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Kevin B. Grier, joint with Rodolfo Cermeño

# Modeling GRACH Processes in Panel Data: Theory, Simulations and Example

# Modeling GARCH processes in Panel Data: Theory, Simulations and Examples

Rodolfo Cermeño
División de Economía
CIDE, México
rodolfo.cermeno@cide.edu

Kevin B. Grier
Department of Economics
University of Oklahoma,
angus@ou.edu

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## Abstract

In this paper we propose and implement a methodology for testing and estimating GARCH effects in a panel data context. We propose simple tests based on OLS and LSDV residuals to determine whether GARCH effects exist and to test for individual effects in the conditional variance. Estimation of the model is based on direct maximization of the log-likelihood function by numerical methods. Monte Carlo studies are conducted in order to evaluate the performance of this MLE estimator for various relevant designs. We also present two empirical applications. We investigate whether investment in a panel of five large U.S. manufacturing firms, or inflation in a panel of seven Latin American countries exhibit GARCH effects. Our panel GARCH estimator satisfactorily captures the significant conditional heteroskedasticity in the data in both cases.

**Key words**: Panel Data Models, OLS, Least Squares Dummy Variables, ARCH and GARCH models, Maximum Likelihood Estimation, Investment, Inflation.

**JEL classification**: C32, C33.

## 1. Introduction

GARCH modeling has proven to be one of the most useful new time series tools of the last 15 years. Modeling the conditional variance of a stochastic process with ARMA techniques has allowed for greatly improved testing of hypotheses about the real effects of risk/uncertainty. Further, while the OLS estimator is still best *linear* unbiased in the presence of conditional heteroskedasticity, the *non-linear* GARCH estimator can provide large efficiency gains over OLS.

In this paper we consider GARCH estimation and testing in panels. In existing multivariate GARCH models, the number of parameters to be estimated grows rapidly with the number of objects (N) under study, making them impractical for applications with even a moderate N. We attack the problem directly from a panel perspective, employing assumptions and techniques common in that literature. A panel GARCH estimator is extremely useful for at least two reasons. First, uncertainty is likely to be more prevalent and have greater real effects in developing countries. However, these countries rarely produce long time series of data for researchers to exploit. To study the determinants and real effects of uncertainty in the developing world, we need a panel GARCH model.

Second, the recent trend of using panel data rather than a single time series to test macroeconomic and financial hypotheses often involves a switch from examining a single long time series to several pooled shorter time series. As we explain below, this sample change can significantly reduce the relative efficiency of the least squares estimator relative to a GARCH estimator. It is therefore valuable to be able to test panel regressions of financial data for GARCH effects and have a more efficient panel estimator available if the error term is found to be conditionally heteroskedastic.

In section 2 below we review the development of GARCH models and discuss how panel GARCH fits into this overall framework. Section 3 derives our basic panel GARCH estimator under the assumption of total parameter homogeneity. Section 4 discusses several generalizations that relax some of the homogeneity assumptions. Section 5 describes a testing and estimation procedure to determine what type of panel GARCH model is appropriate for a given set of data. Section 6 studies the finite sample performance of the GARCH estimator by way of a series of Monte Carlo experiments. In section 7, we provide two empirical examples of our procedure in action, investigating whether either investment in a panel of five large US manufacturing firms, or inflation in a panel of seven countries, exhibit GARCH effects. Finally, section 8 concludes by reviewing our contribution and making some suggestions for future work.

## 2. ARCH and GARCH Models

Volatility clustering, where the occurrence of large residuals is correlated over time, is frequently observed in financial data. Engle's (1982) ARCH paper models volatility clustering by assuming that the conditional variance of today's error term, given the previous errors, follows a moving average process. Engle shows that the efficiency gain from using ARCH estimation instead of least squares can be quite large when the degree of conditional serial dependence in the error variance is strong.[1]

Two important extensions of Engle's model are Bollerslev's (1986) GARCH model, and Engle, Lillien & Robbins' (1987) ARCH-M model. GARCH models the conditional variance of the error term as an autoregressive-moving average (ARMA) process, and the GARCH(1,1)

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[1] The effects of ARCH estimation on empirical results can be dramatic. For example Grier & Perry (1993) show that the sign of the coefficients on money growth in an interest rate regression can depend on accounting for the conditional heteroskedasticity in the data, and Vilauso (2001) shows that the results of Granger causality tests for money and prices also depend on modelling conditional heteroskedasticity.

model has become the most commonly used specification in empirical applications. Bollerslev shows that any arbitrary ARCH model can be well approximated by the GARCH(1,1) specification.

The ARCH-M model permits testing of economic hypotheses about the real effects of risk or uncertainty. Fluctuations in the conditional variance of the error term are tantamount to fluctuations in the predictability of the process. A high conditional variance implies less predictability, or more risk/uncertainty. ARCH-M models are used to measure and test the significance of time varying risk premia in financial data.[2]

Bollerslev, Engle & Wooldridge (1988) introduce the multivariate GARCH model where the conditional covariance matrix $\mathbf{H}$ at time $t$ (for the GARCH(1,1) case) is given as:

$$vech(\mathbf{H}_t) = \mathbf{C} + \mathbf{A}vech(\boldsymbol{e}_{t-1}\boldsymbol{e}_{t-1}^{'}) + \mathbf{B}vech(\mathbf{H}_{t-1}) \tag{1}$$

Here $vech$ refers to the column stacking operator of the lower portion of a symmetric matrix, $\boldsymbol{e}_{t-1}$ is the vector of errors at time $t-1$, and $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are coefficient matrices. In the three variable case, this covariance structure requires estimating 78 coefficients. To simplify, Bollerslev, Engle & Wooldridge assume that matrices $\mathbf{A}$ and $\mathbf{B}$ are diagonal, which in the tri-variate case reduces the number of coefficients to be estimated to 18. Bollerslev (1990) introduces a further simplification, the constant correlation model, further reducing the estimated parameters in the tri-variate case to 12. Given the large number of coefficients to be estimated, even employing extreme simplifying assumptions, existing empirical multivariate GARCH models consider only a small number of variables.

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[2] While the majority of GARCH applications are in finance (see Bollerslev, Chou, and Kroner 1992 for a review), the technique is useful in macro and development economics as well. Recently, Grier and Perry (1996, 2000) use a multivariate GARCH-M model to test for the effects of inflation uncertainty on the dispersion of relative prices and on real output growth in the US.

Our goal is to extend the utility of GARCH modeling in economics by introducing a tractable methodology for GARCH estimation and testing in a panel data setting. The estimator is not designed to study the covariance of errors across entities, but rather to model the common conditional variance in a group of entities. As noted in the introduction, the extension of GARCH modeling to panels is important for two reasons. First, uncertainty is likely to be more prevalent and have greater real effects in developing countries. However, these countries rarely produce long time series of data for researchers to exploit. Thus to test for the real effects of uncertainty where they are likely to be most important requires pooling several countries into a single data set and then estimating the time varying conditional variances. Our panel GARCH estimator can accomplish exactly that.

Second, the recent switch from time series to panel approaches for testing economic theories may well exacerbate conditional heteroskedasticity problems in the data. The shorter time series that typically are pooled to create a panel data set encompass more recent data, which are likely to enjoy greater volatility clustering. As older observations are discarded and the sample becomes more heavily weighted with more recent data, the strength of GARCH effects may grow, as may the efficiency gain from using a GARCH estimator.

For example, suppose that the years 1981 – 85 contain a large amount of volatility clustering in most countries. For a 100 year, single country time series, volatility clustering is severe in 5% of the sample. For a 20 year, 10-country panel covering the 1970s and 1980s, severe volatility clustering occurs in 50% of the sample. To the extent that switching from a time series to a panel approach piles up more observations that are conditionally heteroskedastic, least squares becomes less and less efficient compared to GARCH estimation.

## 3. The Basic Panel GARCH Model

This section describes the specification and estimation of a simple panel data model with a time- varying conditional variance. At this stage we assume complete parameter homogeneity across units in the panel. In the next section this assumption is relaxed to allow for some forms of parameter heterogeneity. We consider the following general pooled regression model:

$$y_{it} = \mu + \phi y_{it-1} + \mathbf{x}_{it}\underline{\beta} + u_{it}, \quad i = 1,\ldots,N, \quad t = 1,\ldots T \tag{2}$$

$$u_{it} = \sigma_{it}\varepsilon_{it}, \quad \varepsilon_{it} \sim NID(0,1), \tag{3}$$

where $N$ and $T$ are the number of cross sections and time periods in the panel respectively, $y_{it}$ is the dependent variable, $\mu$ is the common intercept coefficient, $\mathbf{x}_{it}$ is a row vector of explanatory variables of dimension $k$, $\underline{\beta}$ is a $k$ by 1 vector of coefficients, $u_{it}$ is the disturbance term, and $\phi$ is the AR parameter. We assume that $|\phi| < 1$. We also assume that $T$ is relatively large so that we can invoke consistency of Least Squares estimators[3]. Under the assumption $\phi = 0$, the process given by equation (1) becomes static. A general process for $\sigma_{it}$ is given by the following GARCH ($p$, $q$):

$$\sigma_{it}^{2} = \alpha + \sum_{m=1}^{q} \gamma_m u_{i,t-m}^{2} + \sum_{n=1}^{p} \delta_n \sigma_{i,t-n}^{2}, \tag{4}$$

which can be expressed more compactly as:

$$\sigma_{it}^{2} = \alpha + A(L,\underline{\gamma})u_{it}^{2} + B(L,\underline{\delta})\sigma_{it}^{2}, \tag{5}$$

where $\alpha$ is a common intercept coefficient, $\underline{\gamma}$ and $\underline{\delta}$ are column vectors of dimensions $q$ and $p$ respectively, and $A(L,\underline{\gamma})$, and $B(L,\underline{\delta})$ are polynomials in the lag operator $L$. The previous equations, are simply extensions of Bollerslev's (1986) GARCH process to each cross-section in the panel. Notice that, if $B(L,\underline{\boldsymbol{d}})=0$ we have an ARCH (*q*) process as in Engle (1982), and if $A(L,\underline{\boldsymbol{g}})=B(L,\underline{\boldsymbol{d}})=0$ we have homoskedastic disturbances. The model defined by equations (2) and (5) will be referred to as Model A. From theorem (1) in Bollerslev (1986), the condition $A(1)+B(1)<1$ is sufficient to assure that the GARCH (*p*, *q*) given by equation (5) be stationary for each cross-section in the panel.

Engle (1982) has pointed out that in a pure time series context, each observation is conditionally normally distributed but the vector of *T* observations is not jointly normally distributed. In fact, the joint density is the product of the conditional densities for all *T* observations. The previous statement applies directly to each cross-sectional unit in the panel considered in Model A. Thus, extension to the panel data case is straightforward as long as the disturbances in the model are assumed to be cross-sectionally independent.

For observation (*i*, *t*), the conditional density is:

$$f\,(y_{it}\,/\,x_{it},m,\underline{\boldsymbol{b}},\boldsymbol{a},\underline{\boldsymbol{g}},\underline{\boldsymbol{d}})=(2\boldsymbol{p}\boldsymbol{s}^{2}_{it}(\boldsymbol{a},\underline{\boldsymbol{g}},\underline{\boldsymbol{d}}))^{-1/2}\exp-\frac{(y_{it}-\boldsymbol{m}-\boldsymbol{f}y_{it-1}-\mathbf{x}_{it}\,\underline{\boldsymbol{b}})^{2}}{2\boldsymbol{s}^{2}_{it}(\boldsymbol{a},\underline{\boldsymbol{g}},\underline{\boldsymbol{d}})}, \qquad (6)$$

which implies, under the previous cross-sectional independence assumption, the following log-likelihood function:

$$l=-\frac{NT}{2}\ln(2\boldsymbol{p})-\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\ln(\boldsymbol{s}^{2}_{it}(\boldsymbol{a},\underline{\boldsymbol{g}},\underline{\boldsymbol{d}}))-\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{(y_{it}-\boldsymbol{m}-\boldsymbol{f}y_{it-1}-\mathbf{x}_{it}\,\underline{\boldsymbol{b}})^{2}}{\boldsymbol{s}^{2}_{it}(\boldsymbol{a},\underline{\boldsymbol{g}},\underline{\boldsymbol{d}})}, \qquad (7)$$

Even though the OLS estimator in equation (2) is still consistent and the most efficient among the class of linear estimators, the MLE estimator based upon (7) is a more efficient non-linear

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[3] For dynamic models with fixed effects and *i.i.d.* errors, it is well known that the LSDV estimator is biased in small *T* samples, i.e. see Kiviet (1995). Later in section 6, will asses the bias of this estimator vis a vis the non-linear MLE estimator when we have conditionally heteroskedastic disturbances.

estimator. In addition, by using MLE we can obtain the parameters of both, the conditional mean (equation 2) and conditional variance (equation 5) simultaneously.[4]

From MLE theory we know that under regularity conditions the MLE estimator of the parameter vector $\boldsymbol{q} = (m, \underline{\boldsymbol{b}}', \boldsymbol{a}, \underline{\boldsymbol{g}}', \underline{\boldsymbol{d}}')'$ is consistent, asymptotically efficient and asymptotically normally distributed. These excellent asymptotic properties, however do not directly speak to the properties of the estimator in sample sizes likely to be encountered in practice. We thus provide some evidence on the finite sample performance of this MLE estimator relative its OLS counterpart by Monte Carlo simulations for a few designs. We present these results in Section 6 below.

Finally, it is important to notice that our model can be easily extended to the GARCH-M class of models in which the conditional variance enters into the conditional mean equation and also to models where exogenous regressors enter the variance equation. In the former case, equation (2) becomes:

$$y_{it} = m + f y_{it-1} + \mathbf{x}_{it}\underline{\boldsymbol{b}} + \boldsymbol{r}\boldsymbol{s}_{it} + u_{it}, \qquad t = 1, \ldots T. \tag{8}$$

In the later case, equation (5) can be reformulated as:

$$\boldsymbol{s}_{it}^{2} = \boldsymbol{a} + A(L, \underline{g})u_{it}^{2} + B(L, \underline{\boldsymbol{d}})\boldsymbol{s}_{it}^{2} + \mathbf{z}_{it}\boldsymbol{q}, \tag{9}$$

where $\mathbf{z}_{it}$ is a vector of explanatory variables that may or may not include the variables in $\mathbf{x}_{it}$, and $\boldsymbol{q}$ is a vector of parameters.

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[4] We will pursue direct maximization of (7) by numerical methods as given in the Optimization module of the GAUSS computer program. The asymptotic covariance matrix of the MLE estimator will be approximated as the

## 4. Relaxing the Homogeneity Assumptions

Model A can easily be modified to allow for different forms of parameter heterogeneity. In principle, it is possible to have heterogeneity in intercepts and/or slopes in both the mean and variance regressions. In fact there are 16 distinct combinations. In order to have a manageable number of cases we only allow for heterogeneity in intercepts in the mean and variance equations. In addition to Model A, we consider the following 3 models:

(i) Individual effects in the mean equation and full parameter homogeneity in the variance equation (Model B).

(ii) Individual effects in the variance equation and full parameter homogeneity in the mean equation (Model C)

(iii) Individual effects in both the mean and variance equations (Model D).

The mean and variance equations for Model D are given by

$$y_{it} = m_i + f y_{it-1} + \mathbf{x}_{it}\underline{\boldsymbol{b}} + u_{it}, \qquad i = 1, \ldots, N, \quad t = 1, \ldots T \tag{10}$$

$$\boldsymbol{s}_{it}^{2} = \boldsymbol{a}_i + A(L, \underline{\boldsymbol{g}})u_{it}^{2} + B(L, \underline{\boldsymbol{d}})\boldsymbol{s}_{it}^{2}, \tag{11}$$

with $m_i$ and $\boldsymbol{a}_i$ representing the corresponding individual specific effects. In this case, the full parameter vector $\underline{\boldsymbol{q}} = (\underline{m}', \underline{\boldsymbol{b}}', \underline{\boldsymbol{a}}', \underline{\boldsymbol{g}}', \underline{\boldsymbol{d}}')'$ has $(2N + k + p + q + 1)$ elements since $\underline{m}$ and $\underline{\boldsymbol{a}}$ are vectors of dimension $N$ with typical elements $m_i$ and $\boldsymbol{a}_i$ respectively. If these individual-specific effects are treated as fixed, the basic model given in the previous section applies directly to this case with no modifications other than including dummy variables in both the mean and variance equations. Model B considers individual effects in the mean equation and a common intercept coefficient in the variance equation $(\boldsymbol{a}_i = \boldsymbol{a})$. In this case there are $(N + k + p + q + 2)$

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inverse of the outer product of the gradient vectors of $l$, evaluated at actual MLE estimates.

parameters to be estimated.  The same number of parameters would have to be estimated in the case of Model C.

If the cross-section dimension of the sample (*N*) is not small, however, the number of parameters to estimate can become unusually large.  In this situation it would be convenient to sweep out the individual effects in the mean equation if they are found significant in order to reduce the number of parameters to be estimated.

## 5. Choosing the Correct Panel GARCH model

We propose the following methodology to identify the appropriate statistical model. First, test for the presence of individual effects in the mean equation.  Second, test for ARCH effects using OLS or LSDV residuals depending on the results in the first step.  Third, determine if there are individual effects in the conditional variance process.  Finally, after choosing and estimating a model, check its residuals to ensure that there is no remaining conditional heteroskedasticity.

### 5.1 testing for individual effects in the mean equation

We propose testing for individual effects in the mean equation using the LSDV estimator with heteroskedasticity and autocorrelation consistent covariance matrix, along the lines of White (1980) and Newey and West (1987) estimators applied to panel.[5]

For models A and B, where the variance process is identical across units, the OLS and LSDV are still best linear estimators.  However for models C and D the unconditional variance will be different across units and the previous estimators will no longer be efficient and inference based upon them will not be valid.  Given that we do not know a priori which is the appropriate model and that we can have auto correlation problems in practice, it seems convenient to use a covariance matrix robust to heteroskedasticity and autocorrelation. Specifically we will test the null hypothesis $H_0 : m_1 = m_2 = \cdots = m_N$ by means of a Wald-test, which will follow a $c^2_{(N-1)}$ distribution asymptotically.

## 5.2 testing for ARCH effects and individual effects in the conditional variance

The second step uses either LSDV or OLS squared residuals (according to whether individual effects were found or not in the mean equation) to test for ARCH effects. We can use the estimated autocorrelation and / or partial autocorrelation coefficients to determine the existence and possible order of ARCH effects. Also, the null hypothesis of conditional homoskedasticity or ARCH (0), against ARCH ($j$) can be tested for a few relevant values of $j$. This can be done via *LM*-test statistics based on the previous squared residuals and referred to the $c^2_{(j)}$ distribution. In practice, rejecting ARCH(0) in favor of a large number of significant lags will lead to the estimation of a GARCH model. That is to say, we are testing for ARCH, but given that a GARCH(1,1) approximates quite well ARCH models of arbitrarily large orders, we are considering here as viable alternatives ARCH(1), ARCH(2), and GARCH(1,1).

Finally, we test for individual effects in the ARCH process in two ways. First we can test whether the squared residuals have a constant mean across the cross-sectional units. Second, we can regress the OLS/LSDV squared residuals on an appropriate number of lagged squared residuals with and without individual effects and compare the fits via an F or Chi-square test.

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[5] Arellano (1987) has extended the White's heteroskedasticity consistent covariance estimator to panel data but this estimator is not apropriate here since it has been formulated for small *T* and large *N* panels which is not our case.

### 5.3 selecting the final model

After initially choosing an appropriate conditional variance model, based on an analysis of the squared residuals described above, and estimating the full model via maximum likelihood, it is important to make sure that all conditional heteroskedasticty has been captured in the estimation. We can accomplish this in two ways. First, we can add additional ARCH or GARCH terms and check their significance. Second, we can test the squared normalized residuals for any autocorrelation pattern. If significant patterns remain, alternative specifications of the conditional variance should be estimated and checked.

## 6. Finite Sample Performance of the Panel GARCH Estimator

It is well known that in the context of time series GARCH models, the (non-linear) MLE estimator not only has desirable asymptotic properties but also it is more efficient than the OLS estimator. Little is known, however, on the finite sample performance of the MLE estimator relative to its OLS counterpart in finite samples, particularly in panel data. This section presents some Monte Carlo results that shed light on the performance of the MLE estimator in panels with GARCH errors. In particular, we study the bias and precision of the MLE and OLS estimators of the parameters of the conditional mean equation (equation 2) as well as the performance of the MLE estimator of the parameters in the variance equation (equation 4).

### 6.1 Monte Carlo design

The data-generating model is defined by equation (2) and equation (4) given before. Notice that the error term in the mean equation is drawn from a normal distribution with mean zero and a variance that changes over time according to equation (4). For practical purposes the simulation study is limited to the pooled regression model and to the cases of ARCH (1) and GARCH (1,1) errors.

For the conditional mean, we consider separately the static and dynamic cases. In the static case, only one exogenous regressor with coefficient $b = 1$ enters the mean equation. In the dynamic case, we consider a pure AR (1) process without exogenous regressors. In this case, the AR parameter $f$ takes the alternative values {0.5, 0.8}, representing moderate and high persistence of the AR (1) process respectively. The intercepts of the mean and variance equations are each set to 1.0 in all cases ($m = a = 1$). For the ARCH (1) process, $g_1$ (the coefficient of $u_{t-1}^2$) takes on the alternative values {0.5, 0.9}, representing moderate and high degrees of conditional heteroskedasticity. For the GARCH (1,1) process, we set $g_1 = 0.3$ (the coefficient on the term $u_{t-1}^2$) for all cases, but we allow $d_1$ (the coefficient on the term $s_{t-1}^2$) to take on the values {0.3, 0.6,}.

Finally, we have set the number of trials in each Monte Carlo experiment to 1000 but given that the program that solves the model numerically does not always converge, the final number of (valid) trials is sometimes less than 1000. The results are presented in Tables A1 through A8 in the Appendix.

### 6.2 static panel ARCH results

Tables A1 and A2 contain the results for the static mean, ARCH(1), case. The first observation is that as *T* increases for a given *N,* the OLS and MLE estimators of the intercept and slope coefficients in the mean equation improve on a mean squared error criterion. Although we do not observe a clear pattern in the bias of both estimators, their standard errors clearly diminish with *T*. Second, when comparing the OLS and MLE estimators (for the mean equation), we find that the MLE outperforms the OLS estimator in terms of precision and mean squared error. In fact the MLE estimator consistently has a MSE of about half of that of the OLS estimator in the case $g_1 = 0.5$, and less than half in the case $g_1 = 0.9$.

Turning to the MLE estimator of the variance coefficients $a$ and $g_1$ (intercept and ARCH (1) coefficient respectively), in both cases we observe improvements in precision and mean squared error as *T* increases. However, there is no obvious pattern in the biases. On a mean squared error criterion, the MLE estimator of the variance coefficients appears to be quite acceptable.

Turning back to the mean equation, it is worth mentioning that although the MLE estimator is better than its OLS counterpart, the MLE estimator of the intercept coefficient still shows relatively large mean squared errors as percentage of the true parameter values. In fact, in the case of the smallest sample (*N*=5, *T*=20) the mean squared error of this estimator 17.1% and 21.4% for the cases of $g_1 = 0.5, 0.9$ respectively.

### 6.3 dynamic panel ARCH results

Tables A3 and A4 show the results for the ARCH (1) process in the dynamic mean case. We find that both the OLS and MLE estimators of the coefficients of the (dynamic) mean equation become more precise as *T* increases. Overall, both estimators improve as *T* increases showing smaller mean squared errors in all cases. As opposed to our findings in the static case, in this case we observe that the MLE estimator of the intercept and AR parameters in the mean equation clearly outperforms the OLS estimator in terms of both bias and precision.

Regarding the MLE estimators of the ARCH equation, we find that their precision increases with the sample size and that the biases generally become smaller as *T* increases. On a mean squared error criterion, we find improvement in the MLE estimators of both parameters ($a$ and $g_1$) as the sample size increases in all cases.

When looking at the performance of these estimators as the persistence of the mean process as well as the strength of the ARCH process increase, given the sample sizes, we observe that the biases of the coefficients of the variance equation generally increase. Also, the bias of $a$ becomes negative while the bias of $g_1$ becomes positive. Another pattern that we find is that the standard error of $a$ increases while the standard error of $g_1$ decreases. Overall, on a mean squared error criterion, we find that the results for the MLE estimator of the variance equation are quite acceptable.

### 6.4 static Panel GARCH results

Monte Carlo results for the static mean case with GARCH errors are presented in Tables A5 and A6. We observe that the MLE and OLS estimators of the coefficients of the mean equation ($m$ and $b$) improve as $T$ increases in terms of standard errors and mean squared errors. Also the MLE estimators of the previous coefficients are less biased than their OLS counterparts in most cases. In terms of precision, the MLE estimators outperform the OLS estimators in all cases. A similar result is found in terms of mean squared errors.

Concerning the MLE estimators of the coefficients in the variance equation ($a, g_1, d_1$), we find that their biases generally diminish as $T$ increases. Similar results are found for the standard errors and mean squared errors.

When the persistence of the GARCH process is increased, the performance of OLS and MLE estimators of the mean coefficients worsens in terms of bias and precision, although the MLE is still better than the OLS estimator. Regarding the MLE estimator of the variance coefficients we observe that the bias increases except for the GARCH coefficient ($d_1$). On the other hand, the ARCH and GARCH coefficients improve in terms of dispersion. On a mean squared error criterion, the estimators of these two coefficients improve while the estimator of the intercept coefficient worsens.

### 6.5 dynamic Panel GARCH results

The Monte Carlo results for the model with dynamic mean and GARCH (1,1) errors are presented in Tables A7 and A8. In this case we find that the MLE and OLS estimators of the parameters in the mean equation ($m$ and $f$) improve as $T$ increases given $N$ on any criterion considered. When comparing both estimators we find that in all cases the MLE estimator is less biased and more precise than the OLS estimator.

For the MLE estimators of the variance coefficients, except for the ARCH parameter ($d_1$), we find that they are generally less biased as $T$ increases. Also, all MLE estimators of the variance coefficients perform better both in terms of standard errors and mean squared errors as $T$ increases in all cases considered.

When we allow for more persistence in both the dynamic mean and variance processes, we find that for both the OLS and MLE estimators of the mean equation, the bias of the intercept coefficient ($m$) increases while the bias of the slope coefficient ($f$) decreases. The same pattern is observed in terms of dispersion. In this case we observe again that the MLE outperforms the OLS estimator. Regarding the MLE estimators of the variance coefficients, we find that the intercept coefficient worsens but the ARCH and GARCH coefficients improve on a mean squared error. In fact, the intercept becomes more biased and less precise while the ARCH and GARCH coefficients become less biased and more precise. Overall, as the values of $f$ and $d_1$

coefficients are increased we obtain less reliable estimators of the intercepts in both the mean and variance equations. However, the estimators of the slope coefficients in the mean ($f$) and variance equations ($g_1, d_1$) become more reliable.

## 7. Empirical Examples

In this section we illustrate the applicability of the panel GARCH estimation and testing methodology proposed in section 5. Specifically we investigate whether the uncertainty associated with investment in a panel of five large U.S. manufacturing firms, and inflation in a panel of seven Latin American countries can in fact be captured with ARCH or GARCH models.

### 7.1 investment in a panel of five large U.S manufacturing firms

Here we use the well-known Grunfeld investment data set.[6] This is a panel of 5 large U.S. firms over 20 years. For each firm and for every year we have observations on gross investment (*I*), the market value of the firm (*F*), and the value of the stock of plant and equipment (*C*). The values of the variables *F* and *C* correspond to the end of the previous year. We test whether the conditional variance of the investment process is time dependent.

The model is specified as follows:

$$I_{it} = m_i + b_1 F_{it} + b_2 C_{it} + u_{it}, \qquad i = 1,...,5; t = 1,...,20 \tag{12}$$

$$s_{it}^2 = a_i + \sum_{n=1}^{p} d_n s_{i,t-1}^2 + \sum_{m=1}^{q} g_m u_{i,t-m}^2 \tag{13}$$

Notice that we allow for heterogeneity only through individual effects in both the conditional mean and conditional variance equations. We begin by testing for individual effects in the mean equation. The computed Wald statistic (using a HAC covariance matrix with lag

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[6] These data are taken from Greene (1997, p. 650, Table 15.1).

truncation equal to 2), is $c^{2}_{(4)} = 118.226$, which ishigh enough to clearly reject the null hypothesis of no individual effects in the mean equation. Next we attempt to identify ARCH effects using the squared residuals from LSDV estimation of the mean equationWe compute the partial auto correlation coefficients of the squared residuals (Table 1).

**TABLE 1: Estimated partial auto-correlation coefficients on squared LSDV residuals (investment data)**

| | Coefficient | t-ratio | p-value |
|---|---|---|---|
| PAC(1) | 0.5225* | 4.0785 | 0.0000 |
| PAC(2) | -0.0925 | -0.6633 | 0.7455 |
| PAC(3) | 0.0519 | 0.3542 | 0.3621 |
| PAC(4) | 0.0728 | 0.4862 | 0.3139 |
| PAC(5) | 0.2325° | 1.5168 | 0.0664 |
| PAC(6) | -0.1235 | -0.7816 | 0.7817 |
| PAC(7) | 0.1293 | 0.7731 | 0.2207 |
| PAC(8) | 0.0482 | 0.2828 | 0.3889 |
| PAC(9) | 0.1245 | 0.6978 | 0.2435 |
| PAC(10) | -0.1638 | -0.9763 | 0.8342 |

LSDV estimated squared residuals are used since there is evidence of individual effects in the mean equation. The symbols *, ^ and ° indicate respectively 1%, 5% and 10% significance levels.

In these data, only the first partial autocorrelation coefficient is statistically significant at the 0.05 level. It thus appears that the conditional variance of the error process follows an ARCH(1). Next, we try to determine if the conditional variance equation has individual effects by regressing the LSDV squared residuals on their first lag and a set of firm specific intercepts and then testing whether the intercepts share a common coefficient.

The computed statistics $F_{(4,94)} = 2.960$ and $c^{2}_{(4)} = 11.864$ reject the null hypothesis of no individual effects in the ARCH process at the 5% significance level. The model selection process thus suggests that there are individual effects in the mean equation, and that the conditional variance follows an ARCH (1) with individual effects, which is our model D in section V above.

**TABLE 2: Panel estimation results for Investment with ARCH effects**

| | Constant | ■$_1$ | ■$_2$ | ■$_3$ | ■$_4$ | ■$_5$ | F | C | Log-likelihood |
|---|---|---|---|---|---|---|---|---|---|
| OLS estimates Mean equation | -48.0297<br>(-2.1903)^ | | | | | | 0.1051<br>(8.2850)* | 0.3054<br>(3.8471)* | -624.9927 |
| | | $s^2 = 16194.677$ | | | | | | | |
| LSDV estimates Mean equation | | -76.0668<br>(-0.7990) | -29.3736<br>(-1.8587)° | -242.1708<br>(-5.0185)* | -57.8994<br>(-3.2350)* | 92.5385<br>(1.7389)° | 0.1060<br>(4.7937)* | 0.3467<br>(7.1857)* | -561.8468 |
| | | $s^2 = 4777.2951$ | | | | | | | |
| ARCH(1): Pooled Regression (Model A) | -37.4254<br>(-6.6876)* | | | | | | 0.1087<br>(40.3168)* | 0.3358<br>(15.2096)* | -584.8165 |
| | | $s_t^2 = \underset{(1.5385)°}{796.6344} + \underset{(2.9566)\wedge}{1.5593}\,\hat{u}_{t-1}^2$ | | | | | | | |
| ARCH(1): Individual Effects in mean only (Model B) | | 222.2649<br>(11.6958)* | 20.6421<br>(4.9555)* | -82.6617<br>(-6.4023)* | -4.8258<br>(-0.9006) | 230.9331<br>(21.7843)* | 0.0502<br>(10.4699)* | 0.1699<br>(20.0284)* | -510.6109 |
| | | $s_t^2 = \underset{(3.0355)*}{109.4899} + \underset{(5.3285)*}{2.1890}\,\hat{u}_{t-1}^2$ | | | | | | | |
| ARCH(1): Individual Effects in mean and variance (Model D) | | 256.4222<br>(8.3794)* | 24.7232<br>(3.5760)* | -51.7389<br>(-1.9547)^ | -0.2614<br>(-0.0368) | 275.3949<br>(7.8697)* | 0.0457<br>(3.7677)* | 0.1518<br>(3.0724)* | -503.6508 |
| | | $s_t^2 = \underset{(0.9555)}{2434.819} + \underset{(1.8053)\wedge}{124.2918} + \underset{(2.5735)*}{594.4582} + \underset{(2.1779)\wedge}{74.3458} + \underset{(2.0791)\wedge}{5852.0537} + \underset{(2.8303)*}{0.9004}\,\hat{u}_{t-1}^2$ | | | | | | | |

These results are obtained by direct maximization of the log-likelihood function by numerical methods. For each model we show the mean coefficients followed by the estimated equation for the conditional variance process. Values in parenthesis are t-ratios and the symbols *, ^, °, indicate significance levels of 1%, 5% and 10% respectively. The t-ratios for OLS and LSDV estimates are based on HAC standard errors with a lag truncation of 2.

Table 2 presents maximum likelihood estimates of this model. For comparison we also consider Model A, which corresponds to a pooled regression model whose conditional variance follows an ARCH(1) process, and present OLS and LSDV estimates of the mean equation.

As noted above, the data reject the null hypothesis of no individual effects in the mean equation at the 0.01 level. This can be seen in Table 2 either by comparing either panels one and two (OLS vs. LSDV) or by comparing panels three and four (ARCH(1) pooled vs. ARCH(1) with individual mean effects). The data also reject the null hypothesis of conditional homoskedasticity, also at the 0.01 level. This can be seen either by comparing panels one and three (OLS vs. ARCH(1) pooled), or panels two and four (LSDV vs. ARCH(1) with individual mean effects) in Table 2. Finally the data reject the null hypothesis of no individual effects in the conditional variance equation at the 0.01 level (as seen by comparing panels 4 and 5 in Table 2). The final preferred model is still Model D, the final estimation in Table 2, which can be described as ARCH(1) with individual effects in both the mean and conditional variance equations. We do not find evidence of any significant auto-correlation in the normalized squared residuals from Model D, thus ensuring that this specification captures all conditional heteroskedasticity from these data.

From the reported results, we can see that accounting for the conditional heteroskedasticity in data notably changes the values of the coefficients on the explanatory variables in the mean equation. The coefficients on $C$ (value of the firm's plant and equipment) and $F$ (the firm's market capitalization) are both around 50% smaller and have smaller t-statistics in the preferred model than in the OLS, LSDV or ARCH(1) pooled specifications, though both coefficients are still significant at the 0.01 level.[7] In sum we find significant conditional heteroskedasticity in this well-known panel, and modeling it via panel GARCH materially affects the results of interest.

### 7.2. inflation in a panel of seven Latin American countries

Here we study inflation in 7 countries (Argentina, Brasil, Chile, Colombia, México, Peru and Venezuela) using quarterly observations on inflation rates $(\pi)$ from 1991.1 to 1999.4.[8] As in the first example, we test whether inflation uncertainty, as represented by the conditional variance of the error term, can be well approximated by a GARCH process.

The model for the mean of inflation is specified as a simple *AR(1)* process:

$$\pi_{it} = \mu_i + \beta_1 \pi_{it-1} + u_{it}, \qquad i = 1,...,7; t = 1,...,36 \qquad (14)$$

$$\sigma^2_{it} = \alpha_i + \sum_{n=1}^{p} \delta_n \sigma^2_{i,t-1} + \sum_{m=1}^{q} \gamma_m u^2_{i,t-m} \qquad (15)$$

Again, we allow for heterogeneity only through individual effects in the conditional mean and conditional variance equations. Testing for individual effects in the mean equation yields the computed Wald statistic (using a HAC covariance matrix with lag truncation equal to 2,) of $\chi^2_{(6)} = 7.284$, which is insignificant at any conventional level. In this case, there is no evidence against the null of no individual effects in the mean equation.

Table 3 presents the computed partial auto-correlation coefficients of the squared OLS residuals for the first 10 lags. Only the first autocorrelation is statistically significant, leading again to the preliminary choice of an ARCH(1) model for the conditional variance of inflation in this panel.

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[7] It is also interesting to note that including the individual effect in the conditional variance changes the model from possibly non-stationary (ARCH coefficient > 1.0 ) to stationary (ARCH coefficient of 0.90).

**TABLE 3: Estimated partial auto-correlation coefficients on squared OLS residuals (inflation data)**

| | Coefficient | t-ratio | p-value |
|---|---|---|---|
| PAC(1) | 0.3637** | 5.6554 | 0.0000 |
| PAC(2) | -0.0143 | -0.2083 | 0.4176 |
| PAC(3) | 0.0386 | 0.5640 | 0.2866 |
| PAC(4) | 0.0323 | 0.4720 | 0.3187 |
| PAC(5) | -0.0216 | -0.3147 | 0.3766 |
| PAC(6) | 0.0210 | 0.3060 | 0.3799 |
| PAC(7) | 0.0107 | 0.1566 | 0.4379 |
| PAC(8) | -0.0041 | -0.0601 | 0.4761 |
| PAC(9) | -0.0182 | -0.2661 | 0.3952 |
| PAC(10) | 0.0577 | 0.8969 | 0.1853 |

OLS estimated squared residuals are used since there is evidence of no individual effects in the mean equation. The symbol ** indicate 1% significance levels.

To look for individual effects in the conditional variance equation, we tested the null hypothesis of equality of the average squared residual across the seven countries and tested the significance of country specific intercepts in a regression of the squared residual on its first lag. In neither case was there any evidence found in favor of individual effects in the conditional variance. The model selection procedure here picks an ARCH(1) model with full parameter homogeneity in both equations (Model A). The first two panels in Table 4 show the estimated inflation process using pooled OLS, then using Model A.

We find a strong degree of conditional heteroskedasticity in these data, with a highly significant estimated ARCH(1) coefficient of around .83. Relative to OLS, the MLE estimator finds a larger and more significant intercept term and a smaller AR(1) term in the mean equation. Thus, even in the relatively tranquil 1990s, Latin American inflation exhibits strong, though not very persistent volatility clustering, indicating that there is still substantial inflation uncertainty in the region.

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[8] These data are compiled from the International Monetary Fund's (IMF) International Financial Statistics CD dated March 2000.

**TABLE 4: Panel estimation results for Inflation with ARCH effects**

| | constant | $\pi_{t-1}$ | Log-likelihood |
|---|---|---|---|
| OLS estimates<br>Mean equation | 1.0392<br>(0.5216) | 0.9065<br>(9.4187)* | -1156.3338 |
| | | $\sigma^2 = 570.974$ | |
| ARCH(1):<br>Pooled regression<br>(Model A) | 1.8356<br>(3.3807)* | 0.8195<br>(37.2695)* | -926.1028 |
| | | $\sigma_t^2 = \underset{(8.1457)^{*}}{36.2354} + \underset{(6.1209)^{*}}{0.8297}\,\hat{u}_{t-1}^2$ | |
| ARCH(1):<br>Pooled regression<br>(Model A with<br>lagged inflation in<br>variance) | 1.5561<br>(3.33960)* | 0.8368<br>(29.2266)* | -904.5676 |
| | | $\sigma_t^2 = \underset{(4.0068)^{*}}{9.8704} + \underset{(4.0020)^{*}}{0.4091}\,\hat{u}_{t-1}^2 + \underset{(5.6897)^{*}}{2.4664}\,\pi_{t-1}$ | |

These results have been obtained by direct maximization of the log-likelihood function by numerical methods. For each model we show the mean coefficients followed by the estimated variance or the estimated equation for the conditional variance process. Values in parenthesis are t-ratios and the symbol (*) indicates significance level of 1%. The t-ratios for OLS estimates are based on HAC standard errors with a lag truncation of 2.

## 7.3. is higher inflation less predictable in this panel?

Friedman (1977) and Ball (1992) argue that higher average inflation rates are less predictable than lower rates. Here we test that hypothesis with our 7 country panel, using a simple modification of our panel GARCH estimator. Specifically, we include lagged inflation as a regressor in the ARCH equation. The results appear in the last panel of Table 4 where the lagged inflation variable is positive and significant.[9] Compared to the standard panel ARCH(1) estimates in the middle panel, we find that the ARCH coefficient falls from around .8 to around .4, the intercept in the conditional variance equation falls from around 36 to around 10, and the maximized value of the likelihood function rises from –926 to –904.

## 8. Conclusions

In this paper, we present and implement a methodology to test for and estimate GARCH effects in panel data sets. Our statistical method consists of the following steps: (i) Testing for individual effects in the mean equation. (ii) Testing for ARCH effects using squared LSDV or OLS residuals (depending on whether individual effects are found or not in the previous stage), and testing for the presence of individual effects in the ARCH process. (iii) Estimating the relevant GARCH specification by maximum likelihood and checking the squared residuals.

We present Monte Carlo results showing that the MLE estimator of the coefficients of the mean equation is less biased and more precise relative to their OLS counterpart in practically all cases considered. Also we find that the MLE estimator of the conditional variance coefficients have a quite acceptable performance except for the intercept in the dynamic mean cases with high persistence of the variance process.

Our empirical applications show that the uncertainty associated with investment decisions (in the panel of 5 large U.S. manufacturing firms) as well as inflation (in the panel of 7 Latin American countries), can be well approximated by a pooled conditionally heteroskedastic error process. Our results show that accounting for this volatility clustering in the data can materially change the estimated effects of variables of interest.

Straightforward, but important, extensions of this work include panel GARCH-M models where the effects of uncertainty on the conditional mean can be tested, and the further development of panel GARCH models with exogenous variables in the conditional variance equation.

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[9] It should be remarked that all the parameters of the model (mean and variance) are estimated simultaneously by Maximum Likelihood. Also, for the two ARCH estimations presented here, we have not found evidence of any significant auto correlation when examining their corresponding squared normalized residuals.

# APPENDIX

TABLE 1A: Monte Carlo results for static mean model and ARCH (1) errors ($g_1 = 0.5$)

| | | OLS | | | | | | MLE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | Coeff. | Bias | (%) | Std. Dev. | (%) | MSE | (%) | Bias | (%) | Std. Dev. | (%) | MSE | (%) |
| N = 5 | $m$ | 0.0067 | 0.7 | 0.2787 | 27.9 | 0.0777 | 7.8 | 0.0166 | 1.7 | 0.2266 | 22.7 | 0.0516 | 5.2 |
| T = 20 | $b$ | -0.0019 | -0.2 | 0.5038 | 50.4 | 0.2539 | 25.4 | -0.0252 | -2.5 | 0.4130 | 41.3 | 0.1712 | 17.1 |
| | $a$ | | | | | | | 0.0070 | 0.7 | 0.2483 | 24.8 | 0.0617 | 6.2 |
| | $g_1$ | | | | | | | -0.0223 | -4.5 | 0.2065 | 41.3 | 0.0432 | 8.6 |
| N = 5 | $m$ | -0.0020 | -0.2 | 0.1704 | 17.0 | 0.0290 | 2.9 | -0.0005 | -0.1 | 0.1301 | 13.0 | 0.0169 | 1.7 |
| T = 50 | $b$ | 0.0045 | 0.5 | 0.3130 | 31.3 | 0.0980 | 9.8 | 0.0000 | 0.0 | 0.2363 | 23.6 | 0.0559 | 5.6 |
| | $a$ | | | | | | | 0.0090 | 0.9 | 0.1489 | 14.9 | 0.0223 | 2.2 |
| | $g_1$ | | | | | | | -0.0029 | -0.6 | 0.1245 | 24.9 | 0.0155 | 3.1 |
| N = 5 | $m$ | 0.0039 | 0.4 | 0.1292 | 12.9 | 0.0167 | 1.7 | 0.0035 | 0.4 | 0.1024 | 10.2 | 0.0105 | 1.1 |
| T = 100 | $b$ | -0.0024 | -0.2 | 0.2211 | 22.1 | 0.0489 | 4.9 | -0.0032 | -0.3 | 0.1717 | 17.2 | 0.0295 | 3.0 |
| | $a$ | | | | | | | -0.0030 | -0.3 | 0.0982 | 9.8 | 0.0096 | 1.0 |
| | $g_1$ | | | | | | | -0.0005 | -0.1 | 0.0867 | 17.3 | 0.0075 | 1.5 |
| N = 10 | $m$ | -0.0147 | -1.5 | 0.2010 | 20.1 | 0.0406 | 4.1 | -0.0078 | -0.8 | 0.1617 | 16.2 | 0.0262 | 2.6 |
| T = 20 | $b$ | 0.0342 | 3.4 | 0.3554 | 35.5 | 0.1275 | 12.8 | 0.0225 | 2.3 | 0.2896 | 29.0 | 0.0844 | 8.4 |
| | $a$ | | | | | | | -0.0124 | -1.2 | 0.1673 | 16.7 | 0.0281 | 2.8 |
| | $g_1$ | | | | | | | -0.0017 | -0.3 | 0.1392 | 27.8 | 0.0193 | 3.9 |
| N = 10 | $m$ | 0.0077 | 0.8 | 0.1231 | 12.3 | 0.0152 | 1.5 | 0.0057 | 0.6 | 0.0955 | 9.6 | 0.0092 | 0.9 |
| T = 50 | $b$ | -0.0062 | -0.6 | 0.2228 | 22.3 | 0.0497 | 5.0 | -0.0036 | -0.4 | 0.1708 | 17.1 | 0.0292 | 2.9 |
| | $a$ | | | | | | | 0.0041 | 0.4 | 0.1016 | 10.2 | 0.0103 | 1.0 |
| | $g_1$ | | | | | | | -0.0076 | -1.5 | 0.0883 | 17.7 | 0.0079 | 1.6 |
| N = 10 | $m$ | 0.0042 | 0.4 | 0.0891 | 8.9 | 0.0079 | 0.8 | 0.0010 | 0.1 | 0.0724 | 7.2 | 0.0052 | 0.5 |
| T = 100 | $b$ | -0.0032 | -0.3 | 0.1559 | 15.6 | 0.0243 | 2.4 | 0.0007 | 0.1 | 0.1241 | 12.4 | 0.0154 | 1.5 |
| | $a$ | | | | | | | -0.0014 | -0.1 | 0.0695 | 7.0 | 0.0048 | 0.5 |
| | $g_1$ | | | | | | | -0.0011 | -0.2 | 0.0619 | 12.4 | 0.0038 | 0.8 |
| N = 20 | $m$ | 0.0072 | 0.7 | 0.1345 | 13.5 | 0.0181 | 1.8 | 0.0075 | 0.8 | 0.1062 | 10.6 | 0.0113 | 1.1 |
| T = 20 | $b$ | -0.0064 | -0.6 | 0.2367 | 23.7 | 0.0561 | 5.6 | -0.0113 | -1.1 | 0.1873 | 18.7 | 0.0352 | 3.5 |
| | $a$ | | | | | | | 0.0004 | 0.0 | 0.1162 | 11.6 | 0.0135 | 1.4 |
| | $g_1$ | | | | | | | -0.0047 | -0.9 | 0.0964 | 19.3 | 0.0093 | 1.9 |
| N = 20 | $m$ | 0.0042 | 0.4 | 0.0956 | 9.6 | 0.0092 | 0.9 | 0.0036 | 0.4 | 0.0732 | 7.3 | 0.0054 | 0.5 |
| T = 50 | $b$ | -0.0041 | -0.4 | 0.1576 | 15.8 | 0.0249 | 2.5 | -0.0032 | -0.3 | 0.1224 | 12.2 | 0.0150 | 1.5 |
| | $a$ | | | | | | | -0.0030 | -0.3 | 0.0691 | 6.9 | 0.0048 | 0.5 |
| | $g_1$ | | | | | | | -0.0035 | -0.7 | 0.0602 | 12.0 | 0.0036 | 0.7 |
| N = 20 | $m$ | 0.0024 | 0.2 | 0.0628 | 6.3 | 0.0030 | 0.3 | 0.0008 | 0.1 | 0.0487 | 4.9 | 0.0024 | 0.2 |
| T = 100 | $b$ | -0.0041 | -0.4 | 0.1112 | 11.1 | 0.0124 | 1.2 | -0.0017 | -0.2 | 0.0841 | 8.4 | 0.0071 | 0.7 |
| | $a$ | | | | | | | -0.0015 | -0.2 | 0.0488 | 4.9 | 0.0024 | 0.2 |
| | $g_1$ | | | | | | | 0.0013 | 0.3 | 0.0437 | 8.7 | 0.0019 | 0.4 |

## TABLE 2A Monte Carlo results for static mean model and ARCH (1) errors ($g_1 = 0.9$)

| | | OLS | | | | | | MLE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | Coeff. | Bias | (%) | Std. Dev. | (%) | MSE | (%) | Bias | (%) | Std. Dev. | (%) | MSE | (%) |
| N = 5 | $m$ | -0.0271 | -2.7 | 0.7242 | 72.4 | 0.5253 | 52.5 | -0.0056 | -0.6 | 0.2489 | 24.9 | 0.0610 | 6.1 |
| T = 20 | $b$ | 0.0338 | 3.4 | 1.2232 | 122.3 | 1.4974 | 149.7 | 0.0123 | 1.2 | 0.4626 | 46.3 | 0.2141 | 21.4 |
| | $a$ | | | | | | | -0.0187 | -1.9 | 0.2951 | 29.5 | 0.0875 | 8.8 |
| | $g_1$ | | | | | | | 0.0845 | 9.4 | 0.2315 | 25.7 | 0.0607 | 6.7 |
| N = 5 | $m$ | -0.0083 | -0.8 | 0.3396 | 34.0 | 0.1154 | 11.5 | -0.0037 | -0.4 | 0.1431 | 14.3 | 0.0205 | 2.1 |
| T = 50 | $b$ | 0.0234 | 2.3 | 0.5930 | 59.3 | 0.3522 | 35.2 | 0.0131 | 1.3 | 0.2468 | 24.7 | 0.0611 | 6.1 |
| | $a$ | | | | | | | -0.0165 | -1.7 | 0.1705 | 17.1 | 0.0293 | 2.9 |
| | $g_1$ | | | | | | | 0.0367 | 4.1 | 0.1519 | 16.9 | 0.0244 | 2.7 |
| N = 5 | $m$ | 0.0030 | 0.3 | 0.2369 | 23.7 | 0.0562 | 5.6 | 0.0022 | 0.2 | 0.1008 | 10.1 | 0.0102 | 1.0 |
| T = 100 | $b$ | 0.0095 | 1.0 | 0.4063 | 40.6 | 0.1656 | 16.6 | -0.0013 | -0.1 | 0.1719 | 17.2 | 0.0296 | 3.0 |
| | $a$ | | | | | | | -0.0008 | -0.1 | 0.1189 | 11.9 | 0.0141 | 1.4 |
| | $g_1$ | | | | | | | 0.0127 | 1.4 | 0.1072 | 11.9 | 0.0116 | 1.3 |
| N = 10 | $m$ | 0.0018 | 0.2 | 0.4282 | 42.8 | 0.1834 | 18.3 | -0.0043 | -0.4 | 0.1648 | 16.5 | 0.0272 | 2.7 |
| T = 20 | $b$ | 0.0197 | 2.0 | 0.8894 | 88.9 | 0.7914 | 79.1 | 0.0070 | 0.7 | 0.3024 | 30.2 | 0.0915 | 9.2 |
| | $a$ | | | | | | | -0.0270 | -2.7 | 0.1908 | 19.1 | 0.0372 | 3.7 |
| | $g_1$ | | | | | | | 0.0901 | 10.0 | 0.1561 | 17.3 | 0.0325 | 3.6 |
| N = 10 | $m$ | 0.0003 | 0.0 | 0.2569 | 25.7 | 0.0660 | 6.6 | -0.0004 | 0.0 | 0.1027 | 10.3 | 0.0105 | 1.1 |
| T = 50 | $b$ | 0.0090 | 0.9 | 0.4365 | 43.7 | 0.1906 | 19.1 | 0.0030 | 0.3 | 0.1745 | 17.5 | 0.0305 | 3.1 |
| | $a$ | | | | | | | -0.0087 | -0.9 | 0.1144 | 11.4 | 0.0132 | 1.3 |
| | $g_1$ | | | | | | | 0.0330 | 3.7 | 0.1002 | 11.1 | 0.0111 | 1.2 |
| N = 10 | $m$ | -0.0005 | -0.1 | 0.1836 | 18.4 | 0.0337 | 3.4 | 0.0049 | 0.5 | 0.0679 | 6.8 | 0.0046 | 0.5 |
| T = 100 | $b$ | 0.0064 | 0.6 | 0.3083 | 30.8 | 0.0951 | 9.5 | -0.0044 | -0.4 | 0.1188 | 11.9 | 0.0141 | 1.4 |
| | $a$ | | | | | | | -0.0057 | -0.6 | 0.0815 | 8.2 | 0.0067 | 0.7 |
| | $g_1$ | | | | | | | 0.0136 | 1.5 | 0.0762 | 8.5 | 0.0060 | 0.7 |
| N = 20 | $m$ | -0.0038 | -0.4 | 0.2757 | 27.6 | 0.0760 | 7.6 | 0.0053 | 0.5 | 0.1229 | 12.3 | 0.0151 | 1.5 |
| T = 20 | $b$ | 0.0139 | 1.4 | 0.4968 | 49.7 | 0.2470 | 24.7 | -0.0052 | -0.5 | 0.2147 | 21.5 | 0.0461 | 4.6 |
| | $a$ | | | | | | | -0.0171 | -1.7 | 0.1342 | 13.4 | 0.0183 | 1.8 |
| | $g_1$ | | | | | | | 0.0767 | 8.5 | 0.1086 | 12.1 | 0.0177 | 2.0 |
| N = 20 | $m$ | 0.0073 | 0.7 | 0.1881 | 18.8 | 0.0354 | 3.5 | 0.0024 | 0.2 | 0.0732 | 7.3 | 0.0054 | 0.5 |
| T = 50 | $b$ | -0.0196 | -2.0 | 0.3272 | 32.7 | 0.1074 | 10.7 | -0.0024 | -0.2 | 0.1252 | 12.5 | 0.0157 | 1.6 |
| | $a$ | | | | | | | 0.0006 | 0.1 | 0.0831 | 8.3 | 0.0069 | 0.7 |
| | $g_1$ | | | | | | | 0.0312 | 3.5 | 0.0730 | 8.1 | 0.0063 | 0.7 |
| N = 20 | $m$ | -0.0018 | -0.2 | 0.1218 | 12.2 | 0.0148 | 1.5 | -0.0004 | 0.0 | 0.0484 | 4.8 | 0.0023 | 0.2 |
| T = 100 | $b$ | 0.0023 | 0.2 | 0.2042 | 20.4 | 0.0417 | 4.2 | 0.0014 | 0.1 | 0.0847 | 8.5 | 0.0072 | 0.7 |
| | $a$ | | | | | | | -0.0053 | -0.5 | 0.0574 | 5.7 | 0.0033 | 0.3 |
| | $g_1$ | | | | | | | 0.0205 | 2.3 | 0.0540 | 6.0 | 0.0033 | 0.4 |

TABLE 3A: Monte Carlo results, for the dynamic mean model ( $f = 0.5$ ) and ARCH (1) errors ( $g_1 = 0.5$ )

| | | OLS | | | | | | MLE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | Coeff. | Bias | (%) | Std. Dev. | (%) | MSE | (%) | Bias | (%) | Std. Dev. | (%) | MSE | (%) |
| N = 5 | $m$ | 0.0546 | 5.5 | 0.2769 | 27.7 | 0.0797 | 8.0 | 0.0377 | 3.8 | 0.2112 | 21.1 | 0.0460 | 4.6 |
| T = 20 | $f$ | -0.0243 | -4.9 | 0.1160 | 23.2 | 0.0141 | 2.8 | -0.0168 | -3.4 | 0.0870 | 17.4 | 0.0078 | 1.6 |
| | $a$ | | | | | | | 0.0076 | 0.8 | 0.2518 | 25.2 | 0.0635 | 6.4 |
| | $g_1$ | | | | | | | -0.0306 | -6.1 | 0.2126 | 42.5 | 0.0461 | 9.2 |
| N = 5 | $m$ | 0.0292 | 2.9 | 0.1838 | 18.4 | 0.0347 | 3.5 | 0.0170 | 1.7 | 0.1283 | 12.8 | 0.0167 | 1.7 |
| T = 50 | $f$ | -0.0149 | -3.0 | 0.0813 | 16.3 | 0.0068 | 1.4 | -0.0090 | -1.8 | 0.0533 | 10.7 | 0.0029 | 0.6 |
| | $a$ | | | | | | | 0.0099 | 1.0 | 0.1506 | 15.1 | 0.0228 | 2.3 |
| | $g_1$ | | | | | | | -0.0067 | -1.3 | 0.1274 | 25.5 | 0.0163 | 3.3 |
| N = 5 | $m$ | 0.0124 | 1.2 | 0.1306 | 13.1 | 0.0172 | 1.7 | 0.0078 | 0.8 | 0.0913 | 9.1 | 0.0084 | 0.8 |
| T = 100 | $f$ | -0.0049 | -1.0 | 0.0587 | 11.7 | 0.0035 | 0.7 | -0.0031 | -0.6 | 0.0377 | 7.5 | 0.0014 | 0.3 |
| | $a$ | | | | | | | -0.0023 | -0.2 | 0.0995 | 10.0 | 0.0099 | 1.0 |
| | $g_1$ | | | | | | | -0.0014 | -0.3 | 0.0873 | 17.5 | 0.0076 | 1.5 |
| N = 10 | $m$ | 0.0333 | 3.3 | 0.2029 | 20.3 | 0.0423 | 4.2 | 0.0261 | 2.6 | 0.1442 | 14.4 | 0.0215 | 2.2 |
| T = 20 | $f$ | -0.0149 | -3.0 | 0.0864 | 17.3 | 0.0077 | 1.5 | -0.0109 | -2.2 | 0.0589 | 11.8 | 0.0036 | 0.7 |
| | $a$ | | | | | | | -0.0118 | -1.2 | 0.1670 | 16.7 | 0.0280 | 2.8 |
| | $g_1$ | | | | | | | -0.0049 | -1.0 | 0.1459 | 29.2 | 0.0213 | 4.3 |
| N = 10 | $m$ | 0.0237 | 2.4 | 0.1324 | 13.2 | 0.0181 | 1.8 | 0.0140 | 1.4 | 0.0890 | 8.9 | 0.0081 | 0.8 |
| T = 50 | $f$ | -0.0095 | -1.9 | 0.0584 | 11.7 | 0.0035 | 0.7 | -0.0047 | -0.9 | 0.0375 | 7.5 | 0.0014 | 0.3 |
| | $a$ | | | | | | | 0.0025 | 0.3 | 0.1016 | 10.2 | 0.0103 | 1.0 |
| | $g_1$ | | | | | | | -0.0072 | -1.4 | 0.0867 | 17.3 | 0.0076 | 1.5 |
| N = 10 | $m$ | 0.0112 | 1.1 | 0.1012 | 10.1 | 0.0104 | 1.0 | 0.0065 | 0.7 | 0.0645 | 6.5 | 0.0042 | 0.4 |
| T = 100 | $f$ | -0.0043 | -0.9 | 0.0455 | 9.1 | 0.0021 | 0.4 | -0.0024 | -0.5 | 0.0265 | 5.3 | 0.0007 | 0.1 |
| | $a$ | | | | | | | -0.0016 | -0.2 | 0.0698 | 7.0 | 0.0049 | 0.5 |
| | $g_1$ | | | | | | | -0.0019 | -0.4 | 0.0629 | 12.6 | 0.0040 | 0.8 |
| N = 20 | $m$ | 0.0163 | 1.6 | 0.1472 | 14.7 | 0.0219 | 2.2 | 0.0078 | 0.8 | 0.1033 | 10.3 | 0.0107 | 1.1 |
| T = 20 | $f$ | -0.0065 | -1.3 | 0.0629 | 12.6 | 0.0040 | 0.8 | -0.0023 | -0.5 | 0.0419 | 8.4 | 0.0018 | 0.4 |
| | $a$ | | | | | | | -0.0020 | -0.2 | 0.1174 | 11.7 | 0.0138 | 1.4 |
| | $g_1$ | | | | | | | -0.0073 | -1.5 | 0.0957 | 19.1 | 0.0092 | 1.8 |
| N = 20 | $m$ | 0.0083 | 0.8 | 0.0999 | 10.0 | 0.0100 | 1.0 | 0.0061 | 0.6 | 0.0648 | 6.5 | 0.0042 | 0.4 |
| T = 50 | $f$ | -0.0029 | -0.6 | 0.0443 | 8.9 | 0.0020 | 0.4 | -0.0018 | -0.4 | 0.0271 | 5.4 | 0.0007 | 0.1 |
| | $a$ | | | | | | | -0.0044 | -0.4 | 0.0698 | 7.0 | 0.0049 | 0.5 |
| | $g_1$ | | | | | | | -0.0030 | -0.6 | 0.0610 | 12.2 | 0.0037 | 0.7 |
| N = 20 | $m$ | 0.0031 | 0.3 | 0.0763 | 7.6 | 0.0058 | 0.6 | 0.0002 | 0.0 | 0.0435 | 4.4 | 0.0019 | 0.2 |
| T = 100 | $f$ | -0.0012 | -0.2 | 0.0355 | 7.1 | 0.0013 | 0.3 | -0.0001 | 0.0 | 0.0185 | 3.7 | 0.0003 | 0.1 |
| | $a$ | | | | | | | -0.0006 | -0.1 | 0.0493 | 4.9 | 0.0024 | 0.2 |
| | $g_1$ | | | | | | | 0.0009 | 0.2 | 0.0433 | 8.7 | 0.0019 | 0.4 |

TABLE 4A: Monte Carlo results for dynamic mean model ( $f = 0.8$ ) and ARCH (1) errors ( $g_1 = 0.9$ )

| | | OLS | | | | | | MLE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | Coeff. | Bias | (%) | Std. Dev. | (%) | MSE | (%) | Bias | (%) | Std. Dev. | (%) | MSE | (%) |
| N = 5 | $m$ | 0.0992 | 9.9 | 0.4816 | 48.2 | 0.2418 | 24.2 | 0.0295 | 3.0 | 0.2007 | 20.1 | 0.0411 | 4.1 |
| T = 20 | $f$ | -0.0204 | -2.6 | 0.0750 | 9.4 | 0.0060 | 0.8 | -0.0063 | -0.8 | 0.0308 | 3.9 | 0.0010 | 0.1 |
| | $a$ | | | | | | | -0.0078 | -0.8 | 0.2937 | 29.4 | 0.0863 | 8.6 |
| | $g_1$ | | | | | | | 0.0419 | 4.7 | 0.2540 | 28.2 | 0.0663 | 7.4 |
| N = 5 | $m$ | 0.0927 | 9.3 | 0.3566 | 35.7 | 0.1357 | 13.6 | 0.0277 | 2.8 | 0.1319 | 13.2 | 0.0182 | 1.8 |
| T = 50 | $f$ | -0.0177 | -2.2 | 0.0594 | 7.4 | 0.0038 | 0.5 | -0.0048 | -0.6 | 0.0208 | 2.6 | 0.0005 | 0.1 |
| | $a$ | | | | | | | -0.0115 | -1.2 | 0.1653 | 16.5 | 0.0274 | 2.7 |
| | $g_1$ | | | | | | | 0.0189 | 2.1 | 0.1578 | 17.5 | 0.0252 | 2.8 |
| N = 5 | $m$ | 0.1040 | 10.4 | 0.3385 | 33.9 | 0.1254 | 12.5 | 0.0126 | 1.3 | 0.0971 | 9.7 | 0.0096 | 1.0 |
| T = 100 | $f$ | -0.0194 | -2.4 | 0.0620 | 7.8 | 0.0042 | 0.5 | -0.0021 | -0.3 | 0.0161 | 2.0 | 0.0003 | 0.0 |
| | $a$ | | | | | | | -0.0018 | -0.2 | 0.1148 | 11.5 | 0.0132 | 1.3 |
| | $g_1$ | | | | | | | 0.0096 | 1.1 | 0.1086 | 12.1 | 0.0119 | 1.3 |
| N = 10 | $m$ | 0.0882 | 8.8 | 0.4096 | 41.0 | 0.1755 | 17.6 | 0.0146 | 1.5 | 0.1264 | 12.6 | 0.0162 | 1.6 |
| T = 20 | $f$ | -0.0170 | -2.1 | 0.0734 | 9.2 | 0.0057 | 0.7 | -0.0027 | -0.3 | 0.0183 | 2.3 | 0.0003 | 0.0 |
| | $a$ | | | | | | | -0.0168 | -1.7 | 0.1915 | 19.2 | 0.0370 | 3.7 |
| | $g_1$ | | | | | | | 0.0481 | 5.3 | 0.1639 | 18.2 | 0.0292 | 3.2 |
| N = 10 | $m$ | 0.0700 | 7.0 | 0.2809 | 28.1 | 0.0838 | 8.4 | 0.0078 | 0.8 | 0.0841 | 8.4 | 0.0071 | 0.7 |
| T = 50 | $f$ | -0.0128 | -1.6 | 0.0508 | 6.4 | 0.0027 | 0.3 | -0.0016 | -0.2 | 0.0135 | 1.7 | 0.0002 | 0.0 |
| | $a$ | | | | | | | -0.0083 | -0.8 | 0.1165 | 11.7 | 0.0137 | 1.4 |
| | $g_1$ | | | | | | | 0.0247 | 2.7 | 0.1091 | 12.1 | 0.0125 | 1.4 |
| N = 10 | $m$ | 0.0422 | 4.2 | 0.2624 | 26.2 | 0.0706 | 7.1 | 0.0066 | 0.7 | 0.0667 | 6.7 | 0.0045 | 0.4 |
| T = 100 | $f$ | -0.0081 | -1.0 | 0.0507 | 6.3 | 0.0026 | 0.3 | -0.0008 | -0.1 | 0.0108 | 1.3 | 0.0001 | 0.0 |
| | $a$ | | | | | | | -0.0048 | -0.5 | 0.0802 | 8.0 | 0.0065 | 0.6 |
| | $g_1$ | | | | | | | 0.0095 | 1.1 | 0.0780 | 8.7 | 0.0062 | 0.7 |
| N = 20 | $m$ | 0.0350 | 3.5 | 0.2707 | 27.1 | 0.0745 | 7.5 | 0.0084 | 0.8 | 0.0830 | 8.3 | 0.0070 | 0.7 |
| T = 20 | $f$ | -0.0070 | -0.9 | 0.0442 | 5.5 | 0.0020 | 0.3 | -0.0016 | -0.2 | 0.0115 | 1.4 | 0.0001 | 0.0 |
| | $a$ | | | | | | | -0.0209 | -2.1 | 0.1278 | 12.8 | 0.0168 | 1.7 |
| | $g_1$ | | | | | | | 0.0514 | 5.7 | 0.1156 | 12.8 | 0.0160 | 1.8 |
| N = 20 | $m$ | 0.0417 | 4.2 | 0.2742 | 27.4 | 0.0769 | 7.7 | 0.0043 | 0.4 | 0.0616 | 6.2 | 0.0038 | 0.4 |
| T = 50 | $f$ | -0.0088 | -1.1 | 0.0520 | 6.5 | 0.0028 | 0.4 | -0.0006 | -0.1 | 0.0093 | 1.2 | 0.0001 | 0.0 |
| | $a$ | | | | | | | -0.0014 | -0.1 | 0.0814 | 8.1 | 0.0066 | 0.7 |
| | $g_1$ | | | | | | | 0.0230 | 2.6 | 0.0750 | 8.3 | 0.0062 | 0.7 |
| N = 20 | $m$ | 0.0420 | 4.2 | 0.2783 | 27.8 | 0.0792 | 7.9 | 0.0031 | 0.3 | 0.0443 | 4.4 | 0.0020 | 0.2 |
| T = 100 | $f$ | -0.0084 | -1.1 | 0.0554 | 6.9 | 0.0031 | 0.4 | -0.0005 | -0.1 | 0.0071 | 0.9 | 0.0001 | 0.0 |
| | $a$ | | | | | | | -0.0064 | -0.6 | 0.0575 | 5.7 | 0.0033 | 0.3 |
| | $g_1$ | | | | | | | 0.0164 | 1.8 | 0.0556 | 6.2 | 0.0034 | 0.4 |

# TABLE 5A: Monte Carlo results for static mean model and GARCH (1,1) errors $(g_1 = 0.3, d_1 = 0.3)$

| | | OLS | | | | | | MLE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | Coeff. | Bias | (%) | Std. Dev. | (%) | MSE | (%) | Bias | (%) | Std. Dev. | (%) | MSE | (%) |
| N = 5 | $m$ | -0.0070 | -0.7 | 0.3202 | 32.0 | 0.1026 | 10.3 | -0.0077 | -0.8 | 0.2831 | 28.3 | 0.0802 | 8.0 |
| T = 20 | $b$ | 0.0133 | 1.3 | 0.5585 | 55.9 | 0.3121 | 31.2 | 0.0138 | 1.4 | 0.4790 | 47.9 | 0.2297 | 23.0 |
| | $a$ | | | | | | | 0.1233 | 12.3 | 0.5271 | 52.7 | 0.2931 | 29.3 |
| | $g_1$ | | | | | | | -0.0004 | -0.1 | 0.1661 | 55.4 | 0.0276 | 9.2 |
| | $d_1$ | | | | | | | -0.0716 | -24 | 0.2263 | 75.4 | 0.0563 | 18.8 |
| N = 5 | $m$ | -0.0124 | -1.2 | 0.2027 | 20.3 | 0.0412 | 4.1 | -0.0110 | -1.1 | 0.1812 | 18.1 | 0.0330 | 3.3 |
| T = 50 | $b$ | 0.0287 | 2.9 | 0.3416 | 34.2 | 0.1175 | 11.8 | 0.0240 | 2.4 | 0.3025 | 30.2 | 0.0921 | 9.2 |
| | $a$ | | | | | | | 0.0650 | 6.5 | 0.4035 | 40.3 | 0.1670 | 16.7 |
| | $g_1$ | | | | | | | -0.0026 | -0.9 | 0.1094 | 36.5 | 0.0120 | 4.0 |
| | $d_1$ | | | | | | | -0.0307 | -10 | 0.1971 | 65.7 | 0.0398 | 13.3 |
| N = 5 | $m$ | 0.0021 | 0.2 | 0.1486 | 14.9 | 0.0221 | 2.2 | 0.0020 | 0.2 | 0.1312 | 13.1 | 0.0172 | 1.7 |
| T = 100 | $b$ | -0.0021 | -0.2 | 0.2540 | 25.4 | 0.0645 | 6.5 | -0.0018 | -0.2 | 0.2244 | 22.4 | 0.0504 | 5.0 |
| | $a$ | | | | | | | 0.0454 | 4.5 | 0.3105 | 31.0 | 0.0984 | 9.8 |
| | $g_1$ | | | | | | | -0.0017 | -0.6 | 0.0768 | 25.6 | 0.0059 | 2.0 |
| | $d_1$ | | | | | | | -0.0166 | -5.5 | 0.1526 | 50.9 | 0.0236 | 7.9 |
| N = 10 | $m$ | 0.0019 | 0.2 | 0.2163 | 21.6 | 0.0468 | 4.7 | -0.0010 | -0.1 | 0.1978 | 19.8 | 0.0391 | 3.9 |
| T = 20 | $b$ | 0.0023 | 0.2 | 0.3709 | 37.1 | 0.1376 | 13.8 | 0.0060 | 0.6 | 0.3383 | 33.8 | 0.1145 | 11.4 |
| | $a$ | | | | | | | 0.0906 | 9.1 | 0.4497 | 45.0 | 0.2104 | 21.0 |
| | $g_1$ | | | | | | | -0.0037 | -1.2 | 0.1179 | 39.3 | 0.0139 | 4.6 |
| | $d_1$ | | | | | | | -0.0452 | -15 | 0.2109 | 70.3 | 0.0465 | 15.5 |
| N = 10 | $m$ | 0.0047 | 0.5 | 0.1482 | 14.8 | 0.0220 | 2.2 | 0.0018 | 0.2 | 0.1327 | 13.3 | 0.0176 | 1.8 |
| T = 50 | $b$ | -0.0069 | -0.7 | 0.2523 | 25.2 | 0.0637 | 6.4 | -0.0014 | -0.1 | 0.2254 | 22.5 | 0.0508 | 5.1 |
| | $a$ | | | | | | | 0.0498 | 5.0 | 0.3092 | 30.9 | 0.0981 | 9.8 |
| | $g_1$ | | | | | | | -0.0008 | -0.3 | 0.0739 | 24.6 | 0.0055 | 1.8 |
| | $d_1$ | | | | | | | -0.0198 | -6.6 | 0.1513 | 50.4 | 0.0233 | 7.8 |
| N = 10 | $m$ | -0.0017 | -0.2 | 0.1043 | 10.4 | 0.0109 | 1.1 | -0.0014 | -0.1 | 0.0911 | 9.1 | 0.0083 | 0.8 |
| T = 100 | $b$ | 0.0073 | 0.7 | 0.1752 | 17.5 | 0.0308 | 3.1 | 0.0066 | 0.7 | 0.1526 | 15.3 | 0.0233 | 2.3 |
| | $a$ | | | | | | | 0.0297 | 3.0 | 0.2110 | 21.1 | 0.0454 | 4.5 |
| | $g_1$ | | | | | | | 0.0018 | 0.6 | 0.0527 | 17.6 | 0.0028 | 0.9 |
| | $d_1$ | | | | | | | -0.0132 | -4.4 | 0.1029 | 34.3 | 0.0108 | 3.6 |
| N = 20 | $m$ | -0.0032 | -0.3 | 0.1567 | 15.7 | 0.0246 | 2.5 | -0.0010 | -0.1 | 0.1416 | 14.2 | 0.0201 | 2.0 |
| T = 20 | $b$ | 0.0074 | 0.7 | 0.2628 | 26.3 | 0.0691 | 6.9 | 0.0054 | 0.5 | 0.2338 | 23.4 | 0.0547 | 5.5 |
| | $a$ | | | | | | | 0.0227 | 2.3 | 0.3525 | 35.3 | 0.1248 | 12.5 |
| | $g_1$ | | | | | | | -0.0027 | -0.9 | 0.0858 | 28.6 | 0.0074 | 2.5 |
| | $d_1$ | | | | | | | -0.0081 | -2.7 | 0.1696 | 56.5 | 0.0288 | 9.6 |
| N = 20 | $m$ | -0.0023 | -0.2 | 0.1035 | 10.3 | 0.0107 | 1.1 | -0.0030 | -0.3 | 0.0907 | 9.1 | 0.0082 | 0.8 |
| T = 50 | $b$ | 0.0075 | 0.7 | 0.1734 | 17.3 | 0.0301 | 3.0 | 0.0092 | 0.9 | 0.1523 | 15.2 | 0.0233 | 2.3 |
| | $a$ | | | | | | | 0.0171 | 1.7 | 0.2209 | 22.1 | 0.0491 | 4.9 |
| | $g_1$ | | | | | | | 0.0001 | 0.0 | 0.0545 | 18.2 | 0.0030 | 1.0 |
| | $d_1$ | | | | | | | -0.0063 | -2.1 | 0.1071 | 35.7 | 0.0115 | 3.8 |
| N = 20 | $m$ | 0.0029 | 0.3 | 0.0779 | 7.8 | 0.0061 | 0.6 | 0.0037 | 0.4 | 0.0684 | 6.8 | 0.0047 | 0.5 |
| T = 100 | $b$ | -0.0012 | -0.1 | 0.1311 | 13.1 | 0.0172 | 1.7 | -0.0022 | -0.2 | 0.1143 | 11.4 | 0.0131 | 1.3 |
| | $a$ | | | | | | | 0.0171 | 1.7 | 0.1482 | 14.8 | 0.0223 | 2.2 |
| | $g_1$ | | | | | | | -0.0016 | -0.5 | 0.0378 | 12.6 | 0.0014 | 0.5 |
| | $d_1$ | | | | | | | -0.0061 | -2.0 | 0.0742 | 24.7 | 0.0055 | 1.8 |

TABLE 6A: Monte Carlo results for static mean model and GARCH (1,1) errors

$(g_1 = 0.3, d_1 = 0.6)$

| | | OLS | | | | | | MLE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | Coeff. | Bias | (%) | Std. Dev. | (%) | MSE | (%) | Bias | (%) | Std. Dev. | (%) | MSE | (%) |
| N = 5<br>T = 20 | $m$ | 0.0229 | 2.3 | 0.6099 | 61.0 | 0.3726 | 37.3 | 0.0474 | 4.7 | 0.5120 | 51.2 | 0.2644 | 26.4 |
| | $b$ | -0.0159 | -1.6 | 1.1127 | 111 | 1.2384 | 123 | -0.0603 | -6.0 | 0.9249 | 92.5 | 0.8591 | 85.9 |
| | $a$ | | | | | | | 0.8730 | 87.3 | 1.5831 | 158.3 | 3.2682 | 327 |
| | $g_1$ | | | | | | | 0.0080 | 2.7 | 0.1419 | 47.3 | 0.0202 | 6.7 |
| | $d_1$ | | | | | | | -0.1271 | -21 | 0.2294 | 38.2 | 0.0688 | 11.5 |
| N = 5<br>T = 50 | $m$ | -0.0110 | -1.1 | 0.3851 | 38.5 | 0.1485 | 14.8 | -0.0021 | -0.2 | 0.2847 | 28.5 | 0.0810 | 8.1 |
| | $b$ | 0.0184 | 1.8 | 0.7048 | 70.5 | 0.4971 | 49.7 | 0.0066 | 0.7 | 0.5142 | 51.4 | 0.2645 | 26.4 |
| | $a$ | | | | | | | 0.2291 | 22.9 | 0.7105 | 71.0 | 0.5573 | 55.7 |
| | $g_1$ | | | | | | | 0.0037 | 1.2 | 0.0874 | 29.1 | 0.0077 | 2.6 |
| | $d_1$ | | | | | | | -0.0310 | -5.2 | 0.1290 | 21.5 | 0.0176 | 2.9 |
| N = 5<br>T = 100 | $m$ | 0.0059 | 0.6 | 0.2890 | 28.9 | 0.0835 | 8.4 | 0.0075 | 0.8 | 0.2219 | 22.2 | 0.0493 | 4.9 |
| | $b$ | -0.0007 | -0.1 | 0.4888 | 48.9 | 0.2390 | 23.9 | -0.0067 | -0.7 | 0.3750 | 37.5 | 0.1407 | 14.1 |
| | $a$ | | | | | | | 0.0986 | 9.9 | 0.4131 | 41.3 | 0.1804 | 18.0 |
| | $g_1$ | | | | | | | 0.0023 | 0.8 | 0.0611 | 20.4 | 0.0037 | 1.2 |
| | $d_1$ | | | | | | | -0.0162 | -2.7 | 0.0807 | 13.5 | 0.0068 | 1.1 |
| N = 10<br>T = 20 | $m$ | 0.0011 | 0.1 | 0.4589 | 45.9 | 0.2106 | 21.1 | 0.0067 | 0.7 | 0.3602 | 36.0 | 0.1298 | 13.0 |
| | $b$ | -0.0128 | -1.3 | 0.8453 | 84.5 | 0.7147 | 71.5 | -0.0113 | -1.1 | 0.6624 | 66.2 | 0.4389 | 43.9 |
| | $a$ | | | | | | | 0.2708 | 27.1 | 0.9280 | 92.8 | 0.9345 | 93.4 |
| | $g_1$ | | | | | | | -0.0021 | -0.7 | 0.0931 | 31.0 | 0.0087 | 2.9 |
| | $d_1$ | | | | | | | -0.0347 | -5.8 | 0.1546 | 25.8 | 0.0251 | 4.2 |
| N = 10<br>T = 50 | $m$ | 0.0052 | 0.5 | 0.2860 | 28.6 | 0.0818 | 8.2 | 0.0074 | 0.7 | 0.2248 | 22.5 | 0.0506 | 5.1 |
| | $b$ | 0.0007 | 0.1 | 0.4858 | 48.6 | 0.2360 | 23.6 | -0.0055 | -0.6 | 0.3842 | 38.4 | 0.1476 | 14.8 |
| | $a$ | | | | | | | 0.0886 | 8.9 | 0.4128 | 41.3 | 0.1783 | 17.8 |
| | $g_1$ | | | | | | | 0.0020 | 0.7 | 0.0639 | 21.3 | 0.0041 | 1.4 |
| | $d_1$ | | | | | | | -0.0141 | -2.4 | 0.0844 | 14.1 | 0.0073 | 1.2 |
| N = 10<br>T = 100 | $m$ | 0.0161 | 1.6 | 0.2001 | 20.0 | 0.0403 | 4.0 | 0.0073 | 0.7 | 0.1472 | 14.7 | 0.0217 | 2.2 |
| | $b$ | -0.0184 | -1.8 | 0.3429 | 34.3 | 0.1179 | 11.8 | -0.0043 | -0.4 | 0.2516 | 25.2 | 0.0633 | 6.3 |
| | $a$ | | | | | | | 0.0421 | 4.2 | 0.2647 | 26.5 | 0.0718 | 7.2 |
| | $g_1$ | | | | | | | -0.0028 | -0.9 | 0.0442 | 14.7 | 0.0020 | 0.7 |
| | $d_1$ | | | | | | | -0.0041 | -0.7 | 0.0563 | 9.4 | 0.0032 | 0.5 |
| N = 20<br>T = 20 | $m$ | 0.0067 | 0.7 | 0.3281 | 32.8 | 0.1077 | 10.8 | 0.0015 | 0.2 | 0.2497 | 25.0 | 0.0623 | 6.2 |
| | $b$ | -0.0089 | -0.9 | 0.5485 | 54.9 | 0.3010 | 30.1 | 0.0033 | 0.3 | 0.4267 | 42.7 | 0.1821 | 18.2 |
| | $a$ | | | | | | | 0.0005 | 0.0 | 0.4925 | 49.2 | 0.2425 | 24.3 |
| | $g_1$ | | | | | | | -0.0062 | -2.1 | 0.0677 | 22.6 | 0.0046 | 1.5 |
| | $d_1$ | | | | | | | 0.0022 | 0.4 | 0.0991 | 16.5 | 0.0098 | 1.6 |
| N = 20<br>T = 50 | $m$ | 0.0105 | 1.1 | 0.1973 | 19.7 | 0.0390 | 3.9 | 0.0108 | 1.1 | 0.1488 | 14.9 | 0.0223 | 2.2 |
| | $b$ | -0.0114 | -1.1 | 0.3411 | 34.1 | 0.1165 | 11.6 | -0.0094 | -0.9 | 0.2521 | 25.2 | 0.0636 | 6.4 |
| | $a$ | | | | | | | 0.0183 | 1.8 | 0.2531 | 25.3 | 0.0644 | 6.4 |
| | $g_1$ | | | | | | | -0.0047 | -1.6 | 0.0426 | 14.2 | 0.0018 | 0.6 |
| | $d_1$ | | | | | | | 0.0010 | 0.2 | 0.0538 | 9.0 | 0.0029 | 0.5 |
| N = 20<br>T = 100 | $m$ | 0.0116 | 1.2 | 0.1430 | 14.3 | 0.0206 | 2.1 | 0.0117 | 1.2 | 0.1090 | 10.9 | 0.0120 | 1.2 |
| | $b$ | -0.0146 | -1.5 | 0.2508 | 25.1 | 0.0631 | 6.3 | -0.0162 | -1.6 | 0.1927 | 19.3 | 0.0374 | 3.7 |
| | $a$ | | | | | | | 0.0185 | 1.8 | 0.1737 | 17.4 | 0.0305 | 3.1 |
| | $g_1$ | | | | | | | 0.0011 | 0.4 | 0.0299 | 10.0 | 0.0009 | 0.3 |
| | $d_1$ | | | | | | | -0.0040 | -0.7 | 0.0369 | 6.2 | 0.0014 | 0.2 |

## TABLE 7A: Monte Carlo results for dynamic mean model ( $f = 0.5$ ) and GARCH (1,1) errors ( $g_1 = 0.3, d_1 = 0.3$ )

| | | OLS | | | | | | MLE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | Coeff. | Bias | (%) | Std. Dev. | (%) | MSE | (%) | Bias | (%) | Std. Dev. | (%) | MSE | (%) |
| N = 5<br>T = 20 | $m$ | 0.0628 | 6.3 | 0.2670 | 26.7 | 0.0752 | 7.5 | 0.0422 | 4.2 | 0.2316 | 23.2 | 0.0554 | 5.5 |
| | $b$ | -0.0306 | -6.1 | 0.1031 | 20.6 | 0.0116 | 2.3 | -0.0231 | -4.6 | 0.0925 | 18.5 | 0.0091 | 1.8 |
| | $a$ | | | | | | | 0.1339 | 13.4 | 0.5363 | 53.6 | 0.3056 | 30.6 |
| | $g_1$ | | | | | | | -0.0030 | -1.0 | 0.1654 | 55.1 | 0.0274 | 9.1 |
| | $d_1$ | | | | | | | -0.0718 | -24 | 0.2307 | 76.9 | 0.0584 | 19.5 |
| N = 5<br>T = 50 | $m$ | 0.0284 | 2.8 | 0.1700 | 17.0 | 0.0297 | 3.0 | 0.0194 | 1.9 | 0.1499 | 15.0 | 0.0229 | 2.3 |
| | $b$ | -0.0126 | -2.5 | 0.0690 | 13.8 | 0.0049 | 1.0 | -0.0086 | -1.7 | 0.0591 | 11.8 | 0.0036 | 0.7 |
| | $a$ | | | | | | | 0.0798 | 8.0 | 0.4034 | 40.3 | 0.1691 | 16.9 |
| | $g_1$ | | | | | | | -0.0057 | -1.9 | 0.1117 | 37.2 | 0.0125 | 4.2 |
| | $d_1$ | | | | | | | -0.0357 | -12 | 0.1943 | 64.8 | 0.0390 | 13.0 |
| N = 5<br>T = 100 | $m$ | 0.0169 | 1.7 | 0.1240 | 12.4 | 0.0157 | 1.6 | 0.0113 | 1.1 | 0.1043 | 10.4 | 0.0110 | 1.1 |
| | $b$ | -0.0078 | -1.6 | 0.0516 | 10.3 | 0.0027 | 0.5 | -0.0050 | -1.0 | 0.0415 | 8.3 | 0.0017 | 0.3 |
| | $a$ | | | | | | | 0.0292 | 2.9 | 0.3114 | 31.1 | 0.0978 | 9.8 |
| | $g_1$ | | | | | | | -0.0037 | -1.2 | 0.0766 | 25.5 | 0.0059 | 2.0 |
| | $d_1$ | | | | | | | -0.0083 | -2.8 | 0.1547 | 51.6 | 0.0240 | 8.0 |
| N = 10<br>T = 20 | $m$ | 0.0238 | 2.4 | 0.1893 | 18.9 | 0.0364 | 3.6 | 0.0146 | 1.5 | 0.1671 | 16.7 | 0.0281 | 2.8 |
| | $b$ | -0.0130 | -2.6 | 0.0753 | 15.1 | 0.0058 | 1.2 | -0.0090 | -1.8 | 0.0657 | 13.1 | 0.0044 | 0.9 |
| | $a$ | | | | | | | 0.0878 | 8.8 | 0.4359 | 43.6 | 0.1977 | 19.8 |
| | $g_1$ | | | | | | | 0.0105 | 3.5 | 0.1202 | 40.1 | 0.0146 | 4.9 |
| | $d_1$ | | | | | | | -0.0460 | -15 | 0.2078 | 69.3 | 0.0453 | 15.1 |
| N = 10<br>T = 50 | $m$ | 0.0179 | 1.8 | 0.1275 | 12.7 | 0.0166 | 1.7 | 0.0131 | 1.3 | 0.1064 | 10.6 | 0.0115 | 1.1 |
| | $b$ | -0.0073 | -1.5 | 0.0512 | 10.2 | 0.0027 | 0.5 | -0.0054 | -1.1 | 0.0417 | 8.3 | 0.0018 | 0.4 |
| | $a$ | | | | | | | 0.0398 | 4.0 | 0.3010 | 30.1 | 0.0922 | 9.2 |
| | $g_1$ | | | | | | | 0.0023 | 0.8 | 0.0790 | 26.3 | 0.0062 | 2.1 |
| | $d_1$ | | | | | | | -0.0200 | -6.7 | 0.1482 | 49.4 | 0.0224 | 7.5 |
| N = 10<br>T = 100 | $m$ | 0.0139 | 1.4 | 0.0860 | 8.6 | 0.0076 | 0.8 | 0.0111 | 1.1 | 0.0712 | 7.1 | 0.0052 | 0.5 |
| | $b$ | -0.0052 | -1.0 | 0.0355 | 7.1 | 0.0013 | 0.3 | -0.0038 | -0.8 | 0.0288 | 5.8 | 0.0008 | 0.2 |
| | $a$ | | | | | | | 0.0259 | 2.6 | 0.2201 | 22.0 | 0.0491 | 4.9 |
| | $g_1$ | | | | | | | -0.0034 | -1.1 | 0.0533 | 17.8 | 0.0029 | 1.0 |
| | $d_1$ | | | | | | | -0.0103 | -3.4 | 0.1093 | 36.4 | 0.0120 | 4.0 |
| N = 20<br>T = 20 | $m$ | 0.0195 | 1.9 | 0.1357 | 13.6 | 0.0188 | 1.9 | 0.0124 | 1.2 | 0.1159 | 11.6 | 0.0136 | 1.4 |
| | $b$ | -0.0074 | -1.5 | 0.0544 | 10.9 | 0.0030 | 0.6 | -0.0037 | -0.7 | 0.0458 | 9.2 | 0.0021 | 0.4 |
| | $a$ | | | | | | | 0.0391 | 3.9 | 0.3611 | 36.1 | 0.1320 | 13.2 |
| | $g_1$ | | | | | | | 0.0020 | 0.7 | 0.0867 | 28.9 | 0.0075 | 2.5 |
| | $d_1$ | | | | | | | -0.0234 | -7.8 | 0.1733 | 57.8 | 0.0306 | 10.2 |
| N = 20<br>T = 50 | $m$ | 0.0099 | 1.0 | 0.0885 | 8.8 | 0.0079 | 0.8 | 0.0081 | 0.8 | 0.0753 | 7.5 | 0.0057 | 0.6 |
| | $b$ | -0.0041 | -0.8 | 0.0354 | 7.1 | 0.0013 | 0.3 | -0.0034 | -0.7 | 0.0290 | 5.8 | 0.0008 | 0.2 |
| | $a$ | | | | | | | 0.0164 | 1.6 | 0.2148 | 21.5 | 0.0464 | 4.6 |
| | $g_1$ | | | | | | | -0.0002 | -0.1 | 0.0523 | 17.4 | 0.0027 | 0.9 |
| | $d_1$ | | | | | | | -0.0094 | -3.1 | 0.1064 | 35.5 | 0.0114 | 3.8 |
| N = 20<br>T = 100 | $m$ | 0.0045 | 0.4 | 0.0658 | 6.6 | 0.0044 | 0.4 | 0.0040 | 0.4 | 0.0548 | 5.5 | 0.0030 | 0.3 |
| | $b$ | -0.0015 | -0.3 | 0.0260 | 5.2 | 0.0007 | 0.1 | -0.0014 | -0.3 | 0.0206 | 4.1 | 0.0004 | 0.1 |
| | $a$ | | | | | | | 0.0111 | 1.1 | 0.1525 | 15.3 | 0.0234 | 2.3 |
| | $g_1$ | | | | | | | -0.0026 | -0.9 | 0.0383 | 12.8 | 0.0015 | 0.5 |
| | $d_1$ | | | | | | | -0.0048 | -1.6 | 0.0758 | 25.3 | 0.0058 | 1.9 |

TABLE 8A: Monte Carlo results for dynamic mean model ( $f = 0.8$ ) and GARCH (1,1) errors ($g_1 = 0.3, d_1 = 0.6$ )

| | | OLS | | | | | | MLE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample | Coeff. | Bias | (%) | Std. Dev. | (%) | MSE | (%) | Bias | (%) | Std. Dev. | (%) | MSE | (%) |
| N = 5 | $m$ | 0.0850 | 8.5 | 0.4440 | 44.4 | 0.2044 | 20.4 | 0.0602 | 6.0 | 0.3653 | 36.5 | 0.1371 | 13.7 |
| T = 20 | $b$ | -0.0142 | -1.8 | 0.0572 | 7.2 | 0.0035 | 0.4 | -0.0098 | -1.2 | 0.0465 | 5.8 | 0.0023 | 0.3 |
| | $a$ | | | | | | | 0.8140 | 81.4 | 1.5544 | 155.4 | 3.0787 | 308 |
| | $g_1$ | | | | | | | -0.0005 | -0.2 | 0.1409 | 47.0 | 0.0199 | 6.6 |
| | $d_1$ | | | | | | | -0.1213 | -20 | 0.2378 | 39.6 | 0.0713 | 11.9 |
| N = 5 | $m$ | 0.0680 | 6.8 | 0.3062 | 30.6 | 0.0984 | 9.8 | 0.0434 | 4.3 | 0.2386 | 23.9 | 0.0588 | 5.9 |
| T = 50 | $b$ | -0.0141 | -1.8 | 0.0445 | 5.6 | 0.0022 | 0.3 | -0.0087 | -1.1 | 0.0352 | 4.4 | 0.0013 | 0.2 |
| | $a$ | | | | | | | 0.2021 | 20.2 | 0.6969 | 69.7 | 0.5265 | 52.6 |
| | $g_1$ | | | | | | | 0.0010 | 0.3 | 0.0902 | 30.1 | 0.0081 | 2.7 |
| | $d_1$ | | | | | | | -0.0282 | -4.7 | 0.1326 | 22.1 | 0.0184 | 3.1 |
| N = 5 | $m$ | 0.0520 | 5.2 | 0.2216 | 22.2 | 0.0518 | 5.2 | 0.0312 | 3.1 | 0.1701 | 17.0 | 0.0299 | 3.0 |
| T = 100 | $b$ | -0.0093 | -1.2 | 0.0344 | 4.3 | 0.0013 | 0.2 | -0.0053 | -0.7 | 0.0252 | 3.1 | 0.0007 | 0.1 |
| | $a$ | | | | | | | 0.1176 | 11.8 | 0.4337 | 43.4 | 0.2019 | 20.2 |
| | $g_1$ | | | | | | | 0.0030 | 1.0 | 0.0650 | 21.7 | 0.0042 | 1.4 |
| | $d_1$ | | | | | | | -0.0198 | -3.3 | 0.0862 | 14.4 | 0.0078 | 1.3 |
| N = 10 | $m$ | 0.0540 | 5.4 | 0.3122 | 31.2 | 0.1004 | 10.0 | 0.0399 | 4.0 | 0.2448 | 24.5 | 0.0615 | 6.2 |
| T = 20 | $b$ | -0.0104 | -1.3 | 0.0399 | 5.0 | 0.0017 | 0.2 | -0.0067 | -0.8 | 0.0316 | 3.9 | 0.0010 | 0.1 |
| | $a$ | | | | | | | 0.2665 | 26.6 | 0.9302 | 93.0 | 0.9364 | 93.6 |
| | $g_1$ | | | | | | | 0.0022 | 0.7 | 0.1035 | 34.5 | 0.0107 | 3.6 |
| | $d_1$ | | | | | | | -0.0430 | -7.2 | 0.1650 | 27.5 | 0.0291 | 4.8 |
| N = 10 | $m$ | 0.0436 | 4.4 | 0.2368 | 23.7 | 0.0580 | 5.8 | 0.0276 | 2.8 | 0.1565 | 15.6 | 0.0253 | 2.5 |
| T = 50 | $b$ | -0.0067 | -0.8 | 0.0333 | 4.2 | 0.0012 | 0.1 | -0.0040 | -0.5 | 0.0232 | 2.9 | 0.0006 | 0.1 |
| | $a$ | | | | | | | 0.0960 | 9.6 | 0.4362 | 43.6 | 0.1995 | 20.0 |
| | $g_1$ | | | | | | | -0.0023 | -0.8 | 0.0606 | 20.2 | 0.0037 | 1.2 |
| | $d_1$ | | | | | | | -0.0133 | -2.2 | 0.0872 | 14.5 | 0.0078 | 1.3 |
| N = 10 | $m$ | 0.0292 | 2.9 | 0.1717 | 17.2 | 0.0303 | 3.0 | 0.0187 | 1.9 | 0.1211 | 12.1 | 0.0150 | 1.5 |
| T = 100 | $b$ | -0.0043 | -0.5 | 0.0266 | 3.3 | 0.0007 | 0.1 | -0.0030 | -0.4 | 0.0175 | 2.2 | 0.0003 | 0.0 |
| | $a$ | | | | | | | 0.0366 | 3.7 | 0.2705 | 27.1 | 0.0745 | 7.5 |
| | $g_1$ | | | | | | | 0.0010 | 0.3 | 0.0446 | 14.9 | 0.0020 | 0.7 |
| | $d_1$ | | | | | | | -0.0064 | -1.1 | 0.0570 | 9.5 | 0.0033 | 0.5 |
| N = 20 | $m$ | 0.0273 | 2.7 | 0.2123 | 21.2 | 0.0458 | 4.6 | 0.0182 | 1.8 | 0.1622 | 16.2 | 0.0266 | 2.7 |
| T = 20 | $b$ | -0.0039 | -0.5 | 0.0296 | 3.7 | 0.0009 | 0.1 | -0.0020 | -0.2 | 0.0216 | 2.7 | 0.0005 | 0.1 |
| | $a$ | | | | | | | 0.0559 | 5.6 | 0.5910 | 59.1 | 0.3524 | 35.2 |
| | $g_1$ | | | | | | | -0.0077 | -2.6 | 0.0668 | 22.3 | 0.0045 | 1.5 |
| | $d_1$ | | | | | | | -0.0057 | -0.9 | 0.1111 | 18.5 | 0.0124 | 2.1 |
| N = 20 | $m$ | 0.0155 | 1.5 | 0.1563 | 15.6 | 0.0247 | 2.5 | 0.0108 | 1.1 | 0.1142 | 11.4 | 0.0132 | 1.3 |
| T = 50 | $b$ | -0.0020 | -0.2 | 0.0236 | 3.0 | 0.0006 | 0.1 | -0.0013 | -0.2 | 0.0166 | 2.1 | 0.0003 | 0.0 |
| | $a$ | | | | | | | 0.0472 | 4.7 | 0.2801 | 28.0 | 0.0807 | 8.1 |
| | $g_1$ | | | | | | | -0.0003 | -0.1 | 0.0448 | 14.9 | 0.0020 | 0.7 |
| | $d_1$ | | | | | | | -0.0076 | -1.3 | 0.0594 | 9.9 | 0.0036 | 0.6 |
| N = 20 | $m$ | 0.0160 | 1.6 | 0.1265 | 12.7 | 0.0163 | 1.6 | 0.0074 | 0.7 | 0.0819 | 8.2 | 0.0068 | 0.7 |
| T = 100 | $b$ | -0.0023 | -0.3 | 0.0207 | 2.6 | 0.0004 | 0.1 | -0.0013 | -0.2 | 0.0126 | 1.6 | 0.0002 | 0.0 |
| | $a$ | | | | | | | 0.0254 | 2.5 | 0.1789 | 17.9 | 0.0326 | 3.3 |
| | $g_1$ | | | | | | | -0.0009 | -0.3 | 0.0326 | 10.9 | 0.0011 | 0.4 |
| | $d_1$ | | | | | | | -0.0038 | -0.6 | 0.0398 | 6.6 | 0.0016 | 0.3 |

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