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N Person Wars of Attrition



Introduction and Motivation

In this paper we analyze a war of attrition in which N players fight for one prize. The importance of this topic stems from the fact that there are lots of situations that can be best understood when thought of as multiple player wars of attrition. Some examples are the battles to control new technologies, like high definition TV standards, “group ware” for corporate intranets, interactive videotex, mobile phones, web browsers, or most notably, the market for computer operating systems and word processors. The war of attrition can also be used to describe labor strikes, litigation, R&D tournaments, price wars, bargaining, the supply of public goods, oil exploration, the process of agreement to macroeconomic stabilization, rent seeking activities in general, and lobbying in particular, animal conflict, real wars, and the process of agreement to form winning coalitions in politics. footnote Despite the sizable literature on the topic, there are in the literature very few models for N person wars of attrition (see Bulow and Klemperer (1999) and Kapur (1995)) and none yields predictions consistent with the data on exit times by participants.

There are several contests that can be viewed exactly as a war of attrition with N players and one prize to be won by the player who stays in the game the longest: kissing contests, pole sitting contests, standing contests and dancing marathons are just a few. Also, according to Bulow and Klemperer, Avinash Dixit every year offers a \$20 prize to the student who continues clapping the longest at the end of his Princeton University game theory course.

In each of these contests the data show two salient features. First, it is not the case that all but two players quit immediately, the remaining two entering a standard war of attrition. This disproves the prediction of Bulow and Klemperer that players sort themselves instantaneously, and those with the two highest valuations play a standard war of attrition. Second, in each game there is a time such that the games finish at some point close to that time. This disproves Kapur’s prediction that players draw their quitting times from an exponential distribution.

More importantly, there is evidence (see the references in Hopenhayn, 2001) that the pattern of drop-outs in new industries disproves both Kapur’s and Bulow and Klemperer’s predictions. We do not put too much emphasis on this evidence, since it could be argued that the N person wars of attrition are not good models for these industries.

The Model

There are $N \geq 2$ players. Each must decide, depending on the history, to stay and fight, or quit. When all players but 1 have quit, the game ends. In each period t in which a player stays, he must pay a cost of $c_t \geq 0$. If the game ends in period T , the last player to quit earns $vN^T \delta^{T-1} c_t$, where $N \in (0, 1]$ is the discount factor and $v > 0$ is the value of the prizes. footnote If a player quits in period T and is not the last to quit, he earns $\delta^{T-1} c_t$. If at a certain date, all players quit simultaneously, nobody keeps the prize. Generally, it is either assumed that the cost is constant at 1 per period, or that the prize is normalized to 1. Here we do not normalize neither the prize, nor the cost, because we will perform some comparative statics results.

The Unique Symmetric Stationary equilibrium.

We will now compute the unique symmetric stationary equilibrium (strategies do not depend on the time in which other players quit). We will write SSE for symmetric stationary equilibrium. This game has several asymmetric equilibria, and some non-stationary equilibria. Analyzing all those other equilibria is beyond the scope of this paper. We have chosen to analyze the SSE of the game, because that is the standard usage, and that will make our results comparable with those of the literature.

In what follows, we will propose a profile of strategies, and then show that it is an equilibrium. Finally, we will show that it is the unique equilibrium. The proposed profile is one in which, in a subgame starting in period t , in which there are n players, players play quit with probability

$$p_{n,t} = \left(\frac{c_t}{v} \right)^{\frac{1}{n-1}}$$

if $\frac{c_t}{v} \geq 1$, or quit immediately otherwise. When other players are playing these strategies, the player is indifferent between staying, which has a cost of c_t and gives a payoff of v with probability

$$\underbrace{\left(\frac{c_t}{v} \right)^{\frac{1}{n-1}} \dots \left(\frac{c_t}{v} \right)^{\frac{1}{n-1}}}_{n-1 \text{ times}} = \left(\left(\frac{c_t}{v} \right)^{\frac{1}{n-1}} \right)^{n-1} = \frac{c_t}{v}$$

for an expected utility of 0, or quitting, which yields a utility of 0.

We will now show that the proposed equilibrium is in fact unique among the class of SSE. Suppose we are in a subgame starting in period t , with two players, and that both are quitting in this period with probability $p_{2,t}$. Of course, $p_{2,t} = 1$ is the unique equilibrium if $c_t \geq v$, so assume $c_t < v$. Then, $p_{2,t} = 1$ can’t be an equilibrium, since staying yields a payoff of $v - c_t$

while quitting yields a payoff of 0. Similarly, $p_{2,t} = 0$ is an equilibrium if and only if $c_t = 0$.

Suppose now that $p_{2,t} \in (0, 1)$. We must have that the utility of staying, $p_{2,t}v - c_t$, equals the utility of quitting, 0 (disregarding the sunk costs). We thus have $p_{2,t} = c_t/v$ as the unique SSE. The expected payoff to both players in this game is 0.

Suppose that we have shown that in any subgame starting in period t , with less than n players, the expected payoff to all players in any SSE is 0. Let $p_{n,t}$ be the probability of quitting, for all players, in a subgame starting in period t , with n players. It is again easy to show that $p_{n,t} = 0$ and $p_{n,t} = 1$ are equilibria if and only if $c_t = 0$ ($c_t > v$ respectively). So suppose $p_{n,t} \in (0, 1)$. We must then have that the utility of quitting, 0, must equal the utility of staying. If players are indifferent between quitting and staying, it must be the case that $E_{n,t}$, the expected utility of being in a subgame starting in period t , with n players, is 0. Thus, when all players are following strategy $p_{n,t}$, we obtain

$$v p_{n,t}^{n+1} + n E_{n,t} (1 - p_{n,t})^{n+1} + 0 = n [1 - p_{n,t}^{n+1} - (1 - p_{n,t})^{n+1}] = c_t$$

or, equivalently, $p_{n,t} = \left(\frac{c_t}{v}\right)^{\frac{1}{n+1}}$.

- ¾ We have in fact shown that the proposed equilibrium is subgame perfect, since the strategies form an equilibrium starting in any subgame.
- ¾ Funny enough, discounting plays no role in the equilibrium.
- ¾ The present model can be very easily extended, in the usual way, to $N + K$ players and K prizes, and we actually do that later in the paper.
- ¾ We could extend our model to encompass the case in which, even after quitting you pay a cost, until the game is resolved
- ¾ There are, of course, other non-symmetric and non-stationary equilibria, like, player 1 stays forever, and everybody else quits. See Kornhauser et. al. (1989) on how to select among such asymmetric equilibria on a similar context.
- ¾ If players care about the number of players that quit before them (some psychological utility) then our model has an equilibrium in continuous time, which is not “everybody quits immediately”. That is, we can incorporate Kapur’s model into ours, since in his model the payoff to player i is strictly increasing in the number of players who quit before i .

Comparative Statics

The comparative statics for this model are very easy. First, as the size of the prize increases, the probability of quitting decreases:

$$\frac{dp_{n,t}}{dv} = - \frac{\left(\frac{c_t}{v}\right)^{\frac{1}{n+1}}}{(n+1)v}$$

The standard “bogus” argument for this is that staying is nicer (because the prize is nicer) so players stay with higher probability. The truth is that players must quit with lower probability, so that the other players are indifferent between staying and quitting.

Second, as the cost of staying increases, the probability of quitting increases:

$$\frac{d\left(\frac{c_t}{v}\right)^{\frac{1}{n+1}}}{dc_t} = \frac{\left(\frac{c_t}{v}\right)^{\frac{1}{n+1}}}{(n+1)c_t}$$

Again, standard “bogus” and correct intuitions hold. Finally, as the number of players in a subgame increases, the probability of quitting increases:

$$\frac{d\left(\frac{c_t}{v}\right)^{\frac{1}{n+1}}}{dn} = \frac{\left(\frac{c_t}{v}\right)^{\frac{1}{n+1}}}{(n+1)^2} \ln \frac{c_t}{v}$$

Here, the “bogus” argument could be that, when the number of players is small, you put more effort, than when there are many players, since it is more likely that you will win. The true reason is that with more players, the probability of quitting must be higher, so that the final product of all probabilities of quitting remains the same.

An interesting case that is easy to analyze in this context is what happens to the rate of quitting if costs are decreasing. This may happen if firms are investing in R&D, so that the cost of fighting decreases in each period.

Extensions

In this section we analyze four simple extensions of the previous model. In the first, we analyze the case where the per period cost also depends on the number of players in the game. In the second we incorporate a feature first introduced by Bulow and Klemperer: that after quitting, and until the game finishes, players continue paying a cost f . Finally, we study two related extensions, in which the number of prizes in dispute is $k > 1$.

Cost also depends on the number of players

An interesting extension of our model is to allow the cost to depend on the number of players. For example, if we use the model to predict how the producers of web browsers behave in their competition, the cost of staying in the market will depend on the number of players. That is, if there is a cost of producing that does not depend on the number of players, still the (negative) profits will depend on the number of players still alive.

Also, with constant costs, and letting the time periods go to 0, everybody would quit immediately. We don't have that problem because we can calibrate costs period by period. The problem is that in the calibration costs may jump up when you calibrate models for two players. We can fix that by making the costs depend on the number of players (in addition to them depending on time). In this case, players' total utility at the end of the game will depend on when other people dropped out.

If the cost in period t , when there are n players is $c_{n,t}$, quitting with probability

$$p_{n,t} = \frac{1}{c_{n,t}}$$

constitutes an equilibrium. Note that one can always choose the costs so that not everybody quits immediately, as in Bulow and Klemperer.

What kind of data would push us towards a model where costs depend on the number of players? Well, if in the 9th hour of the contest, the estimated cost (from probability of somebody quitting) when there are 5 players left is different from the estimated cost when there are 2 players left, that would contradict the constant (across number of players) costs. Why is this? Well, because, suppose, that the probability of a player quitting in the 9th hour is 10% if there are 3 players. Normalizing v to 1, this means that $c_9 = (0.1)^2$: that is, that the cost per hour, at the 9th hour, is 0.01. Now suppose that we ask ourselves what would have happened if we had chosen as unit a half hour. Assume that the chance of somebody quitting in the first half hour of the 9th hour is 5% and the probability of quitting in the second half hour is 5%. Then, we obtain $c_{19} = (0.05)^2$ and $c_{20} = (0.05)^2$. Then, $c_{19} + c_{20} = 2 \cdot 0.0025 = 0.005 \neq 0.01$, that is, the sum of the half hour costs does not add up to the one hour cost. With more players this becomes worse, and everything is fine with just two players.

After quitting, and until the end of the game, a cost f is paid.

In each period after a player has quit, until the game finishes, players must pay a cost f . We will first outline some steps of the case where the cost varies over time, and then concentrate on the case of constant costs.

Of course, the strategies for subgames in which there are two players are the same as before, since after one player quits, the game finishes. Consider a subgame with 3 players, and suppose we are in period t . If a player decides to quit, the probability that the game will end in time t is, letting $q_{3,t}$ denote each player's probability of quitting in a subgame starting at t with 3 players,

$$2q_{3,t}(1 - q_{3,t}) + q_{3,t}^2 = 2q_{3,t} - q_{3,t}^2$$

Note that the game ends if either one player quits, or both quit. Let $p_2(t+s)$ denote the probability that the game will end in $t+s$. Then, $p_2(t+s)$ is the probability that the game does not end in t , times the probability that it does not end in any of the $t+r$ periods, for $r = 1, \dots, s-1$, times the probability that it ends in $t+s$, conditional on having reached $t+s$. Therefore we get that

$$\begin{aligned} p_2(t+s) &= (1 - 2q_{3,t} + q_{3,t}^2) \left[2p_{2,t+s} - p_{2,t+s}^2 \right] \prod_{r=1}^{s-1} (1 - 2p_{2,t+r} + p_{2,t+r}^2) \\ &= (1 - 2q_{3,t} + q_{3,t}^2) \left[2 - 2p_{2,t+s} + p_{2,t+s}^2 \right] \prod_{r=1}^{s-1} (1 - 2p_{2,t+r} + p_{2,t+r}^2) \end{aligned}$$

We then obtain that the expected utility of quitting at time t is

$$\sum_{s=1}^{\infty} p_2(t+s) f$$

In the special case in which costs are constant, and letting q_3 denote the probability of quitting when there are three players, we get

$$\begin{aligned} p_2(t+s) &= (1 - 2q_3 + q_3^2) \left[2 - 2\left(\frac{c}{v}\right) + \left(\frac{c}{v}\right)^2 \right] \prod_{r=1}^{s-1} \left(1 - 2\left(\frac{c}{v}\right) + \left(\frac{c}{v}\right)^2 \right) \\ &= \left(\frac{c}{v}\right)^2 (1 - 2q_3 + q_3^2) \left[2 - 2\left(\frac{c}{v}\right) + \left(\frac{c}{v}\right)^2 \right] \left(1 - 2\left(\frac{c}{v}\right) + \left(\frac{c}{v}\right)^2 \right)^{s-1} \end{aligned}$$

and therefore the utility of quitting becomes

$$\frac{f\left(\frac{c}{v}\right) \left(1 - q_3\right)^2 \left[2 - \frac{c}{v}\right]}{\frac{c}{v} \left(2 - \frac{c}{v}\right)} \stackrel{s=K}{>} \left(1 - \frac{c}{v}\right)^2 \sum_{s=1}^{s=K} \left(\frac{c}{v}\right)^s = \frac{f\left(1 - q_3\right)^2}{\frac{c}{v} \left(2 - \frac{c}{v}\right)}$$

It is again easy to show that any SSE must be in mixed strategies, so we obtain that the probability of quitting must be fixed so that the utility of staying equals the utility of quitting. Recalling that the value in the two player war is still 0, the utility of staying becomes: v times the probability that the other two players quit, plus the value of the subgame with three players (the value of quitting, since players are mixing) times the probability that none quits, plus 0 times the probability that one quits, minus the cost c . That is,

$$v q_3^2 \frac{f\left(1 - q_3\right)^2}{\frac{c}{v} \left(2 - \frac{c}{v}\right)} \left(1 - q_3\right)^2 - c$$

Let $a = \frac{f}{\frac{c}{v} \left(2 - \frac{c}{v}\right)}$ and $q = q_3$, we want to show that $g'(q) = v q^2 - a(1 - q)^4 - c + a(1 - q)^2$ equals 0 only once, for $q \in [0, 1]$. We have that $g'(1) > 0$ and $g'(0) < 0$ and since g is continuous, so it must cross 0. To check that it only happens once, we show that $g''(q) > 0$:

$$\frac{dg'(q)}{dq} = (2v - 10a)q + 2a + 12aq^2 - 4aq^3$$

This equation has one root greater than 1, and the other two are imaginary, so $g'(0) > 0$ and $g'(1) > 0$ guarantee the desired result. That is, there is a unique SSE in the game with three players. In principle, one can follow this procedure to show, by induction, that in any subgame with n players there is a unique equilibrium.

There are $k - 1$ prizes.

We now analyze a model in which there are $N + k$ players, and $k - 1$ prizes. The game ends whenever the number of players who have not quit is less or equal than k . There are two different ways of modeling this situation, none of which is better than the other.

Two to Tango

The first possibility is to assume that the remaining players win the objects if and only if, exactly k are left. That is, if at some date t there are $n > k$, and at the next there are $m < k$, the game finishes, but the m remaining players do not receive a prize. The remaining players only receive the prizes if $m = k$. The reason for the title of the subsection is that if what is being fought for is the possibility of dancing tango ($k = 2$) and only 1 player is left, nobody receives the prize. This way of modeling the war of attrition is appropriate whenever there is a “capacity” constraint, such that the final outcome is good only if exactly k players have not quit.

To solve this version of the model one proceeds exactly as before. In a subgame starting in period t , with $n + k$ players, all players will quit with a probability $p_{n,t}$ that makes

$$v p_{n,t}^n \left(1 - p_{n,t}\right)^{k-1} = \frac{c}{v \binom{n+k-1}{n}} \quad \#$$

Then a player will win a prize if exactly n players of the “other” $n + k - 1$ quit, and this happens with probability

$$\binom{n+k-1}{n} v p_{n,t}^n \left(1 - p_{n,t}\right)^{k-1}$$

yielding a utility of staying of 0. The problem with this setup is that the left hand side of (ref: tango) is not monotonic, and therefore the equilibrium is not unique.

Prizes are k objects.

We now turn to the second modeling alternative, in which the $m \geq k$ remaining players after the game has finished get the prizes. This modeling alternative is more natural for an “all pay” auction in which the prizes awarded are, for example, bonds.

Suppose we are in period t , in a game with $k + 1$ players. Then, if a player stays, he will win one of the prizes iff at least one of the players quits. If everybody is mixing with probability q_1 , this happens with probability

$$q_1^k + k q_1^{k-1} \left(1 - q_1\right) + \dots + \binom{k}{k-1} q_1 \left(1 - q_1\right)^{k-1} = \sum_{i=0}^{i=k-1} \binom{k}{i} q_1^{k-i} \left(1 - q_1\right)^i$$

Equating this to c/v we obtain (via maple)

$$q_1 = \left(1 - \frac{c}{v}\right)^{\frac{1}{k}} + 1$$

which, of course, is equal to the main model when $k = 1$. Note that as the number of prizes increases, q_1 decreases, since everybody must be staying “more” to leave everybody indifferent. Again, the bogus intuition is that staying is more profitable, since the chance that you’ll get a prize increases, so you want to quit less. Of course, the value of a subgame with $k + 1$ players is then 0.

Suppose now that we have shown that for all subgames with at least $n + k + 1$ players the equilibrium is unique and the value of that subgame is 0. Assume now that we are in a subgame starting in period t , with $n + k$ players and everybody is quitting with probability q_n . If a player stays he wins one of the prizes iff at least n players quit, and this happens with probability

$$q_n^{n+k+1} + \binom{n+k+1}{1} q_n^{n+k} (1-q_n) + \dots + \binom{n+k+1}{k+1} q_n^n (1-q_n)^{k+1} =$$

$$\sum_{j=0}^{j=k+1} \binom{n+k+1}{j} q_n^{n+k+1-j} (1-q_n)^j = Q_n \mathbb{1}_{q_n \geq \frac{c_t}{v}}$$

We obviously have that $Q_n \mathbb{1}_{q_n \geq \frac{c_t}{v}}$, the probability of at least n players quitting when each is quitting with probability q_n , is strictly increasing in q_n :

$$\frac{d(Q_n \mathbb{1}_{q_n \geq \frac{c_t}{v}})}{dq} = \sum_{j=0}^{j=k+1} \binom{n+k+1}{j} \frac{(n+k+1-j)(1-q_n)^{j-1} q_n^{n+k+1-j}}{q(1-q)^j} q^{n+k+1-j} (1-q)^j$$

$$= (1-q)^{k+1} q^{n+1} \binom{n+k+1}{k}$$

Since $Q_n(0) = 1$ and $Q_n(1) = 0$ and $Q_n \mathbb{1}_{q_n \geq \frac{c_t}{v}}$ is strictly increasing, we obtain that there is a unique q_n such that $Q_n \mathbb{1}_{q_n \geq \frac{c_t}{v}} = \frac{c_t}{v}$ (for $0 \leq c_t \leq v$, since otherwise the game is trivial).

The comparative statics are again very simple for c_t and v , and are therefore omitted. To analyze the comparative statics on k , fix a period t , and a number of players in the subgame, say s . Suppose too that the equilibrium probability of quitting was q when there were k prizes. Suppose that we increase the number of prizes to $l > k$. Again, the continuation payoff in any subgame with less than s players is 0. If players continued to play q , the expected payoff to staying would be strictly positive, since the expected payoff before the change in the number of prizes was 0, and now the payoffs increase from 0 to v whenever the number of quits is between $s + l + 1$ and $s + k + 1$. That is,

number of quits	payoff for k prizes	payoff for l prizes
1	0	0
:	0	0
$s + l + 1$	0	v
:	0	v
$s + k + 1$	v	v
:	v	v
$s + 1$	v	v

Therefore, if the number of prizes increases, the equilibrium probability of quitting must decrease to leave players indifferent. That is, $\frac{dq_n}{dk} < 0$. The bogus intuition is that since there are more prizes, staying is better.

Doing an analogous reasoning, increasing the number of players is harmful: you need more people to quit in order to gain a prize. Therefore, the equilibrium probabilities of quitting must be increasing in the number of players.

Our predictions and those of the literature vs the data

As was argued in the introduction, the data presents two features. First, not everybody quits immediately, as is predicted by Bulow and Klemperer. Our model is better in that respect, because we can calibrate each period’s cost to make the proportion of quitting players in each period the outcome of the equilibrium strategy.

Second, Kapur (1995) predicts that in each subgame with n players, each player chooses his quitting time according to an exponential distribution. This implies that in the data we should observe, with enough observations, that for any time T , there is a proportion of the players still fighting in time T . Of course this is impossible for the contests we analyze. However, a test that does not depend on having “infinitely many” data points is easy to construct. To do so, one just needs to test whether the distribution of quitting times of the players is independent of the time period in which the subgames start, conditional on those distributions being exponential with the same parameter. Another drawback of Kapur’s model (which is not reflected in the data) is that his assumption that the prize obtained by player j depends on the number of players that quit before j does not match the game at hand.

We now show that our model predicts that as the length of each time period goes to 0, we recover Bulow and

Klemperer's result. Although their model is not a special case of our's, we can match their predictions as a special case of our model. The type of analysis carried out, and the driving force, is similar to the calculations at the end of subsection ref: cnt.

Suppose that we have fixed our time periods in 1 hour each, and assume that when there are three players, the chance that one will quit in the first hour of play is 10%. That is, when we observe the data we see that, on average, at the end of the first hour 90% of the players are still there. As before, for a cost of $v = 1$, this means that the cost of the first hour must be 0.01. Now divide the time period in pieces of size A , and let $A \rightarrow 0$. We obtain that, for example, if $A = \frac{1}{100}$

$$p_A = (0.01 D A)^{\frac{1}{A}} = 0.1 A^{\frac{1}{A}} = 0.1 D \left(\frac{1}{100} \right)^{\frac{1}{A}} = 0.01$$

That is, if we keep the costs so that the total cost in the first hour is constant at 0.01, and we divide the hour in 100 periods, we obtain that the cost of each hundredth of the hour must have a cost of $\frac{0.01}{100}$. Then, the probability that a player will quit in each hundredth of the first hour is 0.01. Therefore, the probability that player i quits before the hour, conditional on players $j \neq i$ not quitting is

$$\begin{aligned} \text{prob}(\text{quit in first period}) + \dots + \text{prob}(\text{quit in hundredth period}) = \\ \frac{1}{100} + \frac{1}{100} D \frac{99}{100} + \dots + \frac{1}{100} D \left(\frac{99}{100} \right)^{99} = \frac{1}{100} \sum_{t=0}^{99} \left(\frac{99}{100} \right)^t = 0.63397 \end{aligned}$$

As was remarked before, this implies that as the time periods get shorter, we get the same result of Bulow and Klemperer. This is good and bad. Good, because their model's predictions appear as a special case of our's. It is also bad, because it sort of defeats the whole purpose of our exercise. We believe that as long as time periods are not too short, so as to give estimates of the cost that are, in some sense which we don't know how to describe, "too" low, our model is fine.

As we said before, it is likely that this feature of the model will require the estimated costs to "jump up" when we go from, for example, three players to two players. It would probably be interesting to write down a model in which firms' investments to stay in the market (the cost of fighting) go up when there are fewer players. For example, McAfee (2001) has a model in which firms exert more effort in fighting when they are closer to winning the contest. A jump in the costs for the hardbody setup would be harder to explain. Of course, to get that jump, you need to "estimate" your model with very short periods.

One can get a model in which the cost of fighting is decreasing in the number of players in the following way. This is inspired in the market for word processors, or operating systems. There is a recent book with the story of Microsoft, and Bill Gates said that at first they had to work really hard to keep their clients, developing software that would work on DOS, so that the clients would not switch to other operating systems. Also, they sold their programs and operating systems very cheap, in order to keep their clientele, and enlarge it. The idea in the story I am telling is that the larger your market share, the harder you have to work to keep it, and more market share does not mean more profits (until you are the only one left). Start out with n many firms. Each has a $1/n$ share of the market. The cost they have to pay to "serve" their clients is $c(1/n)$, where $c(\cdot)$ is increasing. Then, just complete the model as before.

A brief discussion of the literature

Our equilibrium does not converge to a continuous time equilibrium. The reason for this is that in our game, players stay in each period only because they hope that everybody will quit in the current period. Of course, as we go to continuous time, it is impossible that all other players will quit at the same time. By making the assumption that payoffs are increasing in the number of players that quit before "you", Kapur ensures that there is an additional incentive to staying around.

Similarly, Bulow and Klemperer (1999), who have asymmetric information, N -players and continuous time, manage to get existence of an equilibrium by postulating that the players have to pay a cost after quitting, until the game ends. So, instead of increasing the payoff of staying (as Kapur) they decrease the payoff to quitting. footnote

Krishna and Morgan (1997) have an N -player war of attrition in continuous time and asymmetric information. However, players choose, before starting, a quitting date. This is weird given that players can not condition their actions on other's behavior.

Maynard Smith (1974) has two players, complete info in continuous time. Bishop et al (1978) has two players asymmetric info, continuous time. Kornhauser et al (1989) have a discrete time version of the war of attrition for two players with asymmetric information, in which players move sequentially.

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