consumables the same resources that they are competing to appropriate. Taken literally, the use of the resources in the common pool to appropriate from the common pool presents a chicken-and-egg conundrum. Thus, as already mentioned, presumably Hirshleifer intends this assumption to be an innocent simplification, not to be taken literally. The question is whether this assumption is an innocent

simplification.

To replicate Hirshleifer's analysis of this model, assume that an agent's consumption is an increasing concave function of the difference between the amount of resources that the agent appropriates from the common pool and the amount of resources that the agent allocates to the appropriative competition. Specifically, assume that

$$(6) \quad c = F(e, r) = (e - r)^\gamma,$$

where $0 < \gamma < 1$. Equation (6) implies that equilibrium consumption, c^* , equals $(E - R^*)^\gamma$. Thus, c^* is a decreasing concave function of R^* . In addition, the relation between c^* and R^* is such that, for any positive value of adequate consumption, \underline{c} , no matter how small, a large enough value of R^* would be inconsistent with the requirement that c^* be as large as \underline{c} . More precisely, for c^* to be as large as \underline{c} , R^* cannot be larger than $E - \underline{c}^{1/\gamma}$.

Equation (6) also implies that the ratio of $-\partial F(E, R^*)/\partial r$ to $\partial F(E, R^*)/\partial e$ equals one. Hence, the equilibrium condition for determining R^* , equation (5), requires that the marginal effect of allocating resources to the appropriative competition on amount of resources appropriated from the common pool equals one. Specifically, equation (5) becomes

$$(7) \quad R^* = m \frac{n}{n+1} E.$$

Equation (7) implies that R^* is proportionate to m . Hence, given that c^* is a decreasing concave function of R^* , equation (7) also implies that c^* is a decreasing concave function of m . In addition, equation (7) implies that c^* is positive if and only if m is smaller than $1 + 1/n$. The possibility that in equilibrium consumption would not be positive arises because in the one-factor model agents can choose to waste the entire resource endowment in appropriative competition. The concave locus in Figure 1 illustrates the relation between c^* and m for the one-factor model.

Let \bar{m}_1 denote the value of \bar{m} , the maximum value of m that is consistent with c^* being as large as \underline{c} , in the one-factor model. By the definition of \bar{m} , for m equal to \bar{m}_1 ,

c^* equals \underline{c} . By varying \underline{c} in Figure 1, we see that \bar{m}_1 is a decreasing concave function of \underline{c} . More precisely, using equation (7), the condition for the viability of anarchy, $c^* \geq \underline{c}$, becomes $m \leq \bar{m}_1$, where

$$(8) \quad \bar{m}_1 = \left[1 - \left(\frac{\underline{c}}{\hat{c}} \right)^{1/\gamma} \right] \frac{n+1}{n}.$$

According to equation (8), as \underline{c} approaches potential consumption, \hat{c} , where \hat{c} equals E^γ , \bar{m}_1 approaches zero. This result implies that, if adequate consumption is almost as large as potential consumption, then anarchy is unlikely to be viable. Even with a small decisiveness parameter, and hence a small equilibrium allocation of resources to appropriative competition, consumption would be smaller than adequate consumption.

Equation (8) also implies that, as \underline{c} approaches zero, \bar{m}_1 approaches $1 + 1/n$. This result says that with a large number of agents competing to appropriate a large amount of resources, even if adequate consumption is small, the maximum value of the decisiveness parameter that is consistent with the viability of anarchy is not larger than one. With a small number of agents competing to appropriate a proportionately smaller amount of resources, the maximum decisiveness parameter that is consistent with the viability of anarchy would be larger. But, even with only two agents, the maximum decisiveness parameter would be only twice what it would be with a large number of agents.

3. Two-Factor Models

To determine the generality of this analysis, consider an alternative setup in which the resources to be appropriated and the resources used in the appropriative competition are distinct. Specifically, assume that each agent is endowed with one unit of an inalienable resource, which we can take to be time and effort. Assume further that agents use time and effort to appropriate resources from the common pool. Thus, in equation (1) r now represents the fraction of its time and effort that an agent allocates to appropriative competition. As agents do not use the resource in the common pool to appropriate from the common

pool, this setup avoids the chicken-and-egg conundrum of the one-factor model.⁹

Assume further that an agent's consumption depends on the amount of resources that the agent appropriates from the common pool and on the amount of time and effort that the agent allocates to production according to a standard constant-elasticity-of-substitution (CES) technology,

$$(9) \quad c = F(e, r) = [\alpha e^\sigma + \beta(1 - r)^\sigma]^{1/\sigma},$$

where $0 < \alpha < 1$, $0 < \beta < 1$, and $\sigma \leq 1$. In equation (9) $1 - r$ is the fraction of its time and effort that an agent allocates to production. The constant elasticity of substitution between e and $1 - r$ in equation (9) is $-1/(1 - \sigma)$.

For σ equal to zero, equation (9) becomes a Cobb-Douglas production technology, $c = F(e, r) = e^\alpha (1 - r)^\beta$. For σ equal to one, equation (9) becomes a linear production technology, $c = F(e, r) = \alpha e + \beta(1 - r)$. With a linear technology resources from the common pool and time and effort, although distinct, are perfect substitutes in production.¹⁰

The economic problem in this model is that the appropriation of resources and the production of consumables are alternative uses of time and effort. Each agent must choose how to allocate its one unit of time and effort between these activities.

Equation (9) implies that equilibrium consumption, c^* , equals $[\alpha E^\sigma + \beta(1 - R^*)^\sigma]^{1/\sigma}$. Thus, c^* is a decreasing concave function of R^* for σ smaller than one and a decreasing linear function of R^* for σ equal to one. Potential consumption, \hat{c} , equals $(\alpha E^\sigma + \beta)^{1/\sigma}$.

⁹One might imagine a hybrid model in which agents have to use both time and effort and the resources in the common pool in the competition to appropriate from the common pool. Such a model would present the same chicken-and-egg conundrum as the one-factor model.

¹⁰Another possible, albeit strained, interpretation of the special case of a linear production technology is that there is only one resource, with each agent having an inalienable endowment of some of it and with the rest of it being initially in a common pool, but that agents can use only their inalienable endowments in the appropriative competition.

Equation (9) also implies that $\partial F(E, R^*)/\partial e$ equals $\alpha(c^*/E)^{1-\sigma}$ and that $\partial F(E, R^*)/\partial r$ equals $-\beta[c^*/(1-R^*)]^{1-\sigma}$. Hence, with two factors and a CES technology the equilibrium condition for determining R^* , equation (5), becomes

$$(10) \quad m \frac{n}{n+1} \frac{E}{R^*} = \frac{\beta}{\alpha} \left(\frac{E}{1-R^*} \right)^{1-\sigma}.$$

3.1. Cobb-Douglas Technology

To explore further the implications of allowing for two factors, we have to distinguish the case of CES technology with σ less than or equal to zero from the case of CES technology with σ positive, but not larger than one. Consider the case of $\sigma \leq 0$. In this case the constant elasticity of substitution is between zero and minus one. For simplicity, and without loss of generality, we can focus on σ equal to zero, the Cobb-Douglas technology with constant elasticity of substitution equal to minus one.

With $\sigma \leq 0$, equation (9) implies that c goes to zero as r goes to one. In other words, with $\sigma \leq 0$, time and effort are essential for production. Accordingly, as in the one-factor model, for any positive value of \underline{c} , no matter how small, a large enough value of R^* would be inconsistent with the requirement that c^* be as large as \underline{c} . More precisely, with the Cobb-Douglas technology, for c^* to be as large as \underline{c} , R^* cannot be larger than $1 - (\underline{c}/\hat{c})^{1/\beta}$, where \hat{c} , potential consumption, equals E^α .

But, with $\sigma \leq 0$, equation (10) implies that R^* is not proportionate to m . This property obtains because, with $\sigma \leq 0$, e and r are not perfect substitutes in the production of consumables. Instead, the ratio of $-\partial F(E, R^*)/\partial r$ to $\partial F(E, R^*)/\partial e$ is an increasing convex function of R^* that goes to infinity as R^* approaches one. As a result, equation (10) implies that R^* is an increasing concave function of m , which only approaches one asymptotically as m goes to infinity.

Given this relation between R^* and m , c^* is a decreasing function of m , but c^* only approaches zero asymptotically as m goes to infinity. These results say that, because the ratio of $-\partial F(E, R^*)/\partial r$ to $\partial F(E, R^*)/\partial e$ goes to infinity as R^* approaches one, for any

finite value of m agents allocate a positive fraction of their time and effort to production. In Figure 1 the convex locus that is asymptotic to the horizontal axis illustrates the relation between c^* and m for the two-factor model with a Cobb-Douglas technology.¹¹

Let \bar{m}_{CD} denote the value of \bar{m} with a Cobb-Douglas technology. By the definition of \bar{m} , for m equal to \bar{m}_{CD} , c^* equals \underline{c} . By varying \underline{c} in Figure 1, we see that \bar{m}_{CD} is a decreasing convex function of \underline{c} . More precisely, using equation (10), with σ equal to zero, the condition for the viability of anarchy, $c^* \geq \underline{c}$, becomes $m \leq \bar{m}_{CD}$, where

$$(11) \quad \bar{m}_{CD} = \left[\left(\frac{\hat{c}}{\underline{c}} \right)^{1/\beta} - 1 \right] \frac{\beta}{\alpha} \frac{n+1}{n}.$$

According to equation (11), and as in the one-factor model, as \underline{c} approaches \hat{c} , \bar{m} approaches zero. Again, if adequate consumption is almost as large as potential consumption, then anarchy is unlikely to be viable.

But, equation (11) also implies that, as \underline{c} approaches zero, \bar{m}_{CD} increases without bound. This result says that a model with a Cobb-Douglas production technology, or, more generally, with a CES production technology with σ less than or equal to zero, is qualitatively different from the one-factor model. The important consequence of equation (11) is that, if adequate consumption is small, relative to potential consumption, then, with a Cobb-Douglas production technology, and, more generally, with a CES production technology with σ less than or equal to zero, only an extremely large decisiveness parameter would threaten the viability of anarchy.

3.2. Linear Technology

Now consider the case of CES technology with σ positive. In this case the constant elasticity of substitution is less than minus one. For simplicity, and without loss of generality,

¹¹In drawing this figure we have normalized potential consumption, \hat{c} , to be the same in the two-factor model with a Cobb-Douglas technology as in the one factor model. For a Cobb-Douglas technology \hat{c} equals E^α . Thus, this normalization implies that α equals γ . This normalization also implies that at the vertical axis the slope of the convex locus equals the slope of the concave locus derived from the one-factor model.

we can focus on σ equal to one, the linear technology in which resources from the common pool and time and effort, although distinct, are perfect substitutes in production and in which the constant elasticity of substitution is minus infinity.

With $\sigma > 0$, equation (9) implies that time and effort are not essential for production, because with $\sigma > 0$, c goes to αe , and not to zero, as r goes to one. Accordingly, if \underline{c} is not larger than αE , then anarchy is viable for any feasible value of R^* . Conversely, a large enough value of R^* would be inconsistent with the requirement that c^* be as large as \underline{c} if and only if \underline{c} is larger than αE . More precisely, with a linear production technology, for c^* to be as large as \underline{c} , R^* cannot be larger than $\min\{(\hat{c} - \underline{c})/\beta, 1\}$, where \hat{c} , potential consumption, equals $\alpha E + \beta$.

With $1 > \sigma > 0$, equation (10) still implies that R^* is an increasing concave function of m , which only approaches one asymptotically as m goes to infinity. With $\sigma = 1$, equation (10) implies that R^* is an increasing linear function of m for $m < (1 + 1/n) (\beta/\alpha) (1/E)$, and that R^* equals one for $m \geq (1 + 1/n) (\beta/\alpha) (1/E)$.

Given these relations between R^* and m , with $1 > \sigma > 0$, c^* is a decreasing convex function of m , which approaches αE asymptotically as m goes to infinity. With $\sigma = 1$, c^* is a decreasing linear function of m for $m < (1 + 1/n) (\beta/\alpha) (1/E)$, and c^* equals αE for $m \geq (1 + 1/n) (\beta/\alpha) (1/E)$. These results say that, because with positive σ time and effort are not essential for production, consumption is positive for any finite value of m . In Figure 1 the piece-wise linear locus illustrates the relation between c^* and m for the two-factor model with a linear production technology.¹²

Let \overline{m}_L denote the value of \overline{m} with a linear technology. By the definition of \overline{m} , for m equal to \overline{m}_L , c^* equals \underline{c} . Using equation (10), with σ equal to one, the condition

¹²Because for a linear technology \hat{c} equals $\alpha E + \beta$, the normalization that potential consumption is the same in the two-factor model with a linear technology as in the one factor model implies that $\alpha E + \beta$ equals E^γ . Also, we have drawn the slope of the linear locus as equal to the slopes of the other loci at the vertical axis. This construct implies that we also have normalized E to equal one.

for the viability of anarchy, $c^* \geq \underline{c}$, becomes $m \leq \overline{m}_L$, where

$$(12) \quad \overline{m}_L = \begin{cases} \infty & \text{for } \underline{c} \leq \alpha E \\ \frac{\hat{c} - \underline{c}}{\alpha E} \frac{n+1}{n} & \text{for } \underline{c} > \alpha E. \end{cases}$$

According to equation (12), and as in the previous models, as \underline{c} approaches \hat{c} , \overline{m} approaches zero. Again, if adequate consumption is almost as large as potential consumption, then anarchy is unlikely to be viable.

But, equation (12) also implies that, if, as depicted in Figure 1, \underline{c} is not larger than αE , then the requirement that c^* be as large as \underline{c} is satisfied for all values of m . Thus, a model with a linear production technology, or, more generally, with a CES production technology with positive σ , also is qualitatively different from the one-factor model. With a linear production technology, and, more generally, with a CES production technology with positive σ , if adequate consumption is not too large, then even an extremely large decisiveness parameter would not threaten the viability of anarchy. Alternatively, if \underline{c} is larger than αE , then the requirement that c^* be as large as \underline{c} would not be satisfied for sufficiently large values of m . But, to threaten the viability of anarchy the decisiveness parameter again would have to be larger than in the one-factor model.

4. Summary

Jack Hirshleifer's one-factor model of anarchy, which assumes that agents use in appropriative competition the same resources that they are competing to appropriate, suggests that anarchy is fragile, because in this one-factor model equilibrium consumption could be adequate for the viability of anarchy only if the decisiveness parameter of the technology of appropriation were small. In this paper, after replicating Hirshleifer's analysis of the one-factor model, we generalized the analysis to allow the resources to be appropriated and the resources used in the appropriative competition to be distinct. Specifically, we assumed that the appropriation of resources requires time and effort. We also assumed that an agent's con-

sumption depends on the amount of resources that the agent appropriates from the common pool and on the amount of time and effort that the agent allocates to production according to a standard constant-elasticity-of-substitution production technology. In this extended model the appropriation of resources and the production of consumables are alternative uses of time and effort.

We found that in any model, if the amount of consumption that is adequate for the viability of anarchy is large, then anarchy is unlikely to be viable. But, our analysis revealed that the one-factor assumption in Hirshleifer's model of anarchy is not an innocent simplification. Specifically, we found that, with an elasticity of substitution between resources and time and effort between zero and minus one, if adequate consumption is not too large, relative to potential consumption, then only an extremely large decisiveness parameter would threaten the viability of anarchy. Furthermore, we found that, with an elasticity of substitution less than minus one, if adequate consumption is not too large, relative to potential consumption, then even an extremely large decisiveness parameter would not threaten the viability of anarchy. In sum, with an apparently realistic distinction between the resources that agents compete to appropriate and the resources that agents use in the appropriative competition, anarchy does not seem to be fragile, at least not for the reason that Hirshleifer's analysis suggests.

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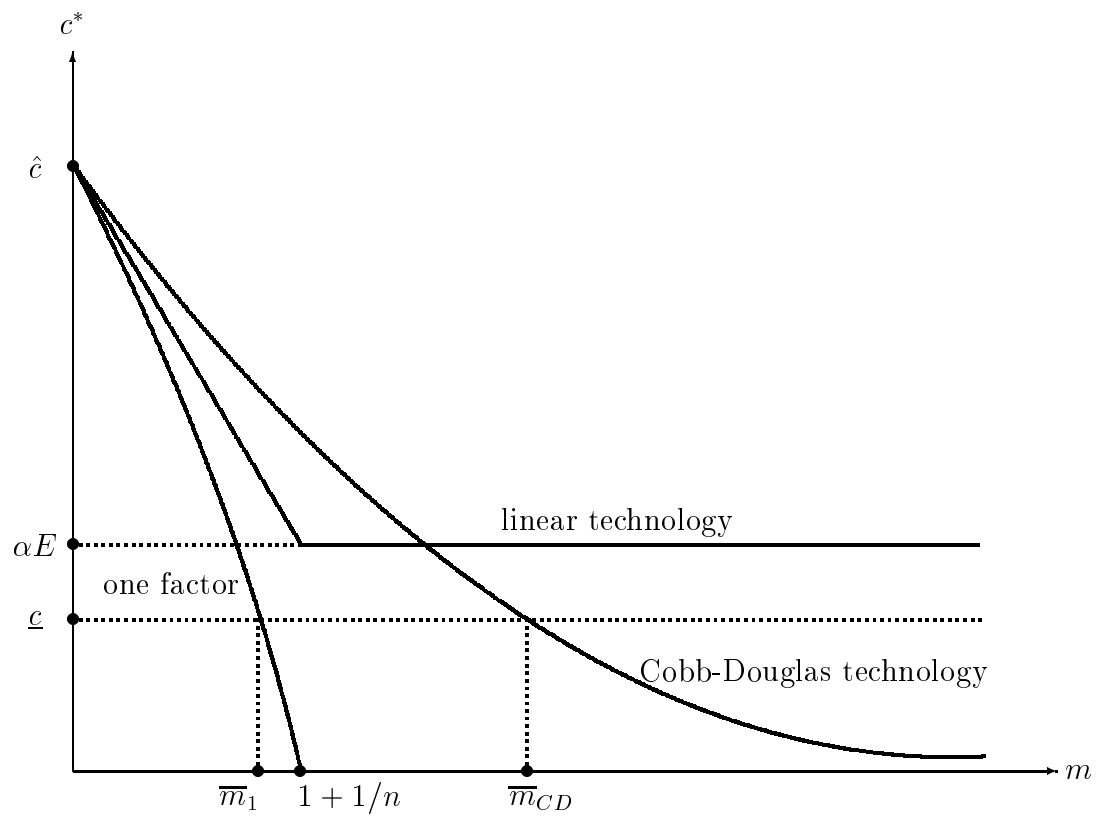


Figure 1: Equilibrium Consumption