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**Campaign Contributions with Swing Voters**



## Campaign Contributions with Swing Voters

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**Abstract:** We analyze contributor behavior when there are two types of voters: *positioned* voters, who care about both the political positions and leadership abilities of candidates; and *swing* voters, who care only about the leadership abilities of candidates. Campaign expenditures are assumed to alter voters' perceptions of a candidate's leadership ability. When the number of swing voters is small, there exists a pure-strategy Nash equilibrium in the campaign contribution game; however, a pure-strategy equilibrium may not exist when the number of swing voters is large. We find that the contributions for both candidates decrease when there are more swing voters.

"Elections are won by doing two things: mobilizing your base and winning the independent swing voters."<sup>1</sup>

Karl Rove, campaign strategist for George W. Bush

## **1. Introduction**

There is general agreement among policy makers, political analysts, and candidates that campaign spending has a significant effect on election outcomes.<sup>2</sup> For example, former presidential candidate Elizabeth Dole cited the financial advantage of George W. Bush as the reason for her decision to drop out of the 2000 Republican presidential primary. At the same time, the behavior of *independent* or *swing* voters (i.e., those who are not loyal to a specific candidate or party) can often be the deciding factor in close elections. Indeed, much campaign advertising is geared toward swaying swing voters late in the campaign (see Berke, 2000). The purpose of this paper is to investigate how swing voters affect campaign contributions.

Our analysis postulates the existence of two types of voters, *positioned* and *swing*. Positioned voters judge candidates based on the candidates' political positions as well as their perceived leadership abilities.<sup>3</sup> In contrast, swing voters hold no political positions;<sup>4</sup> rather, they vote for the candidate who they perceive to have the greatest leadership ability. Here we use the term 'ability' to refer to a composite index of candidate characteristics, which are universally desired by voters (for example, statesmanship, intelligence, and relevant experience). While a candidate's political position is known and fixed, his perceived ability level is a function of

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<sup>1</sup> Quoted in Berke (2000).

<sup>2</sup> Several papers record the correlation between candidate spending and election outcomes; see Erikson and Palfrey (1998), Gerber (1998), Palda and Palda (1998), Nagler and Leighley (1992), Banaian and Luksetich (1991), Johnston (1978). For a survey see Morton and Cameron (1992).

<sup>3</sup> Enelow and Hinich (1982) first introduced "personal characteristics" or "valence issues" as distinct arguments in the utility functions of the voters. Along these lines, Harrington and Hess (1996) assume that each candidate has a valence index.

<sup>4</sup> Feddersen and Pesendorfer (1996) use the term "swing voter" to refer to the "pivotal voter" in an election.

campaign expenditures.<sup>5</sup> Thus, other things equal, political expenditures increase a candidate's votes by increasing the candidate's perceived level of ability among voters.<sup>6</sup>

Candidates rely on a representative contributor to provide financial support for their campaign expenditures. Contributors have an alternate use of funds, so contributions have an *opportunity cost* and are chosen strategically. One may interpret the representative contributors as being political parties who use their resources (e.g., the labor and human capital of volunteers) to raise funds for multiple candidates in various political races. Alternatively, the contributions could come from either a special interest group or a political action committee.<sup>7</sup> These contributors may be concerned with policy issues that only one of the candidates is willing to support.<sup>8</sup> The benefits of such policies are concentrated and accrue primarily to the group members, while the costs are widely distributed and insignificant to individual voters.

We find that the number of swing voters is a critical factor when it comes to predicting the behavior of contributors. For example, as long as the number of swing voters is small, a pure-strategy equilibrium exists. However, a pure-strategy equilibrium may not exist if the number of swing voters is large. One might expect contributors to behave more aggressively if the electorate consists of more swing voters. Contrary to this intuition, we find that campaign contributions decrease when there are relatively more swing voters. Finally, we prove that a mixed-strategy equilibrium generally exists when contributors have quasilinear utility functions.

There is, of course, a large literature on voting and voting behavior. Models related to the one we develop are Baron (1994), Grossman and Helpman (1996) and Potters, Sloof and van Winden (1997). These papers introduce two groups of voters, informed and uninformed, and

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<sup>5</sup> It has been suggested that campaign expenditures may convey information about a candidate by signaling support from well-informed contributors. Helsley and O'Sullivan (1994), Lohmann (1993), and Potters et al. (1997) provide theoretical models along these lines. Morton and Myerson (1998) work with "impressionable" voters.

<sup>6</sup> The assumption that expenditures translate into a higher probability of success is not without precedent. Baron (1994), Magee et al. (1989), and Grossman and Helpman (1996) each make similar assumptions.

<sup>7</sup> Makinson and Goldstein (1992) give a detailed description of groups financing Members of Congress.

<sup>8</sup> See Baron (1994; p. 33) for a taxonomy of policy dimensions.

study the electoral equilibria in the presence of campaign contributions and lobbies. Helsley and O’Sullivan (1994) also consider swing voters, but their use of the term is different from ours. In their case, the swing voter is indifferent and votes for a project if he perceives that the benefits exceed the cost. Harrington and Hess (1996) analyze positive and negative campaigning when candidates have different valence indices, which are formally identical to perceived ability levels in our model. They find that the candidate who has a lower valence index does more negative campaigning.

The paper proceeds as follows. In the next section we present the model. Sections 3 and 4 discuss the existence of pure- and mixed-strategy Nash equilibria, respectively. Section 5 concludes.

## **2. The Model**

There are two candidates who hold fixed political positions along a political spectrum of unit length. Candidate 1 is distance  $\mathbf{a}$  from one end of the spectrum, and candidate 2 is distance  $\mathbf{b}$  from the other end, where  $\mathbf{a} + \mathbf{b} \leq 1$ . We assume that candidate 1 is to the “left” of candidate 2 so that candidates 1 and 2 occupy positions  $x_1 = \mathbf{a}$  and  $x_2 = 1 - \mathbf{b}$ , respectively. The candidates will run against one another in an election.

There are  $P$  ‘positioned’ voters who have political opinions that can be characterized as ideal points along an ideological spectrum. The positioned voters are uniformly distributed along the spectrum whose length is normalized to unity. In addition to the positioned voters, there are  $S$  ‘swing’ voters who have no position on the political spectrum. (Our results would be essentially unchanged if we were to assume that swing voters place very little weight on the political positions of candidates.)

All voters care about the general ability levels of the candidates. We assume that the perceived ability of candidate  $i$  is a function of his expenditure. Let  $c_1$  and  $c_2$  represent the expenditures of candidates 1 and 2, respectively. Then, all voters perceive the general ability levels of candidates 1 and 2 to be  $\mathbf{q} c_1$  and  $\mathbf{q} c_2$ . A larger  $\mathbf{q}$  implies that campaign expenditures have a greater effect on voter perceptions.

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The swing voters vote for candidate 1 if  $q c_1 > q c_2$ , which requires  $c_1 > c_2$ . On the other hand, if  $c_1 < c_2$  the swing voters vote for candidate 2. Finally, if  $c_1 = c_2$  each candidate receives  $S/2$  votes from the swing voters.

A positioned voter 'located' at point  $x$  incurs an “ideological cost” of  $(\alpha-x)^2$  if he votes for candidate 1, and a corresponding cost of  $(1-\beta-x)^2$  if he votes for candidate 2, although perceived ability can compensate for differences between a candidate's chosen platform and the ideal points of voters. More precisely, a positioned voter 'located' at point  $x$  receives utility  $u(1, \mathbf{a}, x) = q_1 - (\mathbf{a}-x)^2$  if candidate 1 is elected, and  $u(2, \mathbf{b}, x) = q_2 - (\mathbf{b}-x)^2$  if candidate 2 is elected.<sup>9</sup> Thus, a person at position  $x$  votes for candidate 1 if

$$q c_1 - (\alpha-x)^2 > q c_2 - (1-\beta-x)^2 \quad (1)$$

Let  $\bar{x}(c_1, c_2)$  be the location of the indifferent positioned voter. Setting an equality in (1) and solving for  $x$  yields:

$$\bar{x}(c_1, c_2) = \frac{(1-\mathbf{b}+\mathbf{a})}{2} + \frac{q(c_1 - c_2)}{2(1-\mathbf{b}-\mathbf{a})} \quad (2)$$

To summarize, in the case that candidate 1 outspends candidate 2 (that is,  $c_1 > c_2$ ) his share of the total vote is  $v_1 = \bar{x} P + S$ , while candidate 2's share is  $v_2 = (1-\bar{x})P$ .

As described, there is no uncertainty in the model, which implies that one should be able to (perfectly) predict who will win the election.<sup>10</sup> However, it would not be difficult to 'inject' uncertainty into the model by assuming that voter turnout for each candidate is a random variable. In such a scenario, the candidate with the largest share of the vote would not necessarily win the election since he might experience a low turnout. As long as the probability of winning is an increasing function of the candidate's vote share, which would typically be true, our results continue to hold.

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<sup>9</sup> Enelow and Hinich (1982), Banks (1990), and Grossman and Helpman (1999) also use a quadratic loss function.

<sup>10</sup> Previous authors have also used deterministic models, including Harrington and Hess (1996) and Grofman (1985).

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There is one representative contributor for each political candidate. Contributor  $i$  has preferences over  $v_i$  and some alternative use of their money,  $z_i$ . These preferences for contributor  $i$  are represented by the utility function  $u_i(v_i, z_i) = f_i(v_i) + g_i(z_i)$  (for  $i=1,2$ ), where both  $f_i$  and  $g_i$  are increasing, strictly concave functions with  $\lim_{t \rightarrow 0} f_i'(t) = \infty$  and  $\lim_{t \rightarrow 0} g_i'(t) = \infty$ .

Contributor  $i$  has wealth  $w_i$ , which must be allocated across contributions and the alternate use of funds,  $z_i$ . Throughout we assume  $w_1 \neq w_2$  which, in general, implies that the game is not symmetric even if the utility functions of contributors are identical. Since candidates must spend all contributions, the contribution of contributor  $i$  is also denoted by  $c_i$  (as is the expenditure of candidate  $i$ ). Normalizing the price of  $z$  to unity, the budget constraint for the contributor is  $w_i = z_i + c_i$ .

Using the above expression for the number of votes, and substituting also the resource constraint of the contributor, we can write the payoffs received by contributor  $i$  as follows:

$$U_1(c_1, c_2) = \begin{cases} f_1(P \cdot \bar{x}(c_1, c_2) + S) + g_1(w_1 - c_1) & \text{if } c_1 > c_2 \\ f_1(P \cdot \bar{x}(c_1, c_2) + S/2) + g_1(w_1 - c_1) & \text{if } c_1 = c_2 \\ f_1(P \cdot \bar{x}(c_1, c_2)) + g_1(w_1 - c_1) & \text{if } c_1 < c_2 \end{cases}$$

and

$$U_2(c_1, c_2) = \begin{cases} f_2(P(1 - \bar{x}(c_1, c_2)) + S) + g_2(w_2 - c_2) & \text{if } c_2 > c_1 \\ f_2(P(1 - \bar{x}(c_1, c_2)) + S/2) + g_2(w_2 - c_2) & \text{if } c_2 = c_1 \\ f_2(P(1 - \bar{x}(c_1, c_2))) + g_2(w_2 - c_2) & \text{if } c_2 < c_1 \end{cases}$$

The payoff functions given above are neither continuous nor globally concave as they jump abruptly when  $c_1 = c_2$ . As a result, a Nash equilibrium, whether it be in pure or mixed strategies, may not exist. The next two sections discuss the existence of pure- and mixed-strategy equilibria.

### 3. Pure-Strategy Equilibria

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A pair of contribution levels  $c_1^*$  and  $c_2^*$  constitute a pure-strategy Nash equilibrium if neither contributor would wish to change his contribution, given the contribution of the other. The next result shows that a pure-strategy equilibrium must be asymmetric.

LEMMA 1: *If  $(c_1^*, c_2^*)$  is a pure-strategy equilibrium of the contribution game, then we must have  $c_1^* > c_2^*$ .*

We do not prove Lemma 1 as the intuition is straightforward: If both contributors donate the same amount of money, it will pay for one of them, say  $i$ , to increase his donation by some arbitrarily small  $\epsilon > 0$ . In this way, candidate  $i$  receives  $S/2$  more votes, which increases the contributor  $i$ 's utility by finite amount. However, the cost of this increase is arbitrarily small. In light of Lemma 1, and without loss of generality, we focus on pure-strategy equilibria in which contributor 1 contributes more than contributor 2, which implies that candidate 1 receives the swing vote (the case when  $c_2 > c_1$  is completely symmetric).

Although the payoff functions are not globally concave, it still must be the case that in any pure-strategy equilibrium neither contributor wishes to increase or decrease his contribution by a small amount. Using the payoffs given above, the following first-order conditions for contributors 1 and 2, respectively, are necessary conditions:

$$\left( \frac{Pq}{2(1-\mathbf{a}-\mathbf{b})} \right) \cdot f_1'(P \cdot \bar{x}(c_1, c_2) + S) - g_1'(w_1 - c_1) = 0 \quad (3)$$

$$\left( \frac{Pq}{2(1-\mathbf{a}-\mathbf{b})} \right) \cdot f_2'(P \cdot (1 - \bar{x}(c_1, c_2))) - g_1'(w_1 - c_1) = 0$$

(4)

A pure-strategy Nash equilibrium  $(c_1^*, c_2^*)$ , if one exists, must simultaneously satisfy equations (3) and (4).

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PROPOSITION 1: *A pure-strategy equilibrium exists if the group of swing voters is sufficiently small.*

According to the above proposition, the swing voters do not necessarily cause the failure of a pure-strategy equilibrium. However, the existence of a pure-strategy equilibrium is assured only when the number of such voters is sufficiently small. To see why a large number of swing voters can cause a pure-strategy equilibrium to not exist suppose that the whole voting population is composed of swing voters ( $P=0$ ). Thus, the candidate who spends the most (receives the largest contribution) gets the entire vote. In this case, it can never be an equilibrium for both contributors to donate positive unequal amounts, since the one who donates the least could do just as well by contributing zero. However, if contributor  $i$  donates zero, then contributor  $j$  should contribute as little as possible (formally,  $j$ 's optimization problem has no solution), which in turn makes it optimal for  $i$  to donate some small positive amount – a contradiction. Suppose now that the number of swing voters is small. Proposition 1 says it will not be optimal for contributor 2 to “leapfrog” contributor 1, which would break the pure-strategy equilibrium.

One may think that a decrease in the swing voter group should cause donors to decrease their contribution levels as the benefit of being the highest contributor is now less. However, as shown by the next proposition, this is not the case.

PROPOSITION 2: *A decrease in the number of swing voters implies an increase in the pure-strategy equilibrium contributions for both candidates.*

We know from Lemma 1 that in any pure-strategy equilibrium we must have  $c_1^* \geq c_2^*$ . It follows that, at equilibrium, a small change in either contribution affects only the number of positioned voters. However, swing voters are perfect substitutes for positioned voters – only the total number of votes matters to contributors. Therefore, when the number of swing voters decreases by a small amount, the indifferent voter at the old equilibrium becomes more valuable

to contributor 1, since his candidate now receives fewer swing votes. Contributor 1 thus finds it optimal to increase his contribution.

Contributor 2, on the other hand, does not alter his contribution level in direct response to the decrease in the number of swing voters. But, the indirect result of an increase in  $c_1$  is to decrease the number of positioned votes received by candidate 2. This, in turn, induces contributor 2 to respond by increasing his contribution as well.

#### **4. Mixed-Strategy Equilibria**

As stated in the introduction, it is not clear that mixed-strategy equilibria necessarily exist in the present model. Results in Dasgupta and Maskin (1986) can often be useful in showing existence in discontinuous games of this sort, however, those theorems require the sum of the payoffs to be upper semi-continuous in contributions. This need not be the case in the game considered here with the utility functions assumed in section 2. We can, nonetheless, show existence for the following special class of utility functions.

*PROPOSITION 3: If the utility functions of the contributors are quasi-linear in votes so that  $u_i(v_i, z_i) = v_i + g(z_i)$ , then a mixed-strategy equilibrium exists.*

#### **5. Conclusion**

In the above analysis, we found that the number of swing voters is critical when it comes to being able to predict the behavior of contributors who fund campaigns. In fact, the contribution game we provide may not possess a pure-strategy equilibrium when the number of swing voters is large. However, a pure-strategy equilibrium necessarily exists as long as the number of swing voters is small. (If contributors have quasilinear utility functions, then a mixed-strategy equilibrium necessarily exists, regardless of the number of swing voters.) Interestingly, we found that, other things equal, campaign contributions in the pure-strategy equilibria will be larger for both candidates when there are relatively few swing voters. This result counters the intuition that swing voters cause contributors to behave more aggressively.

**APPENDIX**

Proof of Proposition 1:

Let  $c_1^*$  and  $c_2^*$  be defined as the simultaneous solution to equations (3) and (4). We focus on  $c_1^* > c_2^*$ , since the proof for  $c_1^* < c_2^*$  is similar and, by Lemma 1, we cannot have a pure-strategy equilibrium in which  $c_1^* = c_2^*$ .

It is useful to define the following functions, which give the payoffs to contributors if candidate 1 receives the entire swing vote for all  $c_1$  and  $c_2$ .

$$\begin{aligned} \mathbf{p}_1(c_1, c_2) &\equiv f_1(P \cdot \bar{x}(c_1, c_2) + S) + g_1(w_1 - c_1) \\ \mathbf{p}_2(c_1, c_2) &\equiv f_2(P(1 - \bar{x}(c_1, c_2))) + g_2(w_2 - c_2) \end{aligned}$$

Note that for all  $c_1$  and  $c_2$  such that  $c_1 > c_2$ , we have  $\pi_1(c_1, c_2) = U_1(c_1, c_2)$  and  $\pi_2(c_1, c_2) = U_2(c_1, c_2)$ . So, the first-order conditions associated with maximizing  $\pi_1$  and  $\pi_2$  are exactly those given by equations (3) and (4). The following calculations indicate that  $\pi_1(c_1, c_2)$  is strictly concave in  $c_1$  and  $\pi_2(c_1, c_2)$  is strictly concave in  $c_2$ .

$$\frac{\partial^2 \mathbf{p}_1}{\partial c_1^2} = \left[ \frac{Pq}{2(1 - \mathbf{b} - \mathbf{a})} \right]^2 f_1''(\cdot) + g_1''(\cdot) < 0 \quad (\text{A1})$$

$$\text{and } \frac{\partial^2 \mathbf{p}_2}{\partial c_2^2} = \left[ \frac{Pq}{2(1 - \mathbf{b} - \mathbf{a})} \right]^2 f_2''(\cdot) + g_2''(\cdot) < 0 \quad (\text{A2})$$

Given that  $c_1^*$  and  $c_2^*$  simultaneously satisfy (3) and (4) it follows that

$$c_1^* = \underset{c_1}{\text{Arg max}} \pi_1(c_1, c_2^*) \quad \text{and} \quad c_2^* = \underset{c_2}{\text{Arg max}} \pi_2(c_1^*, c_2).$$

We now show that  $c_1^*$  is a best-response to  $c_2^*$ . Given that  $\pi_1(c_1, c_2^*) = U_1(c_1, c_2^*)$  for all  $c_1 > c_2^*$ , it is sufficient to show that it is not optimal for contributor 1 to choose  $c_1 \leq c_2^*$ . However,  $\pi_1(c_1, c_2^*) > U_1(c_1, c_2^*)$  for all  $c_1 \leq c_2^*$  since  $U_1(c_1, c_2^*)$  'jumps down' if candidate 1 does not receive the swing vote. Using the fact that  $c_1^* > c_2^*$  and  $c_1^* = \underset{c_1}{\text{Arg max}} \pi_1(c_1, c_2^*)$ , we conclude that  $c_1^* = \underset{c_1}{\text{Arg max}} U_1(c_1, c_2^*)$ , which implies that  $c_1^*$  is a best-response to  $c_2^*$ .

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What remains to be shown is that  $c_2^*$  is a best-response to  $c_1^*$ , for sufficiently small  $S$ . Given that  $c_2^* = \underset{c_2}{\text{Arg max}} \pi_2(c_1^*, c_2)$  and  $\pi_2(c_1^*, c_2) = U_1(c_1^*, c_2)$  for all  $c_2 < c_1^*$ , it is sufficient to verify that for sufficiently small  $S$  contributor 2 will not find it optimal to choose a  $c_2 \geq c_1^*$ . Specifically,  $c_2^*$  is the best response to  $c_1^*$  if

$$\begin{aligned} f_2(P(1 - \bar{x}(c_1^*, c_2^*))) + g_2(w_2 - c_2^*) \\ - f_2(P(1 - \bar{x}(c_1^*, c_2)) + S) - g_2(w_2 - c_2) > 0 \quad \text{for all } c_2 \geq c_1^*. \end{aligned} \quad (\text{A3})$$

Now we can view the left-hand side of (A3) as a function of  $S$ . So define

$$\begin{aligned} \mathbf{f}(S) \equiv f_2(P(1 - \bar{x}(c_1^*, c_2^*))) + g_2(w_2 - c_2^*) \\ - f_2(P(1 - \bar{x}(c_1^*, c_2)) + S) - g_2(w_2 - c_2). \end{aligned}$$

Clearly,  $\mathbf{f}(S)$  is a decreasing, continuous function of  $S$ . Further, at  $S = 0$ , we have

$$\mathbf{f}(0) = \pi_2(c_1^*, c_2^*) - \pi_2(c_1^*, c_2) > 0 \quad \text{for all } c_2 \geq c_1^*, \quad (\text{A4})$$

where the sign follows from the fact that  $c_2^* = \underset{c_2}{\text{Arg max}} \pi_2(c_1^*, c_2)$ . By the continuity of  $\mathbf{f}(S)$

it follows that for sufficiently small  $S > 0$  we have  $\mathbf{f}(S) > 0$ . In words,  $c_2^*$  is a best-response to  $c_1^*$  for sufficiently small  $S > 0$ .

We have shown that  $c_1^*$  is a best-response to  $c_2^*$ , and that  $c_2^*$  is a best-response to  $c_1^*$  for sufficiently small  $S > 0$ . It thus follows that  $(c_1^*, c_2^*)$  constitute a pure-strategy Nash equilibrium for sufficiently small  $S > 0$ . QED.

### Proof of Proposition 2

The following proof sacrifices some rigor for intuition. We start by showing that in the neighborhood of any pure-strategy equilibrium, contributions are strategic complements, which implies that the best-response functions are upward sloping. To see that this is true, differentiate equation (3), which defines the best-response function of contributor 1 in the neighborhood of the equilibrium, with respect to  $c_1$  and  $c_2$ . Then, simplify the resulting expressions to get

$$1 > \frac{\partial c_1(c_2)}{\partial c_2} = \frac{\left( \frac{Pq}{2(1-a-b)} \right)^2 f_1'''(\cdot)}{\left( \frac{Pq}{2(1-a-b)} \right)^2 f_1''(\cdot) + g_1''(\cdot)} > 0 \quad (\text{A5})$$

Similarly, differentiate equation (4), which defines the best-response function of contributor 2 in the neighborhood of the equilibrium, with respect to  $c_1$  and  $c_2$ . Then, simplify the resulting expressions to get

$$1 > \frac{\partial c_2(c_1)}{\partial c_1} = \frac{\left(\frac{Pq}{2(1-a-b)}\right)^2 f_2'''(\cdot)}{\left(\frac{Pq}{2(1-a-b)}\right)^2 f_2''(\cdot) + g_2''(\cdot)} > 0 \quad (\text{A6})$$

Now consider the effects of increasing  $S$  on the best-response functions. Since equation (4) does not involve  $S$ , the best response of contributor 2 in the neighborhood of the equilibrium is not affected by small changes in  $S$ . However, an increase in  $S$  shifts contributor 1's best response inward as can be seen by differentiating equation (3) with respect to  $c_1$  and  $S$ .

$$\frac{\partial c_1(c_2)}{\partial S} = \frac{-\left(\frac{Pq}{2(1-a-b)}\right) f_1'''(\cdot)}{\left(\frac{Pq}{2(1-a-b)}\right)^2 f_1''(\cdot) + g_1''(\cdot)} < 0 \quad (\text{A7})$$

An increase in  $S$  decreases the best-response for contributor 1, which causes a corresponding equilibrium decrease in both  $c_1^*$  and  $c_2^*$ .

Q.E.D.

Proof of Proposition 3.

We show this proposition by verifying that the conditions of theorem 5 in Dasgupta and Maskin (1986) are met.

First, the strategies of the contributors are the contributions  $c_i$  ( $i=1,2$ ) they make to the candidate of their choice. Each contributor will not donate more than his endowment  $w_i$ . Therefore the individual action sets are  $A_1=[0,w_1]$  and  $A_2=[0,w_2]$ , which are closed and subsets of  $\mathbf{R}^1$ .

The payoff functions  $U_i:[0,w_1] \times [0,w_2] \rightarrow \mathbf{R}^1$  are bounded for finite endowments, and are continuous, except on the subset  $\{(c_1,c_2) \in [0,w_1] \times [0,w_2] : c_1=c_2\}$ , which is of lower dimension than  $[0,w_1] \times [0,w_2]$ .

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In the case of quasilinear utility functions, the sum of the payoff functions is simply  $(U_1+U_2)(c_1,c_2) = P + S + g_1(w_1-c_1) + g_2(w_2-c_2)$ , which is continuous for all  $c_1$  and  $c_2$  and, therefore, upper semi-continuous.

Finally, we have to check whether the individual payoff functions are weakly lower semi-continuous (whose definition is given in Definition 6, p. 13 in Dasgupta and Maskin). Adapting their notation to our case at hand, we say that  $U_1(c_1,c_2)$  is weakly l.s.c. in  $c_1$  if for every  $\bar{c} \in [0, w_1]$  there is a  $\lambda \in (0,1)$  such that

$$\mathbf{I} \lim_{c_1 \xrightarrow{-} \bar{c}} \inf U_1(c_1, \bar{c}) + (1 - \mathbf{I}) \lim_{c_1 \xrightarrow{+} \bar{c}} \inf U_1(c_1, \bar{c}) \geq U_1(\bar{c}, \bar{c})$$

We calculate:

$$\lim_{c_1 \xrightarrow{-} \bar{c}} \inf U_1(c_1, \bar{c}) = P \cdot \bar{x} + g_1(w_1 - \bar{c})$$

$$\lim_{c_1 \xrightarrow{+} \bar{c}} \inf U_1(c_1, \bar{c}) = P \cdot \bar{x} + S + g_1(w_1 - \bar{c})$$

$$U_1(\bar{c}, \bar{c}) = P \cdot \bar{x} + S/2 + g_1(w_1 - \bar{c})$$

It is easy to check that the inequality required by the definition of weak l.s.c. holds for any  $\mathbf{I} \leq \frac{1}{2}$ . The same type of argument applies to  $U_2(c_1,c_2)$ . Therefore, by Theorem 5 in Dasgupta and Maskin (1986), the game with the quasilinear utility functions has a mixed-strategy Nash equilibrium.

Q.E.D.

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