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International Trade Under Uncertainty and Disadvantageous Integration

INTERNATIONAL TRADE UNDER UNCERTAINTY AND DISADVANTAGEOUS INTEGRATION

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Abstract

A model of international trade under uncertainty is considered where there are two countries, two periods and a finite set of uncertain states in the second period. The uncertainty of the model is intrinsic in the sense that it does not depend on the characteristics of the agents. Equilibria without trade and after introduction of trade in certain commodities are compared, and it is shown that while the standard results about gains from trade hold as long as there are complete markets in each country, the situation turns out to be more complicated under incomplete markets, where there is no possibility of obtaining insurance against price uncertainty in the second period. In this case, the classical results about gains from trade hold only under additional conditions, and an example is exhibited where the opening up of trade results in a welfare loss to every consumer in one of the countries.

1. Introduction

One of the fundamental results of international trade theory pertain to the advantages of free trade over no trade, and the basic insights about gains from trade have served as a theoretical underpinning of the development of agreements and institutions for the advancement of free international trade. While many other features of international trade – its structure and dependence on characteristics of the participating countries – yet contain obscure points for the researcher, the results about mutual benefits of trade are simple and – it seems - uncontroversial. This impression is strengthened by the rather small number of works in recent time concerned with this topic.

Nevertheless, some situations where more trade is not necessarily better then less trade have been developed over the last decades, most notably in connection with strategic trade theory (see Brander and Krugman and (1983), Krugman and Helpman (1986)) or in connection with questions of environmental effects of pollution over borders. Such reservations to the overall advantageousness of international trade pertain – as indeed they

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must – to market failures, in the cases mentioned related to the presence of monopolies or to external effects. Other forms of market failures may produce the same effects, and indeed those connected with *uncertainty* (combined with incomplete markets and/or asymmetric information) would be an obvious choice given the amount of attention given to these market imperfections in recent years. In this paper, we present a case of market imperfectness resulting where it may be welfare-reducing for a country to enter into international trade.

The consideration of international trade under uncertainty is not new; indeed the seminal contribution of Helpman and Razin (1978) has started much research in this field, and the present paper may be seen as a continuation of some of the ideas put forward in this work. On the other hand, some of the concepts and models for analyzing markets under uncertainty even in the context of a single economy were developed only in later years. Among recent contributions to the field should be mentioned Hoff (1996), discussing extension of the Heckscher-Ohlin type theory to situations with incomplete markets.

In the present paper, we consider a situation where uncertainty influences production decisions, some of which have to be taken before the uncertainty is resolved. While the theory is general, we had a particular application in mind, namely that of developing countries choosing between crops for the market and subsistence crops; the case of the small Mexican farmer producing either corn for the market or beans for own consumption would be an example. If there is sufficient price uncertainty for the market crop, whereby losses due to low prices might entail starvation, the decision made will keep the farmer in subsistence production, cutting off a possible source of economic growth.

While situation where specific conditions in certain sectors of the economy has detrimental effects on trade and welfare have been found widespread treatment in the literature on development economics (a prominent example being the model developed by Harris and Todaro (1970)), our point here is that the problem is inherent in the sense that it will arise also if all reference to underdevelopment and subsistence farming is done away with. Indeed, the model to be considered is totally within the tradition of classical trade theory, so that the countries involved may (or may not) have access to the same technology, and the consumers and producers are utility and profit maximizers (with a minor exception in regard to attitudes towards risk as explained below). Since we are interested in price uncertainty, the characteristics of agents are supposed not to depend on the states of nature, so that any differences in prices across states is attributable to intrinsic uncertainty caused by the market rather than to external factors.

Our treatment of the model proceeds in two steps: In the first step, we assume that markets are complete and show that the results obtained are as one should expect, in the sense that international trade is indeed better (in a certain sense) than no trade. Then we change the setup by assuming that there are no markets for contingent future delivery, so that agents cannot enter into insurance contracts with respect to future states of nature. Consequently, producers decing today about production for tomorrow incur a risk, since they have to choose between uncertain prospects with respect to the profits earned in each future state; we shall assume that producers are risk averse so that they do not necessarily maximize expected profits but rather choose so as to maximize expected utility of profits.

When markets are incomplete in the above sense, the standard results about welfare superiority of trade over no trade cannot be obtained any more; indeed we give an example of an economy where an equilibrium under autharchy is not improved by opening up for trade, since every consumer in this economy is worse off in the equilibrium which obtains under international trade. Trade, thus, has lowered welfare.

That this can happen is perhaps not too surprising since we have departed from the classical assumptions of complete markets. On the other hand, since it may safely be conjectured that markets existing in reality are far from complete, we should pay attention to the situations where the outcome differs from what would be assumed using conventional wisdom. The policy implications are clearly not that trade should be liberalized, but rather that the minimalistic set of policy instruments of the national governments for fostering trade and developments must be extended with certain additional means of regulation (provision of insurance, control of production or investment decisions) in order for the countries to reap the benefits of international trade.

The notions of incomplete markets and insurance markets have also been used in other recent contributions to the the insurance markets, such as Broll and Wahl (1998), Feeney (1999), and Beladi and Chau (2000), where the aims however are different from those of the present paper.

The paper is structured as follows: In the Section 2 to follow, we describe the basic model and state the results holding under the assumption of complete markets. Then, in Section 3 we relax this assumption of complete markets, in which case the results to be obtained about welfare implications of international trade are much weaker in the sense that they either pertain to specific properties or presume a higher than usual degree of government interference with the economy. The following Section 5 gives an example where trade leads to lower welfare at least in one country. We conclude the treatment of the model with some comments in Section 6, and finally, the proofs of theorems in Sections 2 and 3 are given in Section 7.

2. A simple model of international trade with uncertainty

In this section, we introduce a model of international trade with uncertainty, where certain production decisions must be taken before the uncertain state is revealed. The uncertainty is *intrinsic* in the sense that it does not enter into the description of the economic agents.

In our model there are two countries A and B; we assume that commodities are country-specific, so that the possibility of international trade in certain goods will be introduced as a particular technology; there are l_A goods in country A and l_B goods in country B. Economic activity take place over two periods, and in the second period there are S uncertain states of nature, with probabilities $\pi_1, \dots, \pi_S$. Following the standard procedure, we write commodity bundles in each country as $(x^0, x^1, \dots, x^s)$ with $x^s \in \mathbf{R}^{l_k}$ $k = A, B$. Here the subscript 0 indicates the delivery today whereas $s \in \{1, \dots, S\}$ stands for delivery at date 1 in state s.

We assume that the characteristics of the economy are independent of the particular state $s \in \{1, \dots, S\}$. In country A, there are m_A consumers, and each consumer i has as basic characteristics

- a set $C_i \subset \mathbf{R}^{2l_A}_+$ specifying the possible combinations of consumption in period 0 and 1 (in some state), and
- a (von Neumann-Morgenstern) utility function $u_i : C_i \to \mathbf{R}$, and
- a vector (ω^i_0, ω^i_1) specifying the endowment of goods in the first and second period (thus, endowment is state independent).

The consumption set is defined as

$$X_i = \{(x_0, x^1_1, \dots, x^S_1) \mid (x^0, x^s) \in C_i, \text{ each } s\},$$

and the utility function $U_i : X_i \to \mathbf{R}$ is defined by

$$U_i(x_0, x^1_1, \dots, x^S_1) = \sum_{s=1}^{S} \pi_s u_i(x_0, x^s_1).$$

Endowments are given as $(\omega^i_0, \omega^i_1, \dots, \omega^i_1)$ (thus, the preferences and endowments of consumers do not differ between states).

In a similar way, we introduce state-independent characteristics of the producers, whereby we want to allow for production over time, with input at time 0 and output at time 1. Thus, we assume that there are n_A producers, so that producer j has a basic set of technologically feasible transformations $F_j \subset \mathbf{R}^{l_A}_- \times \mathbf{R}^{l_A}$ (only output is available at time 0), from which the production set is found as the set

$$Y_j = \{(y_0, y^1_1, \dots, y^S_1) \mid (y_0, y^s) \in F_j, \text{ each } s\}.$$

The characteristics of the m_B consumers and n_B producers of country B are introduced in a similar way; the details are omitted.

For the study of equilibria with international trade, we need to make explicit the technology of international commodity transfers; this is done by specifying a a set $K = \{1, \dots, k\}$, two injective maps σ_A, σ_B from K to L_A and L_B respectively, and for each k, the set

$$\begin{aligned} T_k = &\{(z^A, z^B) \in \mathbf{R}^{l_A} \times \mathbf{R}^{l_B} \mid z^A_{0h} + z^B_{0h'} \le 0,\ (z^s_{1h})^A + (z^s_{0h'})^B \le 0, \\ &\quad s = 1, \dots, S,\ (h, h') = (\sigma_A(k), \sigma_B(k)),\ k \in K,\ (z^A, z^B) = (0,0) \text{ otherwise }, \end{aligned}$$

Thus, there are certain goods which can be traded between countries, and if so, then they can be traded in each period and in each future state of nature; we maintain this assumption of state independent trade possibilities in order to be able to exhibit consequences of intrinsic uncertainty in international trade.

Now we may proceed to specify allocations and equilibria in our model: an allocation in country A is an array $z^A = ((x_0, y_0), (x^s_1, y^s_1)^S_{s=1})$, where $(x_0, y_0) = ((x^i_0)^{m_A}_{i=1}, (y^j_0)^{n_A}_{j=1})$ specifies consumption and production at time 0, whereas

$$(x^s_1, y^s_1) = (((x^s_1)^i)^{m_A}_{i=1}, ((y^s_1)^j)^{n_A}_{j=1})$$

specifies the consumption and production decisions at time 1 in state s, $s = 1, \ldots, S$. The allocation is (autarchically) feasible if it is individually feasible in the sense that

$$x^i = (x_0^i, (x_1^1)^i, \ldots, (x_1^S)^i) \in X_i,\ i = 1, \ldots, m_A,$$

$$y^j = (y_0^j, (y_1^1)^j, \ldots, (y_1^S)^j) \in Y_j,\ j = 1, \ldots, n_A,$$

and aggregate feasible for country A so that

$$\sum_{i=1}^{m_A} (x_0^i, (x_1^1)^i, \ldots, (x_1^S)^i) - \sum_{j=1}^{n_A} (y_0^j, (y_1^1)^j, \ldots, (y_1^S)^j) - \sum_{i=1}^{m_A} (\omega_0^i, \omega_1^i, \ldots, \omega_1^i) \leq 0.$$

For later use we introduce the notation $(z_0^A, (z_1^1)^A, \ldots, (z_1^S)^A)$ for the expression on the right hand side of the above inequality. With this notation we may write the criterion for aggregate feasibility in country A as

$$z_0^A \in \mathbf{R}_-^{l_A},$$
$$(z_1^s)^A \in \mathbf{R}_-^{l_A},\ s = 1, \ldots, S.$$

An allocation z^A in country A together with a price system, which is a vector $\in \mathbf{R}_+^{(S+1)l_A}$ is an autharchic equilibrium if

(1) z^A is autharchically feasible,

(2) for each consumer i, the consumption x^i maximizes U_i on all consumption plans $\tilde{x}^i \in X_i$ such that

$$p_0 \cdot \tilde{x}_0^i + \sum_{s=1}^{S} p_1^s \cdot (\tilde{x}_1^s)^i \leq p_0 \cdot x_0^i + \sum_{s=1}^{S} p_1^s \cdot (x_1^s)^i,$$

(3) for each producer j, the production y^j maximizes the profit $p_0 \cdot y_0^j + \sum_{s=1}^{S} p_1^s \cdot (y_1^s)^j$ over Y_j.

For country B, we have the similar notions of allocations in B and of autharchic equilibra. We now turn to equilibria sustained by international trade. For this, we introduce an international allocation as a pair (z^A, z^B), where z^A (z^B) is an allocation in country A (B). The international allocation is (internationally) feasible if it is individually feasible for all consumers and producers in each country, and in addition,

$$(z_0^A, z_0^B) \in \sum_{k \in K} T_k,$$

$$((z_1^s)^A, (z_1^s)^B) \in \sum_{k \in K} T_k,\ s = 1, \ldots, S.$$

Now we can define an international equilibrium as a pair $((z^A, z^B), p)$ consisting of an international allocation and a price system, such that (1) (z^A, z^B) is feasible, such that the individual optimization conditions in (2) and (3) above are satisfied for each consumer and producer in each country, and finally such that

(4) there are no international income transfers,

$$\sum_{i\in M_c} p\cdot (x^i)^c \leq \sum_{j\in N_c} p\cdot (y^j)^c + \sum_{i\in M_c} p\cdot \omega^i$$

for $c = A, B$ (where $\cdot$ is the inner product in $\mathbf{R}^{(S+1)(l_A+l_B)}$).

In order for equilibria of the types specified above to exist in our model, we need some standard assumptions on agents' characteristics.

Assumption 1. *The consumers $i \in M_c$ for $c \in \{A, B\}$ satisfy*

(i) C_i is closed, convex, bounded from below and satisfies $C_i + \mathbf{R}^{l_c} \subset C_i$,

(ii) u_i continuous, quasi-concave and monotonic,

(iii) $(\omega_0^i, \omega_1^i) \in$ intC_i,

and the producers $j \in N_c$ for $c \in \{A, B\}$ satisfy

(iv) F_j is closed, convex and satisfies free disposal, $F_j + \mathbf{R}_-^{l_c} \times \mathbf{R}_-^{l_c} \subset F_j$.

The following results may now be proved; the method of proof is entirely standard, and we have therefore chosen to give the details in an appendix.

Theorem 1. *If Assumption 1 is satisfies, then:*

(1) In each country A and B, there exists an autharchic equilibrium.

(2) There exists an international equilibrium.

We close this section with a further result, which extends those of Theorem 1: In our formulation of the model, uncertainty does not pertain to objective characteristics of the model, which are all state-independent. Therefore, if (relative) equilibrium prices among states, then this must be an effect of purely intrinsic uncertainty, caused by the beliefs of the agents; such equilibria are called *sunspot equilibria,* cf. Cass and Shell (1983). Since the sunspot effects are not the main topic of this paper, we shall not pursue this topic any further at present; in a later section, we exhibit an equilibrium where sunspots do matter, although in a context of incomplete markets.

The classical topic to be discussed in the context of general equilibrium models of international trade is the welfare effect of opening up trade, the gains-from-trade problem. Since the shift from one equilibrium (in this case autharchic) to another (here an international equilibrium) in most cases will result in some consumers becoming better off and some other consumers worse off, we shall follow the approach introduced by Grandmont and McFadden (1972), checking whether for given autharchic equilibria there exist ("sagacious") income distribution rules in each country and an international equilibrium such that all consumers are better off. Since we have chosen not to be explicit on the mechanism of income distribution in our definition of an equilibrium, the details will appear only in the proof, which is given in the appendix.

Theorem 2. *Under Assumption I, for any pair (z^A, z^B) of autharchic equilibrium allocations in countries A and B such that $(x_i^c) \in \mathrm{int}X_i$ for each consumer $i \in M_c$,*

$c = A, B$, there exists an international equilibrium making each individual in each country as least as well off.

The result of the theorem, which states that trade is superior to no trade, is essentially a translation of the known results about gains from trade to the present context, and as such it suffers from certain shortcomings, which become more evident when the details of intertemporal allocation are made explicit. In particular, the "minimum-wealth" condition in (iii) tells us that consumers can survive with their endowments alone, so that production decisions are not crucial in this respect. This does not seem right in view of the possible applications of the model, where consumers rarely are endowed with all necessities of life. If we want to study poor economies opening up for trade we should therefore reconsider the formal structure of the model, leaving the model of this section as a case of particular wellbehavedness but with perhaps less relevance for practice.

3. Equilibria with incomplete markets and risk averse decision makers

While in our first model of general equilibrium trade under uncertainty future markets were assumed to exist in all markets (so that trade in a particular good meant that this good could be traded between countries in spot markets in both periods as well as in futures markets and contingent on any future state). This assumption may be somewhat unrealistic, in particular in the case where the model is intended to capture some features of trade between developed and less developed countries. Therefore we now consider what may be considered a counterpart of the previous model with complete markets, namely a situation where there are no markets for contingent commodities; only production and money transfers connect the two periods, and the market offers no insurance contracts for wealth in the different states at period 1.

More specifically, we assume in the following that there is one good that can be transferred between periods (since it has the function as a storage of value, may be thought of as "money", but which in the application to follow is interpreted as "land"), whereas other goods are perishable. In the absence of insurance markets consumers will have a budget constraint for each of the states of nature; however, the periods 0 and 1 are linked due to the possibility of transferring wealth. This causes some changes in the budget constraints of the consumer, since there will be one constraint for each state of nature.

Also the producers are affected by the changes in the setup. When insurance contracts are not available, some input decisions in period 0 may have the effect that the profit earned is negative in period 1 in some states of nature. We shall assume that firms as well as consumers are risk averse, so that firms no longer maximize profits but rather maximize a utility function defined on profits in each of the states of nature. Thus, for each firm $j \in N_c$, $c \in \{A, B\}$, there is given a utility function $v_j : \mathbf{R}^S \to \mathbf{R}$, so that production plans $y^j = (y_0^j, (y_1^1)^j, \ldots, (y_1^S)^j) \in Y_j$ are evaluated by firm j according to the expected utility

$$V_j(y^j) = \sum_{s=1}^{S} \pi_s v_j(y_0^j, (y_1^s)^j).$$

Thus, in our new context of incomplete markets, we must redefine the equilibria: An allocation z^A in country A together with a price system $(p_0, p_1^1, \dots, p_1^S)$ is an autharchic equilibrium (under incomplete markets) if

(1) z^A is autharchically feasible,

(2) for each consumer i, the consumption plan x^i maximizes U_i on the set of all consumption plans $\tilde{x}^i \in X_i$ such that

$$p_0 \cdot \tilde{x}_0^i + p_1^s \cdot (\tilde{x}_1^s)^i \leq p_0 \cdot x_0^i + p_1^s \cdot (x_1^s)^i,\ s = 1, \dots, S,$$

(3) for each producer j, the production plan y^j maximizes $V_j(y^j)$ over Y_j.

As before, we have the similar notion of an autharchic equilibrium in country B, and finally, we may introduce an internation equilibrium (with incomplete markets) as a pair $((z^A, z^B), p)$ such that (z^A, z^B) is internationally feasible, and such that the conditions of individual optimizations stated in (2) and (3) above are fulfilled for all consumers and producers in both countries, and finally, such that

(4) there are no international income transfers in any state of nature,

$$\sum_{i \in M_c} [p_0 \cdot (x_0^i) + p_1^s \cdot (x_1^s)^i] \leq \sum_{j \in N_c} [p_0 \cdot y_0^j + p_1^s \cdot (y_1^s)^j] + \sum_{i \in M_c} [p_0 \cdot \omega_0^i + p_1^s \cdot (\omega_1^s)^i]$$

for $s = 1, \dots, S$.

The consistency of our model of international trade with incomplete markets is assured by the following existence result (the proof of which is in the last section of the paper):

Theorem 3. *If Assumption 1 is satisfied, then each country has autharchic equilbria with incomplete markets, and there exists an international equilibrium with incomplete markets.*

Equilibria with incomplete markets, existing by Theorem 3, do not necessarily have the same welfare properties of those with complete markets; on the contrary, it is one of the main points of this paper that the classical results about gains from trade do not hold under the same conditions as before. In the next section, we present an example in where an autharchic equilibrium cannot be improved by international trade. The intuition behind this example is that the production decisions, which as a result of risk-averseness are rather cautious, may lead to an inferior situation in period 1 if the price uncertainty increases with international trade.

Below we give two weak versions of gains-from-trade results which can be established in the framework of incomplete markets. The first one gives conditions under which the income of all consumers in each state under international trade is such that the old consumption bundles could be afforded at the current prices; in that case the superiority of the international equilibrium follows by the same arguments as those proving our first result about gains from trade.

Theorem 4. *Assume that each country satisfies Assumption I, and let* $(\bar{z}^A, \bar{z}^B)$ *be a pair of autharchic equilibrium allocations in countries* A *and* B *such that* $(x_i^c) \in \mathrm{int} X_i$ *for each consumer* $i \in M_c$*,* $c = A, B$*. Assume further that for each* $p = (p^A, p^B) \in \mathbf{R}^{(S+1)(l_A+l_B)}$*, the are* $\tilde{y}^j \in \{y^j \in Y^j \mid V^j(y^j) \geq V^j(\bar{y}^j)\}$*,* $j \in N_c$*, such that*

$$\sum_{j \in N_c} [(p_0)^c \cdot y_0^j + (p_1^s)^c \cdot (y_1^s)^j] \geq \sum_{j \in N_c} [(p_0)^c \cdot \bar{y}_0^j + (p_1^s)^c \cdot (\bar{y}_1^s)^j],$$

$c \in \{A, B\}$*. Then there exists an international equilibrium making each individual in each country as least as well off.*

In the second one we show that it is possible for a country to secure advantageous international trade provided that production is regulated directly, meaning that the authorities may have to go beyond regulation of income distribution and interfere with the choices of the agents in the economy. The proofs are given in the final section.

Theorem 5. *Assume that each country satisfies Assumption I, and let* $(\bar{z}^A, \bar{z}^B)$ *be a pair of autharchic equilibrium allocations in countries* A *and* B *such that* $(x_i^c) \in$ *int*X_i *for each consumer* $i \in M_c$*,* $c = A, B$*. Then there exist production plans* $\tilde{y}^j \in Y^j$*,* $j \in N_c$*,* $c \in \{A, B\}$ *and an international equilibrium* $((\tilde{z}^A, \tilde{z}^B), p)$*, where producers are constrained to choose the production plans* $\tilde{y}^j$*, such that every consumer in each country is as least as well off as in the autharchic equilibrium.*

Clearly, both of the above results are weak in the sense that there are either additional assumptions on the characteristics of the economy or additional constraints on equilibrium behavior. The example in the following section shows that without such restrictions the results about globally advantageous results may not hold.

4. An example of disadvantageous international trade

In the present section, we present a simple example of a world with two countries and two industries. The technology is the same in the two countries, but where one of them in one of the industries the choice of technique has to be decided upon before the uncertainty is revealed. In the example, one country is rich and the other country poor, and this influences the possibilities of choice, as the rich country can afford a more risky production plan than can the poor one. As a result, the poor country must use a production technology which on the average is less advatageous than that chosen by the rich country.

The main purpose of our example is to show that this type of situations – equilibria under uncertainty inducing comparative disadvantages due to poorness – can happen. The numerical example does not pretend to picture any real situation, but only to illustrate that uncertainty-driven disadvantages occur in models of international trade which differ only slightly from the classical Heckscher-Ohlin-Samuelson model and contains no pathological features.

To keep the example as simple as possible, we assume that the two available techniques in the first sector are given by the production functions

$$y_{j,1} = A_j k_1^{\alpha_j} l_1^{\beta_j} m_0^{\gamma_j},\ \alpha_j + \beta_j + \gamma_j = 1,\ j = 1, 2,$$

so that there is constant returns to scale in the production factors; note however, that the inputs have different dates, so that k_1 and l_1 are inserted in period 1 (when the uncertainty has been revealed) whereas m_0 has to be inserted in period 0. In the most straightforward interpretation, the first sector is agriculture, where the decisions of the future crop to be cultivated must be taken at date 0, whereas the use of capital and labor may be decided upon at a later stage. As is usual, we assume that the produced commodities may be traded (in period 1) but that production factors cannot. The parameter choices are as follows:

j	A_j	α_j	β_j	γ_j
1	$\frac{20}{\sqrt{10}}$	$\frac{1}{12}$	$\frac{10}{12}$	$\frac{1}{12}$
2	8	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{8}{16}$

The technology in sector two is given by a Cobb-Douglas production function

$$y_{3,1} = 2k_1^{0.5} l_1^{0.5},$$

so that production takes place at period 1. which displays constant returns to scale in k_1 and l_1 alone.

We check that the price system given by

state	p_1	p_2	p_3	r	w	ν
1	1	1	1	1	1	1
2	1	1	1	1	1	1

(prices are the same in the two possible states, and as all uncertainty stems from prices, there is none) may occur as price system in an autharchic equilibrium in one of the countries (depending on the specification of consumers' preferences). In this equilibrium, we want that the available stock of $m_0 = 1$ is divided into two parts $m_{0,1}$ and $m_{0,2}$ of the same size $1/2$ to be inserted into the production of good 1 and 2, respectively. It may easily be checked that the profit maximizing productions are then

$$(y_{1,1}, k_{1,1}, l_{1,1}, m_{0,1}) = (2, 0.375, 0.625, 0.5)$$

in the first technology and

$$(y_{1,2}, k_{1,2}, l_{1,2}, m_{0,2}) = (6.5, 0.5, 5, 0.5)$$

in the second. In the third sector, we have a production

$$(y_{1,3}, k_{1,3}, l_{1,3}) = (40.25, 20.125, 20.125).$$

Finally, there is a consumption of l (as leisure time) to the amount of 4.25.

We shall not specify the consumption side, since we need only that preferences of consumers are such that the aggregate consumption is

$$(2, 6, 40.25, 0, 4.25).$$

The equal split of m between the two technologies is consistent with optimal choice of producers since each of the available techniques yields the same profit of 2 if $m_0 = 1$ is inserted (before payment for this factor), and by constant returns to scale this profit is retained in any split of this unit between technologies will be equivalent to the producer.

We next consider an international equilibrium at prices

state	p_1	p_2	p_3	r	w	ν
1	2	1	1	1	1	1
2	0.5	1	1	1	1	1

In this equilibrium prices differ in the two states, so that the economies are subject to risk. More specifically, it may be checked that if some of the factor m_0 is inserted in production of commodity 1, then profits are negative in state 2. Assuming sufficient risk averseness of producers in the country considered, this means that all of m_0 will be inserted in the production of commodity 2 which will give the same nonnegative profit in the two states. In the production of commodity 2, a profit of 2 is earned before paying for m_0 (independent of states), and since production in sector 3 displays constant returns to scale, there are no further profit incomes, so that total income in the country (profits plus value of endowments) is $2 + 51 = 53$.

We note now that this income is insufficient for buying the bundle of the autharchic equilibrium in state 1, where its value at the new equilibrium prices is

$$2 \cdot 2 + 1 \cdot 10 + 1 \cdot 40.25 + 1 \cdot 4.25 = 58.50,$$

and even for buying it in state 2. If consumers' preferences are sufficiently curved at the bundles bought in the autharchic equilibrium, then it follows that whatever they buy in the international equilibrium must be inferior – and if the preferences of all consumers are identical, it is inferior for every consumer in the country considered.

What makes the example work is the of course that producers turn away from the production of commodity 1 due to its risk of loss and concentrate on the production of commodity 2 where the factor m_0 has much less impact, so that overall profits are lower than they could have been (in the average), resulting in low purchasing power in the equilibrium after trade.

5. Concluding remarks

In the preceding sections, it has been shown that if markets are incomplete in the sense that there are no markets available for providing insurance against future (price) uncertainty, then risk averse producers may choose in a way which results in less welfare after than before trade. While the incompleteness of markets are certainly an important prerequisite for this outcome, it is the behavior of producers that makes it happen.

Consequently, the relevant policy measures for improving welfare after trade must pertain to either the incompleteness of markets or to the production decisions. In regard to the first option, that of creating domestic insurance markets, there are some inherent problems, since the unfavorable events may hit everyone in the country; it is therefore not straightforward to create a credible insurance against these events on the level of the individual country. The alternative suggested by Theorem 5 is less risky in the sense that the production plans chosen from an overall welfare consideration actually secures the country a welfare level as least as high as what it had initially.

In this sense, some government interference in production decision might be considered as a legitimate means of indirect insurance against the price uncertainties experienced by an open economy. It might be added that the (not too many) examples of developing countries that have combined economic growth with export orientation and openness seem to support our results in the sense that production has been rather closely controlled by government or by small groups of firms with large influence on government policy.

Alternatively, incompleteness of markets may be counteracted by joint effort of the international community, thus eliminating the need for domestic policy to contain market risks. The fundamental superiority of trade over no trade is certainly at work, it should only be given the proper means (international or domestic) of unfolding itself.

6. Proofs of theorems

In this section, we provide the proofs of the theorems in the body of the paper. The first of this is given only in brief outline since it is a standard application of general equilibrium theory.

Proof of Theorem 1: (1) It suffices to consider country A, since the same argument will show the existence of an autarchic equilibrium in country B. We first choose a positive constant $D > 0$ such that if (z^A, p^A) is an autharchic equilibrium in A, then $(m_c + n_c)\|x_i\| < D$ and $(m_c + n_c)\|y_j\| < D$. Then we define truncated production sets (needed in order that the income distribution rule be well-defined) as

$$Y_j^D = \{y_j \in Y_j \mid y_{jh} \leq D, \text{ all } h\}.$$

Now define an income distribution rule $r_D^A : \mathbf{R}_+^{(S+1)l_A} \to \mathbf{R}^{m_A}$ such that

$$\sum_{i \in M_A} (r_D^A)^i(p) = \max\{p \cdot y \mid y \in \sum_{j \in N_A} Y_j^D\} + \sum_{i \in M_A} p \cdot \omega^i,$$

where we have used the notation $\cdot$ for the inner product in $\mathbf{R}^{(S+1)l_A}$.

Now, as a result of Assumption I the economy

$$((X_i, U_i, \omega^i)_{i=1}^{m_A}, (Y_j^D)_{j=1}^{n_A}, r_D^A)$$

satisfies the standard assumptions for existence of a Walras equilibrium (see, e.g. Gale and Mas-Colell (1975), or the textbook version in Green, Mas-Colell, and Whinston (1995)), and this Walras equilibrium is easily seen to be an autharchic equilibrium.

(ii) To prove existence of an international equilibrium, we again transform the situation to the standard model of generel equilibrium. For this, we define the commodity space as $\mathbf{R}^{(S+1)l_A} \times \mathbf{R}^{(S+1)l_B}$; the consumption sets X_i of consumers $i \in M_A$ or $i \in M_B$, and the truncated production sets Y_j^D for $j \in N_A$ or $j \in N_B$ are naturally embedded in this commodity space; here $D > 0$ is chosen in the same way as above. Moreover, we define the income distribution rule $r_D : \mathbf{R}_+^{(S+1)(l_A+l_B)} \to \mathbf{R}^{M_A \cup M_B}$ by

$$r_D(p^A, p^B) = (r_D^A(p^A), r_D^B(p^B)),$$

where r_D^A and r_D^B were defined in the proof of part (1) of the theorem. Finally, for each $k \in K$, define a producer whose technology is the convex cone $\hat{T}_k^D = \hat{T}_k \cup \{z \in \mathbf{R}^{(S+1)(l_A+l_B)} \mid z_h \leq D\}$ and $\hat{T}_k$ is the set of pairs (z^A, z^B) such that $(z_0^A, z_0^B) \in T_k$, $((z_1^s)^A, (z_1^s)^B) \in T_k,\ s = 1, \ldots, S$. Then the economy

$$(((X_i, U_i, \omega^i)_{i \in M_c})_{c=A,B}, ((Y_j^D)_{j \in N_c})_{c=A,B}, (\hat{T}_k^D)_{k \in K}, r_D)$$

satisfies assumptions for existence of a Walras equilibrium (as given e.g. in Gale and Mas-Colell (1975)). This Walras equilibrium satisfies all the conditions of an international equilibrium. □

In order to prove Theorem 2 about gains from trade, we only have to define the income distribution rule in a particular way such that every consumer is at least as well off as in the initial situation (a "sagacious" income distribution policy in the terminology of Grandmont and McFadden (1972)).

Proof of Theorem 2: Let z^A and z^B be the given allocations belonging to autharchic equilibria in country A and B, respectively. As previously, let $D > 0$ be a (large) constant, and define truncated production sets $Y_j^D = \{y_j \in Y_j \mid y_{jh} \leq D, \text{ all } h\}$, for all $j \in N_c$, $c = A, B$. Next, define the the income distribution rule $\tilde{r}_D^c : \mathbf{R}_+^{(S+1)(l_A+l_B)} \to \mathbf{R}^{M_c}$ by

$$(\tilde{r}_D^c)_i(p) = p^c \cdot (x^c)^i + \frac{1}{m_c}\left[\max\{p^c \cdot y^c \mid y^c \in \Sigma_{j\in N_c} Y_j^D\} + \max\{p \cdot y \mid y \in \Sigma_{k\in K} \hat{T}_k^D\} - \Sigma_{j\in N_c} p^c \cdot (y^c)^j\right],$$

for $c = A, B$, where p^c denotes the projection of p on the coordinates associated with country c. It follows from the definition that $\tilde{r}_D = (\tilde{r}_D^A, \tilde{r}_D^B)$ is indeed an income distribution rule, in the sense that

$$\sum_{c\in\{A,B\}} \sum_{i\in M_c} (\tilde{r}_D^c)^i(p) = \max\left\{p \cdot y \mid y \in \sum_{c\in\{A,B\}} \sum_{j\in N_c} Y_j^D + \sum_{k\in K} T_k^D\right\} + \sum_{c\in\{A,B\}} \sum_{i\in M_c} p \cdot \omega^i$$

for all $p \in \mathbf{R}_+^{(S+1)(l_A+l_B)}$. Now we use as previously that the economy

$$(((X_i, U_i, \omega^i)_{i\in M_c})_{c\in\{A,B\}}, ((Y_j)_{j\in N_c})_{c\in\{A,B\}}, (T_k)_{k\in K}, \tilde{r}_D)$$

has a Walras equilibrium, which defines an international equilibrium in our model. Since for each consumer $i \in M_c$, for $c = A, B$, we have that

$$(\tilde{r}_D^c)_i(p) \geq p^c \cdot (x^c)^i,$$

we may conclude that the bundle obtained in this international equilibrum is at least as good for every consumer as what was obtained in the autharchic equilibrium. This completes the proof of Theorem 2. □

Proof of Theorem 3: To prove the first part, we consider country A; the argument for country B is similar.

For each consumer $i \in M_A$, we introduce S consumers $(i, s),\ s = 1, \ldots S$, where $\omega^{(i,s)} = (\omega_1^s)^i$ for all s, $X^{(i,1)} = C_i$, whereas for $s \neq 1$, the consumption set depends on the bundle of $(i, 1)$ so that $X^{(s,1)}(x^{(s,1)}) = \{(x_0, x_1^s) \in C^i \mid x_0 = x_0^{(s,1)}\}$. For each s, the utility function $U^{(i,s)}$ depends on the bundles of the consumers (i, s'), $s \neq s'$, is defined by

$$U^{(i,s)}(x^{(i,s)}; (x^{(i,s')})_{s\neq s}) = U^i(x^{(i,s)}; (x^{(i,s')})_{s\neq s}).$$

Consider now the economy (with externalities in consumption)

$$(((X^{(i,s)})_{s=1}^S)_{i\in M_A}, (Y^j)_{j\in N_A}, \rho_A),$$

where $\rho_A : \mathbf{R}_+^{(S+1)l_A} \to \mathbf{R}^{m_A S}$ is an allocation-dependent income distribution rule such that

$$\sum_{i\in M_A} \rho_A^{(i,s)}(z, p) = \sum_{j\in N_A} [p_0 \cdot y_0^j + p_1^s \cdot (y_1^s)^j] + \sum_{i\in M_A} [p_0 \cdot \omega_0^i + p_1^s \cdot (\omega_1^s)^i]$$

for each s.

It is easily checked that this economy has an equilibrium (extending the method of proof of Gale and Mas-Colell (1975) or Borglin and Keiding (1976) to the case of allocation-dependent income distribution rules). By our construction, the equilibrium bundles of agents (i, s) in $\mathbf{R}^{l_A} \times \mathbf{R}^{l_A}$ for fixed i all have the same first component and therefore define an autharchic allocation, which is seen to be an autharchic equilibrium.

To prove the second part of the theorem, the above argument is repeated along the same lines as in the proof of Theorem 1. □

Proof of Theorem 4: To prove this theorem we have to repeat the existence proof given above with a specific ("sagacious") income distribution rule, which at any price permits each of the artificial consumers (i, s) to buy the bundle $\bar{x}^{(i,s)}$ obtained in the given autharchic equilibrium. The existence of such an income distribution rule is assured by the additional assumptions of the theorem, and consequently, the statement of the theorem follows by the same arguments as in the proof of Theorem 2. □

Proof of Theorem 5: Let $\tilde{y}^j = \bar{y}^j$ (the production plans of the selected autharchic allocations) for each $j \in N_c$, $c \in \{A, B\}$. Defining new production sets as $\tilde{Y}^j = \{\tilde{y}^j\} - \mathbf{R}_+^{(S+1)l_c}$ for $j \in N_c$, $c \in \{A, B\}$, we have a situation where Theorem 4 applies, and we conclude that there is an international equilibrium for the economies with the production sets $\tilde{Y}^j$ such that every consumer is at least as well off as in the autharchic allocation. This proves the statement of the theorem. □

7. References

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