LACEA 2001
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Pooled Estimation and Error Correction Approach to a Dynamic Panel Solow Growth Model

POOLED ESTIMATION AND ERROR CORRECTION APPROACH TO A DYNAMIC PANEL SOLOW GROWTH MODEL.

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SUMMARY

The purpose of this paper is to estimate a Solow growth model taking into account a specific dynamic panel data approach, which allows for model misspecification and error correction model (ECM) estimation. Our data set compiles information about per capita income, population growth and saving rates for 123 countries during thirty years, from 1960 to 1989. First, we run ADF and PP unit root tests and LS tests to allow possible structural changes for all countries in the sample. This country-by-country analysis is carried out in order to identify the variables integration orders that drive all the variables in the model. Based upon this three samples are endogenously selected: the I(1) *per capita* income countries, the I(2) *per capita* income countries and those which an structural break in the *per capita* income series can be found. Then, we perform panel data unit root and cointegration tests in order to check for the entire panel dynamics. Finally the panel cointegration dynamics is estimated by the Dynamic Ordinary Least Squares (DOLS) method only for I(1) time series. The final model we have achieved is corrected to account for usual misspecifications of traditional panel data growth models. Moreover, it is able to demonstrate evidences that support the original findings on the magnitude of inputs shares of the neoclassical function and approach a more reasonable convergence rates across countries.

Keywords: DOLS, dynamic panels, error correction model, I(1) regressors, economic growth model.

JEL Classification: C22, C23, O4, O57.

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I – <u>Introduction</u>

The neoclassical economic growth theory was revolutionized since Robert Solow has published a seminal paper in 1956. Furthermore, it posited the arguments for a discussion based on the long run dynamics of growth. The essential aspect of the Solow model is the neoclassical assumption on the analytical representation of the production function, which usually assumes constant returns to scale, diminishing returns to each input and some degree of substitution among them. Such a kind of framework, combined to a specification that takes into account a constant saving rate function and an exogenous technical change rate became the cornerstone of the literature concerning the behavior of income growth across countries.

Two important results arise from the basic models sketched above that are worth to emphasize. The first one concerns the conditional convergence of income among countries and the second addresses an issue on how to introduce technical change into this sort of model. Moreover, the first result has been exhaustively discussed in the past decades and implies a negative correlation between the initial level of the real *per capita* GDP and the subsequent rates of growth of the same variable. Indeed, this is an evidence that arises from the assumption of diminishing returns to each input, which also ensures that a less capital-intensive country tends to experiment higher rates of return and, consequently, higher GDP growth rates. Finally, the second result recognizes that the growth of output per worker must eventually cease in the absence of continuous improvement in technology. A realistic modelling of the conception of new ideas considers that these non-rival goods will only be produced if they are at least partially excludable, something that can be reasoned only in the context of imperfect competition. In this case, it is straightforward that the constant returns to scale tends to turn into increasing returns if this input is introduced into the production function.

Once this theoretical framework is taken as a referential, the development of the growth theory that follows has encountered serious obstacles mainly with respect to technical excessiveness and to the steadily loss of sensitiveness to empirical applications. Probably, this lack of empirical support motivated a new wave of research that started in the mid-1980s known as *endogenous growth theory*. These sorts of models permit the researcher to achieve the conclusion that the growth rate of the GDP can go indefinitely on, once the returns to the investments have not necessarily diminished over time. Unfortunately, many models of this paradigm are not compatible with the convergence hypothesis.

Obviously, the crucial difference between the two approaches discussed above relates to the fact that recent research is intimately linked to empirical implications. Considering this issue in a more accurate fashion, the Solow neoclassical growth model encounters its empirical support in MANKIW, ROMER and WEIL (1992)[1], whose purpose is to present a testable counterpart of it. This is so in order to assure Solow neoclassical model can fit better the data once an additional kind of variable – human capital – is included, improving considerably its original ability to explain income disparities across countries.

Recently, after incorporating a remarkable advance in econometric methods and tools, the economic growth theory empirical tests has been indeed improved. Accounting on this trend, an important contribution is due to ISLAN (1995), whose paper reports the estimates for the parameters of a neoclassical model in a panel data approach. For this sake, the author admits the level effects for individual countries as heterogeneous fixed intercepts in a dynamic panel. Although MRW (1992) findings allow us to conclude human capital performs an important role in the production function, ISLAN (1995) reaches an opposite conclusion, once a country specific technological progress is admitted into the model.

In a subsequent development of econometric methods applied to economic growth theory, LEE, PESARAN and SMITH[2] (1997) present an individual *random effect* version of the model developed by ISLAN (1995) making an allowance for heterogeneities in intercepts and in slopes of the production functions for heterogeneous dynamic panel data. The conclusion reached by these authors addresses to the fact that the homogeneity parameter hypothesis can be undoubtedly rejected. Indeed, LPS (1997) rightly pointed out that different growth rates renders the notion of convergence meaningless in an economic sense, because the knowledge of the speed of converge to the steady state provides no insights on the evolution of the cross-country variance of the output over time.

However, much of the classical econometric theory has been predicated on the assumption that the observed data comes from stationary processes. A preliminary glance at graphs of most economic time series, or even at the historical track record of economic forecasting, is suffices to reveal the invalidity of that assumption: economies evolve, economies grow, and they change over time in both real and nominal terms.

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[1] MRW, henceforth.
[2] LPS, henceforth.

BINDER and PESARAN[3] (1996) showed that there exists a way to solve this question if a stochastic version of the Solow model substitutes the traditional one. This is so because technology and labor are explicitly treated as stochastic processes with unit roots, which provides the methodological basis for using *random effects* in the equations to be estimated in the panel data. Once we take this settings as given, BP (1993) infer that the estimate of the convergence parameter can be interpreted purely in terms of the random components dynamics measured in the panel data model with no further information about convergence dynamics itself. BP (1993) also conclude that the stochastic neoclassical growth model developed in their paper is not necessarily a contradiction, independently of the presence of unit roots in the *per capita* output across countries.

One additional question concerns the existence of unit roots not only in the dependent variable data, but also in the population growth and in the saving rates time series as well; i.e. for all the other variables that enter the growth regression of the panel. The purposes of this paper are threefold. First, we present some exploratory results normally observed in an individual countries basis, summarizing the evidences on unit roots and possible structural breaks across countries. Second, there is a specific concern on enlarging the scope of the discussion by applying this methodology to the aggregate time series in an entire panel using different techniques. The next step is test for the existence of cointegration relationships between the variables, based upon the procedures presented by KAO (1997) and PEDRONI (1999). Finally, we estimate an error correction model (ECM) for the panel with special attention to the different orders of integration of the variables. Overall, most of our analysis tries to shed some

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[3] BP, henceforth.

additional light on the meaning of the growth regressions in panel data when unit root and, consequently a cointegration relationship are found significant.

The reminder of the paper is organized as follows: the panel data Solow growth model and the status of the current research is discussed in section 2. Section 3 describes the data set. A country-by-country analysis based upon some general econometric evidences and a data set convenient re-sampling are performed in section 4. Section 5 contains the results for unit roots and cointegration tests for the entire panel. The final Solow growth model, which embodies an error correction mechanism, is presented in section 6. Finally, the concluding remarks are summarized in section 7.

II - The Original Solow Model and a Dynamic Panel Solow Model

Once the original Solow growth model hypotheses are valid, it is perfectly feasible to construct the textbook panel model by admitting a transition equation. The methodology is the same as presented by ISLAN (1995) and restated by LPS (1997). Starting with a Cobb-Douglas production function which admits labor augmenting technological advance, then we can write the following:

$$Y(t) = K(t)^{a} \left(A(t)L(t)\right)^{1-a} \tag{1}$$

where: Y is the output, K is the physical capital stock, L is the labor force and A is the technology parameter.

The neoclassical model assumes the following properties concerning the production function $Y(t)$: K is a scalar and the level of physical capital at a starting point in time is something different from zero, i.e. $K(0)>0$; the growth rates of technology and population are exogenously given and equal to g and n, respectively.

Finally, the neoclassical representation also assumes *A(0)>0* and *L(0)>0*. By these assumptions, the basic model implications are that only physical capital is accumulated over time and population growth rate are exogenous and technical change as well. In addition, the Inada conditions on physical capital are assumed to be valid for any given positive level of labor weighted technology, *LA>0*.

Let *per capita* physical capital variable depreciates at an exponential and constant rate $\boldsymbol{d}>0$. The physical capital accumulation process is taken to follow a law of motion as presented by Solow who admits saving as a constant fraction, $s \in (0,1)$, of income. In terms of *per effective worker*, the dynamic model equation for a given *k*, can be written as follows:

(2) $$\dot{k}/k = sf(k)/k - (n + g + \boldsymbol{d})$$

where: *k=K/AL* and *y=Y/AL*.

After turning equation (1) into a linear representation by taking the logs of both sides of it and substituting the results obtained in equation (2) into equation (1), we achieve the following representative equation for the steady state in terms of *per capita* income:

(3) $$\ln\left(\frac{Y(t)}{L(t)}\right) = \ln A(0) + gt + \frac{\boldsymbol{a}}{1-\boldsymbol{a}}\ln(s) - \frac{\boldsymbol{a}}{1-\boldsymbol{a}}\ln(n + g + \boldsymbol{d})$$

This is the equation obtained by MRW (1992) in order to explain the income disparities across countries. Moreover, the authors take this relationship for the sake of measuring the effects of the saving rate and the population growth over income, assuming that the countries are already in their current steady state.

Apart from the different behavior of the countries concerning population growth and saving rates, there is an additional term [*gt+lnA(0)*] in equation (3) that we should be concerned with. Once more, MRW (1992) assume g as to be the same for all countries, so gt can be nothing else than a deterministic trend. However, the same cannot be said about *A(0)*, since this term reflects the initial endowment of the economies. Then, MRW (1992) also admit the following behavior for *lnA(0)*, $\ln A(0) = a + \varepsilon$, where a is a constant and ε is the country specific shock.

This is a crucial point reinforced by ISLAN (1995) e LPS (1997), who argue that this specification form generates a loss of information concerning the dynamic of the technological parameter. This is so, because the panel data approach is the natural way of specifying all the shifts over the countries specifics shock terms ε. In order to proceed on this, we assume there is a law of motion, which can describe the behavior of the economies *per* capita incomes out of their steady state levels. Let y^* be the equilibrium level for the output *per effective worker* and *y(t)* be its actual value at time t. Approximating y on a neighborhood over the steady state value, the convergence path is given by a differential equation in log of the *per capita* income over the time. After doing some algebra on equation (3), we derive the same equation as MRW (1992) do in order to analyze the path of convergence across countries:

$$\ln y_t - \ln y_{t-1} = (1-e^{-\lambda\Delta t})\frac{\alpha}{1-\alpha}\ln(s) - (1-e^{-\lambda\Delta t})\frac{\alpha}{1-\alpha}\ln(n+g+\delta) - (1-e^{-\lambda\Delta t})\ln y_{t-1} \tag{4}$$

ISLAN (1995) demonstrated that a correlation between *A(0)* and all the included independent variables of the model is surely observable. So, once this model is taken in a panel data fashion, ISLAN (1995) derives the growth regressions in a usual notation concerning this literature as in equation (5), below:

$$y_{it} = e^{-l\,t} y_{i,t-1} + (1 - e^{-l\,t}) \frac{\boldsymbol{a}}{1-\boldsymbol{a}} \sum ln(s_{it}) + (1 - e^{-l\,t}) \frac{\boldsymbol{a}}{1-\boldsymbol{a}} \sum ln(n + g +_{it} + \boldsymbol{d}) + (1 - e^{-l\,t}) ln A(0) + \boldsymbol{h}_t + \boldsymbol{u}_{it}$$

where: $\boldsymbol{h}_t$ is the time trend of the technological change and ν_{it} is the transitory error term that varies across countries and across time with expected value equal to zero.

After applying a panel data estimation method to equation (5), some interesting results come up immediately. Though the cross-sectional results of MRW (1992) produce a 2% annual rate of convergence on the average, the estimates obtained in a panel data fashion is more volatile. This statement is supported by ISLAN (1995), who allows for heterogeneities only in the intercept terms and finds annual convergence rates ranging from 3.8% to 9.1%. On the other hand, LPS (1997) achieved annual rates of converge of approximately 30%, allowing for heterogeneity in all the parameters of the model. Furthermore, CASELLI, ESQUIVEL, and LEFORT (1996) suggested an annual convergence rate of 10%, after conditioning out individual heterogeneity's and introducing instrumental variables to account for the dynamic endogeneity problem. NERVOLE (1996), by contrast, finds estimates of the annual convergence rates that are even lower then those generated by cross-section regressions. NERLOVE (1996) also argues that his findings are due to finite sample biases of the estimator adopted to calculate empirical tests of the neoclassical growth model.

Choosing a panel data approach is a matter of incurring in vantages and in disadvantages[4], as well. A main disadvantage concerns the own nature of a panel structure and the procedure of decomposition of the constant term into two additive countries and a time specific component do not necessarily seems theoretically natural for most of the researches. Indeed, one significant advance comes from solving the problem of interpreting the standard cross-section regressions. In particular, de dynamic equation (10) typically displays correlation between lagged dependent variables and the unobservable residual, for example, the Solow residual. Therefore, the resulting regression bias depends on the number of the observations over time and only disappears when that figure approaches the infinity. This point is one of the most important issues that we have treated in this paper, mainly because it compounds a sort of difficulty on the interpretation of the convergence regression findings in terms of the poor countries catching up the rich ones.

The most important works ever been made on this topic admit time spans in the estimation of the panels against the use of an entire time series. A recent exception is FERREIRA, ISSLER & PESSÔA (2000). One cumbersome question that arises from discussion concerns the fact that nobody considers such a procedure could stay hiding important tricks like unit roots and structural breaks that may arise when the entire set of time series is in use. Moreover, since we have detected that a set of time series are described by a first order-integrated stochastic process, $I(1)$, the possible existence of a panel data error correction representation cannot be discarded. So, there is a trade off between the two estimators given by a panel data approach: the within estimator and the between estimator as econometric tools for estimation purposes.

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[4] This argumentation are based in DURLAUF and QUAH (1999)

It is straightforward to realize that this methodology could lead to another puzzle as one rightly point by ISLAN (1995), that short-term disturbances may loom largest as the shortest the time span is. However, it is appropriate to remind the reader that in this paper we are dealing with the time dimension of the panel in a more precise form, i.e. all the proceedings are taken in order to guarantee the absence of biases in the estimated parameters, such as specifying the right stochastic processes underlying the time series behavior. If this is so, it is correct to state that the most efficient way to solve this problem is starting with an individual investigation involving individually the time series that composes the time dimension of the panel, before estimating the panel parameters.

III - Description of the Data Set and Samples

The data set selected for this paper comes from the Real National Accounting, compiled by SUMMER and HESTON (1988) and actually presented as a software package known as *Penn World Tables Mark 5.6*. The data set includes real income, real government and private investment spending and population growth displayed in a yearly basis starting in 1969 and ending in 1989

Usually, since yearly data on human capital is not available, the most common way to solve this kind of constraint in the preexisting papers is by interpolating geometrically the data. Once the validity of this procedure is a point of disagreement in this paper, the neoclassical model we have chosen in order to explore new possibilities as stated above is the original Solow growth model without human capital as an explanatory variable. Once more, the possible existence of a long run relationship between the real *per capita* incomes and rest of panel variables can fit the model much better, rather than a basic augmented version. Finally, according to ISLAN (1995), the introduction of human capital as an independent variable losses its power of explanation as a determinant of real *per capita* income differences across countries, once a panel data model with fixed effect is framed and estimated[5].

IV – Some Evidences on a Country-by-Country Analysis

At the beginning of this paper we state clearly that our goal is shed some additional light on the nature of economic growth paths across countries. In doing so, it is an undisputable fact that nothing can be done without an accurate country-by-country analysis, like the seminal investigation performed by NELSON & PLOSSER (1982) over the United States macroeconomic time series. However, this revealed purpose is no longer so great, since we have investigated the dynamic structure of five time series for each of the123 countries in the data set.

The first step involves running ADF and PP unit roots tests in their usual frames. Further, for the time series we do suspect there could be structural breaks, LEE & STRAZICICH (1998) unit roots tests are calculated allowing for just one endogenous break. Based upon these procedures, 30 countries are dropped out of the original data set due to fail of matching into the integration degree of the time series tested and the final and definite sample covers the rest of the 93 countries. The empirical results we have reached upon in this paper permit us to state the following evidences[6]:

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[5] In the case of the OECD countries, the signal of this variable is wrong. However, this may be due to collinearity.

[6] The table with the individual results is available upon request.

First Evidence:

The real *per capita* income fits into a first order integrated stochastic process, *I(1)*, for 73 countries, which accounts for 80% of the sample. Furthermore, for 20 countries out of those 73, where the *per capita* is *I(1)*, the null hypothesis of one endogenous structural breaks cannot be rejected. The remained 20 countries are found to be represented by a *I(2)* stochastic process. Additionally, the structural breaks found are easily explained if we take a brief of attention to specific aspects concerning economic history of these countries. Most of the results on testing the existence of structural breaks in countries real *per capita* income time series are contained in table I, as below[7]:

Table I: Breaking Countries

Countries	Crash Model	LM t-Stat	Breaking	LM t-Stat
Angola	**1979**	-2.0732	**1974**	-7.9299
Brazil	**1987**	-4.5651	**1979**	-8.2539
Comoros	**1973**	-2.7859	**1974**	-6.5670
Congo	**1971**	-2.8856	**1982**	-5.3682
Ecuador	**1978**	-3.0760	**1976**	-7.2661
El Salvador	**1986**	-2.6212	**1980**	-5.1650
Gabon	**1978**	-3.4512	**1973**	-4.7454
Guatemala	**1971**	-4.5104	**1976**	-8.8248
Honduras	**1974**	-2.7692	**1975**	-6.7004
Ivory Cost	**1981**	-3.6731	**1980**	-9.5219
Luxembourg	**1979**	-6.1447	**1983**	-7.9134
Mozambique	**1979**	-3.3450	**1974**	-4.8953
Nigeria	**1976**	-6.9731	**1984**	-5.9363
Paraguay	**1986**	-2.9508	**1979**	-9.0979
Portugal	**1980**	-4.6433	**1975**	-4.5498
Puerto Rico	**1978**	-5.6969	**1984**	-5.9634
Rwanda	**1985**	-2.5993	**1974**	-3.7439
Saudi Arabia	**1977**	-5.7163	**1973**	-8.4552
Spain	**1973**	-4.7779	**1974**	-5.3410
Venezuela	**1980**	-2.0553	**1982**	-6.8624

Second Evidence:

The *per capita* physical capital time series, obtained by a proxy variable defined as the sum of private and public spending on investments, is a first order-integrated stochastic process, *I(1)* with no exceptions of countries in the sample. These results are based upon ADF and PP calculated unit roots tests.

Third Evidence:

The growth rates of population, across countries are also first order-integrated stochastic processes, *I(1)*. Moreover, the calculated values for ADF and PP unit roots tests are not able to reject the null hypothesis for 101 countries in the data set.

Overall, the results can be summarized highlighting the following features. First, from an original data set of 123 countries, the real *per capita* income demonstrated to be fitted into a purely *I(1)* stochastic process for 73 countries. Among these 73 countries, we also found evidences that support the existence of one endogenous break for 20 countries[8]. Moreover, there are econometric evidences accounting for a *I(2)* stochastic representation for real *per capita* income for 20 additional countries. The rest of 30 countries in the sample are found to be stationary for the same variable.

Second, the *per capita* physical capital time series are described by a *I(1)* stochastic process over the entire data set. Finally, concerning the population growth rates, we found evidences for fitting a *I(1)* stochastic process into 101 countries. The remaining 22 time series ranging for the same variable in the sample are all stationaries.

One of the major problems related to most of recent empirical works on growth models concerns the adoption of an *ad hoc* procedure for choosing the samples to be analyzed. Therefore,

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[7] The table with the results for all countries is available upon request.

[8] See table I.

though choosing for a specific procedure that classifies the countries into oil producers, industrialized and developed nations and others seems to be consistent, nothing can be said about its reliability.

Then, once this argument is taken as reasonable, we choose a new applicable frame in this paper. It is based upon the stochastic characteristics of the processes that drive the entire variable behavior. Relying on a country-by-country analysis as discussed early in this section, it is clearly possible to aggregate the countries in a entire new binary fashion: countries where the real *per capita* income growth rates are represented by a stationary processes and countries where these time series are integrated of order one. This procedure sums up into two samples: the *Integrated Sample* (IS) and the *Stationary Sample* (SS).

Additionally, an empirical reasoning comes out in order to support this alternative methodology. Once we have reached to the conclusion that the right frame for a growth model is based upon cross-section and time-series dimensions of a panel, the next step involves performing a panel data unit root test. Then, we are supposed to test for the existence of a cointegration relationship among the variables that constitutes the model. If this relationship can be supported by the data, the estimation of an error correction mechanism is referred to be the final step.

V - The Panel Data Unit Root Test and Cointegration Tests

The panel data unit root test calculated in this paper is based on a traditional approach incorporated in the recent econometric literature. For the sake of this test, the null hypothesis refers to a non-stationary behavior of the time series, admitting the possibility the errors terms are serially correlated with different serial correlation coefficients across-sectional units. The test is calculated as an averaged ADF t-statistic as presented in IM, PESARAN & SHIN (1995):

$$(6) \qquad y_{it} = \boldsymbol{a}_i + \boldsymbol{b}_i t + \boldsymbol{r} y_{i,t-1} + \boldsymbol{u}_{it}$$

The estimated results of an IPS (1995) panel data unit root test in table II below, and the null hypothesis of unit root is rejected only in the case of real income growth variable. For the other four variables of the model, the calculated IPS (1995) unit root test estimates are not able to reject the null hypothesis. Moreover, the IPS (1995) test is calculated in a panel representation that accounts for a constant term and a time trend component as well, like in equation (6) above.

Table II: Panel Data Unit Root Test Estimates

Variables	IPS t-bar Test	IPS LM-bar Test
Income Growth	-30.6558	37.4643
Per Capita Income	-1.8544	2.1694
Initial Per Capita Income	-1.6099	2.0872
Saving Rates	-1.0288	1.9242
Population Growth	0.0869	0.9186

The cointegration tests performed in this paper are based upon KAO (1999) and PEDRONI (1995; 1999) and the null hypothesis is that the estimated equation is not cointegrated. Then, four types of cointegration tests are performed in order to do so. First, we calculate Dickey and Fuller t-based test (Kao DF-ρ). Second, an augmented Dickey and Fuller t-based test (Kao DF-tρ) is also calculated. Finally, a panel t-parametric statistic (Pedroni ρ-NT), which is calculated on the basis of pooling along within-dimension, and a group t-parametric test (Pedroni t-ρ-NT), which relies on a pooling along between-dimension is also calculated. The final estimations for all tests are in table III, as below:

Table III: Cointegration Test Estimates for the Solow Model

Test Type	Statistic	Probability
Kao DF-r	-70.0489	0.00000
Kao DF-tr	-40.9482	0.00000
Pedroni r-NT	-1.3538	0.00000
Pedroni t-r-NT	-1.763	0.00000

Once the estimated results for all tests proved to be significant compared to the cut-offs significance values, it means that the null hypothesis of no cointegration is rejected. Therefore, the next step involves the estimation of an error correction model for the Solow growth model, which is the main topic on discussion next section.

VI - The Solow Growth Model in an Error Correction Representation

The error correction model[9] we want to estimate is based upon a reparameritization of an autoregressive distributed lag model represented by ARDL (*p, q, q, q...,q*). If the time series observations can be stacked for each group in the panel, the error correction model, ECM, can be written as follows:

$$\Delta \mathbf{y}_i = \phi_i \mathbf{y}_{i,-1} + \mathbf{X}_i \beta_i + \sum_{j=1}^{p-1} \lambda_{ij}^{*} \Delta \mathbf{y}_{i,-j} + \sum \Delta \mathbf{X}_{i,-j} \delta_{ij}^{*} + \mathbf{D}\gamma_i + \varepsilon_i \tag{7}$$

$\forall\ i = 1, ..., N$ where $\mathbf{y}_i = (y_{i1}, ..., y_{iT})'$ is a $Tx1$ vector of observations on real *per capita* income for the *i*-th group of the panel; $\mathbf{X}_i = (\mathbf{x}_{i1}, ..., \mathbf{x}_{iT})'$ is a $T\ x\ k$ matrix of observations on the independent variables of the model which vary across groups and across time, ie., population growth rates and *per capita* physical capital accumulation rates. $\mathbf{D} = (\mathbf{d}_1, ..., \mathbf{d}_T)'$is a matrix of dimension TxS, that

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[9] ECM henceforth.

includes the observations on time-invariant independent variables as intercepts and time trends variables.

Assuming the disturbances are identical and independently distributed across countries and over time and also the roots of the ARDL model are outside the unit circle, we ensure that there exists a long run relationship between y_{it} and $\mathbf{x}_{it}$, defined by the following equation:

(8) $$y_{it} = -(\boldsymbol{b}_i' / \boldsymbol{f}_i)\mathrm{x}_{it} + \boldsymbol{h}_{it}$$

where the error term, $\boldsymbol{h}_{it}$, is a stationary process. Clearly, we rightly conclude that the order of integration of the variable y_{it} is, at most, equal to the order of integration of the regressors.

In order to write the equation (7) in a more compact and intuitive way, we set the long-run coefficients on $\boldsymbol{X}_{it}$ as $\boldsymbol{q}_i$ = -$\boldsymbol{b}_i/\boldsymbol{f}_i$ to be the same across groups, namely, $\boldsymbol{q}_i$=$\boldsymbol{q}$, which results in the following equation:

(9) $$\Delta \mathbf{y}_i = \phi_i \hat{\boldsymbol{\imath}}_i(\theta) + \mathbf{W}_i \kappa_i + \varepsilon_i$$

and:

(10) $$\hat{\boldsymbol{\imath}}_i(\grave{\mathbf{e}}) = \mathbf{y}_{i,-1} - \mathbf{X}_i \grave{\mathbf{e}}$$

is the error correction component of the entire ECM representation.

Based upon our previous evidences on a country-by-country analysis, there are two cases to be discussed: the growth accounting model and the convergence model. The first model is easier than the former to estimate since all the variables -- *per* capita income, saving rates and population growth – are all represented by an integrated stochastic process of unitary order, or a I(1) process. The second one is clearly the case were the dependent variables are all stationary and the regressors are first order integrated processes. Asymptotically, this is a more complicated alternative, once the velocity of convergence expressed by short run and long run parameters estimated by maximum likelihood estimators are not the same, so an stochastic equi-continuity condition is necessary[10] in order to guarantee these estimates are BLUE along the panel.

The table IV below shows the estimates for the growth model:

<u>Table IV</u>: Dynamic Fixed Effects Estimates for the Growth Model‡

Dependent Variable: ln(y)
Sample Size = 1484

Variables	Long-Run Coefficients
ln(s)	0.4940 (0.1846) [2.6754]*
$\ln(n+g+\delta)$	-1.5553 (0.4561) [-3.4100]*
$\phi\theta$	0.4969
ϕ	0.0591
AIC	3165.03
SC	3011.26
Restricted Regression	
$\text{Ln}(s)-\ln(n+g+\delta)$	0.6726 (0.0094) [-6.0845] *
$\phi\theta$	0.6654
ϕ	-0.0571
AIC	3163.44
SC	3014.97
Implied α	0.3319
LR stat for long-run parameters	141.3811
p-value	0.00000

‡Notes: Figures in parenthesis are the standard error.
Significant at 5% level.

Indeed, after estimating a dynamic fixed effect panel data Solow model in an error correction form, it is possible to state the following remarks. First, the coefficients on saving and population growth show the theoretical predicted signs. Moreover, those magnitudes are highly significant. Second, the estimated parameter $\phi\theta$, which accounts for the impact of savings on

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[10] The details and the derivation of the asymptotic distribution of this kind of estimator can be found in PESARAN, SHIN & SMITH (1997).

income, should equal the capital share of one third as in a Cobb-Douglas neoclassical production function. Additionally, the sign of that coefficient is positive when estimated in its restricted form and it matches the predicted theoretical assumption concerning this matter of signs. Thus, our results support the standard view of $a = 1/3$, once the estimated implied α is close to 0.30.

In contrast to the widespread claim that the Solow model explains cross-country variability in labor productivity largely by appealing to variations in technologies, the two readily observable variables on which the Solow model focuses on accounts, in fact, for most of the variations in *per capita* income.

Following this track, our results clearly contradict the MRW (1992) findings for whom human capital is the model omitted variable that causes the deviations from the expectable values for the dependent variable, the *per capita* income. MRW (1992)'s results pull the estimates of the capital share up, or better, they are biased toward a larger value.

Departing from a diverging view compared to LPS (1997), we restrict our sample in order to be sure about obtaining the estimates long run coefficients, so that the conditional convergence do not losses its meaning. Since the estimated coefficient of the lagged dependent variable equals the speed of adjustment of the ECM, the parameter ϕ corresponds to the conditional velocity of convergence and it is approximately 6% per year for the estimated model. In this way our paper is not an exception, since we cannot reject the null hypothesis of conditional convergence among countries in study.

Finally, concerning the countries' time series which are proved to be represented by a I(2) integrated stochastic process, there is some room to add a concluding remark. The possible absence of diminishing returns to capital, a key property upon which the endogenous growth theory relies, is an assumption that many authors are accounting for in the recent literature. This is the case, for example, of LUCAS (1988), ROMER (1986) and REBELO (1991). Based upon the analysis we have just gone through along this paper, it is still an open question if these sorts of endogenous growth models are able enough to explain the dynamic behavior of those economies which income growth stochastic processes embodies an acceleration component. This is certainly a matter for further research on this issue.

VII – Concluding Remarks

The pooled fixed effects Solow growth model provides a tight theoretical framework within which the stochastic process behind the growth of income can be steadily interpreted. Even though the nature of stochastic processes are taking into account an important issue for estimating the long run behavior of income growth, the results we have reached upon are not quite different from the predictions of the Solow model.

Besides, once panel data cointegration technique assigns a fixed effect to allow country specific heterogeneities, the conditional convergence does not loose its meaning. Despite of the fact that evidences on the existence of a unit root indicates the impossibility of convergence in a time series context; this is not the case for our findings. This is due to the circumstance it is feasible to fit an error correction mechanism in a panel data approach able to capture the long run steady-state behavior of income growth.

Empirically, the estimates provided by the model estimated in this paper support the original Solow model without adding any other variable than the population growth and saving rate, once the capital share on output is exactly the same as predicted by the theory, i.e. one third. Perhaps, this remarkable finding concerning the share of capital output allows us to verify a larger and less restrictive conditional coefficient of the speed of convergence.

Finally, our paper provides enough room for new research on this issue involving income growth, mainly concerning those countries which stochastic process generating their income paths manifest the existence of structural breaks and multiple unit roots.

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