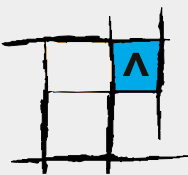




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Claudia Sanchez Vela  
**Profits from Natural Resources  
and Wealth Distribution**



# Allocation of Revenues from Natural Resources and Wealth Distribution

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## 1 Introduction

In many countries, profits from natural resources of the region are used to finance part of the government expenditure under the claim that natural resources belong to the Nation. An example of this is the case of the oil in Mexico. An immediate problem of this policy is the dependence of Government spending on income from oil. The magnitude of this dependence obviously depends on the share of government income financed by natural resources profits. In Mexico, oil profits account for up to one third of total government revenue. A related problem is that easy profits from oil might deter the creation of a strong fiscal policy. Thus, under the need to increase government revenues, it is easy to overexploit the natural resources leading to a faster deterioration of the environment.

Here we will consider an alternative use of the oil revenue. We consider a case in which the total revenue obtained is distributed uniformly among the population. To maintain the same government expenditure level the proportional income tax paid by the households should increase. The amount and the uses of the government expenditure are assumed to be unchanged, and we will just focus on the way this amount can be collected.

Therefore we have two policies to compare. The first one consists in using the total profits from natural resources and a certain level of proportional income tax to finance the government expenditure. And the second one is to redistribute the gains uniformly to the population and increase the proportional tax rate to reach the same level of government expenditure.

The objective of this paper is to compare these two policies analyzing their respective effects in welfare change by type of individual, aggregate welfare change and income distribution of a society composed by heterogeneous agents.

The change of policy implies getting rid of a lump sum tax. It is expected then to have an aggregate welfare loss, but we also expect an improvement in welfare distribution. The issue is, how much is lost, and how big is the change in the welfare distribution. To answer these questions we develop a static model with valued leisure and consumption of infinitely many agents heterogeneous in their productivity level.

Using this model we analyze the effects of the change of policy on labor supply, individual consumption, aggregate production and individual welfare. We can also study the change in welfare distribution and the aggregate cost of the change of policy.

The whole problem can also be interpreted in the opposite direction. Hence, we can consider an efficiency gain from having a lump sum tax (the size of the profits per capita obtained exploiting natural resources) but also implies deterioration in the welfare distribution. We would like to know the magnitude of these two.

## 2 The Model

In this section we present a static model of exploitation of natural resources. There is one type of consumption good, it can be produced using labor or obtained by exploiting the natural resources (belonging to the nation). Labor is provided by a continuum of agents heterogeneous in their productivity level,  $\lambda$ . There is a government in the economy, exploiting the natural resources and taxing the households income.

### 2.1 Profits from Natural Resources

The government exploits certain natural resources, obtaining  $\lambda$  units of the consumption good. A fraction of them,  $t\lambda$  with  $t \in [0; 1]$ , is distributed as a lump-sum transfer to the agents; and the remaining part,  $(1 - t)\lambda$ ; is used to finance some of the government expenditures.

## 2.2 Households

There is a continuum of agents and the total amount of the population is normalized to one. The agents are heterogeneous in the productivity per unit of labor  $\theta$ . The productivity levels are distributed according to the distribution function  $f(\theta)$  (a specific example is shown in figure 1).

Every agent has two sources of income: the lump sum transfer from the government  $t$  and the amount of goods he produces. An agent with productivity level  $\theta$  providing  $l$  units of labor produces  $\theta l$  units of the consumption good.

All agents have the same preferences over consumption and leisure, described by the utility function  $U(c; 1 - l)$ . We assume  $U$  is twice continuously differentiable, increasing, and concave. Each agent maximizes his utility function by choosing labor (restricted to be non negative) and consumption restricted to be less than or equal to his after tax income, taking as given the tax rate  $\tau$  and the lump sum transfer  $t$ . Therefore, an agent with productivity level  $\theta$  solves the following problem

$$\begin{aligned} \text{Max}_{c,l} \quad & U(c; 1 - l) \quad \text{s.t:} \quad c \leq (1 - \tau)\theta l + t; \\ & 0 \leq l \leq 1, \end{aligned} \quad (1)$$

From this problem we obtain the decisions rules for consumption  $c^* = c(\theta; t)$ , and labor,  $l^* = l(\theta; t)$ .

## 2.3 Government

The government spends a fixed amount  $G$ , assumed to be greater than the profits from natural resources,  $\theta$ . A share of the Government's expenditure is financed by a fraction  $(1 - \tau)$  of natural resources profits, the remaining part is financed by taxing the total income of the agents at the proportional rate  $\tau$ . This tax rate is such that the government's budget is balanced. That is, for a given  $t$ ,  $\tau$  has to satisfy:

$$G = \int \tau [\theta l(\theta; t) + t] f(\theta) d\theta + (1 - \tau) \theta. \quad (2)$$

For future reference, let's define  $G_R$  as the level of government expenditure that should be obtained from income taxes i.e.  $G_R = G - (1 - \tau) \theta$ .

### 3 Change of Policy, Analytical Part

In this section we are going to analyze a change of policy. We set the benchmark case as the one in which transfers to the households are equal to zero ( $t = 0$ ), and we will compare this situation with another where some of the profits from natural resources are distributed uniformly among the population ( $t$  takes any value between 0 and 1). We are going to use  $G_{R_0}$  to indicate the government expenditures financed by taxes in the benchmark case ( $G_R$  when  $t = 0$ ) and  $\Phi G_R$  denote the change of this variable after the lump sum transfer is given to the households. This is,  $G_{R_0} = G_i(1 - 0)$ ,  $\Phi G_R = t$  and  $G_R = G_i(1 - t) = G_{R_0} + \Phi G_R$ .

A similar notation is going to be used for labor and consumption:  $l_0$  and  $c_0$  indicate agent's choices for labor and consumption in the benchmark situation, and  $\Phi l$ ,  $\Phi c$  denote the change of value of this variables after the change of policy.

To analyze the change of policy described above we will study the impact of this policy on several variables. First we start with the effect on the optimal choices of the individuals (consumption and leisure) and on their income; by this moment the new income tax,  $\tau$ ; is not known, so the effect on these variables is expressed as a function of  $\tau$ . Then, as a second step, we study how the fiscal policy has to change so the government budget does not change and we get the income tax level for this situation. Third, with the optimal decision rules of the individuals for consumption and leisure and knowing the income tax level and the transfer from natural resources revenues for the new policy, it is possible to obtain the welfare change that the individuals suffer because of the change in policy. Fourth, once we obtain the welfare change each type of individual suffer, we will measure the effect on aggregate variables.

#### 3.1 Consumption, leisure and income

Suppose  $U(c; 1 - l) = c^\alpha (1 - l)^{1-\alpha}$ ;  $\alpha + 1 = 1$ . From the first order conditions of the consumers problem we get the following relations:

$$l^\alpha = \max_{\tau} \left[ \tau \frac{1-l}{l}; 0 \right] \quad (3)$$

$$= \tau \frac{1-l}{l} \quad \text{and} \quad (3')$$

$$c^a = (1 - \tau) \max_{t_i} f^a(w + t_i); t_i g = (1 - \tau) Y \quad (4)$$

where  $Y = Y(w; t)$  is the income before tax of an individual type  $w$ ;

$$Y = w + t(w; t) = \max_{t_i} f^a(w + t_i); t_i g \quad (5)$$

$$) \quad Y_0 = w_0 \text{ \& } \Phi Y = \max_{t_i} (w_0 + t_i; t_i g) \quad (5')$$

Observe that the higher  $w$  the smaller  $\Phi Y$  ( $= \int \min_{t_i} f^a(w + t_i); t_i g$ ) in absolute value. That is, the effect on labor is smaller the greater is the wage of the agent.

From equation (4), we have:

$$\begin{aligned} c^a &= c_0 + \Phi c = (1 - \tau) (c_0 + \Phi Y) \quad (6) \\ &= (1 - \tau) Y_0 + (1 - \tau) \Phi Y - \tau (Y_0 + \Phi Y) \\ &) \quad \Phi c = (1 - \tau) \Phi Y - \tau (Y_0 + \Phi Y) \end{aligned}$$

Thus  $\Phi c$  can be positive or negative depending on the initial income and the change in income. The greater  $Y$  the bigger the decrease in consumption; the greater  $\Phi Y$ , the greater the increase in consumption.

$\Phi Y$  is the same for the whole population except for those who have low productivity. For those agents with  $w < \frac{t}{\tau}$ ,  $\Phi Y$  is greater than for the other agents since  $Y$  is small,  $\Phi c$  is also greater for low productivity people.

### 3.2 Income tax rate

The next step is to obtain  $\tau$  as a function of  $t$  for a given level of government expenditures  $G$ . Using equations (2) and (3) we get:

$$G = \int (1 - \tau) w \, dF(w) = \int \tau \max_{t_i} (w + t_i; 0) \, dF(w) \quad (7)$$

$$= \int \tau Y(w; t) \, dF(w) \quad (8)$$

Since we assumed a proportional income tax,  $\tau$  does not depend on  $w$ , implying that,

$$\tau = \frac{G = \int (1 - \tau) w \, dF(w)}{\int Y \, dF(w)} = \frac{G}{\bar{Y}} \quad (9)$$

where  $\bar{Y}$  is the average income.

### 3.3 Individual welfare change

Now we are able to give a measure of the change in welfare when there is a change of policy from no transfers ( $t = 0$ ) to a situation with lump sum transfers equal to  $t_i$ . The first step is to obtain the level of utility  $U(c^i; 1_i | I^i)$  associated to the new value of  $t$ . Then we get  $\epsilon$  such that  $U(c^i; 1_i | I^i) = U(\epsilon; 1_i | I_0)$ . That is,  $\epsilon$  is the level of consumption that associated with the initial amount of labor yields the new level of utility. The change in welfare for each type of agent, in terms of consumption, will be given by the difference between  $\epsilon$  and  $c_0$ . This welfare measure is going to be positive for some agents (the "poor" ones) and negative for others. So  $\epsilon$  is such that:

$$U(c^i; 1_i | I^i) = (c^i)^\alpha (1_i | I^i)^{-\alpha} = \epsilon^\alpha (1_i | I_0)^{-\alpha} \quad (10)$$

implying,

$$\epsilon = \frac{\bar{A} (1_i | I^i)^{\frac{1}{\alpha}}}{c^i} \quad (11)$$

### 3.4 Aggregate welfare change

The total change in social welfare  $\Phi W$  is given by

$$\Phi W = \int \mu_i [\epsilon(i; t) - c_0(i)] f(i) di, \quad (12)$$

where  $\mu_i$  is the weight given to an agent with productivity level  $i$ .

If  $\mu_i = 1$  for every  $i$ , then the aggregate welfare change is expected to be negative since the change of policy implies to replace a lump sum tax by a higher proportional income tax rate.

## 4 Change of Policy, a Numerical Example

In this section we will use specific functional forms and specific values for the functions and parameters used in the previous section. We will pick these values in such a way that broadly describe the case of allocation of revenues from the Oil Industry in the Mexican Economy.

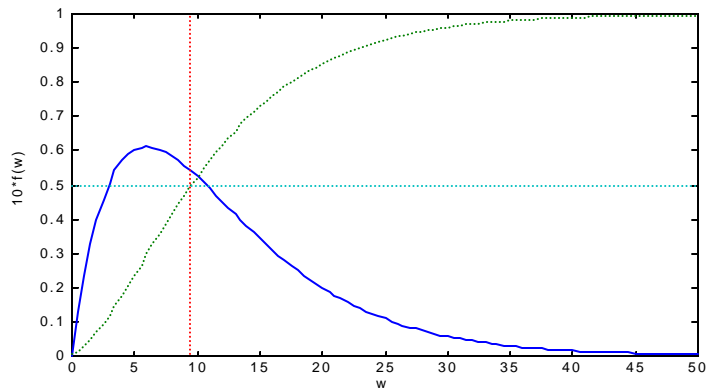


Figure 1: Productivity Distribution Function,  $10 \times f(w)$ .

#### 4.1 Functional Forms and Parameters

To represent the productivity distribution of the population we looked for a probability distribution function (p.d.f.) with two main characteristics. First, we need a p.d.f. with a left tail such that the mean of the population is greater than the median. Second, we require the variance of the population to be six times its mean. Therefore we propose a gamma function with mean equal to twelve (without loss of generality) and its variance six times its mean to be the productivity distribution function, this is  $f(w) = \frac{1}{\Gamma(2; 6)} e^{-w/6} w^{11}$ . Figure 1 shows a graph of ten times this function ( $10 \times f(w)$ ) and its cumulative distribution function (c.d.f.) ( $F(w)$ ). From the figure we can observe that the median of the population is lower than the mean. We scale the p.d.f. by a factor of ten to be able to show these functions together, in some of the following graphs we scale the functions so we can show the cumulative distribution function too.

The profits from exploiting natural resources are assumed to represent one third of the government expenditure,  $\tau = \frac{1}{3}G$ ; and government expenditure is set to one third of the initial gross national income,  $3G = \tau + \tau$ . The elasticity of consumption is set to one third,  $\alpha = \frac{1}{3}$  and  $\beta = \frac{2}{3}$ ; matching these values and taking the mean of the population,  $\bar{w} = 12$  we get that  $GNP_0 = \frac{3}{8}\bar{w} = \frac{9}{2}$ ,  $G = \frac{1}{8}\bar{w} = \frac{3}{2}$ ,  $\tau = \frac{1}{24}\bar{w} = \frac{1}{2}$ ; and  $\lambda_0 = 0.25$  (independent of  $\bar{w}$ ).

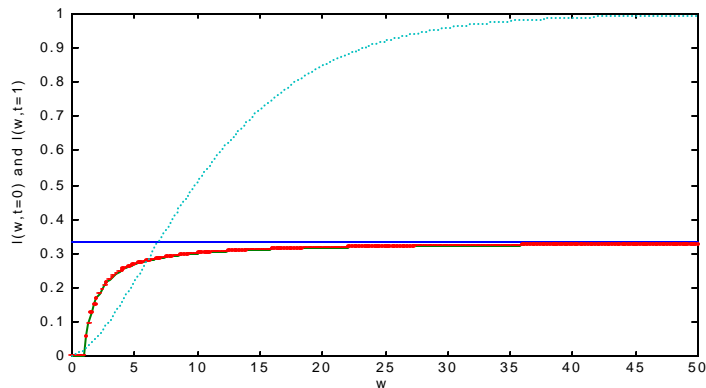


Figure 2: Labor Supply per type of agent

## 4.2 Effects of the Change of Policy

With this setup, and using the equations derived in the previous section, we will analyze the case in which the policy changes from  $t = 0$  to  $t = 1$ . We will focus on the changes in individual consumption and labor supply, consumption distribution, and ...nally, individual and aggregate changes in welfare.

### 4.2.1 Effect on Labor and National Product

Labor supply, as a function of the productivity level, is obtained using equation (3'). Figure 2 shows these functions under both policies and the productivity cumulative distribution, the continuous line is the initial labor supply per type of agent and the dotted is the one after the change. All agents work less due to the transfer they receive (see equation (3')), though the effect is smaller for high productivity people.

The total labor supply changes from 0.333 to 0.2822 units of time. Since the amount of the population is normalized to 1, the aggregate of a variable is the same as its mean. This reduction in labor supply implies a reduction in total production from  $4 = E[l(w, 0)]$  to  $3.667 = E[l(w, 1)]$  units of consumption. Including production from exploitation of natural resources, the gross national product falls from 4.5 to 4.167 units of consumption, which represent a 7.4% of  $GNP_0$ .

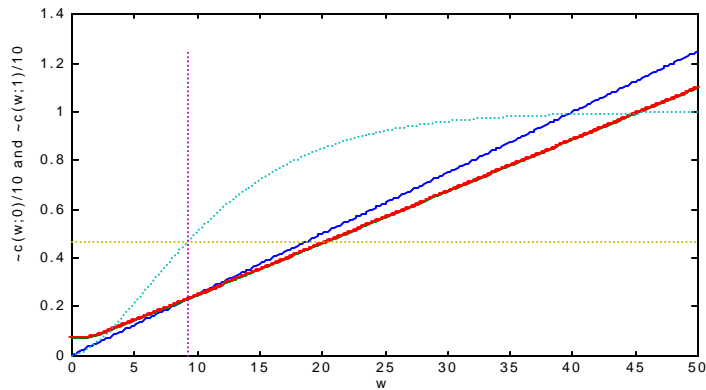


Figure 3: Consumption per type of agent under different policies

#### 4.2.2 Increment on Proportional Income Tax

When the government is not using anymore the profits obtained through natural resources, it needs to increase the proportional income tax rate to be able to balance its budget. The new rate is obtained from equation (9) when  $t = 1$ . Equation (5) is used to get the expected value of the pre-tax income. In this numerical exercise the income tax rate has to increase from 25% to 36%.

#### 4.2.3 Individual Welfare Change

Using  $c^a = c(!; 1)$  and  $I^a = I(!; 1)$  from equations (3) and (4), we got  $\epsilon$  as defined in equation (11). Figure 3 shows the level of  $c_0$  (continuous line) and  $\epsilon$  (line with dots) for each type of agent and the productivity c.d.f. Figure 4 shows the difference between them, representing the change in welfare in consumption terms.  $F(!)$  is also plotted there. To have an idea of how much this represents for each agent, in figure 5 we can see the change in welfare as a percentage of the initial level. From this figures we can see that approximately the "lower" 46% of the population are better off under this new policy and that the change in welfare is more significant for those with lower wages.

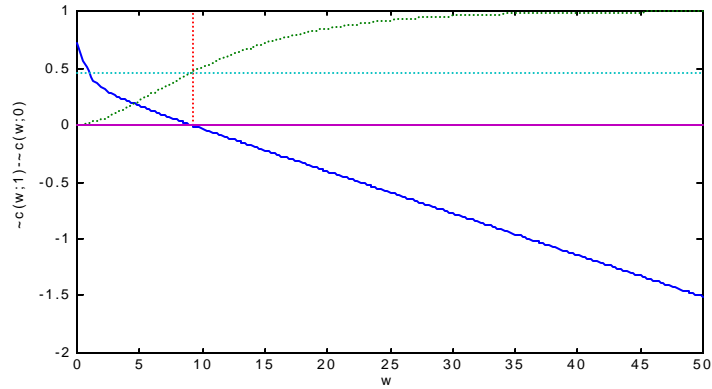


Figure 4: Welfare change per type of agent  $i$  vs:  $e(i; 1)$  ;  $e(i; 0)$

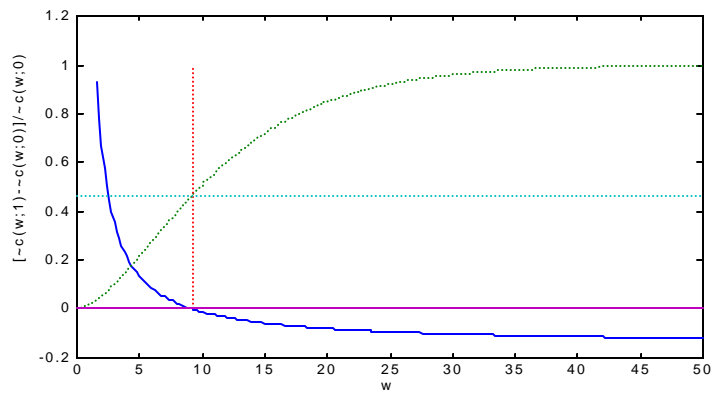


Figure 5: Percentage of Welfare Change per type of agent.  $i$  vs:  $e(i; 1)$  ;  $e(i; 0)$

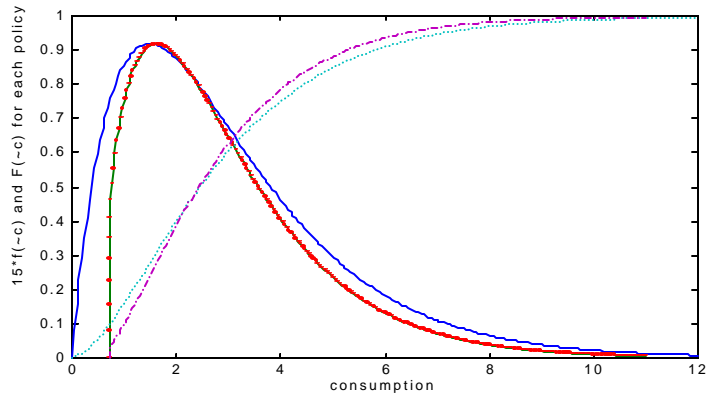


Figure 6: Change in consumption distribution

#### 4.2.4 Change in Welfare Distribution

We use  $c_0(!)$  and  $\epsilon(!)$  as measure of welfare for each type of agent under the initial and ...nal situation: The change in welfare distribution is shown in Figure 6, the dotted line represents the case for  $t = 1$ , we can observe that the people with lower wages consume a bigger proportion of the total consumption under the new policy.

Figure 7 is the Lorenz Curve, which we can use to visualize the change in consumption distribution.

#### 4.2.5 Aggregate Welfare Change

Equal weighting each agent ( $\mu_i = 1$ ) equation (12) gives a measure of the efficiency change. The total loss in welfare is 0.097 consumption units, representing the 3.14% of total initial consumption. The cumulative welfare change is shown in ...gure 8.

## 5 Conclusions

The objective of this project was to analyze a change of policy concerning the allocation of profits obtained exploiting natural resources. The initial situation is such that these profits are used to ...nance part of the government expenditure while the rest of this expenditure is ...nanced by a proportional

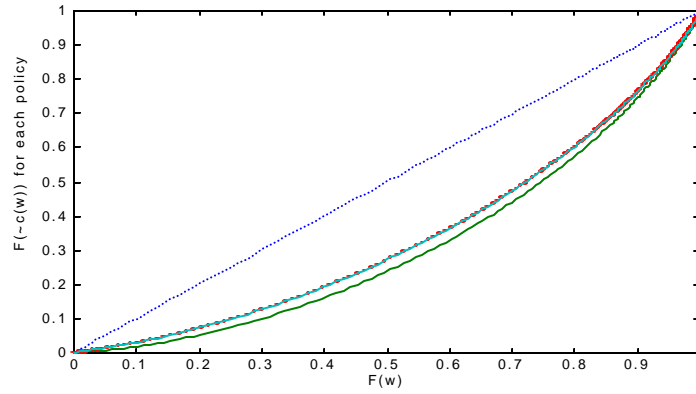


Figure 7: Lorenz Curve under each policy

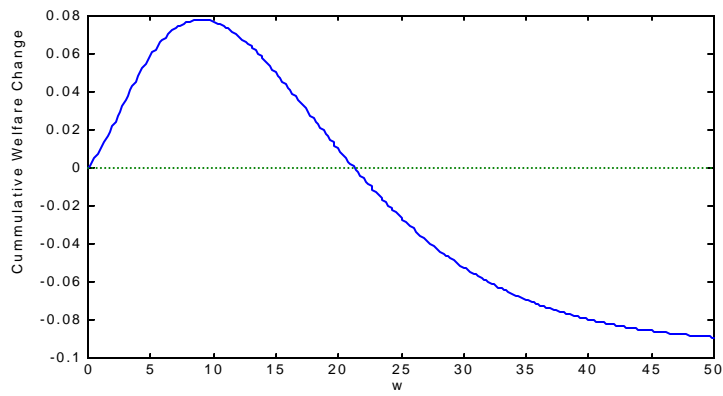


Figure 8: Cumulative Welfare Change

income tax to the population. In the final situation the national profits are distributed uniformly among the population and the proportional income tax has to be higher than before to balance the government budget.

Several issues were considered: the effect on labor supply, the change in consumption among the population, and the aggregate cost of the change of policy.

The amount of labor supplied by each agent is reduced. Low productivity agents experience a higher drop in labor supply. The effects on consumption differ according to the different income levels. Low income agents increase their consumption while high income agents decrease it. Under the new policy approximately 46% of the population (the less productive part) is better off and the 54% more productive part is worse off. If we ponder one unit of consumption the same for rich and poor people, the gain obtained by the poorer 46% of the population is less than the loss suffered by the rest of the population. Therefore there is an aggregate welfare loss. This result is not surprising since the second policy implies getting rid of a lump-sum tax and then recover that amount of money with a distorting tax. Though there is an aggregate welfare loss, the wealth distribution improves as transpires from the Lorenz Curve.

## 6 Possible Extensions

Another issue that could be analyzed is the impact of each policy in social mobility. The model here presented is not helpful, but modifications can be done so that cover this point. We could introduce heterogeneity in wealth and in the ability to learn instead of imposing productivity heterogeneity. A production function for human capital requiring time, wealth and individual abilities will generate difference in productivity levels.