Humberto Moreira, joint with Juliano Assunçao
Towards a Truthful Land Taxation Mechanism in Brazil
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Abstract

This paper shows that the asymmetric information present in the relationship between the government and agricultural producers (farmers and ranchers) has led to persistent problems in the application of land taxes (Imposto Territorial Rural – ITR) in Brazil. When the asymmetric information is taken into account, some use of output taxes in the optimal tax scheme may be more desirable than a pure land tax regime. We consider a model of optimal taxation in which government maximizes the expected tax revenue minus the farmer’s yields from land speculation. There is a continuum of farmers using land to produce an homogeneous agricultural output and to speculate. They have a private information on the parameters of agricultural production and land speculation activities. Also, there is no land rental market and the harvested area cannot exceed the farm size. The main result is that a pure land tax regime is optimal only if there is complete information or there is no idle land in equilibrium. In this fashion, the model presented tries to fill a gap between the models of optimal taxation and the specific models of land taxation.

Resumo

O objetivo do artigo é mostrar que a assimetria de informação presente na relação entre governo e produtores agropecuários pode constituir a origem dos problemas que ainda persistem na aplicação do Imposto Territorial no Brasil. Através da construção de um modelo teórico simples, que se baseia no problema de taxação ótima sob informação assimétrica, é possível analisar limitações inerentes à aplicação do Imposto Territorial Rural que ainda não se incorporaram à análise da taxação de terras. Diante de uma situação onde há terra ociosa, como ocorre no Brasil, o modelo teórico desenvolvido mostra que o uso do ITR como único instrumento tributário não é capaz de implementar o esquema ótimo. E a solução apontada pelo modelo envolve a utilização de um esquema misto que considera o Imposto sobre a Circulação de Mercadorias e Serviços (ICMS) e o ITR. Dessa forma, o modelo apresentado tenta preencher uma lacuna existente entre os modelos de taxação sob informação assimétrica e os modelos mais específicos de taxação de terra. E, como implicações de política, os resultados sugerem um redirecionamento do estudo do ITR, atualmente centrado em questões de ordem operacional, como determinação de alíquotas e outras regras.

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I. Introduction

Rural Property Tax (ITR) in Brazil, since its creation through the Land Statute of 1964, has been used to support public policies for land redistribution. Nevertheless, there is a high level of evasion and default which hinders its efficiency as an instrument of landholding policy. Two large-scale reforms of ITR were carried out, in 1979 and 1996, but the results have not sufficed to overcome the associated problems.

The objective of the paper is to demonstrate that the asymmetry of information present between the government and agricultural producers (farmers and ranchers) causes problems that still persist in the application of the tax. Faced with a situation of idle land as in Brazil, the theoretical model developed shows that the use of ITR alone is not capable of implementing the optimal scheme. The solution pointed out by the model involves the use of a mixed scheme based on an output tax (Tax on Circulation of Merchandise and Services - ICMS), along with the land tax (ITR).

The main contribution of this work is the construction of a theoretical model that concentrates on an analysis of the problem of optimal taxation under asymmetric information, considering specific aspects of agriculture, particularly attuned to the Brazilian situation.

The model presented here attempts to fill a gap between optimal taxation models under asymmetric information and traditional land taxation models. On the one hand, optimal taxation models basically involve taxes on consumption and income, ignoring issues that are relevant to agricultural activity [Mirrlees (1971, 1986)]. On the other hand, papers dealing with land taxation normally concentrate on other questions, described briefly below.
Henry George (1839-1897) was the first to establish an economic rationale for land taxes, in *Progress and Poverty*, published in 1879. He attributed unemployment and low wages to an artificial dearth of land and the poor operation of the market. This artificial scarcity was the result of unequal land distribution and speculative activities. In this context, George proposed land taxation to make the land market more dynamic so as to induce full soil use without distorting marginal incentives. Arnott and Stiglitz (1979) analyzed the general case of what has become known as the Henry George theorem, becoming a classic reference work in this sense.

Other authors have also pointed out the advantages inherent in the use of taxes on land as a source of government revenue [Deininger (1998) and Skinner (1991b)]. Land taxation does not distort the allocation of resources and constitutes one of the few examples of a lump-sum tax in aggregate terms that can ensure a minimum level of revenue collection, since the supply of land is inelastic. Besides this, the farm size is observable, mainly in regions where land ownership is individual and there is accessible and reliable information on the size of holdings.

Hoff (1991), on the other hand, qualifies the use of land taxation. The author argues that in an uncertain activity such as agriculture, in which producers are risk-averse, the exclusive use of land taxation promotes an inefficient allocation of risk. In the Hoff (1991) model, the parallel use of an output tax is shown to be Pareto-superior. The optimal composition of output tax and land tax is determined by the tradeoff between distortion (introduced by output tax) and risk sharing.

Carter and Mesbah (1993), using a multiple-equilibria model, show that the use of a land tax is inefficient in overcoming the “accumulation barrier”. This barrier is established by the critical size of the farm that determines whether the producer will be a small or large landholder in equilibrium. However, this result arises from the linear
scheme for land tax adopted by these authors. Rules with progressive rates, as in the case of ITR, can significantly affect the accumulation barrier in this model.

Skinner (1991a) emphasizes the informational costs required to administer this type of tax. Nevertheless, despite considering the possibility of different types of producers, he does not deal with the problem of mechanism design faced by the government. In the relationship between the government and producers, only the behavior of the latter is strategic. The government has only a positive probability of incorrectly appraising land values.

Another question addressed by Skinner (1991a) establishes that the loss of capital resulting from the application of the tax is transitory, affecting only the current landowners. When the agents have access to other assets, a non-arbitrage condition ensures that the tax is completely absorbed by a reduction in the price of land.

The model developed in this paper is closely tied to the Brazilian situation, in light of the articles already mentioned. The problem of informational costs raised by Skinner (1991a), present in the Brazilian case, can at least be partially resolved by using an output tax. The employment of this tax, besides transferring less risk to producers (as in the model of Hoff (1991)), constitutes an essential instrument to obtain reliable declarations of productivity parameters and the extent of cultivated land.

Land ownership in the model has two basic purposes. It can be used for agricultural production or for speculation, in the latter case considering its value as collateral in an imperfect credit market or as a hedge against macroeconomic instabilities, particularly in periods of high inflation.

The model also considers a continuum of types of producers who are differentiated by their proclivity for speculative activity and/or agricultural productivity. These producers determine the extent of their holdings and the area under cultivation.
Each producer’s type constitutes private information and there is no rental market, i.e., the amount of land under cultivation cannot surpass the farm size.

The government maximizes its utility function, which considers the tax receipts and the speculative benefits of land, observing the farm size and the total amount of output. The supposition that the amount of production is observed by the government tries to include in the model the fact that collection of ICMS is much more efficient than of ITR in Brazil. Also, for the sake of simplicity, the model assumes that the observed output is equivalent to ICMS collection.

The basic results are: relying only on land tax (ITR) is optimal when the government’s information is complete or when in equilibrium there is no idle land; otherwise, the optimal scheme is a linear combination of output and land taxes.

In other words, if there is idle land in equilibrium, there are no tax rates capable of implementing an optimal tax system solely with ITR. But for small producers who operate in equilibrium without idle land, ITR can be implemented, which is compatible with some empirical evidence indicated in Section II.

The article is organized in six sections. Section II presents a brief overview of the application of ITR in Brazil. The theoretical model is laid out in Section III, which establishes the basic notation for Sections IV and V and also analyzes cases of complete and asymmetric information, respectively. Sections IV and V then present the basic results of the model in heuristic form. The formal version of the arguments is contained in the Appendix. Finally, the main implications and contributions of the work are summarized in the Conclusion.
II. The Brazilian Experience with ITR

Land taxation was instituted in Brazil by the Republican Constitution of 1891, delegating competence to the state governments. The responsibility of the states for administering and collecting the tax was maintained by the subsequent Constitutions of 1934, 1937 and 1946. In 1961, with the enactment of Constitutional Amendment No. 5, ITR was transferred to the municipalities, and in 1964, under Amendment No. 10, it was again transferred, this time to the federal government, where it has remained (confirmed in the 1988 Constitution). The passage of the Land Statute in 1964 imposed extra-fiscal functions on land taxation, which in principle took on the job of assisting public land-redistribution policies. (For more information, see Oliveira (1993) and Reydon et al. (2000)).

Below is a brief description of the three successive phases after implementation of the Land Statute. This history is necessary to explain the reasons for the main modifications, and insofar as possible, the scope of the solutions adopted. At the end of this section, the reader should be convinced that despite this series of changes, there is still a chronic problem of tax implementation that hinders the attainment of the desired revenue level. This is the central objective of the theoretical model contained in Section III.

First Phase: 1964-1979

The Land Statute (Law No. 4504 of November 30, 1964) made collection of ITR the responsibility of INCRA (National Agrarian Reform Agency). As described by Oliveira (1993), the basic rate was 0.2%, corrected by coefficients related to size (A), location (B), social conditions (C) and productivity (D), which together determined a tax burden given by:
\[ \text{ITR} = (0.002 \times A.B.C.D) \times VTN, \]

where \( VTN \) represents the value of the unimproved land. Given the intervals for variation of each coefficient, the effective tax rate ranged from 0.24% to 3.456%.

Nevertheless, the original objectives of the tax were not achieved. Oliveira and Costa (1979), cited by Oliveira (1993), concluded that ITR had never constituted a good source of revenue and further, hardly managed to achieve any of the desired changes in the rural environment. The authors’ main conclusions were:

1. “Given the small impact of ITR (and parallel taxes) on the profit and rate of return of rural properties, and given the tax evasion by a large majority of owners, it can be inferred that the referred tax has not contributed, and likely will not contribute, to changing the socioeconomic relations in Brazilian agriculture.

2. From the standpoint of real estate categories, ITR is in many respects incoherent because it falls more heavily on small rather than large landowners and, in many cases, treats rural companies more rigorously than large landowners. The reason for such a regressive outcome is the system for calculating the tax, which does not discriminate between taxpayers according to their category of holding (smallholder, rural company, large landholder).

3. The categorization of rural properties adopted by INCRA to define smallholdings, rural companies and large holdings does not jibe with reality.

4. The intended variation in the legally set rates is not observed. This is due to the fact that the coefficients for size, location, social conditions and productivity are not adequate to the reality of the Brazilian rural structure.

5. The problem of evasion is widespread and serious.

6. The system of updating raw land values in the years between reassessments, according to a monetary restatement (inflation) index, does not reflect the real behavior of land values over time.”
Hence, the situation in the 1970s was, as summarized above, beset by a host of problems with the way ITR was implemented. In this period, due to the importance of operational problems (responsible for huge distortions), questions of a more structural nature were not the focus of debate. It was believed – and this feeling still persists – that the problems involving ITR were of a purely operational nature.

**Second Phase: 1979-1996**

The complications pointed out above led to the first important reform of ITR legislation. The most significant changes for questions related to this work involved Article 49 of the Land Statute, according to which “the general rules for setting Rural Property Tax shall henceforth obey progressive and regressive criteria, taking into account the following factors: the raw land value; the size of the rural property; the degree of use for farming, ranching or forestry exploitation; the level of efficiency obtained from the various uses; the total area country-wide of rural properties held by the same owner; the land classifications and their form of use and profitability.”

Relying again on the description of Oliveira (1993), this overhaul still maintained VTN as part of the tax basis. The applicable tax rate became a function also of the degree of land use (GUT) and the level of efficiency (GEE), such that:

\[ ITR = \left[ f(GUT, GEE) \right] VTN. \]

According to this scheme, the rate would vary from 0.2% to 3.5% (for properties above 100 fiscal modules\(^1\)).

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\(^1\) The definition of the fiscal module of the municipality considers the following factors: type of predominant economic activity (truck farming, permanent cultivation, temporary cultivation, herding and forestry), productivity per crop or activity, and a consideration for smallholders/family farmers.
Figure 1.1 – ITR Revenues (1972-1991)

The data presented by Oliveira (1993) shows that efforts to increase tax collections were frustrated. The levels rose in the years immediately after the reform, but by 1983 had returned to their previous levels, as shown in Figure 1.1. Even in 1990, when there was another peak, the level of revenue collected was an insignificant US$ 20.30 per rural property. In January 1992, revenues corresponded to about 25% of the minimum monthly wage.

According to the Presidential Press Office, the percentage of VTN declared in relation to the real price of land in the 1980s varied from 20% for properties of less than 10 hectares\(^2\) down to only 1.2% for large properties of over 10 thousand hectares. The area reported as usable was far short of the true figure as well, with large landowners declaring around 50% and small ones 94% of the real measure. This declaration of utilization was even more unrealistic in some cases accepted by INCRA in which the actual productivity was more than tenfold the expected value, as calculated by the Brazilian Bureau of Geography and Statistics (IBGE).

The different impact of the ITR collection scheme on small versus large landholders can be seen in the context of the model in Section III. In this model, systems that use only ITR have the desired effects on small producers. For large land-
holders with idle land, it becomes necessary to resort to an additional instrument, ICMS.

Despite the problems of under-reporting and evasion encountered, the question of asymmetric information between the government and landowners still was not incorporated into the analysis. To the contrary, analyses such as that of Sayad (1982) considered hypotheses that summarily dismissed this fundamental aspect of the problem of taxation on the land market. Among the hypotheses enunciated by this author, a standout was that: “farmers and non-farmers alike have the same expectation of value, i.e., both are equally optimistic or pessimistic with relation to the future behavior of prices for rural real estate, and there is no growing of crops just as window dressing, i.e., planting just enough for show in order to minimize the tax burden.”

Again the debate focused on operational questions, mainly the complex calculation criteria of the tax and difficulty of administrative control. The high levels of evasion were attributed to inefficient collection by the tax authorities. As a consequence of these ideas, the administration of ITR was transferred to the Federal Revenue Secretariat (SRF) in 1990.

**Third Phase: Post-1996**

In response to the continuing problems, reform was again undertaken in December 1996, including, among others, the following changes:

- An increase in the tax rate for large and unproductive holdings – the maximum limit of 4.5% for properties larger than 15 thousand hectares rose to 20% on properties above only 5 thousand hectares;
- A reduction in the number of tax-rate brackets from 12 to 6;
- An end to regional differences in rates;

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2 A hectare equals 10,000 square meters or about 2.47 acres.
• The value declared by the owner for ITR purposes would henceforth be considered the appraised value to be paid in the event of expropriation.

Table 1.1 – Rates for Calculation of ITR

<table>
<thead>
<tr>
<th>Total area (in hectares)</th>
<th>Degree of Use (%)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>&gt;80 a 80 a 65 a 60 a 50 a 50 a 30 a 30 a 50 a 20 a 20 a 10 a 10 a 5 a 5 a 3 a 3 a 1 a 1 a 0.5 a 0.5 a 0.25 a 0.25 a 0.125 a 0.125 a 0.0625 a 0.0625 a 0.03125 a 0.03125 a 0.015625</td>
</tr>
<tr>
<td>Up to 50</td>
<td>0.20 0.40 0.60 0.85 1.60 3.00</td>
</tr>
<tr>
<td>50 to 200</td>
<td>0.40 0.80 1.30 1.90 3.40 6.40</td>
</tr>
<tr>
<td>200 to 500</td>
<td>0.80 1.60 2.60 3.90 7.80 15.6</td>
</tr>
<tr>
<td>500 to 1000</td>
<td>1.30 2.60 4.50 6.70 13.3 26.6</td>
</tr>
<tr>
<td>1000 to 5000</td>
<td>2.30 4.60 7.80 11.7 23.4 46.8</td>
</tr>
<tr>
<td>Over 5000</td>
<td>3.30 6.60 10.8 15.4 30.7 61.4</td>
</tr>
</tbody>
</table>


The rates differ only by degree of utilization and total area, as shown in Table 1.1. There are sharply progressive rates in relation to area and regressive ones for land use, so that productive properties are benefited.

Figure 1.2 – ITR Collection (1992-1999)

Reydon et al. (2000) point out the discontinuity in the rates adopted, observing that a property of 50.1 ha with 80.1% of the land utilized can pay 13 times the tax of one of 50.0 ha and 80.0% degree of use. The solution proposed by many is the use of reduction factors, as occurs for income tax.

Besides this, Reydon et al. (2000) show that despite administrative and legal improvements, the high hopes for the reform have not been met. The main reasons are associated with the difficulty of accurately appraising the value of raw land
and the imprecise nature of the concept of area utilized. Table 1.2 shows on the one hand the improvement obtained with the 1996 reform and, on the other, the low level of collection.

According to calculations by Oliveira (1993), the potential revenue from ITR would range between 1.4 and 2.8 billion dollars per year if rates between 0.5% and 1.0% were used. Although these calculations do not consider the effect of the effective application of these rates on landowners’ decisions, the magnitude of the estimates makes it clear that much can be done to increase ITR revenues.

Thus, even after ameliorating a series of operational problems, ITR continues to be largely ineffective. The description in this section is indicative of the incapacity of the Brazilian government to correctly apply this tax scheme and reduce the high level of evasion and under-taxation. The data furnished also point to the fact that this incapacity is more chronic for larger properties.

III. Basic Structure of the Model

The model concentrates on the problem of a government that maximizes its utility by designing a taxation mechanism based on the quantity produced and the farm size. This mechanism is capable of implementing the system of equilibrium transfers to the government. The observation of the amount of output is considered, for purposes of simplification, as the observed ICMS itself, since evasion of ICMS is much lower than for ITR and not significant.

Speculative activity is considered in ad hoc form, as a reduced form from other models in which land is used for non-productive ends, whether as collateral [Hoff, Braverman and Stiglitz (1993)] or as a hedge against inflation [Brandão and Rezende (1992), Feldstein (1980)].
The model considers, then, an economy in which the types of rural producers are indexed by \((\theta, \eta) \in \Theta\), where \(\Theta = [\underline{\theta}, \bar{\theta}] \times [\underline{\eta}, \bar{\eta}]\), distributed according to the distribution function \(F_{\eta\theta}\) with support in the entire rectangle. The parameters \(\theta\) and \(\eta\) refer respectively to agricultural productivity and speculative activities.

The price obtained for agricultural output is normalized at 1, an unlimited quantity of land is available at price \(r\) and each planted hectare costs \(3w\). Both production and lands are perfectly homogeneous.

A farmer of type \((\theta, \eta)\) who buys a property of size \(T\), grows \(A\) and pays a transfer of \(t\) to the government, obtains profits given by:

\[
\Pi = \theta Q(A) - wA + \eta \phi(T) - rT - t
\]

where \(Q\) and \(\phi\) are respectively the production and speculative benefit functions, with \(Q' > 0, Q'' < 0, Q'(0) = \infty, Q'(\infty) = 0, \phi' > 0, \phi'' < 0, \phi'(0) = \infty\) and \(\phi'(\infty) = 0\). This transfer is determined by the government, which can condition it on the amount produced and the size of the holding, which are observable. It can be noted that the greater \(\eta\) is, the larger the benefit from speculation, and likewise, as \(\theta\) increases, so does agricultural productivity.

Assume that there is no rental market and hence the choice of each producer must respect the scarcity condition \(A \leq T\). In this fashion, based on a taxation scheme \(\tau\), farmers are faced with the program

\[
\max_{A,T} \Pi \text{ s.a. } A \leq T. \quad (P)
\]

The government’s utility is dependent on tax revenues and the speculative use of land. With \(\lambda \in [0,1]\) being the “shadow price” attributed to speculative activity, the government utility function is defined by:

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3 Implicitly, to simplify, the productive technology includes fixed proportions, in which \(w\) represents the costs of labor and intermediate inputs per hectare.
\[ U = t - \lambda \eta \phi(T). \]

Therefore, the government decides to oppose speculative activity, which appears to be the Brazilian case. Opposition to speculative land use, incorporated in ad hoc form, can be based on the existence of another sector that needs agricultural products but does not share in any benefit arising from unproductive activity, in a model of equilibrium in the land market, with inelastic land supply.

In considering the utility of individuals in this sector, the government can be motivated to discourage the maintenance of idle land. In an environment with scarce land, the existence of idle land in equilibrium generates an artificial rise in land costs, which winds up restricting agricultural output. So the government, in considering the utility of all individuals, tries to implement taxation to reestablish a socially optimal price of land, besides trying to provide as many public services as possible, which revert to individuals in lump-sum form.

The presence of tax revenue in the utility function can be justified in different ways. A sufficient reason would be that individuals in the economy demand public goods, which are not provided optimally by the private sector. So the government, motivated by its desire to remain in power, has an incentive to maximize its tax receipts in case the provision of resources involves a sufficiently large volume.

To simplify the analysis and bring the problem closer to Brazilian reality, only the case in which there is a deterministic relationship between the types \( \theta \) and \( \eta \) is considered, i.e., \( \theta = \theta(\eta) \), with \( \theta(\cdot) \) continuous and differentiable. In this form, the producers can be completely specified by the parameter \( \eta \), and \( \Theta = [\underline{\eta}, \bar{\eta}] \). The distribution of \( \eta \) in \( \Theta \) is given by the distribution function \( F \). Moreover, the results derived
consider only the case where $\theta < 0^4$. In other words, producers with greater access to speculative activity are not involved totally in agriculture, and faced with imperfections in the labor market, they become less productive, and vice-versa.

The results presented in the next two sections are rigorously enunciated and demonstrated in the Appendix. To illustrate these results graphically, a numerical example is considered.

**IV. Results with Complete Information**

Initially, the choice of a taxation mechanism occurs in an environment of complete information, i.e., the government can exactly observe the type of agents and manages to establish tax collection rules that consider the willingness of each to use land either productively or speculatively.

The government, under complete information, can determine the allocations for each producer via a punitive taxation scheme. The sole conditions that restrict the government’s choice are those of scarcity (SC) and participation (IR) for each producer. The producers accept any government taxation system that produces a non-negative profit level in equilibrium. The optimal taxation mechanism in this case is defined by the program (P.FB) for a producer of type $\eta$:

$$\max_{\{U,A,T\}} U$$

subject to

$$\Pi \geq 0, \quad \text{(IR)}$$

$$A \leq T. \quad \text{(SC)}$$

The solution to this program is defined by the following result.

---

$^4$ The qualitative results obtained are maintained for the case in which $\theta > 0$. The use of ICMS in this case would also be necessary for producers that operate with idle land, in equilibrium, when the government does not take account of the type of producers. This result is only not verified for the case in which $\theta = 0$. 

Proposition 1: Under complete information, the optimal taxation mechanism can involve two categories of producers: (i) those who operate without idle land and are restricted by (SC): $\eta \in \Theta_R$; and (ii) those for whom the restriction (SC) is not active: $\eta \in \Theta_I$.

a) In $\Theta_R$, the equilibrium allocation is such that the cultivated area is equal to the property size. Both are determined by the equality between the total marginal benefit $[\theta Q(A^+_\eta) + \eta \phi(T^-_\eta)]$ and the marginal cost of each cultivated hectare $[w_r + r + \lambda \eta \phi(T^-_\eta)]$.

b) In $\Theta_I$, the producers are not found to be restricted by (SC). The cultivated area is determined by the equality between the marginal benefit $[\theta Q(A^+_\eta)]$ and the marginal cost of each cultivated hectare, $w$. The size of the holding is such that it equals the marginal benefit from speculative activity $[\eta \phi(T^-_\eta)]$ at its marginal social cost $[r + \lambda \eta \phi(T^-_\eta)]$.

c) The government manages to appropriate all the profit of the producers.

Note that the marginal social cost differs from the marginal individual cost by $\lambda \eta \phi(T^-_\eta)$, which suggests a natural interpretation for $\lambda$. When $\lambda = 0$, the government is not bothered by the speculative benefit of land ownership and the levels of $A$ and $T$ determined by the government’s design of the optimal taxation scheme (P.FB) coincide with those determined by the agents (P). In this case, the shadow price from speculation is zero and the government, in maximizing its tax revenue, maximizes the individual profit of each producer, which is fully appropriated. On the other hand, if $\lambda = 1$, the government completely inhibits idle land, since the shadow price from the scarcity restriction becomes constant and equal to $r$ for all $\eta \in \Theta$. 
The transfers required by the government represent all the producers’ profits. In the model with complete information, all restrictions (IR) are binding in equilibrium. Figure 4.1 qualitatively illustrates the shape of the allocations associated with the optimal taxation scheme.\(^5\)

**Figure 4.1 – Optimal Allocation with Complete Information**

The result that follows shows that this scheme can be implemented by a system analogous to ITR.

**Proposition 2:** Under complete information, the solution to the optimal taxation problem can be decentralized by a menu of linear taxes of the form:

\[
T^*_\eta = \beta^*_\eta T + \gamma^*_\eta,
\]

where \( \beta^*_\eta \) corresponds to the difference between the marginal social cost and the marginal individual cost of land ownership and \( \gamma^*_\eta \) is a fixed parcel that adjusts the level of tax collection.

In the implementation offered by the above result, the government offers a pair \( (\beta^*_\eta, \gamma^*_\eta) \) to a producer of type \( \eta \). In solving (P), each farmer chooses the amount of \( A \) and \( T \) determined by the government’s optimal solution. Note that there is a di-

\(^5\) The figures from Sections IV and V were constructed based on a numerical example that considered:
rect analogy between ITR, in performing its function on land distribution, and the parameter $\beta_\eta^*$. The tax rate goes up along with higher $\lambda$ and greater willingness of the farmer to use his land unproductively, which is measured by $\eta$. If $\lambda = 0$, the government does not distort the choice by producers, not taxing ownership of the land.

In $\Theta_r$, we have that $\beta_\eta^* = \frac{\lambda}{1-\lambda}$ and therefore the ITR rate does not vary according to the type of producer. With this, the model shows that in a context of complete information, one may use a single rate for all producers operating with properties above a determined size. Figure 4.2 illustrates the shape that the optimal ITR rates must obey in the case of complete information.

**Figure 4.2 – Optimal Rates with Complete Information**

This result shows that if the government could precisely observe landowner productivity parameters, there would be rates capable of implementing an optimal taxation scheme. Therefore, ITR would be able to implement this optimal solution. And in this solution, if $\lambda > 0$, the government discourages the speculative use of land, and with $\lambda = 1$, there would be no idle land in equilibrium.

\[ Q = \log(A), \phi = k \log(T), \theta = m - \eta \in [0,1], \text{ where } k \text{ and } m \text{ are constants, as are } \nu, \ r \text{ and } \lambda. \]
V. Results with Asymmetric Information

Consider a more realistic model in which there is asymmetric information about productivity and the benefits of land speculation. The problem of choosing the optimal taxation scheme then becomes a typical problem of mechanism design.

By the Revelation Principle, it is enough for the government to concentrate on those mechanisms that directly reveal the truth [Mirrlees (1971)]. In this fashion, besides considering the constraints of scarcity (SC) and participation (IR) for each type of producer, the tax scheme is conditioned by the incentive compatibility constraints (IC).

The IC constraints for each type of producer can be understood as
\[ \eta \in \arg \max \Pi(\hat{\eta} | \eta), \] where \( \Pi(\hat{\eta} | \eta) \) is the profit of a producer of type \( \eta \) who declares himself to be type \( \hat{\eta} \). This is to say, we can focus in an optimal taxation mechanism in which a producer of type \( \eta \) prefers the allocation \((t_\eta, T_\eta, A_\eta)\) over all others.

Determination of the optimal taxation mechanism under asymmetric information is performed by the government through resolution of the following maximization program:

\[
\max_{\{t_\eta, A_\eta, T_\eta, \hat{\eta}\}} \int_{h} t_\eta \text{d}F_\eta(t_\eta) - \lambda \eta \varphi(T_\eta) \quad \text{(P.SB)}
\]

subject to
\[
\Pi(\hat{\eta} | \eta) \geq 0, \quad \text{(IR}_\eta) \]
\[
\Pi(\eta | \eta) \geq \Pi(\hat{\eta} | \eta), \quad \text{(IC}_\eta) \]
\[
A_\eta \leq T_\eta, (\eta, \hat{\eta}) \in \left[\eta, \hat{\eta}\right]^T. \quad \text{(SC}_\eta) \]

Note that the government’s maximization program is written in the variables \( t, A \) and \( T \), notwithstanding that in the model the government observes only \( T \) and \( Q \) in determining \( t \). Nevertheless, as will be proven in the demonstration of Proposi-
tion 4 (in the Appendix), a mechanism based on $t$, $A$, and $T$ can be implemented by a mechanism in $t$, $T$ and $\theta Q$, and vice-versa. In choosing an optimal mechanism, the government incorporates the restrictions $(SC_{\eta})$ in its decision, since the restriction of scarcity and rationality of the agents is common knowledge.

The solution of the model with asymmetric information can be given by the following proposition.

**Proposition 3:** The optimum tax scheme $(\tilde{r}_\eta, \tilde{A}_\eta, \tilde{T}_\eta)$, under asymmetric information, of the program (P.SB) has the following characteristics:

i) There are at most two categories of producers: $\tilde{\Theta}_R = [\underline{\eta}, \bar{\eta}]$ and $\tilde{\Theta}_I = [\bar{\eta}, \bar{\eta}]$ such that $\tilde{A}_\eta = \tilde{T}_\eta$ for all $\eta \in \tilde{\Theta}_R$ and there is an increase in the number of producers restricted by (SC), i.e., $\Theta_R \subset \tilde{\Theta}_R$;

ii) In $\tilde{\Theta}_R$, $(\tilde{A}_\eta, \tilde{T}_\eta)$ are determined by the equality between the marginal benefit and total “virtual” marginal cost of each cultivated hectare, which corresponds to a marginal social cost added to marginal informational income:

$$\left(\frac{1-F(\eta)}{f(\eta)} \phi Q'(\tilde{A}_\eta) + \phi'(\tilde{T}_\eta)\right);$$

but still, $\dot{A}_\eta = \dot{T}_\eta > 0$;

iii) In $\tilde{\Theta}_I$, $(\tilde{A}_\eta, \tilde{T}_\eta)$ are determined by the equality between the marginal benefit and marginal “virtual” cost of each cultivated hectare and each hectare of total land:

$$\frac{1-F(\eta)}{f(\eta)} \phi Q'(\tilde{A}_\eta) \quad \text{and} \quad \frac{1-F(\eta)}{f(\eta)} \phi(\tilde{T}_\eta),$$

respectively, also with $\dot{T}_\eta > 0$ and $\dot{A}_\eta < 0$;

iv) Because of asymmetric information, the transfers to the government are deducted from the informational rent obtained by each agent;

v) Only agents of type $\underline{\eta}$ fail to receive informational rent – for the others, the restrictions (IR) are not active.
This result demonstrates the effect of asymmetric information on the choice of the amount of land used productively and that left idle. If there exists an \( \tilde{\eta} \in \left[ \eta^{-}, \eta^{+} \right] \), the optimal tax scheme of the government cannot fully inhibit idle land, even if \( \lambda = 1 \).

**Figure 5.1 – Optimal Allocation with Complete and Incomplete Information**

![Optimal Allocation Diagram](image)

Even for producers without idle land, \( \eta \in \left[ \eta^{-}, \eta^{+} \right] \), there is a distortion in determining the size of the enterprise in relation to the case with complete information. For these producers, this distortion means the cultivating of smaller plots. Figure 5.1 shows a comparison between the allocations obtained in the cases of complete and incomplete information. Note that except for those producers of type \( \tilde{\eta} \), the choices of cultivated area and establishment size are distorted.

**Proposition 4:** Under asymmetric information, the solution to the optimal taxation problem can be decentralized by a menu of linear taxes of the form

\[
\alpha \theta Q(A) + \beta T + \gamma_{(A,T)},
\]

where \( \alpha \) = 0 for all \( A \in \left[ A_{\lambda}, A_{\tilde{\eta}} \right] \). In this scheme, the government offers \( \left( \alpha, \beta, \gamma_{(A,T)} \right) \) observing production \( \theta Q(A) \) and farm size \( T \). Owners, maximizing their profit, determine \( A \) and \( T \) in accordance with the conditions of Proposition 3.
The above propositions show that the scheme put forward as a solution for the model with complete information cannot be implemented under asymmetric information. This inability to implement ITR as a solution to the problem with asymmetric information lends theoretical support to what the government found in the 1980s in comparing declared against actual data. It is also consistent with the fact that small producers, in general with no idle land, are more likely to correctly declare their land use.

In an economy such as Brazil's, where producers operate with idle land and the government is often in the dark as to the true productivity parameters of the productive and speculative activities available to various landholders, Proposition 4 shows that there are no tax rates able to make ITR into an optimal taxation scheme. The use of ICMS becomes necessary so that those producers with better access to speculative activities will pay their fair share of the tax burden.

Figure 5.2 shows the format for optimal ICMS and ITR rates to implement the optimal taxation scheme. ICMS is zero for producers who are restricted by (SC), operating without idle land. As the level of idle land rises, the ICMS rate rises as well, while the ITR rate decreases. Even though the tax on output causes a distortion in resource allocation, its use is justified by its ability to compose an implementable (or self-revealing) taxation mechanism.
VI. Conclusion

The need to evaluate taxation on land arises from its importance as an instrument for agrarian reform, especially in Brazil.

Agriculture in developing countries like Brazil is characterized by a series of market imperfections that make the price of land higher than the discounted value of the revenue flows generated by agricultural activity and in which the optimal farm size is artificially high. This situation leads to equilibria in which large unproductive landholdings contrast starkly with artificially high land prices that seriously limit the ability of poor family farmers to reach their full productive potential.

Without serious efforts to combat the unproductive use of cultivatable land, agrarian reform (via redistribution or land banks) will only have transitory effects. ITR constitutes an important instrument for land redistribution, besides financing other rural antipoverty programs. In this sense, intensified agrarian reform programs are justified by the need to catalyze the adequate functioning of market forces.

The model presented shows that because ITR is a declaratory tax, the optimal taxation scheme in a context of asymmetric information involves a combination of taxes on land and production. The current rules, even after the 1996 reformulation, would be implementable only if there were complete information or no idle land. Even without problems in administering the tax, the model demonstrates the inability with ITR to combat the chronic high levels of evasion and under-reporting.

Hence, the discussion of the effectiveness of ITR needs to be redirected. Besides considering the operational aspects, vital to obtain the desired results, the scope of the instrument in an environment such as Brazilian agriculture also needs to be questioned. This work attempts to contribute to expanding the debate on land taxation in Brazil, in complement to the other studies mentioned in Section II.
Furthermore, the suggestion to use a mixed system of land and production taxes fits perfectly in the discussion of tax reform that Brazil is currently engaged in. The proposal to return competence for administering ITR to the states could facilitate a cross-checking of data regarding the two taxes.

There is one closing caveat. The analysis presented here used a simple theoretical structure that is adequate only to establish the limits of applying a scheme such as ITR. But, faced with the prospect of using ICMS as an important source of information, discussion of the operational aspects again comes to the forefront. Only a careful and detailed empirical investigation can determine the parameters for a new reformulation of rural taxation policy and mechanisms.

**Bibliografia**


Appendix

This appendix contains the formal arguments presented in sections IV and V. The starting point will be the assumptions used to prove the results. These assumptions aim both to tie the analysis to the Brazilian reality and to simplify the technical matters.

Assumptions:

- **(Monotone Likelihood Ratio):** the distribution function is such that:

  \[
  \frac{d}{d\eta} \left( \frac{1 - F(\eta)}{f(\eta)} \right) < 0. \tag{A1}
  \]

- **(Technologies):** the functions $\theta$, $\phi$ and $Q$ are such that, for $A=T$ and $k$ sufficiently high\(^6\),

  \[
  \dot{\theta} < 0 \text{ e } \ddot{\theta} = 0, \tag{A2}
  \]

  \[
  \theta Q + \phi > 0, \tag{A3a}
  \]

  \[
  k\theta Q' + \phi' > 0, \tag{A3b}
  \]

  \[
  \frac{d}{d\eta} \left( \frac{1 - F}{f} \right) \phi' > 1. \tag{A3c}
  \]

The assumption A2 determines that those who use land for speculation have a lower agricultural productivity, which is observed in Brazil. In most cases, these speculators are only a part-time agricultural producers and, facing to the imperfections in labor markets due to moral hazard problems, become less efficient.

\[^{6}\text{In the prove of proposition 3', a lower bound to } k \text{ will be determined.}\]
The other conditions are technical and ensure a easier characterization of the optimal solution. Note that, given (A3a) and (A3b), the assumptions (A1) and (A3c) are satisfied to a uniform distribution.

**Complete Information**

The optimal mechanism design problem, under complete information, is defined by the following program to each producer \( \eta \in \Theta \):

\[
\max_{\{P.A.T\}} U(T, A) \quad \text{(P.FB)}
\]

subject to

\[
\Pi \geq 0, \quad \text{(IR)}
\]
\[
A \leq T. \quad \text{(SC)}
\]

The solution is characterized by the Proposition 1', which is a formal version of Proposition 1.

**Proposition 1':** Under complete information, the optimal taxation mechanism can involve two categories of producers, i.e., there is at most one \( \eta^* \) such that \( \Theta_\eta = [\eta^*, \eta^*] \) and \( \Theta_I = [\eta^*, \eta^*] \).

a) **Small farmers** (without idle land): for every type \( \eta \in \Theta_\eta, A_\eta^* \) and \( T_\eta^* \) are defined by:

\[
A_\eta^* = T_\eta^*, \\
\theta Q(A_\eta^*) + (1 - \lambda) \eta \phi(T_\eta^*) = w + r. \quad \text{(FB)}
\]

b) **Large landholders** (with idle land): for every type \( \eta \in \Theta_I, A_\eta^* \) and \( T_\eta^* \) are defined by:

\[
\theta Q(A_\eta^*) = w, \\
(1 - \lambda) \eta \phi(T_\eta^*) = r. \quad \text{(FB)}
\]

c) **Transfers:** every type \( \eta \in \Theta, t_\eta^* \) is defined by:

\[
t_\eta^* = \theta Q(A_\eta^*) - wA_\eta^* + \eta \phi(T_\eta^*) - rT_\eta^*.
\]
**Proof.** Initially, note that the \((IR)\) are always binding. If it is not the case for a \(\eta \in \Theta\), the government would increase the tax collection, \(t^{*}_\eta\), satisfying the constraints in \((P.FB)\). Solving \((IR)\) to \(r\) and substituting in the objective function, \((P.FB)\) can be written, for each \(\eta \in \Theta\), as

\[
\max \theta Q(A_\eta) - wA_\eta + (1 - \lambda) \eta \phi(T_\eta) - rT_\eta \quad \text{s.t.} \quad A_\eta \leq T_\eta.
\]

The first order condition is given by:

\[
\theta Q'(A_\eta) = w + \mu_\eta,
\]
\[
(1 - \lambda) \eta \phi'(T_\eta) = r - \mu_\eta,
\]
\[
\mu_\eta (A_\eta - T_\eta) = 0, \quad \mu_\eta \geq 0.
\]

By differentiating \(\mu_\eta\), we get

\[
\mu_\eta' = \theta Q'' + \theta Q' \hat{A}_\eta^{*}
\]
\[
= \frac{(1 - \lambda) [\eta \theta Q'' - \theta Q' \phi']} {\theta Q'' + (1 - \lambda) \eta \phi''} < 0
\]

and, under \((A2)\), there is at most one \(\eta^{*}\) such that \(\mu_\eta^{*} = 0\). Thus, depending on the technologies of agricultural and speculative activities, we can identify the mentioned categories of producers. The first-order conditions for each group are given by \((FB_R)\) and \((FB_I)\), respectively.

The next result shows that the optimal scheme can be implemented by a system analogous to ITR under complete information.

**Proposition 2':** Under complete information, the solution to the optimal taxation problem can be decentralized by a menu of linear taxes of the form:

\[
t^{*}_\eta = \beta^{*}_\eta T + \gamma^{*}_\eta,
\]

where \(\beta^{*}_\eta = \lambda \eta \phi(T^{*}_\eta)\) and \(\gamma^{*}_\eta = t^{*}_\eta - \beta^{*}_\eta T^{*}_\eta\).

**Proof.** Consider \(t_\eta = \beta_\eta T + \gamma_\eta\), where \(\beta_\eta = \lambda \eta \phi(T^{*}_\eta)\) and \(\gamma_\eta = t^{*}_\eta - \beta^{*}_\eta T^{*}_\eta\). The first-order conditions of \((P)\) can be written as:
\[ \theta Q(A) \]
\[ \eta \phi ( ) \quad \beta \]

\[ \Pi(\eta | \eta) \] denote the profits of producer \( \eta \) that declares being \( \eta^* \), i.e.,
\[ \Pi(\eta | \eta) = 0 Q(A_\eta) - w_{A_\eta} + \eta \phi(T_\eta) - r T_\eta - t_\eta, \quad \forall(\eta, \eta) \in [\underline{\eta}, \bar{\eta}]^2. \]

Thus, we can define:

**Definition:** An allocation \( (t_\eta, A_\eta, T_\eta) \) is implementable if, and only if
\[ \Pi(\eta | \eta) \geq \Pi(\eta^* | \eta) \quad \forall(\eta, \bar{\eta}) \in [\underline{\eta}, \bar{\eta}]^2. \] (IC)

The next lemma relies on Guesnerie and Laffont (1984) establish sufficient conditions to make an allocation implementable.

**Lemma:** If the allocation \( (t_\eta, A_\eta, T_\eta) \) is implementable and piecewise \( C^1 \) then:
\[ \frac{d}{d \eta} \Pi(\eta | \eta) = \theta Q(A_\eta) + \phi(T_\eta), \] (IC\(^1\))

and
\[ \theta Q'(A_\eta) A_\eta + \phi'(T_\eta) T_\eta \geq 0 \text{ a.s. in } [\underline{\eta}, \bar{\eta}]. \] (IC\(^2\))
Proof. Let \((\eta, A, T)\) an implementable allocation piecewise \(C^1\). Thus, for every 
\(\eta \in [\underline{\eta}, \overline{\eta}]\),
\[
\Pi(\eta | \eta) \geq \Pi(\underline{\eta} | \eta) = [\theta - \hat{\theta}]Q(A) + (\eta - \overline{\eta})\phi(T)
\]
where \(\hat{\theta} \equiv \theta(\eta)\). Hence,
\[
\Pi(\eta | \eta) - \Pi(\underline{\eta} | \eta) \geq (\theta - \hat{\theta})Q(A) + (\eta - \overline{\eta})\phi(T)
\]
and, switching \(\eta\) and \(\overline{\eta}\), we get
\[
(\theta - \hat{\theta})Q(A) + (\eta - \overline{\eta})\phi(T) \geq \Pi(\eta | \eta) - \Pi(\eta | \overline{\eta}) \geq (\theta - \hat{\theta})Q(A) + (\eta - \overline{\eta})\phi(T).
\]
Dividing by \((\eta - \overline{\eta})\) and taking the limit \(\eta \to \overline{\eta}\), the function \(\Pi\) is proved to be piecewise \(C^1\) and
\[
\frac{d}{d\eta} \Pi(\eta | \eta) = \theta Q(A) + \phi(T) \text{ a.s.}
\]
and, doing the same to \((\eta - \overline{\eta})^2\), we get
\[
\theta Q'(A)\dot{A} + \phi'(T)\dot{T} \geq 0 \text{ a.s.} \star
\]

Under (A3a), \(\Pi(\eta | \eta)\) is increasing in \(\eta\) and only the \((IR)\) is binding in equilibrium, which determines that \(\Pi(\underline{\eta} | \underline{\eta}) = 0\). Integrating \((IC^1)\) by parts and substituting into the objective function, the maximization program can be rewritten as:
\[
\max_{\{\eta, A, T\} | \eta \in [\underline{\eta}, \overline{\eta}]} \int \left[ \theta Q(A) - wA + (1 - \lambda)\phi(T) - rT - \frac{1 - F(\eta)}{f(\eta)} (\theta Q(A) + \phi(T)) \right] dF(\eta)
\]
subject to
\[
\theta Q'(A)\dot{A} + \phi'(T)\dot{T} \geq 0, \quad (IC^2)
\]
\[
A \leq T, \eta \in [\underline{\eta}, \overline{\eta}], \quad (SC)\]

The solution of (P.SB) is characterized by the following proposition.
Proposition 3': Under (A1)-(A3) the optimum taxation scheme \( (\tilde{r}_n, \tilde{A}_n, \tilde{T}_n) \) of the program (P.SB), ignoring the (IC\(^2\)) constraint, has the following characteristics:

i) there are at most two categories of producers: \( \bar{\Theta}_R = [\tilde{\eta}, \tilde{\eta}] \) and \( \bar{\Theta}_I = [\tilde{\eta}, \tilde{\eta}] \);

ii) in \( \bar{\Theta}_R \), \( (\tilde{A}_n, \tilde{T}_n) \) is determined by

\[
\tilde{A}_n = \tilde{T}_n, \\
\theta Q'(\tilde{A}_n) + (1 - \lambda) \eta \phi'(\tilde{T}_n) = w + r + \frac{1 - F(\eta)}{f(\eta)} (\theta Q'(\tilde{A}_n) + \phi'(\tilde{T}_n));
\]

with \( \tilde{A}_n = \tilde{T}_n > 0 \).

iii) in \( \bar{\Theta}_I \), \( (\tilde{A}_n, \tilde{T}_n) \) is determined by

\[
\theta Q'(\tilde{A}_n) = w + \frac{1 - F(\eta)}{f(\eta)} Q'(\tilde{A}_n), \\
(1 - \lambda) \eta \phi'(\tilde{T}_n) = r + \frac{1 - F(\eta)}{f(\eta)} \phi'(\tilde{T}_n);
\]

with \( \tilde{T}_n > 0 \) and \( \tilde{A}_n < 0 \).

iv) \( \tilde{r}_n = \theta Q(\tilde{A}_n) - w \tilde{A}_n + \eta \phi(\tilde{T}_n) - r \tilde{T}_n - \int_{\tilde{\eta}}^{\tilde{\eta}} \theta Q(\tilde{A}_n) + \phi(\tilde{T}_n) \eta d\eta; \)

v) \( \Pi(\eta) = \int_{\tilde{\eta}}^{\tilde{\eta}} \theta Q(\tilde{A}_n) + \phi(\tilde{T}_n) \eta d\eta; \)

vi) for every \( \eta \in [\tilde{\eta}, \tilde{\eta}] \) we have \( \tilde{A}_n < A_n \) and \( \Theta_R \subset \bar{\Theta}_R \), i.e., because of asymmetric information, there is an increase in the proportion of binding producers.

vii) \( (\tilde{r}_n, \tilde{A}_n, \tilde{T}_n) \) is implementable, i.e., the (IC\(^2\)) is satisfied.

Proof. The first-order conditions of (P.SB), ignoring (IC\(^2\)), are given by:

\[
\theta Q'(A_n) = w + \frac{1 - F(\eta)}{f(\eta)} Q'(A_n) + \frac{\mu_n}{f(\eta)}, \\
(1 - \lambda) \eta \phi'(T_n) = r + \frac{1 - F(\eta)}{f(\eta)} \phi'(T_n) - \frac{\mu_n}{f(\eta)},
\]
\[ \mu_\eta (A_\eta - T_\eta) = 0, \quad A_\eta \leq T_\eta. \]

a) Define \( \Theta_R = \{ \eta : \mu_\eta > 0 \} \), hence, for every \( \eta \in \Theta_R \), \( \tilde{A}_\eta = \tilde{T}_\eta \) and the first-order conditions can be written as

\[ \theta Q'(\tilde{A}_\eta) + (1 - \lambda) \eta \phi'(\tilde{T}_\eta) = w + r + \frac{1 - F(\eta)}{f(\eta)} \left[ \theta Q'(\tilde{A}_\eta) + \phi'(\tilde{T}_\eta) \right], \]

\[ \frac{\mu_\eta}{f(\eta)} = \theta Q'(\tilde{A}_\eta) - w - \frac{1 - F(\eta)}{f(\eta)} \theta Q'(\tilde{A}_\eta). \]

The differentiation the first condition with respect to \( \eta \) implies that:

\[ \dot{A} = \dot{T} = \left[ \frac{d}{d\eta} \left( \frac{1 - F}{f} \right) - 1 \right] \phi - \left( \theta - \frac{1 - F}{f} \theta \right) Q^* \dot{A} + \left( 1 - \frac{d}{d\eta} \left( \frac{1 - F}{f} \right) \right) \phi \dot{Q}' > 0 \]

under (A1), (A2) and (A3), since the second-order condition guarantees that the denominator is negative. And, after differentiating \( \mu_\eta \), these assumptions imply

\[ \left( \frac{\mu_\eta}{f(\eta)} \right) = \left( \theta - \frac{1 - F}{f} \theta \right) Q^* \dot{A} + \left( 1 - \frac{d}{d\eta} \left( \frac{1 - F}{f} \right) \right) \phi \dot{Q}' < 0, \]

which demonstrate the parts (i) and (ii).

b) Let \( \Theta_J = \Theta - \Theta_R \). For every \( \eta \in \Theta_J \), the first-order conditions become:

\[ \theta Q'(\tilde{A}_\eta) = w + \frac{1 - F(\eta)}{f(\eta)} \theta Q'(\tilde{A}_\eta), \]

\[ (1 - \lambda) \eta \phi'(\tilde{T}_\eta) = r + \frac{1 - F(\eta)}{f(\eta)} \phi'(\tilde{T}_\eta). \]

Thus, the item (iii) is demonstrated with

\[ \dot{A}_\eta = -\frac{\left[ 1 - \frac{d}{d\eta} \left( \frac{1 - F}{f} \right) \right] \phi \dot{Q}'}{\left( \theta - \frac{1 - F}{f} \theta \right) Q^*} < 0, \quad (\ast) \]
The items (iv) and (v) can be easily derived from \((IC^1)\).

The item (vi) involve a comparison between the conditions that determines \(A^*_\eta\) and \(\widetilde{A}_\eta\) in \(\Theta_R\) and \(\bar{\Theta}_R\), respectively. Under (A2), \(\theta < 0\) and, then, \(A^*_\eta > \widetilde{A}_\eta\) in \([\eta, \min(\eta^*, \bar{\eta})]\). Considering this inequality in the conditions that establish \(\Theta_R\) and \(\bar{\Theta}_R\), we conclude that \(\Theta_R \subset \bar{\Theta}_R\).

To prove that \((\bar{T}_\eta, \widetilde{A}_\eta, \bar{T}_\eta)\) is implementable, we start by noting that

\[
\Pi(\eta | \eta) - \Pi(\bar{\eta} | \eta) = \int \left[ \bar{\phi}Q(\bar{\eta}) + \phi(\bar{T}_\eta) \right] d\bar{\eta} + (\bar{\theta} - \theta) Q(\bar{A}_\eta) + (\bar{\eta} - \eta) \phi(\bar{T}_\eta) \]

\[
= \int \left[ \bar{\phi}Q(\bar{A}_\eta) - Q(\bar{\eta}) + (\bar{\eta} - \eta) \phi(\bar{T}_\eta) \right] d\bar{\eta} \]

\[
= \int \left[ \bar{\phi}Q'(A)dA + \phi'(T)dT \right] d\bar{\eta},
\]

where the last equality resulting from (A2). Considering \(\bar{\eta} < \eta\), there are three possibilities:

- \(\eta, \bar{\eta} \in \Theta_R\): \(\widetilde{A}_\eta = \bar{T}_\eta\) and then

\[
\Pi(\eta | \eta) - \Pi(\bar{\eta} | \eta) = \int \left[ \bar{\phi}Q'(A) + \phi'(A) \right] dA d\bar{\eta}
\]

which is positive under (A3b).

- \(\eta, \bar{\eta} \in \Theta_f\): the condition (*) implies that \(Q(\widetilde{A}_\eta) < Q(\bar{A}_\eta)\) and \(\phi(\bar{T}_\eta) > \phi(\bar{T}_\eta)\) for every \(\bar{\eta} \in [\eta, \bar{\eta}]\) and, thus, \(\Pi(\eta | \eta) > \Pi(\bar{\eta} | \eta)\).

- \(\bar{\eta} \in \Theta_R, \eta \in \Theta_f\): in this case, we have

\[
\bar{T}_\eta = \left[ 1 - \lambda \right] \frac{d}{d\eta} \left( \frac{1 - F}{f} \right) \phi' - \left[ 1 - \lambda \right] \frac{1 - F}{f} \phi'' > 0.
\]
\[
\Pi(\eta | \eta) - \Pi(\tilde{\eta} | \eta) = \int_{\tilde{\eta}}^{\eta} \int_{\tilde{\eta}}^{\eta} \hat{\phi}(A) dA + \int_{\tilde{\eta}}^{\eta} \hat{\phi}(T) dT \] 
\[
= \int_{\tilde{\eta}}^{\eta} \left[ \hat{\phi}(A) + \phi'(A) \right] d\tilde{\eta} + \int_{\tilde{\eta}}^{\eta} \left[ \phi(\tilde{\eta}) - \phi(\tilde{\eta}) \right] d\tilde{\eta} \geq 0,
\]

since, using the same arguments above, the two integrals are non-negative.

The case in which \( \tilde{\eta} > \eta \) is analogous. ♦

Note that the assumption (A3) implies that the (IC^2) constraint is not binding in equilibrium. If this condition is not satisfied, the (IC^2) is binding and the “ironing principle” have to be used. However, it is just a technical issue that will not be addressed in this paper.

**Proposition 4':** Under asymmetric information, the solution to the optimal taxation problem can be decentralized by a menu of linear taxes of the form,

\[
\alpha_A \theta Q(A) + \beta_T T + \gamma_{(A,T)},
\]

where

\[
\alpha_A = \begin{cases} 
0 & \text{se } A \in [\tilde{A}_2, \tilde{A}_1] \\
-1 & \text{se } A \in [\tilde{A}_1, \tilde{A}_2].
\end{cases}
\]

\[
\beta_T = \begin{cases} 
\frac{\tilde{T}_T}{T} + \frac{1}{T} \int_{\tilde{T}_T}^{T} \left[ \lambda_T(T) \phi(T) + \frac{1-F(\tilde{T})(\phi(T))}{\tilde{T}_T(T)} \phi'(T) \right] dT & \text{se } T \in [\tilde{T}_T, \tilde{T}_T] \\
\frac{\tilde{T}_T}{T} + \frac{1}{T} \int_{\tilde{T}_T}^{T} \left[ \lambda_T(T) \phi(T) + \frac{1-F(\tilde{T})(\phi(T))}{\tilde{T}_T(T)} \phi'(T) \right] dT & \text{se } T \in [\tilde{T}_T, \tilde{T}_T].
\end{cases}
\]

and \( \eta(T) \) and \( \eta(A) \) are the inverses of \( \tilde{T}_T \) and \( \tilde{A}_T \), respectively.

In this scheme, the government offers \( (\alpha_A, \beta_T, \gamma_{(A,T)}) \) observing the agricultural output \( \theta Q(A) \) and the farm size \( T \). Producers, maximizing their profit, determine \( A \) and \( T \) in accordance with (SB_1) e (SB_2).
Proof. Define \( q = \theta Q(A) \). Initially, observe that the mechanisms in \((q,T,t)\) and \((A,T,t)\) are equivalent. Since \( Q' > 0 \), the inverse \( A = Q^{-1}(\frac{q}{\theta}) \) is well-defined and, designing a mechanism in \((q,T,t)\), the maximization program faced by the government is given by

\[
\max \{ \eta, q, T \} \text{subject to} \quad \frac{q}{\theta} = Q^{-1}(\frac{q}{\theta}) \
\]

The first-order condition to \( q \) is

\[
1 - w - \frac{1}{Q'(A)} - \frac{1 - F(\eta)}{f(\eta)} = 0
\]

which can be rearranged to

\[
\theta Q'(A) = w + \frac{1 - F(\eta)}{f(\eta)} Q'(A) + \frac{\mu}{f(\eta)}.
\]

Thus, we can focus in mechanisms written in \((A,T,t)\), which are algebraically simpler. Indeed, considering \( q \) or \( A \) is a convenience matter, once there is a well-defined inverse of \( Q \).

Consider a mechanism in which, for each \((A,T)\) announced by the producer, the tax collection is defined by

\[
t(A,T) = \alpha_A \theta Q(A) + \beta_T T + \gamma_{(A,T)},
\]

where \( \alpha_A \equiv \alpha(A) \), \( \beta_T \equiv \beta(T) \) and \( \gamma_{(A,T)} \equiv \gamma(A,T) \) establish the scheme designed by the government. In this scheme, \( \alpha_A \) guarantees \( A = \bar{A} \) (or \( q = \theta Q(\bar{A}) \)), \( \beta_T \) is such that \( T = \bar{T} \) and \( \gamma_{(A,T)} \) is determined by
where $\eta(T)$ is the inverse of $\tilde{T}_t$ (well-defined under (A1)-(A3)).

Facing this scheme, the producer chooses $(A,T)$ so as to maximize his profits, following the program

$$
\max_{A,T} \left( (1-\alpha_A) \theta Q(A) + \eta\phi(T) - wA - (r+\beta_T)T - \gamma(A,T) \right)
$$

subject to

$$
A \leq T.
$$

Denoting by $\mu$ the Lagrange multiplier, the first-order conditions are given by

$$
(1-\alpha_A) \theta Q'(A) = w + \left( \frac{\partial}{\partial A} \alpha_A \right) \theta Q(A) + \frac{\partial}{\partial A} \gamma(A,T) + \mu,
$$

$$
\eta\phi'(T) = r + \beta_T + \left( \frac{\partial}{\partial T} \beta_T \right) T + \frac{\partial}{\partial T} \gamma(A,T) - \mu.
$$

Suppose, for instance, that $\frac{\partial}{\partial A} \gamma_{(\eta,\tilde{\eta})} = \frac{\partial}{\partial T} \gamma_{(\eta,\tilde{\eta})} = 0$ in the curve $\eta \rightarrow (\tilde{\eta}_t, \tilde{T}_t)$.

a) Case $\mu > 0$: $A = T$ and the first-order conditions imply

$$
(1-\alpha_A) \theta Q'(A) + \eta\phi'(T) = w + \left( \frac{\partial}{\partial A} \alpha_A \right) \theta Q(A) + r + \beta_T + \left( \frac{\partial}{\partial T} \beta_T \right) T + \frac{\partial}{\partial A} \gamma(A,T) + \frac{\partial}{\partial T} \gamma(A,T)
$$

and $A = T = \tilde{A}_t = \tilde{T}_t$ whether

$$
\alpha_A = 0
$$

$$
\left( \frac{\partial}{\partial T} \beta_T \right) \tilde{T}_t + \beta_T = \lambda \eta\phi'(\tilde{T}_t) + \frac{1-F(\tilde{T}_t)}{f(\tilde{T}_t)} \left[ \theta Q'(\tilde{A}_t) + \phi'(\tilde{T}_t) \right].
$$

Or, integrating the last condition and considering $\beta_{\tilde{T}_t} = \beta_t$, 

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\[ \beta_T = \frac{\beta}{T} \frac{T_{\eta}}{T} + \frac{1}{T} \int_{\eta}^{T} \left[ \lambda \eta \phi(T) + \frac{1 - F(\eta(T))}{f(\eta(T))} \left( \phi'(T) + \phi(\tilde{T}) \right) \right] dT. \]

b) Case \( \mu = 0 \): to ensure that \( A = \tilde{A}_\eta \) and \( T = \tilde{T}_\eta \) we need

\[ \alpha_A Q(\tilde{A}_\eta) + \left( \frac{\partial}{\partial A} \alpha_A \right) Q(\tilde{A}_\eta) = \frac{1 - F(\eta)}{f(\eta)} \phi'(\tilde{T}_{\eta}), \]

\[ \left( \frac{\partial}{\partial T} \beta_T \right) \tilde{T}_{\eta} + \beta_T = \lambda \eta \phi(\tilde{T}_{\eta}) + \frac{1 - F(\eta)}{f(\eta)} \phi'(\tilde{T}_{\eta}). \]

Or, integrating and using \( \alpha_{\tilde{A}_\eta} = 0 \) and \( \beta_{\tilde{T}_{\eta}} \) given by the case \( \mu > 0 \), we have

\[ \alpha_A = \frac{1}{Q(A)} \int_{\tilde{A}_\eta}^{\tilde{A}_0} \frac{F(\eta(T))}{f(\eta(T))} \phi'(\tilde{T}) d\tilde{T}, \]

\[ \beta_T = \frac{\beta}{T} \frac{T_{\eta}}{T} + \frac{1}{T} \int_{\eta}^{T} \left[ \lambda \eta \phi(T) + \frac{1 - F(\eta(T))}{f(\eta(T))} \phi'(T) \right] dT, \]

where \( \eta \) can be expressed in terms of \( T \) or \( A \).

Finally, we have to show that \( \frac{\partial}{\partial A} \gamma(\tilde{A}_\eta, \tilde{T}_{\eta}) = \frac{\partial}{\partial T} \gamma(\tilde{A}_\eta, \tilde{T}_{\eta}) = 0 \) in the curve \( \eta \to (\tilde{A}_\eta, \tilde{T}_{\eta}) \) in both cases. This is a consequence of the envelope's theorem. Since there is a bijection between \( \eta \) and \( A \), and between \( \eta \) and \( T \), in each case, it is enough to show that \( \frac{\partial}{\partial \eta} \gamma(\tilde{A}_\eta, \tilde{T}_{\eta}) = 0 \). Differentiating \( \gamma \) we obtain

\[ \frac{\partial}{\partial \eta} \gamma(\tilde{A}_\eta, \tilde{T}_{\eta}) = \left[ \theta Q' - w - \theta \frac{\partial}{\partial A} (\alpha_A Q) \right] \tilde{A} + \left[ \eta \phi' - r - \frac{\partial}{\partial T} (\beta_T T) \right] \tilde{T}. \]

And in the points \( A = \tilde{A}_\eta \) and \( T = \tilde{T}_{\eta} \) the two brackets vanish. \( \blacksquare \)