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in Brazil**



Monetary Rules and Macroeconomic Dynamics in Brazil*

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Abstract

In this article, we estimate and simulate an open rational expectations macro model for the Brazilian economy. Our goal is to identify the features of optimal monetary rules and their consequences for the model's short-term dynamics. We compare the performance of three parametrizations of the monetary rule that differ with respect to the inflation variable: a Taylor rule, which is based on past inflation; a rule that combines past inflation and real exchange rate (Ball[5]); and a rule based on inflation forecasts (Bank of England [3]). We solve the model numerically and we use stochastic simulations with iid and correlated shocks to construct efficient frontiers on the inflation variance and output variance space. The sets of optimal rules for the two versions are qualitatively distinct. Since there is uncertainty about the economy's forward-lookingness, we propose a ranking of rules based on an equal weighted average of each model's objective function. The best ranked rules according to this criterion have performance moderately inferior to the optimal rules, but prevent much larger losses which would occur when rules are chosen according to the wrong model.

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1 Introduction

In recent years a New Keynesian framework has replaced traditional IS-LM-AS in the short-run macroeconomic analysis (Clarida, Gali and Gertler [12]). The main modifications are the inclusion of a forward-looking term in the IS equation and the substitution of the LM curve for an interest rate rule. Furthermore, as the equations can be derived from microfoundations, the parameters have structural interpretations, related to technology, preferences and market structure. The resulting small macroeconomic models are simple, but offer a stylized representation for the macroeconomic environment. The small number of equations and variables make its estimation and calibration easier, as well as the investigation of its properties. Those features turn those small macro models into useful tools for policy analysis. Our aim in this paper is to specify, estimate and simulate a small open rational expectations model for the Brazilian economy.

We first specify and estimate an open forward-looking model based on Batini and Haldane [4], as well as its backward-looking restriction. The structural model is composed of five equations, which combined are reduced to three: a forward-looking IS equation; an exchange rate equation based on the uncovered parity; and a New Keynesian Phillips curve, with both forward and backward-looking components, and which embeds some inflationary inertia.

The backward-looking version restricts both the coefficients of future output gap in the IS and expected inflation in the Phillips curve to be zero. As a consequence there are important lags in the effect of monetary policy. The only contemporaneous impact on inflation is through the exchange rate. In the forward-looking version, monetary policy may affect inflation contemporaneously even in a closed economy, since the expectation of a future reduction in the output gap has a negative impact on inflation now.

The model is closed by a monetary rule which specifies how the interest rate reacts to certain macroeconomic variables. We tested three types of rules and an enormous set of parameter values. In the traditional Taylor rules, the interest rate reacts to inflation and output gap. Ball [5] proposes a rule for an open economy where inflation is replaced by a combination between inflation and real exchange rate. The intuition is that when an exchange rate is overvalued there will be exchange devaluations in the future, causing future inflation. The third type of rule uses inflation expectation instead of past inflation. Batini and Haldane [4] argue that this type of rule has superior

performance when the economy is backward-looking.

The system was estimated by 3SLS. We also estimated the model with the parameter restrictions imposed to turn it backward-looking. We chose two calibrations with base on the forward-looking and backward-looking estimations, respectively. In solving the model we use the method proposed by Klein[18], which is based on generalized eigenvalues in Schur form and improves upon Blanchard & Kahn [9].

To compare the long run performance of the many rules of different parametrization, we have run stochastic simulations of the backward-looking and of the forward-looking models. Efficient frontier have been built for the two models, using the output variance and the inflation variance generated by the 4719 tested monetary rules. Since macroeconomists disagree about the real economy forward-lookingness, we have examined the rules performance between models in accordance with sum of the loss function of two versions. This criterion attributes equal probabilities to the backward and to the forward environment. We have compared the performance of the rules that minimize this criterion with the model's specific optimal rules, assuming that the true model is either forward or backward-looking. We conclude that the robust rules, which minimize the above criterion, present moderate losses in relation to the model-specific optimal rules, but avoid the huge losses that would arise in case the rule is chosen based on the wrong model.

Finally, we calculate the impulse response functions of the two models and five rules in four different experiments: inflation target reduction, transitory demand shock, permanent supply shock, foreign exchange rate equilibrium shock.

The article has the following structure. In section 2 we present the forward-looking model by Batini & Haldane [4], as well as its backward-looking restriction. The third section presents the methodology of numerical solution of liner rational expectation models. The fourth describes the data and presents the estimates of the forward and of the backward-looking versions. The fifth section presents the stochastic simulations under alternative monetary policy rules, builds efficient frontiers and obtains the efficient rules for each model. The sixth section deals with the impulse response functions, showing the dynamics implied by the different models and rules. The last section presents the final considerations and points the directions of the future research.

2 The structural model

We chose Batini & Haldane [4] model (B&H from now on) as the reference model. B&H model is suitable for being an open economy forward-looking model, able to replicate the empirical dynamics of the main macroeconomic variables. In the model, the microfoundations are separate rules from agents and firms, and not the explicit dynamic optimization of a representative agent. However, the decision rules are motivated by a rational behavior and make use of the same variables that shows up in the explicit derived equations.

The model comprises five linear equations that represents a stochastic dynamic general equilibrium with rational expectations, where the nominal short-run nominal stickiness is caused by staggered wages (like in Fuhrer & Moore [14]).

The first equation is a forward looking IS curve, given by:

$$h_t = \alpha_1 h_{t-1} + \alpha_2 E_t(h_{t+1}) + \alpha_3 [i_{t-1} - E_{t-1}\pi_t] + \alpha_4 x_{t-1} + \varepsilon_{1t}; \quad (1)$$

where: h_t is the output gap; E_t is the expectation conditional on the information available at t (that includes the variables determined at t)¹; $\pi_t = p_t^c - p_{t-1}^c$ is the inflation between $t-1$ and t ; $[i_{t-1} - E_{t-1}\pi_t] = r_{t-1}$ is the expected real interest rate between $t-1$ and t ; and x_t is the real exchange rate (defined as the nominal exchange rate e_t plus the rest of the world consumer price index p_t^{cf} minus the national consumer price index p_t^c); all variables in natural logarithms.

The main difference in relation to the traditional IS is that today's output is dependent upon the expectation of future output. This last term in the IS arises from a fully optimizing general equilibrium macro-model, like in Nelson & McCallum [21]. The expectation of a bigger future consumption is related to the contemporaneous consumption due to the optimal behavior of consumption smoothing.

The supply side has its wage determination equation due to Fuhrer & Moore [14]. The nominal wage is contracted today so that the real wage is an average of the past real wage and the expectation of the future real, plus an excess demand term:

¹It is common in the literature the denomination "forward-looking expectation" for E_t and "current-value expectation" for E_{t-1} . Wallis [32] discusses the difference between E_t and E_{t-1} for estimation purposes.

$$w_t - p_t^c = \chi_0 [E_t(w_{t+1}) - E(p_{t+1}^c)] + (1 - \chi_0) [w_{t-1} - p_{t-1}^c] + \chi_1 (h_t) + \varepsilon_{4t}; \quad (2)$$

where: w_t is the nominal wage. This equation introduces real stickiness and modifies the traditional equation of wage contracting by Taylor [28]. Such a change is sponsored by the fact that nominal stickiness is not enough to cause the observed inflation inertia. Another alternative to generate inflation inertia is to relax the hypothesis of rational expectation².

The price determination is completed by the markup equation:

$$p_t^d = \frac{1}{2} [w_t + w_{t-1}]; \quad (3)$$

where: p_t^d is the domestic output price. By specification (3) the wage contracts last for two periods and domestic output prices are a constant markup over the weighted average of the wage contracts.

The equation:

$$p_t^c = \phi p_t^d + (1 - \phi) (e_t + p_t^{cf}) \quad (4)$$

defines the national consumption price index, where ϕ is the domestic product share in the consumption price index.

Equation (2) in t and in $t - 1$ should be added and equations (3) and (4) substituted in the resulting sum to derive a New-Keynesian open-economy Phillips curve³. If external inflation is zero, we have the following Phillips curve:

$$\pi_t = \chi_0 E_t(\pi_{t+1}) + (1 - \chi_0) \pi_{t-1} + \chi_1 [h_t + h_{t-1}] + \mu [(1 - \chi_0) \Delta x_t - \chi_0 E_t(\Delta x_{t+1})] + \varepsilon_{4t}; \quad (5)$$

²One way is to assume that part of the agents have adaptative expectation, like in Amato e Laubach [1]. Roberts [23] motivates inflation inertia by direct use of inflation expectation data, which the analysis has shown not to be fully rational. Bonomo, Carrasco and Moreira [11] assume that the share of non-rational agentes learns gradually with the profit losses.

³É importante notar que termos constantes não afetam os resultados da simulação do modelo, permitindo-nos eliminá-lo. Então, para os nossos propósitos, o realismo de supor alguma variável igual a zero deve ser avaliado de acordo com o realismo de supor que esta variável seja constante.

where: $\mu = \frac{2(1-\phi)}{\phi}$, and $\Delta x_t = x_t - x_{t-1}$. In this equation, μ is determined from ϕ^4 , implying full pass-through into consumption prices.

The exchange rate is determined through the uncovered interest rate parity of the interest rates, given by:

$$e_t = E_t(e_{t+1}) - \left(i_t - i_t^f\right) + \varepsilon_{3t}; \quad (6)$$

where: i_t^f is the external nominal interest rate, and does not explicitly include an exchange risk premium. This means that the variations in the risk premium are captured in ε_3 , as well as other noises from the foreign exchange market. Pastore & Pinotti [22] argue that this representation is a good approximation for the pattern of the exchange rate in Brazilian economy near past. Rearranging (6) and assuming i_t^f and the external inflation equal zero, we get to an uncovered interest rate parity in terms of the expected variation of the real exchange rate:

$$E_t(\Delta x_{t+1}) - \varepsilon_{3t} = i_t - E_t(\pi_{t+1}). \quad (7)$$

According to this equation, the expectation of real depreciation equals the real interest rate, after discounting the contemporaneous exchange rate shock.

Summing up, the simplified model is:

$$h_t = \alpha_1 h_{t-1} + \alpha_2 E_t(h_{t+1}) + \alpha_3 [i_{t-1} - E_{t-1}\pi_t] + \alpha_4 z_{t-1} + \varepsilon_{1t};$$

$$E_t(\Delta x_{t+1}) = i_t - E_t(\pi_{t+1}) + \varepsilon_{3t};$$

$$\pi_t = \chi_0 E_t(\pi_{t+1}) + (1 - \chi_0)\pi_{t-1} + \chi_1 [h_t + h_{t-1}] + \mu [(1 - \chi_0)\Delta x_t - \chi_0 E_t(\Delta x_{t+1})] + \varepsilon_{4t}.$$

Note that the monetary policy can affect the contemporaneous inflation, since the expectation of tomorrow's output gap decrease may be enough to reduce today's inflation. This is due to two mechanisms: one associated to the forward looking term in the IS and the other associated to the wage equation.

⁴Alternativamente, o coeficiente μ pode ser estimado.

2.1 The backward looking version

By imposing $\alpha_2 = \chi_0 = 0$, we get the backward-looking, similar to Ball [5]. The differences are small: (i) in Ball's Phillips curve, only h_{t-1} matters; (ii) the relation between the foreign exchange and the interest rate allows a coefficient different from unity. Moreover, Ball conceive the time period as one year, while the above model assumes one quarter.

In this backward version there are two transmission mechanisms of the monetary policy: i) an interest rate increase today reduces the level of activity tomorrow and reduces inflation in the two periods following next; ii) an interest rate increase appreciate today's foreign exchange rate and reduces inflation today. Thus, in the backward-looking version, the foreign exchange rate is the only channel through which monetary policy can affect inflation contemporaneously.

2.2 Monetary policy operational rules

As this study is aimed to prescribe policy rules, we have chosen to analyze rules that are operational, this meaning that the Central Bank does not have access to the information available at t when choosing the nominal interest rate from t to $t + 1$ ⁵.

In an application of B&H model, the Bank of England [3] uses a monetary policy rule based on projections of future inflation:

$$r_t = \gamma r_{t-1} + (1 - \gamma) r_t^* + \theta [E_{t-1} \pi_{t+j} - \pi_t^*] + \varphi h_{t-1}; \quad (8)$$

where: $r_t = [i_t - E_t \pi_{t+1}]$ is the expected real interest rate; γ is the smoothing parameter; r_t^* is the equilibrium interest rate (or neutral); θ is the policy response parameter; π_{t+j} is the inflation between $t + j$ and $t + j - 1$; and π_t^* is the inflation target. The rule (8) is a slight change in the rule proposed by Batini & Haldane [4], which uses $E_t \pi_{t+j}$ instead of $E_{t-1} \pi_{t+j}$. A policy that follows (8), react to the forecasts of future inflation.

We are also going to examine the particular case of the above rule in which $j = -1$, that results in the traditional Taylor rule:

$$r_t = \gamma r_{t-1} + (1 - \gamma) r_t^* + \theta [\pi_{t-1} - \pi_t^*] + \varphi h_{t-1} \quad (9)$$

⁵The Central Bank expectation is $E_{t-1} [\cdot]$. Se footnote no. 3.

Ball [5] argues, in the context of a backward-looking small open-macroeconomic model, that the inclusion of the foreign exchange rate in the policy rule increases its efficiency. we consider these kinds of rules as well, which has the form:

$$r_t = \gamma r_{t-1} + (1 - \gamma) r_t^* + \theta [(\pi_{t-1} - \pi_t^*) - \mu x_{t-1}] + \varphi h_{t-1} \quad (10)$$

Each of the above rules, together with (1),(5) and (7) form a linear system of stochastic difference equations with rational expectations to be studied in the following sections⁶.

To allow the comparison among rules, we have experimented among the same subset of policy parameters (γ , θ , φ) for the different rules. We vary φ and θ from 0 to 2,5 with 0,25 steps for all rules. With respect to the smoothing parameter of the interest rate we have tested three values, γ : 0, 0,5 and 1, since there is evidence that values different from zero worsen the rule performance if interest rate smoothness does not enter the authority objective function, which is going to be the case. In the rules that use the forecast of future inflation, j goes from 0 to 10. In total, 4719 were tested: 363 parametrizations of the Taylor rule, 363 Ball versions and 3993 inflation forecast rules.

3 Model Solution

All linear multivariate structural model with rational expectation can be written in the form:

$$\sum_{i=0}^{n_1} \sum_{j=0}^{n_2} M_{ij} E_{t-i} [x_{t+j-i}] = v_t; \quad (11)$$

where: M_{ij} are matrices of coefficients and x_t the vector of variables; $E_{t-i} [\cdot]$ is the expectation conditional on the information available in $t - i$, and v_t is a random vector free of serial correlation.

It is important to note that included in x will be exogenous variables that follow any movement rule, as happens to be the case with some policy rules or exogenous random shocks. The vector v only includes the variables

⁶Note that the LM curve has not been presented, since the quantity of money adjusts endogenously following the nominal interest rate determined by the rule.

fully determined out of the system, like policy variables not endogenized and that don't present serial correlation, or exogenous random shocks.

The most famous method for the solution of linear rational expectation models is by Blanchard & Kahn [9] which proposes to build the system in the form:

$$\begin{bmatrix} P_{t+1} \\ E_t[J_{t+1}] \end{bmatrix} = A \begin{bmatrix} P_t \\ J_t \end{bmatrix} + B \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}; \quad (12)$$

where: P_t is the vector that contains the variables pre-determined at t , and J_t is the vector of endogenous variables at t .

Although its generalized use, the method had some limitations which were eliminated by improvements like Binder & Pesaran[6], King & Watson[16] and [17], Klein [18], or Sims [26]. Except for Sims [26], who has a distinct approach⁷, the others approach is quite close to Blanchard & Kahn [9].

Two questions that arise once we compare (11) to (12) are: (i) how to transform a system that relate many periods like (11) in a two-period system like (12)?; and, given the transformation, (ii) what guarantees that the matrix multiplying the vector in $t + 1$ is an identity matrix (that means, it was a invertible matrix)?

Binder & Pesaran [6] show that, given a structural multivariate model with rational expectations (11) we are able to define:

$$y_t = \begin{pmatrix} x_t \\ E_t[x_{t+1}] \\ \vdots \\ E_t[x_{t+n_2}] \end{pmatrix}, u_t = \begin{pmatrix} v_t \\ 0_{px1} \\ \vdots \\ 0_{px1} \end{pmatrix}, \Gamma_0 = \begin{pmatrix} M_{00} & M_{01} & \cdots & M_{0n_2} \\ 0_p & I_p & \cdots & 0_p \\ & & \ddots & \\ 0_p & 0_p & \cdots & I_p \end{pmatrix},$$

$$\Gamma_i = \begin{pmatrix} M_{i0} & M_{i1} & \cdots & M_{in_2} \\ 0_p & 0_p & \cdots & 0_p \\ & & \ddots & \\ 0_p & 0_p & \cdots & 0_p \end{pmatrix}, \Gamma_{-1} = \begin{pmatrix} 0_p & 0_p & \cdots & 0_p & 0_p \\ -I_p & 0_p & \cdots & 0_p & 0_p \\ & & \ddots & & \\ 0_p & 0_p & \cdots & -I_p & 0_p \end{pmatrix},$$

and to represent (11) in the form:

$$\sum_{i=0}^{n_1} \Gamma_i y_{t-i} + \Gamma_{-1} E_t[y_{t+1}] = u_t. \quad (13)$$

⁷Sims approach consists in treating the conditional expectation and the expectational error associated as additional endogenous variables, and add to the original system an equation that defines de expectational error.

Using the system (13) above, the reduction to a first-order system is possible by defining:

$$z_t = \begin{pmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-n_1+1} \end{pmatrix}, \Lambda_0 = \begin{pmatrix} \Gamma_0 & \Gamma_1 & \cdots & \Gamma_{n_1-1} \\ 0_{(n_2+1)p} & I_{(n_2+1)p} & \cdots & 0_{(n_2+1)p} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{(n_2+1)p} & 0_{(n_2+1)p} & \cdots & I_{(n_2+1)p} \end{pmatrix},$$

$$\Lambda_1 = \begin{pmatrix} 0_{(n_2+1)p} & 0_{(n_2+1)p} & \cdots & 0_{(n_2+1)p} & \Gamma_{n_1} \\ -I_{(n_2+1)p} & 0_{(n_2+1)p} & \cdots & 0_{(n_2+1)p} & 0_{(n_2+1)p} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{(n_2+1)p} & 0_{(n_2+1)p} & \cdots & -I_{(n_2+1)p} & 0_{(n_2+1)p} \end{pmatrix},$$

$$\Lambda_{-1} = \begin{pmatrix} \Gamma_{-1} & 0_{(n_2+1)p} & \cdots & 0_{(n_2+1)p} \\ 0_{(n_2+1)p} & 0_{(n_2+1)p} & \cdots & 0_{(n_2+1)p} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{(n_2+1)p} & 0_{(n_2+1)p} & \cdots & 0_{(n_2+1)p} \end{pmatrix}, \tilde{u}_t = \begin{pmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

and noting that:

$$z_t = -\Lambda_0^{-1}\Lambda_1 z_{t-1} - \Lambda_0^{-1}\Lambda_{-1} E_t [z_{t+1}] + \Lambda_0^{-1}\tilde{u}_t \quad (14)$$

is equivalent to (13).

Finally, define $\tilde{z}_t = \begin{bmatrix} z_{t-1} \\ z_t \end{bmatrix}$ and rewrite (14) as:

$$\begin{bmatrix} I & 0 \\ 0 & \Lambda_0^{-1}\Lambda_{-1} \end{bmatrix} E_t [\tilde{z}_{t+1}] = \begin{bmatrix} 0 & I \\ -\Lambda_0^{-1}\Lambda_1 & -I \end{bmatrix} \tilde{z}_t + \begin{bmatrix} 0 \\ \Lambda_0^{-1}\tilde{u}_t \end{bmatrix} \quad (15)$$

or simply:

$$CE_t [\tilde{z}_{t+1}] = D\tilde{z}_t + \tilde{w}_t, \quad (16)$$

which has the form of (12), where C is invertible by construction.

In our specific case, only the expectations conditional on t and on $t-1$ show up, what means that (13) reduces to:

$$\Gamma_0 y_t + \Gamma_1 y_{t-1} - u_t = -\Gamma_{-1} E_t [y_{t+1}], \quad (17)$$

and the transformation done in (15) can be anticipated. In (17), we define $\tilde{y}_t = \begin{bmatrix} y_{t-1} \\ y_t \end{bmatrix}$ and rewrite:

$$\begin{bmatrix} I & 0 \\ 0 & -\Gamma_{-1} \end{bmatrix} E_t [\tilde{y}_{t+1}] = \begin{bmatrix} 0 & I \\ \Gamma_1 & \Gamma_0 \end{bmatrix} \tilde{y}_t + \begin{bmatrix} 0 \\ -u_t \end{bmatrix}$$

or

$$CE_t [\tilde{y}_{t+1}] = D\tilde{y}_t + \tilde{u}_t. \quad (18)$$

which can be solved by the methods proposed by Binder & Pesaran[6], King & Watson[16][17], Klein [18], or even Blanchard & Kahn [9].

In this work, we have chosen Klein[18] approach, which solves the system (11) by generalized eigenvalues-eigenvectors in the Schur form, after put the system in the recursive form (18). Given C and D from (18), it is possible a QZ decomposition, which results in Q , Z , S and T such that:

$$Q'SZ' = C \quad \text{and} \quad Q'TZ' = D;$$

where S and T can be arranged such that the stable generalized eigenvalues ($t_{ii}/s_{ii} < 1$) show in the upper-left and the unstable generalized eigenvalues ($t_{ii}/s_{ii} > 1$) show in the inferior part.

(18) can be rewritten as:

$$SZ'E_t [\tilde{y}_{t+1}] = TZ'\tilde{y}_t + Q\tilde{u}_t. \quad (19)$$

what allows the definition of $n_t = Z'\tilde{y}_t$.

If the system has as many unstable eigenvalues as endogenous variables, (19) has the form:

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \left(E_t \begin{bmatrix} n_{t+1}^s \\ n_{t+1}^u \end{bmatrix} \right) = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} n_t^s \\ n_t^u \end{bmatrix} + Q\tilde{u}_t. \quad (20)$$

and it is possible to decouple the system. Since S_{11} and T_{22} are invertible by construction, n^u is independent of n^s and the lower part of the system can be solved forward. Once solved for the series in n^u , n^s is obtained from any initial date of interest. Since $\tilde{y}_t = Zn_t$, the final solution is:

$$y_t = (Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}) y_{t-1} - Lu_t; \quad (21)$$

where $L = Z_{11}S_{11}^{-1} \{ -T_{11}Z_{11}^{-1}Z_{12} (-T_{22}^{-1}Q_{22}) + [T_{12} (-T_{22}^{-1}Q_{22}) + Q_{12}] \}$.

It is important to note that for a stable solution to exist, the upper-left block Z_{11} from matrix Z should be non-singular⁸.

⁸Detailed explanations of the Binder & Pesaran recursive form and the solution of the

4 Data, estimation and calibration

The data used has quarterly frequency and consist of the deseasonaled GDP, the national consumer price index (INPC), the index of the effective real foreign exchange rate (an average of the 15 most important countries in terms of Brazilian trade balance) from the IPEA data. The three-month fixed interest rate was calculated from the value of the third contract to expire in the interest rate future market⁹. The proxy for the potential GDP was calculated by an adjusted quadratic trend, as has been usual in empirical applications¹⁰.

The system were estimated from 1994:3 to 2000:1, since the Real Plan was an structural change, specially with respect to the Phillips curve.

4.1 The estimation problem

Models like the one studied are not trivial to estimate. Besides being a simultaneous equations model with rational expectations, the expectations in the equations are subject to different information sets. Additionally, the economic relations can imply a considerable number of restrictions about the parameters.

As we are interested in the aspects related to the reactions of the economy to changes in regimes, as well as in the efficiency of alternative monetary policy rules, it is not suitable to proceed the direct estimation, simply substituting the expectations by the future values of the variables ($E_{t-1}[x_{t+j}]$ for x_{t+j}). It is important to explicit model the expectations. One alternative is to use the market expectation series of the variables (see Roberts[23]). Another is to solve the model with rational expectations by one of the usual methods and to estimate it using the corresponding technique, like in McCallum [19],

stochastic difference system with rational expectations by the generalized Schur form can be found in Binder and Pesaran [6] and in Klein [18].

⁹The Brazilian interest rate future market has the peculiarity that the DI-future contracts are indeed spot fixed rate contracts. The three-month fixed interest rate was calculated using the third DI contract to expire observed in the first business day of the quarter.

¹⁰Recently, some works (see Gali e Gertler [15]) have raised doubts about the validity of the use of the Hodrick-Prescott filter, or the adjusted quadratic function as proxies for the potential product in the New Keynesina context of this work. However, due to the incipient stage of the discussion, we have chosen to follow the usual practice.

Wallis [32] or Wickens [33]¹¹.

Given the lack of market expectation series, we were left with the rational expectation hypothesis, approached by the error-in-variable method and estimated by three-stage least square (3SLS). The error-in-variable method has the advantage of imposing simpler restrictions than the substitution method. More, as we have a forward-looking rational expectation system, the substitution of the future variable by a sum of its realization and an expectational error imposes a stable solution. The 3SLS estimator is consistent, as shown by Wickens [33].

The chosen strategy was to eliminate $E_t(\Delta x_{t+1})$ and $E_{t-1}(\pi_t)$ from (1) and (5) and to obtain a 2 equation sub-system with forward-looking expectations of the two endogenous variables ($E_t(h_{t+1})$ and $E_t(\pi_{t+1})$). To eliminate $E_t(\Delta x_{t+1})$, we use the uncovered interest parity (6). To eliminate $E_{t-1}(\pi_t)$, we have substituted lagged (7) in lagged (5), and used the expression for $E_{t-1}(\pi_t)$ in the resulting equation to substitute in (1), obtaining the sub-system:

$$h_t = \alpha_1 h_{t-1} + \alpha_2 E_t(h_{t+1}) + \alpha_3 \left(\frac{1}{1 + \mu} \right) i_{t-1} \quad (22)$$

$$- \frac{\alpha_3}{(1 + \mu) \chi_0} \left\{ \begin{array}{l} \pi_{t-1} - (1 - \chi_0) \pi_{t-2} - \chi_1 [h_{t-1} + h_{t-2}] \\ - \mu (1 - \chi_0) \Delta x_{t-1} + [\mu \chi_0 \varepsilon_{3t-1} - \varepsilon_{4t-1}] \end{array} \right\} + \alpha_4 x_{t-1} + \varepsilon_{1t};$$

and

$$\pi_t = \chi_0 (1 + \mu) E_t(\pi_{t+1}) + (1 - \chi_0) \pi_{t-1} + \chi_1 [h_t + h_{t-1}] + \mu [(1 - \chi_0) \Delta x_t - \chi_0 i_t] - [\mu \chi_0 \varepsilon_{3t} - \varepsilon_{4t}]; \quad (23)$$

in which only the expectations $E_t(h_{t+1})$ and $E_t(\pi_{t+1})$ have impact on the present variables.

Due to the possibility of serial correlation in the errors, the instruments should be variables determined before the expectations. Lagged endogenous variables are not admissible. Only exogenous variables are legitimate candidates.

The backward sub-system is simpler, given we only have one expectation:

$$h_t = \alpha_1 h_{t-1} + \alpha_3 [i_{t-1} - E_{t-1} \pi_t] + \alpha_4 z_{t-1} + \varepsilon_{1t}; \quad (24)$$

¹¹Blake[8] is a nice survey of the estimation of rational expectation models.

$$\pi_t = \pi_{t-1} + \chi_1 [h_t + h_{t-1}] + \mu (1 - \chi_0) \Delta x_t + \varepsilon_{4t}. \quad (25)$$

The systems (22)-(23) and (24)-(25) were estimated for the period 1994:3 to 2000:2 by 3SLS, using as instruments: a dummy for the foreign exchange rate regime change in January 1999, a trend for the deseasonaled GDP, the first and second lags of the variation of the nominal wage changes and of the unemployment level, the nominal interest rates (i_{t-1} , i_{t-2}) and the real foreign exchange rate index (x_{t-1} and x_{t-2}). The Table 1 below reports the estimates for the forward-looking and the backward-looking versions. The parameter ϕ was not estimated, but calculated from the national accounts to be approximately 0,95, what implies the reported μ .

Param.	For. Estim.	Back. Estim.
α_1	1,06 (3,75)	0,91 (5,67)
α_2	0,07 (0,28)	0
α_3	-0,44 (-2,96)	-0,51 (-4,32)
α_4	0,08 (1,71)	0,08 (1,97)
χ_0	0,85 (4,84)	0
χ_1	0,09 (0,69)	0,16 (0,99)
μ	0,1	0,1

Table 1: Estimativa dos Parâmetros

The estimated parameters have the expected sign. The interest rate coefficient α_3 is statistically significant and its value indicates that the interest rate has an important effect on the activity level. The foreign exchange coefficient is not significant, probably due to the predominance in the sample of the fixed regime. However, the coefficient of the output gap in the Phillips curve (χ_1) is not significantly different from zero. This last result is annoying, since the effect of the activity level over the inflation is an important part of

the transmission mechanism of the monetary policy. These are the common characteristics of the forward and backward estimates.

In the forward-looking version, the IS is mainly backward-looking while the Phillips curve is quite forward-looking. We have interpreted the high degree of forward-lookingness of the Phillips curve as the result of the flexibility of the Brazilian labor market, what is consistent with the origin of the χ_0 coefficient in the wage equation of the model.

Given the spread belief among Brazilian macroeconomists that the economy is backward-looking, we have also estimated a backward version, restricting the forward-looking coefficients α_2 and χ_0 to zero. The first restriction has no practical effect, given the small value of α_2 in the forward-looking estimates. But the second restriction, on the other hand, has important effects on the model operation. Due to this reason, we have chosen to work with two calibrations based on both estimates of the model.

In the backward-looking estimation, both coefficients of the interest rate in the IS (α_3) as well as of the output gap in the Phillips curve (χ_1) increase, although the last one still not-significant. The expressive magnitude of the interest rate in the IS with high level of significance, and the low significance of the output gap in the Phillips are in accordance with the results got by Andrade & Divino [2] and Pastore & Pinoti [22]¹², who apply backward-looking models. In our case, the measured semi-elasticity of the interest rate is slightly superior in the short-run (their goes from $-0,42$ to $-0,46$), but far superior in the long run (approximately 5 in our case, against 1,5 in theirs), due to our high output gap persistency (α_1). The main differences between those models and ours is that Andrade e Divino[2] make use of one-month output gap lag, while Pastore & Pinoti [22] use data that precede the Real Plan (since 1980).

4.2 Calibration

We have done two calibrations, one forward-looking and another backward-looking, based on the respective estimations. In the forward looking model a small change in relation to the estimated coefficients was necessary to guarantee the stability of the model: the coefficients α_1 and α_2 where proportionally reduced, such that their sum is smaller than 1. The Table 2 show the calibrations we have worked with. for effect of comparison, Batini & Haldane's

¹²This work does not estimate the Phillips curve.

[4] calibration for the United Kingdom is also included.

Param.	B&H Calib.	For. Calib.	Back. Calib.
α_1	0,8	0,93	0,91
α_2	0	0,06	0
α_3	-0,5	-0,44	-0,51
α_4	0,2	0,08	0,08
χ_0	0,2	0,85	0
χ_1	0,2	0,09	0,16
μ	0,5	0,1	0,1

Table 2: Calibração dos Parâmetros

Note that the backward calibration is qualitatively similar to Batini & Haldane's. In theirs, the IS is fully backward-looking ($\alpha_2 = 0$) and the Phillips is mainly backward-looking ($\chi_0 = 0,2$). The main difference is that the parameters related to the foreign exchange rate in the IS and in the Phillips are bigger for the Britain calibration, what was expected given the United Kingdom economy is more open to international trade than the Brazilian economy.

5 Efficient Rules

Assume that the objective function of the monetary authority is to minimize the undesirable fluctuations of inflation and output, by choosing a sequence of expected real interest rates. That means:

$$\min_{\{r_i\}_{i=0}^T} E_0 \sum_{i=1}^T [\lambda (\pi_i - \pi_i^*)^2 + (1 - \lambda) (h_i)^2]; \quad (26)$$

where: $r_t = [i_t - E_t\pi_{t+1}]$ is the expected real interest rate and π^* is the inflation target. The solution to this problem is an efficient monetary policy rule¹³.

Due to the difficulty in obtaining an analytical solution to this model and to the uncertainty about the best way to model the economy (backward or forward), we have adopted the strategy to restrict our universe set to the above mentioned 4719 rules and to compare their performance in the forward and backward versions, based on the objective function (26). Given the constants r^* and π^* , we implement the comparison in accordance with the following sample criterion:

$$\min_{\{r_i\}_{i=0}^T} \lambda \overline{var}(\pi) + (1 - \lambda) \overline{var}(h); \quad (27)$$

where $\overline{var}(x)$ is the mean variance of the sequence $\{x_i\}_{i=0}^T$; and calculate the efficient frontier of the variance of the inflation and the variance of the output for each version, varying λ between 0 and 1. In the simulations that follow, we have calculated the mean variance of the resulting inflation and output for the set of 4719 monetary policy rules in the forward-looking and in the backward-looking versions. The reported values are the mean of 100 replications of 200 periods ($T = 200$)¹⁴.

Stochastic simulations require the definition of the distribution of random shocks, what raises some questions. If on one hand, the basic structural model does not have enough dynamic structure to estimate the residual, the VAR methodology imposes an orthogonality of the primitive disturb, something we would like to avoid. The simulations below were done under the assumption that the shocks are white noise (i.i.d). The case of correlated shocks is performed in the end of this section, using Black et al. [7] approach.

5.1 I.I.D. Shocks

Reported in this subsection are the simulation results when the IS, the Phillips and the foreign exchange rate equation shocks are white noise. Figures 1 and 3 present full pictures of the performance of the set of experimented rules in plane variance of output-variance of inflation in the forward and

¹³Discussions upon discretion or rules can be found in Blinder [10] (informal), Taylor [29] and Taylor [31] (historical evidence) and Clarida, Gali & Gertler[12].

¹⁴As McCallum&Nelson [20], we run simulations of 253 periods and ignore the first 53 periods to abstract from the problems of initial violation of the system solution restrictions.

backward-looking models, respectively. Figures 2 and 4, show the efficient frontier for these models, calculated by varying λ from 0 to 1. Additionally, from one point on the frontier, we show how the effect of the variation of one of the parameters j , θ and φ , keeping the other fixed. In Figures 2 and 4, the notation $R(j, \theta, \varphi, \gamma)$ identifies the rules, where R denotes the type of rule ($R \in \{T, B, F\}$ meaning one of the equations (8)-(10)); j is the lead ($j = -1, 0, 1, \dots, 10$), θ is the reaction to inflation ($\theta \in [0; 2, 5]$), φ is the reaction to the output gap ($\varphi \in [0; 2, 5]$) and γ is the smoothing factor ($\gamma \in [0; 1]$).

5.1.1 The Forward-looking Economy

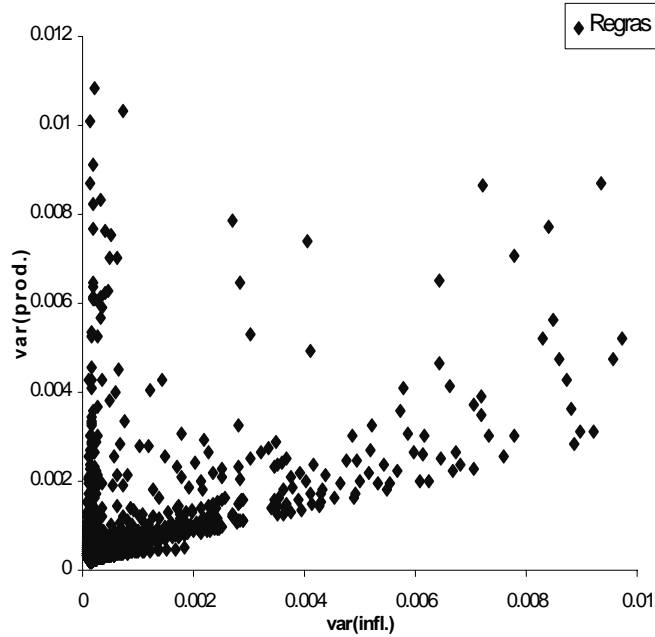


Figura 1: Nuvem de regras para a versão forward i.i.d.

Figure 1 show that in the forward-looking model many monetary rules are associated to small inflation and output variances. The frontier in figure 2 shows that the efficient rules are not very forward-looking, given they are of the Taylor-, Ball- or forward-type with small lead ($j = 0$ or 1). Additionally, the efficient rules put relative more weight on inflation than on output. In fact, the weight on inflation is the maximum among the experimented ($\theta = 2, 5$) for most of the efficient rules, while the weight on the output has

moderated values (φ is 0,5 or 0,75). We can also verify that the efficient rules do not change too much with the efficiency criterion. For example, a Taylor rule with $\theta = 2,5$ and $\varphi = 0,5$ is very close from being efficient, whatever the relative weights of the monetary authority between inflation and output. Finally, there is no smoothing in the interest rate ($\gamma = 0$) in the frontier rules.

The figures 2.A and 2.B demonstrate the effect of the variation in j , θ and φ , from $F(0 ; 2,5 ; 0,75 ; 0)$. A reduction in θ initially increase $var(infl.)$ and decreases $var(prod.)$, this trade-off being less efficient than the one in the frontier. Reducing θ below 1 increases not only the inflation variance, but the output variance as well.

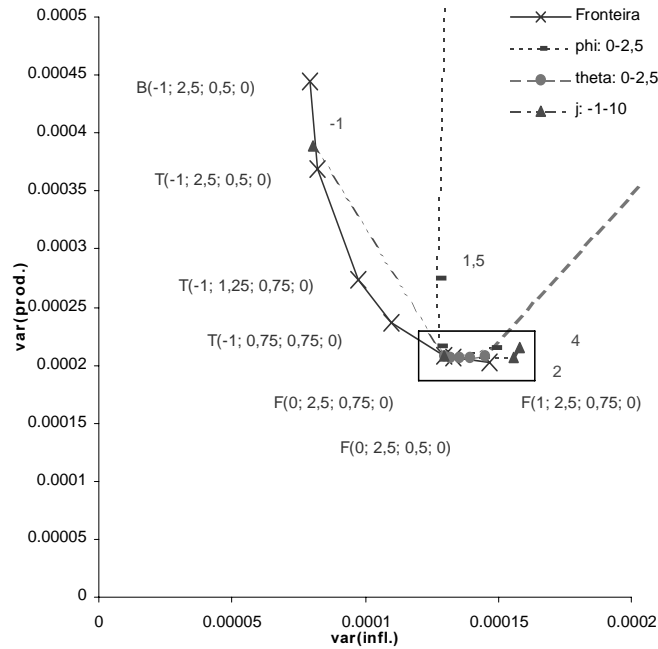


Figura 2.A: Fronteira da versão forward com choques i.i.d.

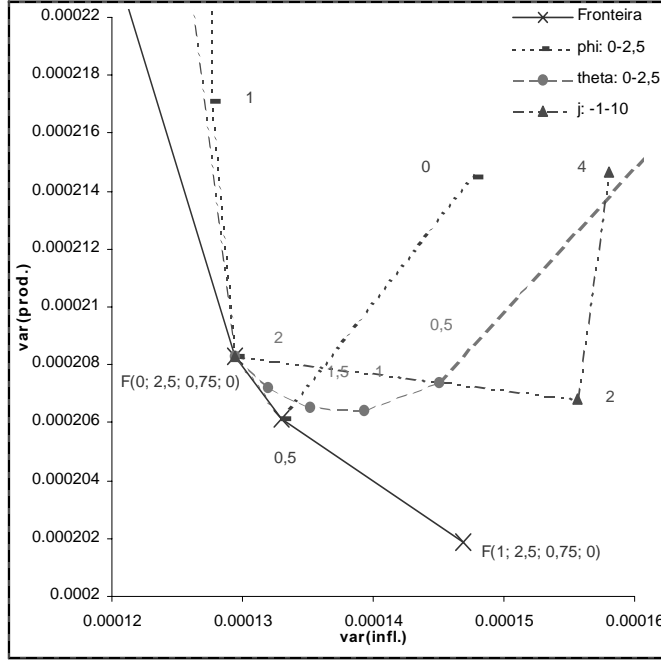


Figura 2.B: Ampliação da fronteira da versão forward

The pictures show that the effect of the variation of φ in the output variance is not monotone. Beginning at $\varphi = 0$, the variance of the output decreases up to $\varphi = 0,5$ and then increases again from then on. A increase in φ has negative effect in the inflation variance, even after the inflection.

The increase in j is effective to reduce the fluctuations in the output up to $j = 2$, although this movement happens with the simultaneous increase in the volatility of inflation. The lengthening of the inflation horizon further than this point causes an increase in the volatility of both variables. For $j > 4$, the system is unstable. It is clear that if a too short horizon causes too much response to current inflation shocks, a very long horizon cause inflation causes too little response and may become destabilizing.

5.1.2 The Backward-looking Economy

In Figure 3, the cluster of rules for the backward-looking model is more distant from the origin than in the forward-looking model. Figure 4 shows that, in general, the backward-looking optimal rules are forward-looking, like in Batini & Haldane [4]. The lead is small ($j = 2$) when the objective is to minimize the variance of the inflation, but increases as the weight in

the output increases. The efficient rules weight in the output is the same for all points in the frontier ($\varphi = 0,75$), but the inflation weight increase monotonically between the minimum ($\theta = 0$), when the inflation weight is zero ($\lambda = 0$), and the maximum tested ($\theta = 2,5$) when $\lambda = 1$. All rules apply zero weight to the interest rate smoothing.

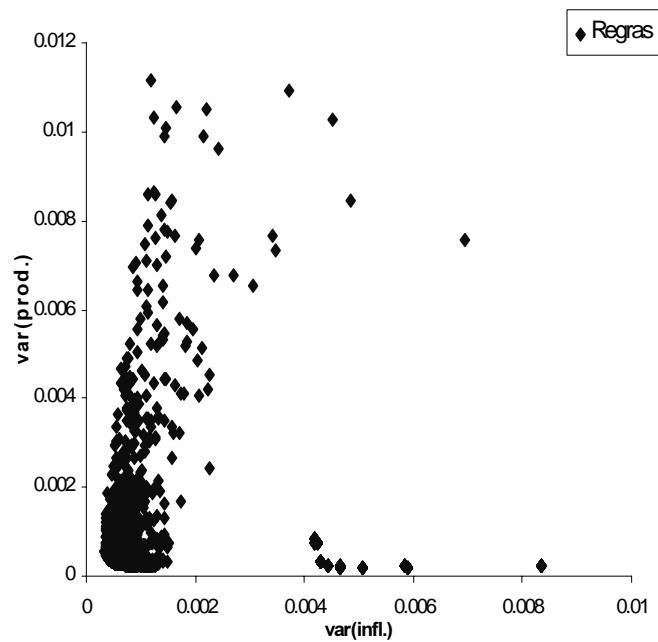


Figura 3: Nuvem de regras para a versão backward. i.i.d.

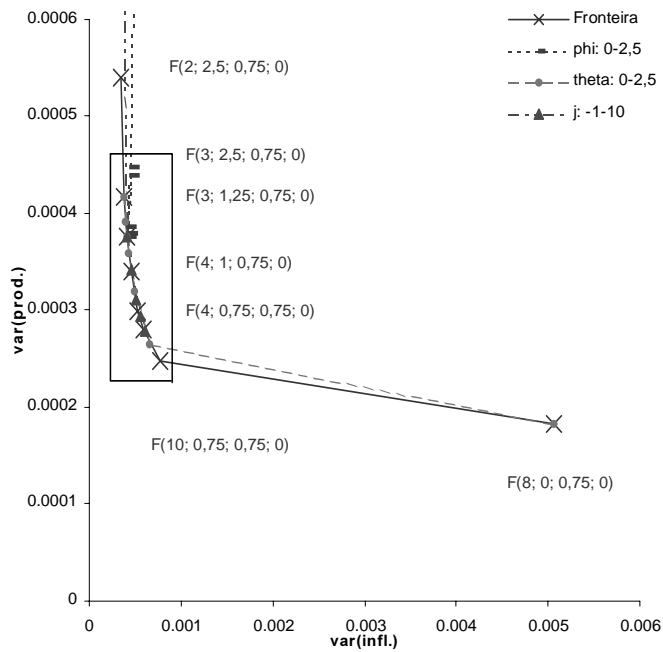


Figura 4.A: Fronteira da versão backward com choques i.i.d.

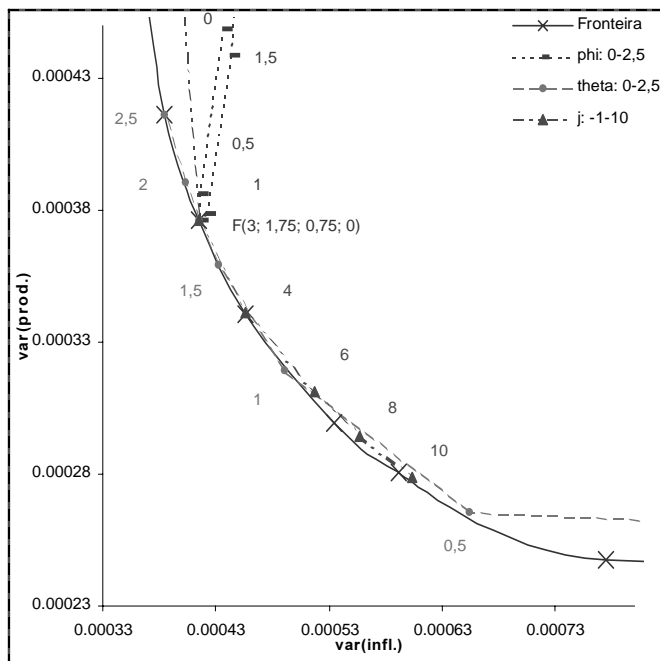


Figura 4.B: Ampliação da fronteira da versão backward

The exercises of parameter variation from the point $F(3; 1, 75; 0, 75; 0)$ on the frontier confirm the figure 3 impression, that there is a huge number of rules close to the frontier. In figures 4A and 4B the lines drawn by the variation of each parameter are always very close from the frontier. The only exception is the effect of the variation in φ , whose value of 0,75 is optimal for every objective function weights. From $\varphi = 0$, both the variances of output and inflation decrease up to $\varphi = 0,75$ and increase from then on. If some concern with output directly contributes to reduce its variance, a violent reaction has opposite effects.

It is shown that an increase in j or a reduction in θ causes an increase in inflation variance and a reduction in the output variance, almost tangent to the frontier. About j , once reacting to the expectation of a future variable, The monetary authority tries to avoid that the future variable deviates from the target. Since some of the present shocks will have been dissipated by the economy in the monitoring date, a forward rule will react less than a backward rule, and the lower interest rate volatility resulting will cause smaller variation in the output. By the other hand, a smaller θ implies a smoother reaction to inflation shocks, smaller interest rate variation and consequently smaller output volatility. That means, a system that is less reactive to short-run inflation has a more stable output.

The comparison between the forward- and the backward-looking models allows some conclusions. If the economy is forward-looking, it is possible to attain higher stability standards. Another difference, also pointed by Batini & Haldane [4] is that if the economy is forward-looking the optimal rule is more backward-looking than if the economy were backward-looking. This result is rationalized by the differences in the transmission mechanisms of both systems. In the backward-looking system, the lag of the interest rate effect in inflation makes reaction to temporary inflation fluctuations worthless, since the reaction effects comes to happen when the shock has been dissipated. In the forward-looking system however, the faster transmission in the monetary policy allows that even transitory shock be smoothed, with beneficial stabilizing and consequences. With a similar argument we can rationalize the smaller weight that the efficient rules put in inflation in the backward-looking system¹⁵.

¹⁵All the above exercises were performed for correlated shocks as well, but have not resulted in qualitative different results. They can be made available upon request.

5.1.3 Robust Rules

The uncertainty about how forward-looking is the Brazilian economy has motivated a study regarding robustness of the efficient rules to the degree of forward-lookingness. One way to implement this is suggested by Taylor [30], who ordinales the rules in each model with respect to the objective function, ranks them in each model and then add up the many models ranking. However, this criterion does not account, for example, for the fact that inflation reductions are more expressive and important in schemes of high volatility. In our case, as the backward version is more volatile, we should favor rules that perform better in it. We suggest a criterion of optimality that to our view fulfill this concern. For each λ , we look for the monetary rules that minimize:

$$\sum_i [\lambda var(\pi) + (1 - \lambda)var(y)]_i; \quad (28)$$

where: $i = For. Calib., Back. Calib.$

The results are shown in Table 3 below, which show the twenty more efficient rules in accordance to our criterion for $\lambda = 0; 0,5; 1$. We have reported as well the loss functions, normalized by the most efficient rule. Note that the set of robust rules has small intersection with the efficient sets of the forward- or backward-looking models.

lambda*var(infl.)+(1-lambda)*var(prod.)					
lambda=0					
posição	regra*	theta	phi	gam	função
1	F10	0.25	0.75	0	1.0000
2	F9	0.25	0.75	0	1.0032
3	F8	0.25	0.75	0	1.0068
4	F7	0.25	0.75	0	1.0107
5	F6	0.25	0.75	0	1.0150
6	F5	0.25	0.75	0	1.0197
7	F4	0.25	0.75	0	1.0253
8	F10	0.25	1	0	1.0275
9	F9	0.25	1	0	1.0306
10	F3	0.25	0.75	0	1.0324
11	F8	0.25	1	0	1.0347
12	F7	0.25	1	0	1.0389
13	F6	0.25	1	0	1.0415
14	F2	0.25	0.75	0	1.0421
15	F5	0.25	1	0	1.0427
16	F10	0.5	0.75	0	1.0429
17	F4	0.25	1	0	1.0451
18	F9	0.5	0.75	0	1.0493
19	F3	0.25	1	0	1.0513
20	F1	0.25	0.75	0	1.0547

lambda=.5					
posição	regra	theta	phi	gam	função
1	F3	1.75	1	0	1.0000
2	F3	2	1	0	1.0009
3	F3	1.5	1	0	1.0016
4	F3	1.5	0.75	0	1.0022
5	F4	2.25	1	0	1.0023
6	F3	1.75	0.75	0	1.0025
7	F3	2.25	1	0	1.0032
8	F4	2	1	0	1.0035
9	F2	1	0.75	0	1.0046
10	F3	2	0.75	0	1.0047
11	F4	2.25	0.75	0	1.0051
12	F4	2.5	0.75	0	1.0055
13	F4	2	0.75	0	1.0056
14	F3	1.25	0.75	0	1.0056
15	F3	2.5	1	0	1.0064
16	F4	1.75	1	0	1.0065
17	F2	1.25	0.75	0	1.0068
18	F4	1.75	0.75	0	1.0076
19	F3	1.25	1	0	1.0078
20	F3	2.25	0.75	0	1.0080

lambda=1					
posição	regra	theta	phi	gam	função
1	T	1.25	1.25	0	1.0000
2	T	1.5	1.5	0	1.0151
3	T	1.25	1.5	0	1.0170
4	B	1.25	1.25	0	1.0173
5	B	1.25	1	0	1.0179
6	B	1	1	0	1.0216
7	T	1.5	1.25	0	1.0286
8	F2	2.5	1	0	1.0330
9	F1	2	0.75	0	1.0369
10	F2	2.5	1.25	0	1.0397
11	T	1	1.25	0	1.0409
12	F1	2.25	0.75	0	1.0421
13	F1	1.75	0.75	0	1.0425
14	B	1	1.25	0	1.0435
15	F2	2.5	0.75	0	1.0443
16	T	1	1	0	1.0478
17	F2	2.25	1	0	1.0493
18	F1	1.75	1	0	1.0511
19	T	1.25	1	0	1.0540
20	F1	2	1	0	1.0552

Notas:
*: FX denota a regra (10) com $j=X$; T a regra (11); e B a regra (12)

Tabela 3: Colocação segundo o critério (28) para choques i.i.d.

Table 4 below compares the performance of the robust rules with the spe-

cific models optimal rules, when the economy is indeed forward- or backward-looking. We have calculated the ratios between the values of the loss functions resulting from the various optimal rules and the model-specific optimal rule.

Regras Ótimas para:	Modelo Verdadeiro	
	forward	backward
lambda = 0		
incerta	1.04	1.18
na versão forward	1	7.12
backward	inf.	1
lambda = 0,5		
incerta	1.06	1.01
na versão forward	1	inf.
backward	1.10	1
lambda = 1		
incerta	1.25	1.10
na versão forward	1	inf.
backward	1.97	1

Tabela 4: Ineficiência relativa

As expected, the above criterion imply some losses when compared to efficient rule of the true model, but it is still justifiable given the huge losses caused by the choice of the efficient rule of the wrong model.

6 Impulse Response

To illustrate the dynamics of the models, some monetary policy rules were selected based on the stochastic simulation performance and on their different characteristics.

Figures 7 to 12 illustrate the models dynamics in four experiments: inflation target reduction (π_t^*), transitory demand shock (ε_{1t}), permanent supply shock (potential output reduction) and overvalued real foreign exchange ($x_0 < 0$) (all in 1%).

In Figure 7 we show the impulse response function of the forward-looking economy with a monetary rule that looks at inflation 10 quarters ahead and with weights 0, 25 in inflation and 0, 75 in the output. This rule was the one that minimizes the output variance in the robust study. It is clear the preventive action of the monetary authority, who moves based on future inflation

forecasts. The inflation target reduction is fast fulfilled by the transitory increase of the real interest rate, with a small and transitory output loss¹⁶. A transitory increase in the demand sponsors the contemporaneous increase in the output, but is confronted with the simultaneous real interest increase to fight inflation, what causes a small recession before convergence. A potential output reduction (or a permanent supply shock) means a temporary inflation increase, in spite of the preemptive increase in the interest rate, and a permanent reduction in the output. Note the permanent supply shock causes a negative overshooting. Finally, the path of an overvalued real foreign exchange rate to the equilibrium generates a devaluation with overshooting, what causes a preemptive increase in the real interest rate and a transitory output loss. A common feature of the presented impulse response functions is that interest rate movements have almost immediate effect in inflation. More, the effects on output are greatly buffered.

¹⁶The real interest rate increases during the adjustment process. Remember the real interest rate refers to the next period inflation.

lambda*var(infl.)+(1-lambda)*var(prod.)					
lambda=0					
posição	regra	theta	phi	gam	função
1	T	0.25	1	0	1.0000
2	T	0.5	1.25	0	1.0076
3	T	0.25	1.25	0	1.0135
4	T	0.5	1	0	1.0184
5	F10	0.25	1	0	1.0375
6	F9	0.25	1	0	1.0377
7	F8	0.25	1	0	1.0388
8	F3	0.25	1	0	1.0399
9	F2	0.25	1	0	1.0406
10	F7	0.25	1	0	1.0411
11	F4	0.25	1	0	1.0421
12	T	0.75	1.25	0	1.0429
13	F6	0.25	1	0	1.0434
14	F5	0.25	1	0	1.0437
15	F10	0.5	1	0	1.0483
16	F9	0.5	1	0	1.0487
17	F1	0.25	1	0	1.0494
18	F8	0.5	1	0	1.0509
19	B	0.25	1	0	1.0537
20	F7	0.5	1	0	1.0561

lambda=,5					
posição	regra	theta	phi	gam	função
1	T	0.75	1	0	1.0000
2	T	1	1.25	0	1.0051
3	T	1	1	0	1.0175
4	F2	2.5	1	0	1.0218
5	T	0.75	1.25	0	1.0229
6	F2	2.25	1	0	1.0231
7	F2	2	1	0	1.0260
8	F2	1.75	1	0	1.0313
9	F2	2.5	0.75	0	1.0376
10	F2	2.25	0.75	0	1.0395
11	F2	1.5	1	0	1.0403
12	F2	2	0.75	0	1.0426
13	F2	2.5	1.25	0	1.0459
14	F3	2.5	1	0	1.0466
15	F2	1.75	0.75	0	1.0477
16	F2	2.25	1.25	0	1.0491
17	F3	2.25	1	0	1.0496
18	F3	2	1	0	1.0543
19	F2	2	1.25	0	1.0544
20	F2	1.25	1	0	1.0553

lambda=1					
posição	regra	theta	phi	gam	função
1	B	1.25	1	0	1.0000
2	F1	2.5	0.75	0	1.0220
3	B	1.5	1	0	1.0227
4	F1	2.25	0.75	0	1.0244
5	T	1.25	1	0	1.0256
6	T	1.5	1.25	0	1.0286
7	F1	2.5	0.5	0	1.0307
8	F1	2	0.75	0	1.0345
9	F1	2.25	0.5	0	1.0349
10	B	1.5	1.25	0	1.0383
11	B	1.25	1.25	0	1.0412
12	T	1.25	1.25	0	1.0435
13	F1	2	0.5	0	1.0457
14	B	1	1	0	1.0473
14	B	1	0.75	0	1.0500
16	F1	1.75	0.75	0	1.0534
17	T	1	1	0	1.0581
18	F1	1.75	0.5	0	1.0645
19	F1	2.25	1	0	1.0677
20	B	1.25	0.75	0	1.0678

Notas:

*: FX denota a regra (10) com j=X; T a regra (11); e B a regra (12)

Tabela 5: Colocação segundo o critério (28) para choques correlacionados

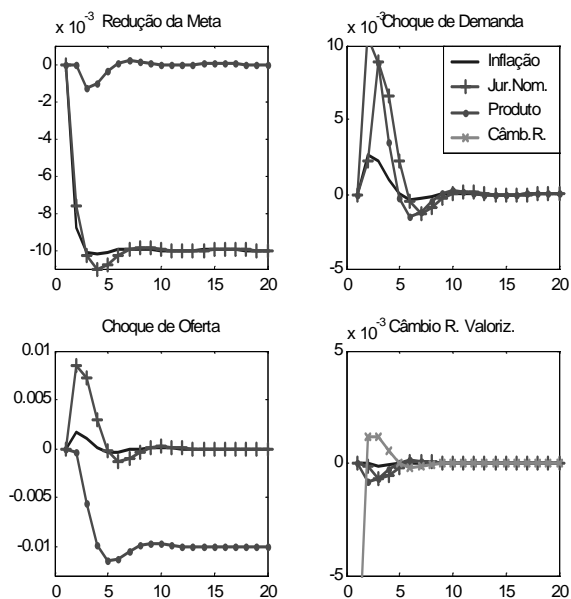


Figura 7: Resposta a impulso da regra $F(10;0,25;0,75,0)$ no modelo forward-looking

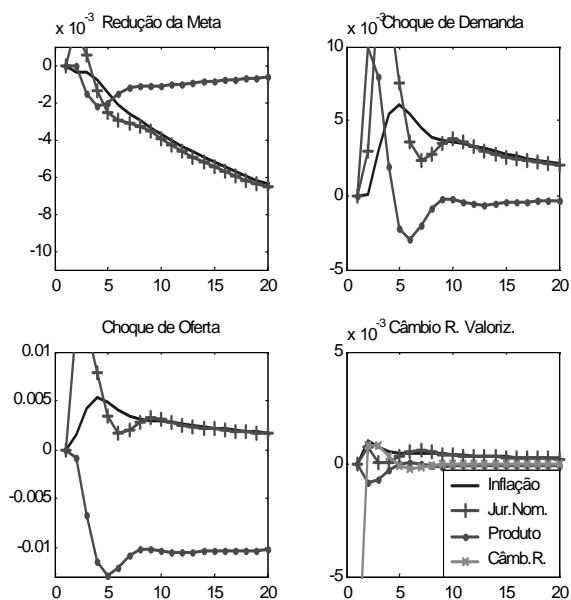


Figura 8: Resposta a impulso da regra $F(10;0,25;0,75,0)$ no modelo

backward-looking

The figure 8 illustrates the effect of this same rule in a backward-looking economy. The inflation target is more costively and slower. The effect of a transitory demand shock can only be neutralized with higher costs, due to the smaller degree of forward-lookingness and to the higher bigger effects of the interest rate over the output. A permanent supply shock also causes a more negative output overshooting and more inflation in this economy. Notwithstanding the bigger overshooting of the foreign exchange rate in the forward-looking economy, its effects over inflation and the recession caused by the temporary interest rate increase are bigger here. It is clear in all simulations that in this economy the inflation reaction to the monetary policy is lagged.

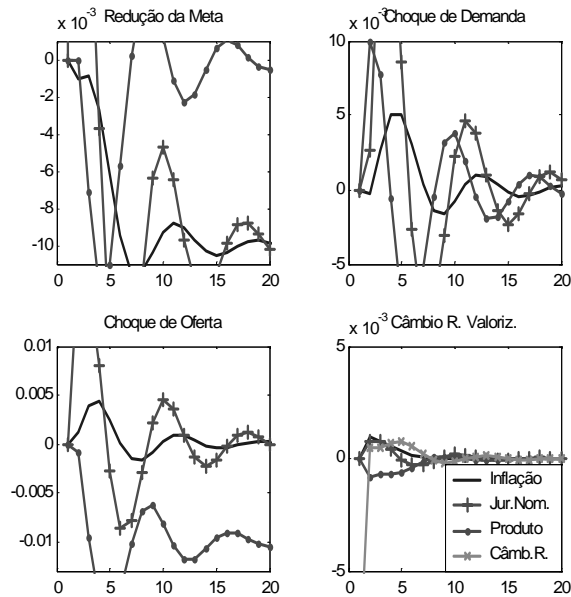


Figura 9: Resposta a impulso da regra $T(-1;1,25;1,25,0)$ no modelo backward-looking.

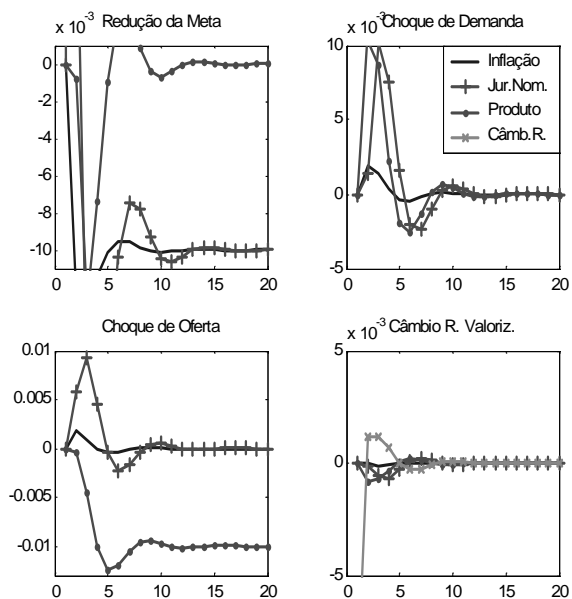


Figura 10: Resposta a impulso da regra $T(-1;2,5;0,5;0)$ no modelo forward-looking.

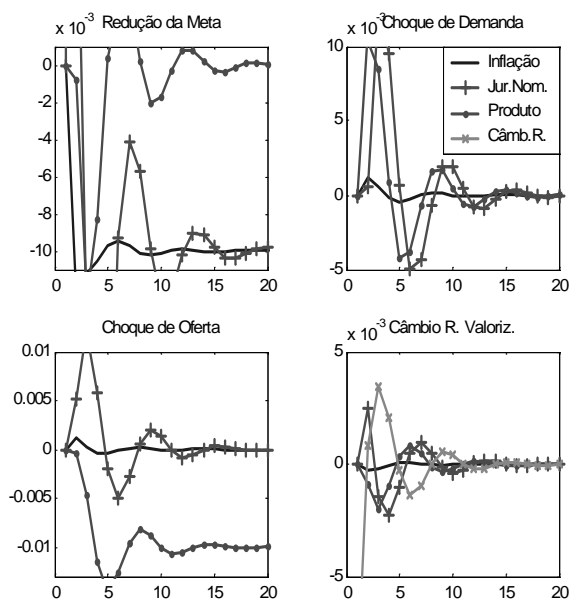


Figura 11: Resposta a impulso da regra $B(-1;2,5;0,5;0)$ no modelo forward-looking.

forward-looking.

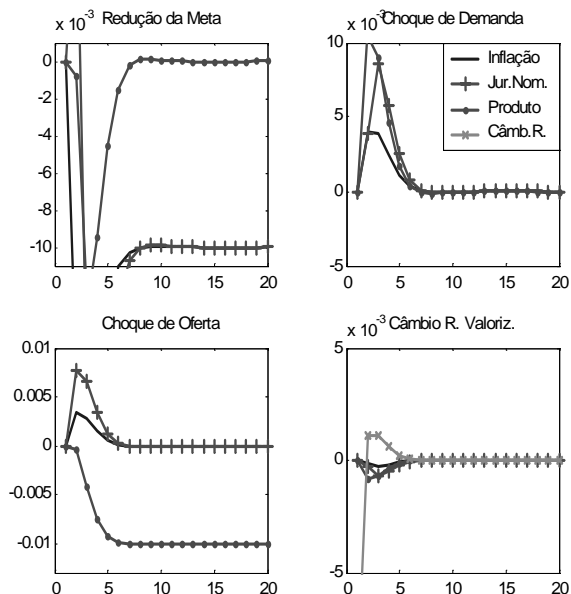


Figura 12: Resposta a impulso da regra $F(3;2,5;0,5;0)$ no modelo forward-looking.

7 Final Considerations and Extensions

Our work tries to identify the characteristics of the optimal monetary policy rules and their short-run dynamics in two versions of a small macroeconomic model estimated to the Brazilian economy. The main conclusions in relation to the optimal rules are that if the economy is forward-looking, we should give more importance to inflation than to output, and the policy rule should be less forward-looking. If the economy is backward-looking, both inflation weight and the degree of the rule forward-lookingness are dependent upon the relative importance the monetary authority puts on the inflation and the output variability. The bigger the relative concern with the variability of the output, the more forward should be the rule and smaller the weight on the inflation. In the backward-looking economy, an increase in the degree of forward-lookingness has the same effect that a reduction in the inflation weight.

The impulse responses confirm the result of the stochastic simulations. However, some results are noticeable. The forward-looking economy reacts to shocks with less oscillation in the output than the backward-looking economy. The monetary policy is able to contemporaneously affect inflation in the forward version, while there is some lag in the backward one. The corrective devaluation of the foreign exchange rate causes a bigger overshooting in the forward economy, but the following oscillations of inflation and output are smaller. Another important result is the low power of forward rules to reduce inflation, even if in a backward-looking economy. In this case, the inflation reduction goal is promptly reached by use of a Taylor rule with similar weights in the inflation and in the output, what causes substantial variation in the output.

Future research should extend the present work in many directions. The most immediate is the deeper investigation of the transmission mechanisms of monetary policy, explicitly dealing with the term structure and the credit channels. Another direction is the construction of an aggregate supply curve that explicitly models the nominal and real stickiness in the Brazilian economy: monopoly power and economics of information are promising routes. These aspects are relevant to understand the Phillips curve functioning, which is still an open issue in Brazilian applied macroeconomics.

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