LACEA 2001
MONTEVIDEO URUGUAY

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Endogenous Normative Indices for the Industry Concentration

ENDOGENOUS NORMATIVE INDICES FOR THE INDUSTRY CONCENTRATION (¤)

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(¤) preliminary version (May, 2001)

ABSTRACT

This paper presents new indices for measuring the industry concentration. The indices proposed (C_n) are of a normative type because they embody (endogenous) weights matching the market shares of the individual …rms to their Marshallian welfare shares. These indices belong to an enlarged class of the Performance Gradient Indexes introduced by Dansby&Willig(1979). The de…nition of C_n for the consumers allows a new interpretation for the Hirschman-Her…ndahl index (H), which can be viewed as a normative index according to particular values of the demand parameters. For homogeneous product industries, C_n equates H for every market distribution if (and only if) the market demand is linear . Whenever the inverse demand curve is convex (concave), H underestimates(overestimates) the industry concentration measured by the normative index. For these industries, H underestimates (overestimates) the concentration changes caused by market transfers from a small …rm to a large …rm according to the convexity (or the concavity) of the demand curve. For heterogeneous product industries, an explicit normative index is obtained for a linear market demand system derived from a quasi-linear utilility function. Under symmetric preferences among the goods, the index C_n depends on the market distribution and on the degree of the product di¤erentiation. C_n equates H if the goods are independent. Under the gross substitution assumption, the H-index often overestimates the normative concentration index and C_n converges to H from below as the degree of the product di¤erentiation increases. Under asymmetric preferences, a mean preserving spread of the product di¤erentiation making the smaller (larger) …rms to produce the more di¤erentiated products may improve (worst) the competition. A mean preserving spread of the substitution among the goods increases the concentration in industries with unequal market distributions. Under asymmetric preferences, the H-index often underestimates the normative measure of the concentration.

Key words: Concentration indices, market shares, Marshallian surplus, product di¤erentiation and substitution.

JEL Classi…cation: D63, L5, L11.

[1] An early version of this paper received useful comments from Afonso A.M. Franco Neto and the participants of the research seminary at EPGE-FGV. Thanks are also given to Marcelo Resende (IE-UFRJ) for encouragement. All erros are indebted to the author.

I - INTRODUCTION

This paper deals with the indices suggested in the literature for measuring the concentration (see Encaoua&Jacquemin, 1980 (hereafter, EJ), among others). When the n firms of the industry are competing according to the Cournot assumption, a natural measure for the industry performance is given by the following concentration index:

$$C \equiv \sum_{i=1} \theta_i s_i \gamma_i \qquad (1.0)$$

where the θ's are nonnegative functions, $s_i = q_i/Q$ is the market share for a firm i; $\frac{dQ_{(i)}}{dq_i}$ is the conjectural variation of a producer, $\gamma_i \equiv 1+\frac{dQ_{(i)}}{dq_i}$ and q_i are the equilibrium quantities supplied by the producer i: ($Q_{(i)} = \sum_{j=1(\neq i)} q_j$). As usual, market shares are ranked in a decreasing order: $s_1 > s_2 > ... > s_n$.

The problems related with the implementation of such indices were discussed in Donsimoni&Geroski&Jacquemin,1984 (hereafther, DGJ). However, as we know, the problems pointed out there still waiting a solution.

The first point deals with the normative judgement embodied in the choice of the weights θ_i. Often, the use of concentration indices is justified by assessing market imperfections in order to make comparisons between different industries, or to provide theoretical basis for market regulatory policies (on behalf the competition or the consumers).

The positive solution given to this problem sets the θ_i's as nonnegative functions of the market shares s_i : $\theta_i = h(s_i)$ which leads to the usual indices like the concentration ratio C^k (for the k greater firms), the Hirschman-Herfindahl(1950) index $H = \sum_{i=1}^{n} s_i^2$, the entropy index C^E, a.s.o.[2] So, mathematicaly C is an application from the positive unitarian simplex S_{n-1}^{+} into $\Re_+$.

According to EJ (pp.88-92), besides the symmetry property, good concentration indices must verify the following properties:

(P1) Transfer principle: if $s_2 < s_1$ and $0 < \Delta < s_2$, then $C(s_1+\Delta, s_2-\Delta, s_3, ..., s_n) > C(s_1, s_2, s_3, ..., s_n)$;

[2] In absence of conjectures ($\gamma_i = 1$), one obtains different expressions for C defined in (1.0) : (i) $C^k \equiv \sum_1 s_{(i)}$, for $\theta_i = 1$ for $i = 1, 2, ..., k$, and $\theta_i = 0$ otherwise; (ii) $H \equiv \sum_1 s_i^2$, for $\theta_i = s_i$, $i = 1, 2, ...n$; (iii) $C^E \equiv -\sum_1 s_i \log s_i$, for $\theta_i = -\log s_i$, $i = 1, 2, ...n$.

(P2) Minimality under symmetry: $C(\frac{1}{n}, ..., \frac{1}{n}) = \min_{\{s_1, ..., s_n\} \in S^+_{n-1}} C(s_1, ..., s_n)$;

(P3) Lorenz criterium: Let $r_1, ... r_n$ and $s_1, ... s_n$ be the market shares from two industries both with size n and C^k_r e C^k_s be the market shares for the k greater firms in each industry, respectively. If $C^k_r \geq C^k_s$, for $k = 1, 2, ... n$, then $C(r_1, ... r_n) \geq C(s_1, ... s_n)$. By this property, the index obtained is a sufficient measure for the inequality occurring in the industry (possibly exempting one defining a welfare function for ordering different distributions).

Similar though it may appear, concentration and inequality concepts are not equivalent one another (as EJ noted, p.89) so that it seemed sound to require two further properties:

(P4) Nondecreasing property facing horizontal mergers: if s' is the market share of the new firm issued from a merger of firms 1 and 2 (of any size) then $C_{n-1}(s', s_3, ... s_n) \geq C_n(s_1, ..., s_n)$;

(P5) Nonincreasing property under symmetry: $C_n(\frac{1}{n}, ..., \frac{1}{n})$ does not increase w.r.t. to n.

D.Encaoua and A.Jacquemin have shown that if P1 is true, then P2 and P3 also will be true, and that the satisfaction of P2 and P4 ensures that P5 holds. Further, if h is some function defined on $[0, 1]$, the authors show that the convexity of the function $sh(s)$ is a sufficient condition for $C = \sum_1^n s_i h(s_i)$ verifies P1 and that the nondecreasing property of $h(s)$ is sufficient for C verifies also P4.

However, some market transfers benefitting larger firms need not always be bad from a normative view on the concentration. On the other hand, since a horizontal merger implies a change in the industry size and structure, property $P4$ may be questioned on the grounds of the productive sinergies that may follow a merger[3] or, alternatively, the existence of uncertainty averse producers in the industry.[4]

This paper specifies weights leading to the definition of a normative concentration index from the consumers' point of view (noted C_n) that checks properties $P1 - P5$ but only for homogeneous product industries. For differentiated industries, property $P1$ is not necessarily verified. Despite its normative trait, such index assume positive expressions, as applications from S^+_{n-1} into $\Re_+$ as it will be shown ahead.

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[3] See WILLIAMSON(1968), FARRELL&SHAPIRO(1990), among others.

[4] Boff(1999) shows that merges that do not increase the productive efficiency may benefit the consumers if all producers are strongly uncertainty averse.

The second problem raised in DGJ's paper refers to the observation and the measurement of the market conduct, which is represented in the formula of C by the parameters °: This problem will not be treated here, so that we assume $° = 1$ throughout the paper.

A natural way to avoid arbitrary choices of the coefficientes μ^0s would be to specify an utility function for the industrial product, as suggested in DGJ(p.424). In partial analysis, one may use the economic surplus generated by the industry which, under the assumption that price changes do not generate revenu effects, can be estimated by the Marshallian surplus calculated from the demand and cost functions. For defining the weights in this paper, we introduce a surplus measure calculated from the residual demand of the firms.

Motivation.

What does motivate a call for normative concentration indices ? In normative analysis, it is relevant to know the ease as the industry activity is able to generate surplus and, in particular, to transfer significant economic values for the consumers, while succeeding to preserve a sufficient degree of competition among the producers. In this sense, it is useful having at hand an index that matches any change in the market position of the firms to the implied changes in their relative contribution to the total welfare generated in the industry. The indices usually employed for guiding industrial policies (particularly, C^k and H) are unsatisfactory for this aim. The shortcomings appear more visibly in the context of mergers or entries. For example, it is easy to show that under Cournot competition the acquisition of an inefficient firm by an efficient incumbent may lead to an increase in the total welfare, meanwhile the H index shows an increase in the measured concentration. This is so because such index does not mitigate the increased market share by taking into account the efficiency gains internalized by the new entity. On the other hand, a privately profitable entry of a new firm into the market may often be socially undesirable under some circumstances. That may be the case when there is setup costs (i.e. scale economies) in the industry (Weizsäcker,1980). In presence of scale economies, Perry(1984) shows that there is a tendency for excessive entry (w.r.t. to the number of firms that maximizes total welfare). A reduction in welfare may also occur when the entrant has an absolute cost disadvantage w.r.t. the incumbents (Klemperer,1988). In all these (and other) cases, we may find that an increase in the competition (signalled by a decrease in H) is accompaigned by a reduction in total welfare[5]. From the point-of view of the consumers, it is well-konwn that the entry of new …rms into the industry (or mergers among incumbents) make room for changes in the actual di¤erentiation and substitution structures prevailing among the goods supplied. Since the usual indices do not re‡ect the map of preferences, they are neither able to capture possible transfers of utility values for the consumers that may follow mergers or entries. A related argument was pointed out recently for assessing the market power of …rms (Young, 2000). It is advocated the need of endogenizing the preferences of the consumers in order to capture the ability of …rms to gain advantage over their rivals by creating asymmetries in the demand for their products.

The normative indices presented below do not give answers for all normative requirements referred above. For the cost economies saved by a particular merger be taken into account, one need using total surplus and de…ning a general index instead of an index for the consumers only, as we did in the paper. However, we believe that the method presented here may open the way for building up other normative indices.

In order to be useful in policy, a concentration index should be calculated directly from the observed data, disregarding any assumption on the (misobserved) conduct of the producers. The paper proposes a theoretic-based index displaying such characteristics. Do it re‡ects abolute changes in the economic surplus ? No, only relative changes are captured. Facing a market transfer or a merger among the incumbents, or the entry of a new …rm or even changes in costs or in tastes, the induced variation in the concentration index is the result of a tradeo¤ between a (undesirable) market position effect and a (desirable) welfare e¤ect. If the index increases, it means that the market position e¤ect dominates; if the index decreases, the welfare e¤ect prevails.

The paper is organized as follows. Section II built up the normative concentration index after the de…nition of a weighted Industry Performance Gradient Index, from a concept introduced by Dansby&Willig(1979). Section III presents a method enabling to measure the welfare contribution

[5] The possibility of a nondecreasing and nonmonotonic relationship between H and the welfare was emphasized by DAUGHETY(1990) in the framework of the Stackelberg equilibrium. On page 1234 he wrotes: "This suggests that in and of themselves, such mesures [H and C^4] do not reveal what one would most like to know: when competition has increased and welfare has improved."

of the individual …rms from their residual demand. A class of endogenous concentration indices for homogeneous product industries is obtained. The sensibility of such indices to changes in the preferences of the consumers are analysed and comparisons are made with the Hirschman-Her…ndhal index, that also belongs to this class. In Section IV we apply our de…nition in order de…ning a class of concentration indices for heterogeneous product industries. The role played in the concentration measure by the substitution degree among the goods is analysed by using a linear class of demand functions. In the symmetric case, comparisons are made with the H index and a sensibility analysis of the normative index to changes in the degree of the product di¤erentiation is carried out by simulation with two hypothetical 5-…rms industry.

II- NORMATIVE CONCENTRATION INDICES

This section focus on the criteria used for de…ning a general normative concentration index for the industry.

2.1 Industry performance gradient indices (IPGI)

In order getting a theoretical suggestion for the formula of our concentration index, we refer here to the Industry Performance Gradient Index $(Á)$; an outstanding concept introduced by Dansby&Willig(1979). This index measures the instantaneous rate of change in the industry welfare function V if the actual output vector q is adjusted in the optimal path $q^{¤}(t)$: $Á = \lim_{t ! 0} \frac{V(q^{¤}(t)) ¡ V(q)}{t}$: The function V is assumed to be concave and C^2 in a open neighborhood of q: De…ning $q^1 = [q^1_1; q^1_2; :::; q^1_n]^0$ and $\Delta_q = q^1 ¡ q$ for the quantity adjustment vector, the optimal vector $q^{¤}(t)$ maximizes V for quantity adjustments Δ_q within a closed hypersphere centered at q with radius t: However, for making q^0_is commensurable (in money value) Dansby&Willig use the actual price vector $P = [P_1; P_2; :::; P_n]$ as scaling factors for the adjustments.

The basic idea justifying such a restriction is that promoting adjustments in the industry structure are costly for the regulator. So, if the adjustments costs to be payed for bringing the actual industry position to a socially better position are larger than the budget, the regulator will be con…ned to look for optimal adjustment paths that are feasible regarding the resources available to him.

Note now $y = [y_1; y_2; :::; y_n]^0$ with $y_i = P_i q_i$ and, analogously, y^1 for the column vector with elements $y^1_i = P_i q^1_i$: The path of the adjustments are outlined by the authors through the usual Euclidean metric applied to the price-based money value of the adjustments $\Delta_y = y^1 - y$: $d(y;y^1) = \sqrt{\Delta_y' \Delta_y}$: So, $V(q^*(t))$ is the maximum of $V(q^1)$ under the restriction $\sum_{i=1}^{n} (P_i \Delta_{q_i})^2 \leq t^2$: By assuming $\frac{\partial V}{\partial q_i} = P_i(q) - \phi_i'(q_i)$; and by using the envelope theorem Dansby&Willig obtain: $\phi^2 = \sum_{i=1}^{n} \lambda_i^2$; where $\lambda_i = [P_i(q) - \phi_i'(q_i)]/P_i(q)$ is the Lerner index for the firm i:

The value of the IPGI depends on the choice of the metric d. Of course, one need not be confined by the usual Euclidean metric. For instance, if one choses a weighted metric like $d(y;y^1) = \sqrt{\Delta_y' \Sigma \Delta_y}$; where Σ is any positive definite and symmetric (pds) matrix of $n-$ order, he can easily check that the weigthed index obtained $(IPGWI)$ is:

$$\phi_\omega = \sqrt{\lambda' \Sigma^{-1} \lambda} \qquad (2.0)$$

where $\lambda = [\lambda_1; \lambda_2; ....; \lambda_n]'$ is the vector of the actual Lerner indices.[6]

The $IPGWI$ is an inverse measure for welfare. Indeed, since V is concave, the larger ϕ is, the more remote from the welfare optimum the industry position (q) is. Notice that Dansby&Willig use an unweigthted metric for the money value of the quantity adjustments of the firms: $\Sigma = I$:

In order to calculate the normative index, we need to estimate the matrix Σ: For this aim, we will use the actual welfare shares ω_i (or ω_{ij}) which will be defined in section 3.2 and section IV:

[6] Define the matrix $D_p = Diag(P_1; P_2; ....; P_n)$: So, $\Delta_y = D_p \Delta_q$. Therefore: $d^2(y;y^1) = \Delta_y' \Sigma \Delta_y = \Delta_q' D_p \Sigma D_p \Delta_q \leq t^2$: By assumption, we have: $\frac{\partial V}{\partial q^1}(q) = D_p \lambda$ (a)

The Lagrangean is: $L(q^1; \mu) = V(q^1) + \mu[t^2 - \Delta_q' D_p \Sigma D_p \Delta_q]$

The first-order conditions for a maximum gives: $\frac{\partial V}{\partial q^1}(q^*(t)) = 2\mu D_p \Sigma D_p \Delta_{q^*}$ (b) and $\Delta_{q^*}' D_p \Sigma D_p \Delta_{q^*} = t^2$ (c) where $\Delta_{q^*} = q^*(t) - q$, with $\mu > 0$:

On the other hand, by the envelope theorem we have:

$\frac{dV}{dt}(q^*(t)) = \frac{\partial L}{\partial t} = 2\mu t$ (d)

Since matrix Σ is positive definite and prices are positive we may write, from (b) : $[D_p \Sigma D_p]^{-\frac{1}{2}} \frac{\partial V}{\partial q^1}(q^*(t)) = 2\mu [D_p \Sigma D_p]^{\frac{1}{2}} \Delta_{q^*}$: Squarring both sides of this equation and by using (c) we arrive to:

$[\frac{\partial V}{\partial q^1}(q^*(t))]' [D_p \Sigma D_p]^{-1} [\frac{\partial V}{\partial q^1}(q^*(t))] = (2\mu t)^2$ (e). Let squarre now equation (d). By equating then its lhs with the lhs of (e) and taking the limit $(\lim_{t \to 0} q^*(t) = q)$ on both sides of the resulting equation (since V is C^2 in a open neigborhood of q) we obtain:

$\phi_\omega^2 \equiv \lim_{t \to 0} [\frac{dV}{dt}(q^*(t))]^2 =: [\frac{\partial V}{\partial q^1}(q)]' [D_p \Sigma D_p]^{-1} [\frac{\partial V}{\partial q^1}(q)]$:

By using now the definition (a) we arrive to: $\phi_\omega^2 = \lambda' \Sigma^{-1} \lambda$: ¤

Two basic ideas are driving the choice of the present metric:

(i) the speed of an overall adjustment in the firms' quantities should be higher as the number of firms in the industry is higher;

(ii) for an overall adjustment of a given size, the weight allocated to the money value of a symultaneous adjustment in the quantities of the firms i and j $(P_i(q_i^1 - q_i)P_j(q_j^1 - q_j))$ should be higher as the welfare share ω_{ij} is higher $(\; i, j = 1, 2, \ldots, n)$).

These two ideas suggest the following structure for the matrix Σ :

$$\Sigma = n\Omega;$$

where $\Omega = [\omega_{ij}]$ is a $n-$ order matrix of the firm's relative share in the total surplus generated in consumption through changes in their own production (ω_{ii}) or through changes in the supply of a rival (ω_{ij}). Notice that the ω's are calculated for a given market structure s. So Ω will depend on this particular market structure, so that we will note Ω_s.

2.2 Concentration indices

We define the normative concentration index by:

$$C_n \equiv \frac{1}{n} s' \Omega_s^{-1} s \qquad (2.1)$$

where $s = [s_1, \ldots, s_n]'$ and, for each $s \in S_{n-1}^{+}$, Ω_s is some positive definite and symmetric (pds) matrix of $n-$ order whose entries ω_{ij} are function of the contribution of the firm i to the economic surplus through the supply of the firm j.

For homogeneous product industries, formula (2.1) can be obtained directly from the IPGWI (2.0); under the Cournot competition assumption, since the first order condition for the industry equilibrium implies $\lambda = \frac{1}{\varepsilon} s$ (where ε is the modulo of the price elasticity of the market demand at the equilibrium point).

The above equilibrium condition does not hold for heterogeneous product industries so that for these industries formula (2.1) is to be taken as a definition. Obviously, such a definition suffers from some drawbacks, because in this case firms get a natural market power by selling differentiated products so that, there is not necessarily an increasing and monotonic relation between the price-cost margin λ_i of a firm i and its market share s_i.[7] However, we are looking for an aggregate index; so, we expect we can normally observe a regular (and positive) correlation between ϕ^2_ω and C_n.

Another expression for (2.1) emphasizes a particular relation with the concentration ratios C^k. Indeed, from the positive definiteness of Ω_s it is known that there is a lower triangular matrix $T = [t_{ij}]$ such that $\Omega_s^{-1} = T'T$ (see Rao,1973, p.74). This allows to write (2.1) as:

$$C_n = \frac{1}{n}\sum_{k=1}^{n}\left(\sum_{j=1}^{k} t_{kj}s_j\right)^2$$

where the term in parenthesis is a weighted concentration ratio of order k. Thus, the normative index C_n is also a mean of squares of all weighted C^k $(k = 1, 2, ..., n)$.

An appealing trait of the definition (2.1) is that it allows inserting the study of the industry concentration in the analysis of positive quadratic forms. In particular, by a classical theorem of Linear Algebra, there is a linear orthogonal transformation of the market share coordinates $s_1, s_2, ..., s_n$, say $m = R's$, allowing to write (2.1) in the new (orthogonal) coordinates $m_1, m_2, ..., m_n$ as:

$$C_n = \sum_{i=1}^{n}\left(\frac{m_i}{\sqrt{n\nu_i}}\right)^2 \qquad (2.1a)$$

where $R = [r_1, r_2, ..., r_n]$ is the orthogonal matrix of the eigenvectors r_i associated to the positive eigenvalues ν_i of the matrix Ω_s. Notice that $m_i = r_i's$.

The next step examines what additional conditions should be imposed on the matrix Ω_s for C_n verifying property $P5$. Property $P2$ requires $C_n(s)$ achieves a minimum for a market share vector $s = \frac{1}{n}1$ (where 1 is a vector of one's). Take for a moment $\Omega_s = \Omega$, a constant matrix. In this case, the positive definitness of the matrix ensures that $C_n(s)$ is a convex function, so that $C_n(s)$ will achieve a minimum in the simplex S^+_{n-1} at $s^* = (\frac{1}{1'\Omega 1})\Omega 1$, with $C_n(s^*) = (\frac{1}{1'\Omega 1})\frac{1}{n}$. Property $P5$ requires this value should be decreasing in n. So, we will impose the condition:

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[7] Recall that the first order condition for the industry equilibrium requires $\lambda_i = 1/\varepsilon_i$, where ε_i is the modulo of the price-elasticity of the demand q_i, whatever the strategic variable is ($\varepsilon_i = -\frac{p_i}{q_i(p)}\frac{\partial q_i(p)}{\partial p_i}$ if prices and $\varepsilon_i = -(\frac{q_i}{p_i(q)}\frac{\partial p_i(q)}{\partial q_i})^{-1}$ if quantities) .

$$c1: \qquad \mathbf{1}'\Omega_s\mathbf{1} = 1.$$

Condition $c1$ means that the elements of the weighting matrix Ω_s should sum to 1. The argument for the satisfaction of property $P2$ is developed below.

Comparisons with the $H-$index

The $H-$index is a normative index with weighting matrix $\Omega = \frac{1}{n}I$.

Once the matrix $\Sigma = n\Omega_s$ is given, there is an upper bound for the normative index in (2.1):

$$C_n(s) \leq \frac{1}{n}(\max_{1\leq i\leq n} \omega^{ii})$$

where ω^{ii} is the i^{th} element of the diagonal of Ω_s.[8] This upper bound does not restraint C_n to be smaller than H, because (since Ω_s is pd): $\omega^{ii} \geq 1/\omega_{ii} \geq 1$. However, the upper bound is minimal when Ω_s is a diagonal matrix (because $\omega^{ii} = 1/\omega_{ii}$ iff $\omega_{ij} = 0$ for all j, $j \neq i$).

The paragraphs in the sequel make clear that the $H-$index may overestimate as well as underestimate the normative concentration measure defined in (2.1).

Let $q^H(t)$ and $q^\omega(t)$ be the optimal paths passing through the industry position q, under the unweighted and weighted values of the adjustments, respectively. Recall that the concentration indices H and C_n are defined as the derivative of the welfare function at q, in the gradient direction that is, when the quantity adjustments move along the optimal paths $q^H(t)$ or $q^\omega(t)$ as $t \to 0$, respectively. Let $|x|$ be the Euclidian norm of a vector x. By using the notations of the previous section, for a given adjustment size $t > 0$, the points $q^H(t)$ and $q^\omega(t)$ lie on the boundary of the elypsoides $|\Delta y| \leq t$ and $\|\Delta y\| = \sqrt{(\Delta y)'\Sigma^{-1}(\Delta y)} \leq t$, respectively. The performance gradient index is obtained by taking Σ as a constant matrix. It is known that, for each t, the Euclidian hyperesphere $|\Delta y| \leq t/M$ lies inside the elypsoide $\|\Delta y\| \leq t$, where $M = \max_{1\leq i\leq n} \omega^{ii}$.

If the matrix Ω_s is "ill-shaped" (e.g., terms $\omega_{ii}/(\min_j \omega_{ij})$ are large and with high variance) the optimal path $q^\omega(t)$ will not be smooth. So, in the

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[8]Note the norm of a vector x by $\|x\| = \sqrt{x'\Sigma^{-1}x}$, and e_i for the i^{th} vector of the standard basis on R^n. For the market share vector s we have: $s = \sum_i s_i e_i$, so that $\|s\| \leq \sum_i s_i\|e_i\| \leq (\frac{1}{n}\max_{1\leq i\leq n}\omega^{ii})^{1/2}\sum_i s_i = (\frac{1}{n}\max_{1\leq i\leq n}\omega^{ii})^{1/2}.$ ∎

neighborhood of $t = 0$; it may be that the directional derivative along the path q^{ω} is higher than that along q^H; so that $\phi^{\omega} > \phi^H$. The reversed condition ($\phi^{\omega} \leqslant \phi^H$); may occurs when the matrix of the weights is "well-shaped" . Technically, this implies that, as t approaches 0; the speed of the adjustments along the weigthed adjustment path is smaller (in mean) than that along the unweigthed path.[9]

For property P5 be satisfied, C_n must be bounded from below by $1/n$; with $C_n(\frac{1}{n}1) = 1/n$. This is so because an equalitarian market structure ($s = \frac{1}{n}1$) can only be soustained if the technology employed by the firms and the preferences of the consumers are both symmetric. As we know, this is the case when the economic surplus is maximum. In order to show that the value of C_n is $1/n$ in this case, let consider that the symmetry of firms and consumers implies that the diagonal and off diagonal elements of Ω_s must be constant, say: $\omega_{ii} = \omega_1$ and $\omega_{ij} = \omega_o$. In this case, Ω_s takes the following form: $\Omega = [aI + \omega_o 11']$; where $a = \omega_1 - \omega_o$. The inverse Ω^{-1} can be performed straightforwardly; it gives: $\Omega_s^{-1} = n\frac{(a+n\omega_o)}{a^2}[aI - \frac{\omega_o}{1+n\omega_o/a}11']$. Then, it is easy to check that $1'\Omega_s^{-1}1 = n^2$; and therefore, $C_n(\frac{1}{n}1) = 1/n$ for all admissible values of ω_1 and ω_o.

As we will see later, the overestimation bias of the $H-$index happens under particular features of the consumer's preferences: in homogeneous product industries it happens when the aggregate market demand is linear or concave. In heterogeneous product industries facing linear demand functions, it occurs when the preferences among the goods are symmetric. Under asymmetric preferences, the $H-$index often underestimates the normative measure of the concentration.

III - HOMOGENEOUS PRODUCT INDUSTRIES

This section starts introducing a measure for the welfare generated by the firms' activity.

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[9] Note $DV(q)$ for the gradient of the welfare function at q; $Dq(t)$ for the vector of speeds and $\Delta q(t) = q(t) - q$. Assume that for $t > 0$ close to zero the following inequality holds: $DV(q) \cdot \Delta q^H(t) > DV(q) \cdot \Delta q^{\omega}(t)$.

From the concavity of V; we have: $V(q^{\omega}(t)) - V(q) \leqslant DV(q) \cdot \Delta q^{\omega}(t)$. Dividing both sides of the former inequality by t and taking the limit we get sucessively: $\phi_{\omega} \leqslant \lim_{t \to 0} DV(q) \cdot \frac{\Delta q^{\omega}(t)}{t} \leqslant DV(q) \cdot \lim_{t \to 0} \frac{\Delta q^H(t)}{t} = DV(q) \cdot Dq^H(0) = \phi_H$.

3.1 The firm contribution to the economic surplus

Assume that the inverse demand $P(Q)$ is a positive and continuous function defined over a given closed and bounded set $[\underline{Q}\,;\,\overline{Q}]$, strictly decreasing w.r.t. the industry output $Q = \sum_{i=1}^{n} q_i$. Assume also that the cost functions ϕ_i are continuously differentiable w.r.t. the firm i quantity q_i $(i = 1; 2; ...; n)$: Define the economic surplus generated by the firm i (W_i) by:

$$W_i \equiv \int_0^{q_i} [P(Q_{(i)} + x) - \phi_i'(x)]dx \qquad (3.0)$$

where $Q_{(i)} = Q - q_i$: The contribution share of firm i (ω_i) for the total surplus will be: $\omega_i = W_i / \sum_1^n W_i$:

Alternatively, if one focus the analysis of the concentration from the point-of-view of the consumers he is led to consider the firm's i share in the consumer surplus, (ω_i^c) which is defined analogously from the consumer surplus generated by the firm:

$$W_i^c \equiv \int_0^{q_i} P(Q_{(i)} + x)dx - P(Q)q_i \qquad (3.1)$$

Notice that the integrand in (3.0) is nonnegative, so ω_i (or ω_i^c) are positively related with s_i. However, the question knowing wether or not the market share of a firm is larger or smaller than its welfare share is an open question. A priori, s_i/ω_i may be both greater or smaller than 1. Indeed, since the functions P and ϕ_i are assumed to be continuous, the mean value theorem applies for the equation (3.0), yielding a linear relationship between the firms supply and the welfare generated by it: $W_i = k_i q_i$; with

$$k_i \equiv P(Q_{(i)} + x_i) - \phi_i'(x_i) \qquad (3.2)$$

where $0 < x_i < q_i$:

$k_i = (W_i / q_i)$ is the mean welfare value (in money units) generated by one unit produced by the firm i. It is a social indicator for the productivity of the firm. Notice that k_i depends on the parameter x_i:

An analogous relation may be set for the consumer surplus: $W_i^c = k_i^c q_i$; with

$$k_i^c \equiv P(Q_{(i)} + x_i^c) - P(Q); \qquad (3.3)$$

where $0 < x_i^c < q_i$: Here, k_i^c is the mean value generated for the consumers by one unity produced by the firm i: The analysis led in the sequel for ω also applies to ω^c (and k^c). From the previous relations we have: $s_i/\omega_i = (\sum_i^n k_i s_i)/k_i$ or, putting $\overline{k}_s \equiv \sum_i^n k_i s_i = W/Q$:

$$s_i/\omega_i = \bar{k}_s/k_i \qquad (3.4)$$

This equation shows that the equality $s_i = \omega_i$ holds for all firms if and only if $k_i = \bar{k}_s$; i.e., if all firms are identical.

One unit produced by a firm i borning a low marginal cost generate a mean welfare value (k_i) above the weighted mean value of the industry ($\bar{k}_s$) and thus, $s_i/\omega_i < 1$. Analogously, a firm with high marginal cost generates a mean welfare value lower than the weighted mean value for the industry $(k_i < \bar{k}_s)$ and therefore its contribution to the aggregate supply is larger than that for the economic surplus ($s_i/\omega_i > 1$).

3.2 Normative indices

For building up a concentration index along (2.1) for the homogeneous product case, we can neglect the cross effects and define the weighting matrix as: $\Omega_s = \mathrm{Diag}[\omega_1, \omega_2, ..., \omega_n]$.

However, for C_n be a positive index, the simplex S^+_{n-1} must be its domain, which in turn needs the welfare shares ω must having an unique representation as a function of the market distribution $s = (s_1, s_2, ..., s_n)$ and the parameters of the demand. If one wants assessing the concentration from a broad point-of-view for homogeneous product industries, this feature is not achieved generally, because the shares ω in (3.0) depend explicitly on the cost functions and the profit shares. But if one bounds the evaluations to a consumer point of view, he can often obtain an exhaustive description of the consumers' welfare shares ω^c defined in (3.1) by the market distribution only.

Example: Let the inverse demand function be:

$$P(Q) = \alpha - \beta Q^{\rho}; \quad \alpha, \beta, \rho > 0 \text{ and } Q < (\tfrac{\alpha}{\beta})^{\frac{1}{\rho}}.$$

The demand is concave for $\rho > 1$; convex for $\rho < 1$ and linear if $\rho = 1$. The use of (3.1) leads to:

$$W_i^c = \frac{\beta Q^{1+\rho}}{1+\rho}[(1+\rho)s_i + (1-s_i)^{1+\rho} - 1]. \qquad \text{So,}$$

$$\omega_i^c = [(1+\rho)s_i + (1-s_i)^{1+\rho} - 1]/[1+\rho+\textstyle\sum_{j=1}^n (1-s_j)^{1+\rho} - n] \qquad (3.5)$$

If ρ tends to zero ($\rho \to 0$), the demand becomes perfectly elastic w.r.t. the price. The consumer's welfare share of the firm i will tend to ω_i^o, with:

$$\omega_i^o = [s_i + (1-s_i)\log(1-s_i)]/[1+\textstyle\sum_{j=1}^n (1-s_j)\log(1-s_j)] \qquad (3.6)$$

(here, log stands for the natural logarithm). If $\rho \to +\infty$ the condition $P > 0$ requires $0 < Q \leq 1$. If $Q = 1$, the elasticity is not defined; if $0 < Q < 1$ the elasticity ε_n ($= \frac{1}{\rho}(\frac{\alpha}{Q^{\rho}} - 1)$) tends to infinity (as $P \to \alpha$). Then, the consumer's welfare share of the firm i will tend to:

$$\omega_i^{\infty} = s_i.$$

With a diagonal matrix Ω_s formula (2.1) gives the following index for homogeneous product industries:

$$C_n \equiv \frac{1}{n} \sum_1^n \left(\frac{s_i}{\omega_i^c}\right) s_i \qquad (3.7)$$

As it was expected, the simulations made in section 3.5 for a particular family of demand functions show that C_n verifies the transfer principle property P1. Hence, property P3 is also checked. Simulations made also show that property P4 is fulfilled.

3.3 A class for normative indices

Different specifications for the demand function e.g., $P(Q;\gamma)$, will allow different classes of normative indices $C_n(\gamma)$ to be defined according to the definition (3.7), where γ is a given parameter vector.

A large class of such indices is generated by the three parameters demand function used in the previous example i.e., $P = \alpha - \beta Q^{\rho}$, with $\gamma = (\alpha, \beta, \rho)$. As it is shown below, the representative index for such a class only depends on the parameter ρ. This particular class will allow to obtain a general property for the normative indices given in (3.7) when the demand is linear ($\rho = 1$).

By substituting in (3.7) the values of ω^c given in (3.5), the following index is obtained:

$$C_n(\rho) = \frac{\left[1 + \rho + \sum_{j=1}^n (1 - s_j)^{1+\rho} - n\right]}{n} \sum_{i=1}^n \frac{s_i^2}{(1+\rho)s_i + (1 - s_i)^{1+\rho} - 1} \qquad (3.8)$$

Notice that $C_n(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}; \rho) = \frac{1}{n}$ and, for $n = 1$, $C_n(1;\rho) = 1$ for every ρ.

By taking the derivative of (3.8), given the constant market distribution $(s_1, \ldots, s_n)$, one can show that $C_n(\rho)$ is a decreasing function of ρ[10]:

[10] By taking the derivative of ω_i w.r.t. ρ and evaluating it at $\rho = 1$ we get $\frac{\partial \omega_i}{\partial \rho}|_{\rho=1} \gtrless 0 \Leftrightarrow \omega_i^c \lessgtr \omega_i^1 = \frac{s_i + (1 - s_i)^2 \log(1 - s_i)}{1 + \sum_j (1 - s_j)^2 \log(1 - s_j)}$. Ceteris pribus, a firm not too efficient ($\omega_i^c < \omega_i^1$) gets a greater welfare share when the inverse demand function is concave than when it is convex (conversely if the firm is highly efficient $\omega_i^c > \omega_i^1$).

For $\rho \to 0$; the use of $(3{:}6)$ leads to an upper bound for $C_n(\rho)$:

$$C_n^o = \frac{\sum_{j=1}^{n}[s_j + (1 - s_j)\log(1 - s_j)]}{n} \sum_{i=1}^{n} \frac{s_i^2}{s_i + (1 - s_i)\log(1 - s_i)} \quad (3{:}9)$$

This is the normative concentration index in the convex limit case for this particular market function: the consumers are willing to match any output supplied at the constant price $P = \alpha - \beta$: Notice that C_n^o only depends on the market distribution s: Also, we can check that $C_n^o(\frac{1}{n}; \frac{1}{n} ; \ldots; \frac{1}{n}) = \frac{1}{n}$ and that, for the monopoly case $(n = 1) : \lim_{s \to 1} C_n^o(s) = 1$:

By making $\rho \to +\infty$; $C_n(\rho)$ converges to:

$$C_n^{\infty} = \frac{1}{n} \quad (3{:}10)$$

which is the minimal value of the index in the concave limit case (symmetric firms). This is also the industry concentration index under perfect competition.

Since the upper bound $C_n^o(s)$ decreases as the variance of the market shares decreases, from the point-of-view of the consumers the range $r(s) = C_n^o(s) - C_n^{\infty}$ gives a measure of the allocative and productive inefficiencies associated with the market distribution $s = (s_1; s_2; \ldots; s_n)$, out of the number of firms and the preferences of the consumers.

The Hirschman-Herfindahl index is $H = \sum_{j=1}^{n} s_j^2$: So, for a linear demand $(\rho = 1)$, the relations obtained in the previous example lead to:

$$s_i/\omega_i^c = H/s_i \quad (3{:}11)$$

The rhs of the above equation allows one identify the firms sharing a larger amount of the welfare than of the output $s_i < \omega_i^c$. These are the firms enjoying a market share higher than the H-index $(H < s_i)$: The converse applies for the firms socially less efficient. Notice that this important result is obtained without any previous hypothesis on the cost functions of the firms.

By evaluating equation $(3{:}8)$ at $\rho = 1$ one arrives to the concentration index for the linear demand case:

$$C_n(1) = H$$

The following proposition establishes a general and important result for homogenous product industries:

Proposition 1: In a homogeneous product industry with continuous and decreasing inverse market demand function P, the $H-$ index is a normative index for every market distribution $(s_i; s_2; ...; s_n) \in S^+_{n-1}$ according to (3.7) if and only if P is linear.

Proof: For the sufficiency, the use of equation (3.11) in connection with (3.7) ensures that the linearity of P leads to $C_n = H$. For the necessity, by using (3.4) we can check that, $(\frac{s_i}{\omega_i})s_i = (\frac{s_i}{k_i^c})\overline{k}_s^c = (\frac{s_i}{k_i^c})\frac{W^c}{Q} = (\frac{q_i}{k_i^c})(\sum_{i=1} \frac{k_i^c q_i}{Q^2})$. So, if P is not linear, each k_i^c in (3.3) cannot be written as a constant proportion of the quantities (say; $k_i^c = \tau q_i$; $\tau > 0$; $i = 1; 2; ...; n$). Therefore, the rhs of the last equality cannot be equal to $\sum_{i=1} (\frac{q_i}{Q})^2 = H$; for every output vector $q = [q_1; q_2; ...; q_n]$. ¤

Other demand functions generate indices that overvalue the indices from the class $C_n(\rho)$. This is the case for the convex demand function with constant elasticity : $P = AQ^{-\delta}$; (Q; A; $\delta > 0$), which generate an index $C_n(\delta)$ increasing in δ. The lower bound $\lim_{\delta \to 0^+} C_n(\delta)$ is identical to C_n^o given in (3.9). [11] The functions belonging to the exponential family (e.g., $P = Be^{-\eta Q}$; $Q; B; \eta > 0$) generate indices $C_n(\eta; Q)$ that overvalue H. However, these indices depend on the industry output Q. [12]

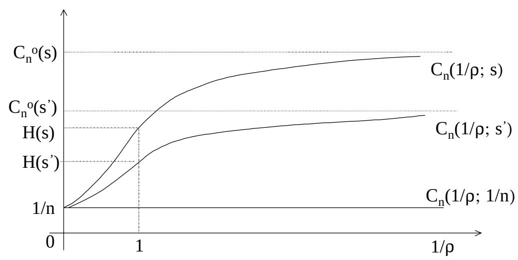
3.4 Nonlinearities and market share transfers

Since $1/\rho$ is a price-elasticity parameter, the figure 1 depicts the curve $(\frac{1}{\rho}; C_n(\frac{1}{\rho}))$ of an industry composed by n firms for three different market structures : a less equalitarian distribution, $s = (s_1; s_2; ...; s_3)$; a more equalitarian one $s^0 = (s^0_1; s^0_2; ...; s^0_3)$; and the perfectly equalitarian distribution $(\frac{1}{n}; ...; \frac{1}{n})$.

---

[11] In this case, the use of (3.1) leads to: $\omega_i^c = [1 - (1-\delta)s_i - (1-s_i)^{1-\delta}]/[n - (1-\delta) - \sum_{j=1}^{n}(1-s_j)^{1-\delta}]$. One can easily check that by taking the limit for $\delta \to 0$, the index obtained by using (3.7) is C_n^o which is given in (3.9).

[12] In this case, $W_i^c = \frac{1}{\eta}P[e^{\eta q_i} - 1 - \eta q_i]$. By using the Taylor series expansion of $e^{\eta Q s_i}$ around $s_i = 0$ and defining $\sigma_l = \sum_{i=1}^{n} s_i^l$, we obtain: $\omega_i^c = [\sum_{l=2}^{\infty} \frac{(\eta Q)^l}{l!} s_i^l]/[\sum_{l=2}^{\infty} \frac{(\eta Q)^l}{l!}\sigma_l]$. By retainning only the first term of the sum ($l = 2$) the substitution of this welfare share in (3.7) leads to the Hirschman-Herfindhal index: $C_n(\eta; Q) \simeq H$.



…g.1: Industry concentration and price-elasticity

So, the presence of nonlinearities in the demand biases the use of the H ¡ index for normative pourposes: from the consumers' point-of-view, H underestimates the magnitude of the industry concentration whenever the inverse market demand is convex (1=½ > 1) and overestimates it if the inverse demand is concave (1=½< 1).

Market share transfers. These features a¤ect also the way as the index works under market share transfers among the …rms. Consider an industry composed by $n = 5$ …rms. In the initial position, the market shares are: $s = (0{:}4; 0{:}3; 0{:}1; 0{:}1; 0{:}1)$: The …rst column in Table I gives the value of the normative index for ½ $= 1; 2; 1; \frac{1}{2}; 0$. The second column gives this value (and the percent of the variation w.r.t. the initial position) resulting from a 5% market share transfer from …rm 5 (its share falls to $0{:}05$) to …rm 3 (its share increases from $0{:}10$ to $0{:}15$). The column 3 reports this value when such transfer bene…ts …rm 1 (its share jumps to $0{:}45$):

Table I		market shares transfers	
	1:inital position	2 (between small …rms)	3(from a small to a large …rm)
C_n^1	$= 0{:}200$	$0{:}200$	$0{:}200$
$C_n(2)$	$= 0{:}267$	$0{:}271$ (1,78%)	$0{:}295$ (10,65%)
$C_n(1)$	$= 0{:}280 = H$	$0{:}285$ (1,78%)	$0{:}315$ (12,50%)
$C_n(\frac{1}{2})$	$= 0{:}287$	$0{:}292$ (1,76%)	$0{:}327$ (13,60%)
C_n^o	$= 0{:}296$	$0{:}301$ (1,73%)	$0{:}340$ (14,85%)

The Table I shows normative indices larger than H in the concave case and smaller in the convex case. However, the way as the nonlinearity of the demand a¤ects the impact of a market share transfer over the concentration hinges on the relative sizes of the incumbent …rms.

The second and third columns in table I show, for the convex demand case, that the Hirschman-Her…ndahl index overestimates the concentration increases resulting from the transfer between two small …rms ($1;78\%$ against $1;76\ \%$ or $1;73\%$) and underestimates the variation of the concentration caused by a transfer bene…tting a large …rm ($+12;50\%$ against $+13;6\%$ or $+14;85\%$). The converse holds if the demand is concave. The di¤erence in the e¤ects according to the demand shape may be explained by the fact that a non dominant …rm have a welfare share relatively greater in the concave case.

IV - HETEROGENEOUS PRODUCT INDUSTRIES

Let start introducing some notations. $q_{(i)} = [q_1; ...; q_{i-1}; q_{i+1};...;q_n]$ is the vector of the positive quantities supplied by the competitors of a producer i; and $P_i(q_i; q_{(i)})$ is the inverse demand function he faces. P_i is assumed to be a positive and continuous function de…ned over a bounded polyedron $B_i \subset R^n_+$; di¤erentiable in its arguments and decreasing in q_i.

Let W_{ij} be the economic surplus generated by the demand for good i through changes in the supply of good j. So,

$$W_{ii} \equiv \int_0^{q_i} [P_i(x; q_{(i)}) - \phi'_i(x)]dx \qquad \text{and}$$

$$W_{ij} \equiv \int_0^{q_j} P_i(x; q_{(j)})dx - \phi'_i(q_i)q_j \ ; \ i \neq j = 1; 2; ...; n.$$

Analogously to (3.1), if one focus on the consumer surplus, the corresponding values are:

$$W^c_{ij} \equiv \int_0^{q_j} P_i(x; q_{(j)})dx - P_i q_j \ ; \ (i; j = 1; 2; ...; n) \qquad (4.0)$$

4.1 Concentration indices

In the sequel we will assume that the quantities q_i are commensurable accros di¤erent …rms. The common measure used will permit aggregation on the quantities and give a meaning for the industry output $Q = \sum_i q_i$ and the market shares $s_i = q_i/Q$.

The measure of the concentration in this case relies on the degree of the product di¤erentiation among the goods supplied in the industry. Several oligopoly models bear out results supporting the so-called di¤erentiation principle: …rms want to di¤erentiate to soften price competition. As Tirole(1989,p.295) notes, this idea …ts well with the observation that …rms often search for market niches when positioning their product. An explicit theoretic support for the differentiation principle is given by Shaked&Sutton(1982, 1987). [13] However, their model aims to get qualitative results for the sources of the concentration only, and not to provide an index for measuring it.

For building up the concentration index along the formula (2.1) we have to specify the weighting matrix $\Omega_s = [\omega_{ij}]$, where each ω_{ij} should be a suitable function of W_{ij}. For the inner product be well defined, Ω_s must be symmetric $(\omega_{ij} = \omega_{ji})$ and positive definite. The first condition should be overcome by taking the mean between the cross terms $\omega'_{ij} = \omega'_{ij} = \frac{1}{2}(\omega_{ij} + \omega_{ji})$. The second condition restraints the admissible industry structures, as we will see below.

4.2 Classes of normative indices

For obtaining an explicit class of normative concentration indices for heterogeneous product industries, assume that the preferences of the consumers may be represented by the quasi-linear utility function:[14]

$$U(q; q_o) = q_o + q'\alpha - q'Aq \qquad (4.1)$$

where $q = [q_1, q_2, ..., q_n]'$, q_o is the amount of a numeraire good, α is a vector of positive parameters $\alpha = [\alpha_1, \alpha_2, ..., \alpha_n]'$, and A is a symmetric matrix of the form: $A = [\Lambda + \gamma\gamma']$, where $\gamma = [\gamma_1, \gamma_2, ..., \gamma_n]'$ and $\Lambda = \mathrm{Diag}[(\beta_1 - \gamma_1^2), (\beta_2 - \gamma_2^2), ..., (\beta_n - \gamma_n^2)]$, where all β_i are positive parameters such that $\beta_i > \gamma_i^2$. Moreover, it is assumed that values of $\beta's$ and $\gamma's$ are such that A is a positive definite matrix. So, if a rational representative consumer maximizes the utility U under the budget constraint $q_o + P'q \leq m$ (revenue) the inverse demand system only depend on prices and is obtained by: $P = \frac{\partial U}{\partial q} = \alpha - 2Aq$, that is, for the good i $(i = 1, 2, ..., n)$:

$$P_i = \alpha_i - 2\beta_i q_i - 2\gamma_i \left(\sum_{j(j \neq i)} \gamma_j q_j\right) \qquad (4.2)$$

[13] By assuming the existence of an industry equilibrium with differentiated products, they show in the latter paper that the fragmentation principle of the market fails to hold if some conditions on the technology and on tastes are fulfilled. More precisely, concentration will emerge if the burden of a quality improvement of the product falls primarily on fixed costs and if, as the market becomes large (by replicating the number of consumers) all consumers demand the highest quality good, whenever quality is priced at its marginal cost (at any quality level). So, the concentration depends basically on the shape of the technology and on the consumers's preferences.

[14] Quasi-linear utility functions are usually assumed in differentiated product models when revenue effects are to be neglected. See for instance, SINGH&VIVES(1984) and YARROW(1985);

The goods i and j are substitute, complement or independent one each other according to $\gamma_j\gamma_i > 0$, $\gamma_j\gamma_i < 0$ or $\gamma_j\gamma_i = 0$. (The quasi-concavity property required for the utility function implies there is not too much complementarities among the goods). High values of $\gamma_i\gamma_j$ means that customers can easily substitute goods i and j one another.

The specification (4.2) deserve some comments.

(i) The direct demand system can be computed straightforwardly by using the inverse matrix: $A^{-1} = [\Lambda + \gamma\gamma']^{-1} = \Lambda^{-1}[\Lambda - \frac{1}{1+d}\gamma\gamma']\Lambda^{-1}$, where $d = \gamma'\Lambda^{-1}\gamma = \sum_{i=1}^{n} \frac{\gamma_i^2}{\beta_i - \gamma_i^2}$. Hence, $q = \frac{1}{2}A^{-1}(\alpha - P)$. The matrix $-\frac{1}{2}A^{-1}$ is the Slutsky matrix of the substitution effects.

(ii) The demand function properties for differentiated oligopolies are discussed in Friedman(1989, p.62). Linear demand functions are commonly used in the literature. See Singh&Vives(1984); Deneckere&Davidson(1985), Shapiro(1989), among others.

Although we must assume rational preferences and utility maximization for obtaining the demand functions that the producers will face, it is worth noting that once formula (2.1) is accepted, no hypothesis on the nature of the producers' competitive conduct is needed for deriving the normative industry concentration index.

In the sequel, we set explicitly an expression for the weighting matrice Ω_s implied by the inverse demand system (4.2):

Influential welfare directions

We are looking at a normative index for the consumers. So, we will evaluate the consumers' surpluses W^c_{ij} in (4.0) for P_i given by (4.2). This gives:

$$W^c_{ii} = \beta_i q_i^2 \qquad (4.3a)$$

$$W^c_{ij} = \gamma_i\gamma_j q_j^2 \qquad (4.3b)$$

For imposing the symmetry condition we define $W^{c\prime}_{ij} = \frac{1}{2}(W^c_{ij} + W^c_{ji})$ and $W^c = \sum_i W^c_{ii} + \sum_{i,j(i\neq j)} W^c_{ij}$. From equations $(4.3a,b)$ we obtain the shares $\omega^c_{ii} = W^c_{ii}/W^c$ and $\omega^c_{ij} = W^{\prime c}_{ij}/W^c$. By dividing the numerator and the denominator by Q^2, the welfare shares become:

$$\omega^c_{ii} = \frac{\beta_i s_i^2}{\sum_{i=1}^{n} \beta_i s_i^2 + \frac{1}{2}\sum_{i,j(i\neq j)} \gamma_i\gamma_j(s_i^2 + s_j^2)} \qquad (4.4)$$

$$\omega^c_{ij} = \frac{\frac{1}{2}\gamma_i\gamma_j(s_i^2 + s_j^2)}{\sum_{i=1}^{n} \beta_i s_i^2 + \frac{1}{2}\sum_{i,j(i\neq j)} \gamma_i\gamma_j(s_i^2 + s_j^2)} = \omega^c_{ji}. \qquad (4.5)$$

The present structure for $\Omega_s = [\omega^c_{ij}]$ does not necessarily verifies the positive definiteness condition.

Let then $A_n = \{\phi = (\beta; \gamma; s) \in R^n_{++} \times R^n_{+} \times S_{n-1} : \Omega = [\omega_{ij}]$ is positive definite matrix of order $n\}$ be the set of the admissible preference parameters and market distributions leading to a positive definite matrix Ω; whose elements are given in (4.4) and (4.5). A_n may be called the measurable n-market structure for the consumers.

4.3 Symmetric preferences among the goods

Consider first the case where the representative consumer has symmetric preferences among the goods: $\beta_i = \beta$ and $\gamma_i = \gamma$ $(i = 1, 2, ..., n)$. The welfare shares in (4.4) and (4.5) become:

$$\omega^c_{ii} = \frac{s_i^2}{[1 + (n-1)\xi]H} \qquad (4.4a)$$

$$\omega^c_{ij} = \frac{\frac{1}{2}\xi(s_i^2 + s_j^2)}{[1 + (n-1)\xi]H} = \omega^c_{ji} \qquad (4.5a)$$

Where H is the Hirschmann-Herfindahl index and $\xi = \frac{\gamma^2}{\beta}$; $\beta > 0$.

This parameter is the squarred root of the coefficient known in the literature as the degree of the product differentiation (Singh&Vives,1984, p.548). The differentiation increases as ξ decreases to 0 for finite values of β (that is for $\gamma^2 \to 0$); meaning that the products become independent. On the other hand, the measurability condition imposes γ^2 be bounded from above to β; so that $0 \leq \xi < 1$.

An element $\phi \in A_n$ is now written simply as $\phi = (\xi; s)$; where A_n is defined as a subset of $[0; 1) \times S_{n-1}$.

Under the equalitarian market distribution $(s = \frac{1}{n}1)$ we should have the higher welfare value, because with symmetric preferences the consumer surplus is maximized if the firms have equal sizes. So, the minimal value of the gradient index ϕ_ω leads in this case to a minimal value for C_n. As we have already proved in section 2.2, we have $C_n(\xi; \frac{1}{n}1) = 1/n$ for all parameter $\xi \in [0; 1)$. Moreover, $C_n(\xi; s) > 1/n$ for all $(\xi; s) \in A_n$ (both conditions make properties $P2$ and $P5$ holding).

Concentration and product differentiation

From the equations $(4.5a)$ and $(4.4a)$; when the goods are independent $(\xi = 0)$; the share ω^c_{ii} equals s_i^2/H and ω^c_{ij} vanishes. Thus, it is easy to check that $C_n(s) = H(s)$ for every $s \in S_{n-1}$. Therefore, if the preferences are symmetric and the goods are independent the $H-$ index can be viewed as a normative index for heterogenous product industries.

Let $\overset{o}{A}_n$ be the inner set of A_n. We have then the following proposition.

Proposition 2. In a differentiated products industry where the producers face linear market demand functions, and the consumers have symmetric preferences among the goods, the normative concentration index $C_n(\phi_j)$ converges to $H(s)$ for every measurable and decreasing sequence of market structures $\phi_j = (\lambda_j; s)$, $j = 0; 1; 2; \ldots$ converging to $\phi = (0; s)$.

Proof: Given $s \in S_{n-1}$, the elements of $\Omega_s(\lambda)$ defined in $(44a)$ and $(45a)$ are continuous functions of λ. For market structures $(\lambda; s) \in \overset{o}{A}_n$, the same can be said about the elements of $\Omega_s^{-1}(\lambda)$. So, for a given s, $C_n(.; s)$ is a continuous function of λ. Since $C_n(0; s) = H(s)$, every sequence $\phi_j = (\lambda_j; s)$ in $\overset{o}{A}_n$ converging to $\phi = (0; s)$ makes the sequence of indices $(C_n(\phi_j))$ converging to $H(s)$. $\square$

We would like to show that the convergence is monotonic. Unfortunately, the sign of $\dfrac{\partial C_n}{\partial \lambda}$ is ambiguous.[15]. In any case, the result obtained in the proposition shows that the $H-$ index does not take into account the consumers' substitution of the goods supplied by the industry.

Under the present assumptions, the matrix Ω_s resulting from $(4.4a)$ and $(4.5a)$ is well-shaped, for a large set of industry structures $\phi \in A$. So, for these industries, the $H-$ index overestimates de industry concentration, from the consumers' point of view. This is what the simulations we made have shown. Also, for the market distributions used, the index $C_n(\lambda; n)$ converges monotonically to $H(s)$ (by lower values), as $\lambda \to 0$, showing that

[15] Write the matrix Ω_s as: $\Omega_s = \frac{1}{[1+(n-1)\lambda]H} B$, where the elements b_{ij} of B are obtained from $(4.4a)$ and $(4.5a)$: $b_{ij} = s_i^2$, and $b_{ij} = \frac{1}{2}\lambda(s_i^2 + s_j^2)$. Notice now that $\frac{\partial B}{\partial \lambda} \equiv D_s$ (say), is a matrix whose elements on the diagonal are zeros and those off the diagonal are $\frac{1}{2}(s_i^2 + s_j^2)$. Notice also that $\dfrac{\partial B^{-1}}{\partial \lambda} = -B^{-1}D_sB^{-1}$. Since $\Omega_s^{-1} = \frac{1}{n}H[1+(n-1)\lambda]B^{-1}$, by derivating the index (2.1) w.r.t. λ, we obtain:

$$\frac{\partial C_n}{\partial \lambda} = \frac{1}{n}Hs'B^{-1}[(n-1)(B-\lambda D_s) - D_s]B^{-1}s$$

By assumption, the matrix B^{-1} is positive definite but the genre of the matrix into brackets is indefinite (its elements are: $(n-1)s_i^2$ on the diagonal and $-\frac{1}{2}(s_i^2 + s_j^2)$ off the diagonal). Thus, the sign of the derivative is ambiguous.

the product di¤erentiation degree harms the competition $(\frac{@C_n}{@»} < 0$; in (4.6)): This agree with rare empirical estimates for this relation that one can ...nd in the literature (Wright,1978).

In order providing a report of the results obtained, the calculations presented below consider a 5- ...rms industry with two ...xed market distributions: an equalitarian distribution,

$s = (0{:}22; 0{:}195; 0{:}195; 0{:}195; 0{:}195)$, with standard deviation $¾(s) = {:}01$ and a less equalitarian one :

$s^0 = (0{:}4; 0{:}3; 0{:}1; 0{:}1; 0{:}1)$, with standard deviation $¾(s^0) = {:}126{:}$

The values of the normative concentration index have been calculated from the cononical representation of C_n (given in $2{:}1a$); for a large range of values of the di¤erentiation parameter » (values of $»^{¡1}$ were picked up between $1{:}5$ and 1000 for the ...rst distribution and between 4 and 1000 for the second distribution). The calculations runned with Mathematica. For an increasing sequence of $»^{¡1}$ values (meaning increasing product di¤erentiation) the Table II displays values of the normative index lower than the H ¡ index in both cases, $H(s) = {:}2005$ and $H(s^0) = {:}280$ respectively.

Table II Concentration and product di¤erentiation

$»^{¡1}$:	1:5	2	4	5	10	20	25	100	1000
$C_n(»; s)$	.2000$^+$	.2001	.2002	.2002$^+$	.2003$^+$	.2004	.2004$^+$	.2004^{++}	.2005$^{¡}$
$C_n(»; s^0)$	*	*	.2252	.2294	.2432	.2570	.2606	.2742	.2793

* : the structures $(»; s^0)$ are not measurable.

As it is shown in Table III ahead, the same features keep holding for a 4- ...rms industry.

Market transfers and horizontal mergers

Our de...nition for the normative index assumes that the concentration is lessen when an industry adjustment involving a market transfer from a small ...rm to a larger one is taken nearby of the optimal path. Since the money value of the adjustments are weigthed according to the relative contributions of the ...rms to the consumers' welfare, our measurement concern brings any adjustment in the industry output vector as close as possible to the optimal path. So, it may be that some market transfers lead to a fall in the measured concentration instead of a raise, as required by property $P1{:}$ This occurs often when the amount transfered and the ...rm benen...tted by the transfer are not large enough for the inequality e¤ect on the market distribution overtaking the positive e¤ect on the consumers' surplus.

For the above market structures $Á = (»; s); (»; s^0)$ we calculate $C_n(Á)$ under a 5% market share transfer from a small ...rm. In the ...rst case, the ...rm bene...tted is the equal sized ...rms (2) and (3); generating the new market distributions:

$\mathbf{b} = (:245; 0:22; 0:195; 0:195; 0:145)$ and $\mathbf{b}^0 = (0:4; 0:3; 0:15; 0:1; 0:05)$;
with standard deviations $0:033$ and $0:130$; respectively. In the second case, the ...rm (1) is bene...tted, and the new distributions are:

$\mathbf{e} = (:270; 0:195; 0:195; 0:195; 0:145)$ and $\mathbf{e}^0 = (0:45; 0:3; 0:1; 0:1; 0:05)$,
with standard deviations $0:033$ and $0:151$; respectivelty.

The results are shown in the columns 2 and 3 of the Table III, for three di¤erent values of $» : 0:2$; $0:1$, and $0:05$: The two last columns of Table III report the values of $C_n(Á)$ under two horizontal mergers. In the ...rst, the merger concerns ...rms of equal market shares: ...rms (5) and (2) in one industry and ...rms (5) and (3): This leads to a pos-merger distribution $\bar{s} = (0:39; 0:22; 0:195; 0:195)$ in the more equalitarian distribution and $\bar{s}^0 = (0:4; 0:3; 0:2; 0:1)$ in the less equalitarian distribution. The standard deviation (sd) are $0:081$ and $0:112$; respectively. The second merger concerns the smaller and the larger ...rms in the industry, and the new distributions are $s^¤ = (0:415; 0:195; 0:195; 0:195)$ with sd $0:095$ and $s^{¤0} = (0:5; 0:3; 0:1; 0:1)$ with sd $0:166$:

Table III Concentration, market transfers and horizontal mergers

initial position	market transfers		horizontal mergers	
low inequality : $C_n(»; s)$	$(5) \; ! \; (2)$	$(5) \; ! \; (1)$	$(5) + (2)$	$(5) + (1)$
$¾(s)$: $0:01$	$0:033$ (230%)	$0:033$(230%)	$0:081$ (710%)	$0:095$(850%)
$» = 0:2$: $0:2002^+$	$0:2022$ (1.1%)	$0:2036$(1.7%)	$0:262$(30.8%)	$0:271$(35.3%)
$» = 0:1$: $0:2003^+$	$0:2032$(1.4%)	$0:2051$(2.4%)	$0:269$(34.3%)	$0:277$(38.3%)
$» = :05$: $0:2004$	$0:2041$(1.8%)	$0:2063$(2.9%)	$0:272$(35.7%)	$0:281$(40.2%)
H : $0:2005$	$0:2055$ (2.4%)	$0:208$ (3.7%)	$0:276$(38%)	$0:286$ (42.8%)
initial position	**market transfers**		**horizontal mergers**	
high inequality : $C_n(»; s)$	$(5) \; ! \; (3)$	$(5) \; ! \; (1)$	$(5) + (3)$	$(5) + (1)$
$¾(s^0)$: $0:126$	$0:130$ (3.2%)	$0:151$ (19.8)	$0:112$(-11.1%)	$0:166$(32%)
$» = 0:2$: $0:230$	$0:211$(-8.2%)	¤	$0:263$(14.3%)	$0:282$(22.6%)
$» = 0:1$: $0:243$	$0:232$(-4.5%)	$0:248$(2.1%)	$0:275$(13.1%)	$0:306$(25.9%)
$» = :05$: $0:257$	$0:250$(-2.7%)	$0:270$(5%)	$0:291$(13.2%)	$0:327$(27.2%)
H : $0:280$	$0:285$ (+1.8%)	$0:315$(12.5%)	$0:300$(7.1%)	$0:360$(28.6%)

¤ : Á 2 A

Values in parenthesis are the percent changes w.r.t. the initial position.

The results showed in table III elicit the following coments:

1. More differentiation among the goods harms the competition;

2. Small market transfers taking place in industries with unequal market distributions may be socially beneficial, whenever they do not benefit large firms, because they create welfare effects for the consumers that overtake the market distribution effects on the concentration (see column 2 on the lower part of the table). However, small transfers may harm the competition in equalitarian industries even if they benefit a small firm.

Notice that the $H-$ index overestimates the concentration changes caused by market transfers, in both types of industries (see columns 2 and 3);

3. The mergers considered in the example always increase the normative concentration in the industry, meaning that the size effect (reduction in n) overweighs the two other effects (welfare and market distribution effects). However, the size effect reduces as the number of firms increases, so that a fall in the concentration index cannot be excluded for some mergers that reduce the variance of the actual market distribution in large industries.

Notice that the $H-$ index overestimates the concentration effect of mergers in the equalitarian distribution industry considered in the example.

4.4 Asymmetric preferences among the goods

Here vectors $\beta = (\beta_1, \beta_2, ..., \beta_n)$, $\gamma = (\gamma_1, \gamma_2, ..., \gamma_n)$ may be considered, accounting for the asymmetry existing in the preferences among the goods of the representative consumer. The other way round of the symmetric case, here the $H-$ index often underestimates the normative measurement of the industry concentration. Three cases must be considered, according we assume symmetric or asymmetric substitution assumptions.

a) Gross substitution

For a fixed value of γ $(= \bar{\gamma}$, say) we can define a product differentiation vector $\xi = (\xi_1, \xi_2, ..., \xi_n)$, where $\xi_i = \bar{\gamma}^2/\beta_i$, to be used in connection with equations (4.4) and (4.5) for obtainning the normative concentration values under asymmetric preferences for any market distribution s.

This has be done for the former 5-firms industries examined above, which have market distributions: $s = (0.22, 0.195, 0.195, 0.195, 0.195)$ and $s' = (0.4, 0.3, 0.1, 0.1, 0.1)$. In order to obtain values directly comparable with those got in the symmetric case, we have taken two product differentiation vectors (one for each industry) both with mean $\bar{\xi} = \bar{\xi}' = 0.1$, for matching the values $C_n(0.1; s) = 0.2003^+$ and $C_n(0.1; s') = 0.243$ obtained under symmetric preferences in each industry. The product differentiation vectors are: $\xi = (.05; .1125; .1125; .1125; .1125)$ and $\xi' = (.05; .09; .12; .12; .12)$. The implicit hypothesis assumed in these choices is that the larger firms produce more differentiated goods than the smaller ones. In order to assess the implications of such a hypothesis for the measured concentration we have also calculated the normative index value under the reverse differentiation profiles: $\xi^r = (.1125; .1125; .1125; .1125; .05)$ and $\xi'^r = (12; .12; .12.; 09; 05)$. The results obtained are given below:

low market inequality industry:

$C_n(\bar{\xi} = 0.1 : s) = 0.2240$ (11.8%) ; $C_n^r(\bar{\xi} = 0.1 : s) = 0.2156$ (7.6%)

high market inequality industry:

$C_n(\bar{\xi}' = 0.1 : s') = 0.361$ (48.5%) ; $C_n^r(\bar{\xi}' = 0.1 : s') = 0.222$ (-8.4%)

Two main facts shall be emphasized from the above results:

1. A mean preserving spread of the product differentiation making the smaller (larger) firms to produce the more differentiated products, may decrease (increase) the measured concentration in industries with higher market inequality;

2. The concentration effects of a mean preserving spread of the differentiation among the goods seems to be smaller in industries with lower market inequalitiy.

b) Asymmetric substitution

Under the asymmetric substitution assumption we consider the vector $\gamma = (\gamma_1; \gamma_2; ...; \gamma_n)$. If we assume the parameters β_i are identical across the firms $(= \beta$; say), we can define the cross differentiantion parameters $\xi_{ij} = \gamma_i \gamma_j / \beta$; for $i \neq j$; to be used in conection with formulas (4.4) and (4.5). For the two above market distributions, we set $\beta = 1$ and chose substitution vectors in such a way that the mean value $\bar{\xi}$ equals 0.1 $(\frac{1}{5} \sum_{i \neq j} \xi_{ij} = 0.1)$ in each case, in order to make the results comparable with those obtained under the gross substitution assumption. The vectors are $\gamma = (.25; .1371; .1371; .1371; .1371)$ for the industry with distribution s; and $\gamma' = (.25; .20; .1175; .1175; .1175)$ for the industry with distribution s'. As before, the notation C_n^r stands for the index values calculated under the reversed substitution vectors. The results are presented below. (In parenthesis are given the percent change w.r.t. to the values obtained in the symmetric case: $C_n(0.1; s) = 0.2003^+$ and $C_n(0.1; s') = 0.243$).

low market inequality industry:

$C_n(\bar{\xi} = .1; s) = 0.2008$ (0.24.%) ; $C^r_n(\bar{\xi} = .1 : s) = 0.2004$ (0^+%)

high market inequality industry:

$C_n(\bar{\xi}' = .1; s') = 0.273$ (12.3%) ; $C^r_n(\bar{\xi}' = .1 : s') = 0.264$ (8.9%)

Two main facts shall be emphasized from the above results:

1. A mean preserving spread of the product substitution increases significantly the concentration only in industries with unequal market distribution;
2. The increase in the concentration is higher (lower) when the substitution spread benefits the larger (smaller) firms.

c) Overall asymmetry

We calculate the index C_n for the previous market distributions s and s' under the substitution vectors (γ) used in (b) and those (β) considered in (a), that are implyied in the choice of the differentiation vectors ξ and ξ': These vectors are: $\beta = (2; .888; .888; .888; .888)$ and $\beta' = (2; 1.111; .833; .833; .833)$: The equations (4.4) and (4.5) were used. The results are given below. (Again, the percent change w.r.t. to the values obtained in the symmetric case are given):

low market inequality industry:

$C_n(\beta; \gamma : s) = 0.223$ (11.3.%) ;

high market inequality industry:

$C_n(\beta; \gamma : s') = 0.426$ (75.6%) ;

The values show that the concentration increasing is larger in the unequal market distribution. Notice also that, in both cases, the $H-$ index underestimates the normative view of the concentration, by (11.2%) in the equalitarian market distribution and by (52.1%) in the unequal market distribution case.

V - <u>FINAL COMMENTS</u>

This paper presents a new method for obtaining industry concentration indices for the consumers in both cases, homogeneous and differentiated product industries. The major contribution of the paper consists obtaining endogenous classes of such indices based on an Industry Performance Gradient Weighted Index (IPGWI) that relate the characteristics of the consumers' preferences. The results obtained on the relationship between concentration and product differentiation should now be confronted with the predictions of theoretic models emphasizing the role of the research and innovation like in Shaked&Sutton(1982, 1987) or the competitive conduct as in

Deneckere(1983) or Chang(1991) (for collusion), among others. Empirical ...ndings supporting the negative relation are obtained by Wright(1978).

The method developed here can be easily extended for obtaining an enlarged normative index by using total welfare shares ! instead of $!^c$ as did here. The derived index would re‡ect also changes in costs incurred by the ...rms.

Although Dansby&Willig's theoretical contribution for homogeneous product industries emphasizes the welfare aspects embodied in the de...nition of the H-index, the introduction and the adoption in empirical researches of this index have been justi...ed by arguments mainly dealing with its statistical properties (see, Adelman,1969; Kelly,1981, among others). As we know, the attempts to improve the indices belonging to the same class (by correcting the weights applied to the number n and to the inequality in the market distribution $s_1; :::s_n)$ have been limited handling with weigths derived from particular distribution functions previously assumed for the shares (see Davies,1980 and references). Moreover, no links between H and speci...c concentration indices built up for di¤erentiated product industries are reported in the literature.

In the normative approach, the Hirschman-Her...ndahl index plays a critical role for homogeneous product industries. For these industries, H splits in two parts the range for the normative index increases or decreases over H according to the convexity or the concavity of the market demand. By using H; underestimations or overestimations of the e¤ects on the concentration of market share transfers among the ...rms are implied in these cases. For these industries, the normative index exhibits properties $P1$ ¡ $P5$ proposed by Encaoua&Jacquemin(1980).

For heterogeneous product industries facing linear maket demand functions and symmetric preferences among the goods, the H -index is a normative index for all market distributions if and only if the goods are independent. An important result obtained in this context sets H as a limiting value for the normative indices, as the products become more and more di¤erentiated. For a large class of industry structures, the H- index underestimates the industry concentration. Increases in the di¤erentiation among the goods increases the concentration. Under asymmetric preferences and gross substitution assumptions, examples are given showing that, for a ...xed market distribution, a mean preserving spread of the di¤erentiation degree making the small ...rms to produce higher di¤erentiated goods improve the competition, if the market distribution is unequal. On the other hand, an increase in the concentration is observed when the differentiation spread benefits the larger firms. Moreover, a mean preserving spread of the substitution increases significantly the concentration only in industries with unequal market distribution. The increase in the concentration is higher (lower) when the substitution spread benefits the larger (smaller) firms. Under asymmetric preferences, the $H-$index often underestimates the normative measure of the concentration.

This paper presents several formulas useful in empirical researches. For example, the consumer's measure defined along formulas $(3.9;10)$: $r(s) = C_n^o(s) - C_n^1$ is useful for comparing the allocative or productive inefficiencies related with a given market distribution (s) accross different homogeneous product industries. By putting $\delta_i = m_i/\sqrt{n\sigma_i}$ in the canonic representation $(2.1a)$ of C_n for differentiated industries, one can calculate the weights (δ_i^2/C_n) of the principal welfare directions in the normative assessement of the concentration. The higher eigenvector elements corresponding to the higher weights enable one identifying the firm positions susceptible for local interventions.

For estimating C_n; formulas given in (3.8) for homogeneous industries and $(4.4)-(4.5b)$ for heterogeneous product industries call for careful econometric estimations of the underlying market demand parameters. In the first case, the Hirschman-Herfindahl hypothesis $(\rho = 1)$ can be tested statistically. For differentiated industries facing linear market demand, the hypothesis of symmetric preferences (with their implications for over or underestimation of $H(s)$ w.r.t. C_n) can also be carried out for all measurable industry structures $(\xi; s)$ as defined in the paper. To this aim, accurate estimations of the product differentiation vector ξ from the observed data are needed.

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