LACEA 2001
MONTEVIDEO URUGUAY

Pedro Dal Bó

Tacit Collusion Under Interest Rate Fluctuations

Tacit Collusion under Interest Rate Fluctuations

Pedro Dal Bó*

UCLA

May 31, 2001

Abstract

When collusive agreements are illegal, oligopolies must rely on the threat of future price wars to sustain collusion. If firms' objective is to maximize the present value of profits, the shadow of that threat changes with the interest rate. That is, the interest rate determines how much firms discount the future, and this affects the level of collusion that can be supported. In an environment of identical firms, price competition and changing interest rate (and hence changing discount factor) I characterize the optimal tacit collusion equilibrium and show that it displays the following properties: 1) the higher the discount factor the higher the equilibrium prices and profits, 2) the higher the volatility of the discount factor the lower the equilibrium prices and profits, and 3) as expected, an increase in the number of firms reduces the scope of collusive behavior.

Keywords: tacit collusion, interest rate, random discount factor, repeated games.

JEL Classification: C7, L13.

*I am grateful to David Levine and John Riley for invaluable guidance and ideas. I also want to thank Anna Aizer, Harold Demsetz, Philip Leslie, David Rahman and seminar participants at UCLA for very useful comments and discussions. Any remaining errors are mine.

1 Introduction

It is well known that oligopolies can use the threat of future price wars to sustain prices above the perfect competitive levels if they care sufficiently enough about the future, Friedman [2]. But if firms' objective is to maximize the present value of profits, how much firms care about the future depends on the interest rate. That is, the interest rate determines how much firms discount the future, and this in turn affects the level of collusion that can be supported. For simplicity I concentrate on changes in the discount factor keeping in mind that these changes correspond to changes on the interest rate[1].

I characterize the maximum symmetric tacit collusion prices and profits that can be supported as a function of the discount factor when it is randomly and independently drawn every period, firms are identical and they compete on price. From this characterization, the three main points of this paper are derived.

First, the lower the discount factor (the higher the interest rate) in a period, the lower the prices and profits that can be supported in equilibrium in that period. The intuition behind this is simple and resembles the intuition from Rotemberg and Saloner [5]. If the discount factor is relatively low, the shadow of future retaliation for a present deviation is also low. Thus, to prevent deviations today, it may be necessary to decrease prices and profits today in order to lower the gains of deviation. While in Rotemberg and Saloner [5] the changes of the relative importance of the present and the future profits were driven by changes in the size of the market (fluctuations in demand), in this paper the changes are driven by fluctuations in the discount factor.

Second, this paper shows that the higher the volatility of the discount factor, the lower the prices and profits that can be supported in equilibrium. There are two reasons for this. First, an increase in the volatility of the discount factor reduces the range for which perfect collusion (i.e. monopoly prices and profits) can be supported. Second,

[1] The discount factor may also change for other reasons such as changes on the probability that the product may became obsolete or that the government may nationalize the industry -things that are relatively rare.

while for some range of discount factors the profits depends positively on the discount factor, for some higher range the profits are constant and hence, increases in the volatility of the discount factor reduce the expected profits.

The third result is that an increase in the number of firms reduces the scope of collusive behavior, as expected. For a given distribution of the discount factor it is possible to find a sufficiently large number of firms such that it is impossible for them to support any level of tacit collusion. The reason is that by increasing the number of firms the short run incentives to deviate can be made arbitrarily large, relative to the future punishment. This follows from the fact that a firm may capture all the market by an infinitesimal decrease in price and, hence, the size of the gain of deviation increases with the number of firms.

While most of the literature on repeated games has studied environments with very little discounting, this paper offers an example of the phenomena that appear under the more realistic assumption of significant discounting which varies over time.

The results of this paper have interesting implications for the applied industrial organization literature on tacit collusion. Most of those papers, such as Porter [4], Rotemberg and Saloner [5] and Ellison [1], do not include the interest rate in the regressions, thereby excluding an important explanatory variables in the study of tacit collusion.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 characterizes the optimal tacit collusion profits as a function of the discount factor and the main results of the paper. Section 4 presents examples in which the discount factor is distributed uniformly and Section 5 concludes.

2 Model

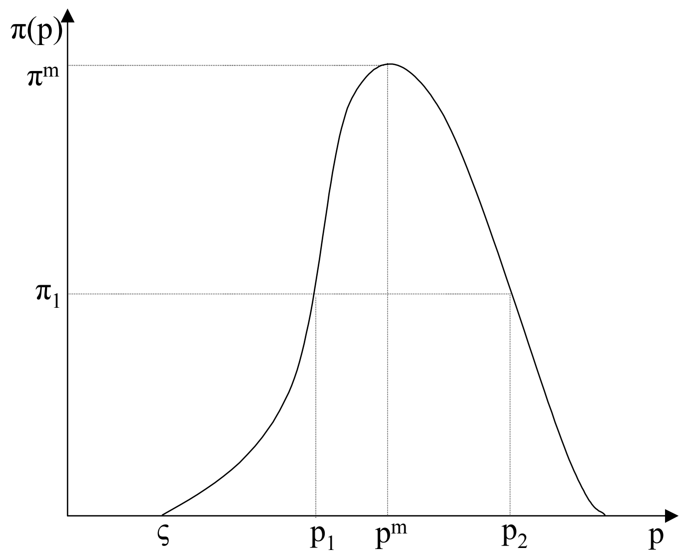
Consider a market with N identical firms with a constant marginal cost of ς and facing an inverse demand function $D(p)$ $(D'(p) < 0)$. The firms compete on price and the demand is divided equally among the firms charging the lowest price. I restrict my attention to equilibria in which all the firms charge the same price p. In this symmetric case I can write the profits of each firm as $\pi(p) = \frac{(p-\varsigma)D(p)}{N}$ and total industry profits as $\Pi(\delta) = (p-\varsigma)D(p)$. I assume that there exists a price p^m that maximizes the total industry profits, that is, p^m is the monopoly price. Denote $\pi^m = \pi(p^m)$ as the monopoly profit per firm and $\Pi^m = N\pi^m$ as total industry monopoly profit.

The firms play an infinitely repeated game with the discount factor δ_t (which discounts earnings from $t+1$ to t) changing on time: δ_t is i.i.d. with p.d.f. $f(\delta_t)$ and c.d.f. $F(\delta_t)$. Call a an b the infimum and supremum, respectively, of the support of $f(\delta_t)$. In period t, the firms observe the realization of the discount factor, δ_t, before choosing the prices for that period. The firms only care about profit and are risk neutral and, hence, their objective is to maximize the discounted stream of profits. I also assume that the firms can not commit to charge a given price or sign contracts between them or with third parties regarding prices. Therefore, any equilibrium of the model must be a subgame perfect equilibrium of the infinitely repeated Bertrand oligopoly game.

3 Optimal tacit collusion with a random discount factor

I look for symmetric optimal tacit collusion strategies. That is, strategies that in equilibrium support the maximum symmetric present value of profits. Since the environment in which the firms interact does not change over time, with the exception of the discount factor, the optimal tacit collusion solution will consists on a function that relates the highest equilibrium price that the firms can charge in one period with the realized discount factor in that same period. Therefore the solution will consist of a function $p^*(\delta) : [a,b] \to [0,p^m]$. This in turn defines a function $\pi^*(\delta) : [a,b] \to [0,\pi^m]$, which denotes the optimal tacit collusion equilibrium profits as a function of the period discount factor.

Fortunately, in the search for the optimal tacit collusion behavior it is enough to work with $\pi^*(\delta)$. As we see in Figure 1, a given level of profits, for example π_1, can result from different prices, such as p_1 and p_2. Given that I am interested on the optimal levels of profits that can be supported under tacit collusion and the fact that π_1 can more easily be supported by p_1 than by p_2, I only consider the increasing part of the profit function. In this way for every profit lower than π^m corresponds one and only one price lower than p^m. Therefore, I can define the function $\phi(\pi) = \pi^{-1}(\pi) : [0, \pi^m] \rightarrow [0, p^m]$, and once I solve for $\pi^*(\delta)$, I can recover $p^*(\delta)$ as $p^*(\delta) = \phi(\pi^*(\delta))$. Note that $\phi'(\pi) > 0$ from the fact that $D'(p)$ exists.



The profit function

Using the recusivity of the problem, for every firm the present value at $t = 0$ of its stream of profits can be written as: $V(\delta_0) = \pi(\delta_0) + \delta_0 \int_a^b V(\delta_1) f(\delta_1) d\delta_1$ where for every t:

$$V(\delta_t) = \pi(\delta_t) + \delta_t \int_a^b V(\delta_{t+1}) f(\delta_{t+1}) d\delta_{t+1} \tag{1}$$

and $\pi(\delta_t)$ denotes the profits that the firms receive at time t if the discount factor is δ_t.

Since these firms can not commit to a given price, in equilibrium they must be unwilling to charge a price different from the equilibrium price. How much can a firm gain from deviating? If all the firms are charging the same price above the marginal cost, a single company can decrease its price by a penny and capture all the market. Therefore, if the equilibrium profit is $\pi(\delta_t)$, a single company can gain $(N-1)\pi(\delta_t)$ by deviating (if we forget about pennies). For firms to be unwilling to deviate, punishment must follow a deviation. How much can a firm lose from being punished? In this case, the best punishment is to revert forever to the Bertrand-Nash equilibrium of the one-stage game. Since the Bertrand-Nash equilibrium gives a payoff of zero it is not possible to force a firm to a lower payoff. Under this threat if one firm deviates it will earn the total industry profit the period of deviation but then it will earn zero profits for ever. Then, for no firm to have incentives to deviate the following must hold:

$$\pi(\delta_t) \leq \frac{\delta_t}{N-1} \int_a^b V(\delta_{t+1}) f(\delta_{t+1}) d\delta_{t+1} \forall \delta_t \tag{2}$$

Next, I simplify equations (1) and (2) by eliminating $V(\delta_t)$ and expressing them as a function of $\pi(\delta_t)$. Integrating over (1) and rearranging we have $\int_a^b V(\delta_t) f(\delta_t) d\delta_t = \frac{1}{1-\overline{\delta}} \int_a^b \pi(\delta_t) f(\delta_t) d\delta_t$, where $\overline{\delta}$ is the expected value of δ_t. Plugging this into (1) and (2) we have:

$$V(\delta_t) = \pi(\delta_t) + \frac{\delta_t}{1-\overline{\delta}} \int_a^b \pi(\delta_{t+1}) f(\delta_{t+1}) d\delta_{t+1} \tag{3}$$

and

$$\pi(\delta_t) \leq \frac{\delta_t}{(N-1)\left(1-\overline{\delta}\right)} \int_a^b \pi(\delta_{t+1}) f(\delta_{t+1}) d\delta_{t+1} \forall \delta_t \tag{4}$$

In addition, the profits per firm can not be greater than under monopoly pricing:

$$\pi(\delta_t) \leq \pi^m \tag{5}$$

Therefore it is clear that to maximize $V(\delta_0)$ the firms have to choose profits as large as possible without violating the incentive compatibility constraint (4) and the feasibility constraint (5). Then, dropping the subindexes for simplicity, the optimal tacit collusion profits levels $\pi^*(\delta)$ is a function from $[a,b]$ to $[0,\pi^m]$ subject to the following equation[2]:

$$\pi^*(\delta) = \min\left\{\pi^m, \frac{\delta}{(N-1)\left(1-\overline{\delta}\right)}\int_a^b \pi^*(\delta')f(\delta')d\delta'\right\} \forall\delta \tag{6}$$

The following proposition fully characterizes the function $\pi^*(\delta)$.

Proposition 1 *The function $\pi^*(\delta)$ depends on $f(\delta)$ and N in the following way:*

1) if $\overline{\delta} \geq 1 - \frac{a}{N-1}$, $\pi^*(\delta) = \pi^m$;

2) if $\frac{N-1}{N} \leq \overline{\delta} < 1 - \frac{a}{N-1}$, $\pi^*(\delta) = \pi^m$ for $\delta \geq c$ and $\pi^*(\delta) = \frac{\delta}{c}\pi^m$ for $\delta < c$, for a number $c \in (a,b]$ that solves the following equation: $c = (N-1)(1-\overline{\delta}) + \int_a^c F(\delta)d\delta$;

3) if $\overline{\delta} < \frac{N-1}{N}$, $\pi^*(\delta) = 0$.

Proof. Case 1): $\overline{\delta} \geq 1 - \frac{a}{N-1}$ implies that $\pi^m \leq \frac{a}{(N-1)\left(1-\overline{\delta}\right)}\pi^m \leq \frac{\delta}{(N-1)\left(1-\overline{\delta}\right)}\pi^m \ \forall\delta$ and perfect collusion, $\pi^*(\delta) = \pi^m$, can be supported.

Case 2): I use repeatedly the fact that (integrating by parts):

$$\int_a^s \delta f(\delta)d\delta = sF(s) - \int_a^s F(\delta)d\delta \tag{7}$$

Consider the case in which the two terms inside the brackets in equation (6) are binding for different ranges of δ. Given that the second term is increasing in δ, the first term would be binding for, say, $\delta > c$ and the second one for $\delta < c$, and both equal and binding for $\delta = c$. In this case, integrating over (6) and denoting $A = \int_a^b \pi^*(\delta)f(\delta)d\delta$, I have the following:

$$A = \int_a^c \frac{\delta}{(N-1)\left(1-\overline{\delta}\right)}Af(\delta)d\delta + (1-F(c))\,\pi^m \tag{8}$$

and

$$\pi^m = \frac{c}{(N-1)\left(1-\overline{\delta}\right)}A \tag{9}$$

[2]It could be argue that equation (6) must not hold for every δ since having profits lower than posible in a finite subset does not affect the expected value. But if we want the solution to be independent of δ_0 the equation must hold for every possible value of the discount factor.

From (9) I have that $A = \frac{\pi^m (N-1)\left(1-\overline{\delta}\right)}{c}$, and substituting in (8) and rearranging: $(N-1)\left(1-\overline{\delta}\right) = \int_a^c \delta f(\delta) d\delta + (1-F(c))\, c$. By (7) and rearranging:

$$c = (N-1)\left(1-\overline{\delta}\right) + \int_a^c F(\delta) d\delta \tag{10}$$

It remains to be shown that the number c that solves equation (10) exists and is unique. Write $H(r) = (N-1)\left(1-\overline{\delta}\right) + \int_a^r F(\delta) d\delta - r$. Then $H(c) = 0$. If $\frac{N-1}{N} < \overline{\delta} < 1 - \frac{a}{N-1}$, it can be easily seen that $H(a) = (N-1)\left(1-\overline{\delta}\right) - a > 0$ and $H(b) = (N-1)\left(1-\overline{\delta}\right) - \overline{\delta} < 0$. In addition, $H(r)$ is continuous[3] and strictly decreasing ($\frac{\partial H(r)}{\partial r} = F(r) - 1 < 0$ for $a \leq r < b$). Then, there exists a unique number c, between a and b, that makes $H(c) = 0$. If $\overline{\delta} = \frac{N-1}{N}$, $H(b) = 0$ and $c = b$ is the unique solution since $H(.)$ is strictly decreasing.

Case 3): If $\overline{\delta} < \min\left\{\frac{N-1}{N}, 1 - \frac{a}{N-1}\right\}$, from the analysis of the previous two cases it follows that the only possible solution to equation (6) is $\pi^*(\delta) = 0$. Since $\frac{N-1}{N}$ can be greater than $1 - \frac{a}{N-1}$ only if $a > \frac{N-1}{N}$, in which case $\overline{\delta}$ can never be lower than $\frac{N-1}{N}$, it follows that $\pi^*(\delta) = 0$ if $\overline{\delta} < \frac{N-1}{N}$. ■

In case 1), $\overline{\delta} \geq 1 - \frac{a}{N-1}$, any possible realization of the discount factor is high enough for each firm to value the future monopoly profits more than the one stage profits of deviation, and, hence, perfect collusion is an equilibrium. On the contrary, in case 3), $\overline{\delta} < \frac{N-1}{N}$, all the realizations of the discount factor are too low to be able to support any level of collusion. In between these two cases, $\frac{N-1}{N} \leq \overline{\delta} < 1 - \frac{a}{N-1}$, perfect collusion can be supported for a range of high realizations of the discount factor while only lower levels of profits can be supported for a range of low realizations. The reason for this is that while for some realizations it is not possible to support full collusion it may

[3] $H(r)$ is continuous only if $\int_a^r F(\delta) d\delta$ is continuous which is true. Proof: Given $\varepsilon > 0$, take $s \in [a, b]$ such that $0 < r - s < \varepsilon$. Then, $\left|\int_a^r F(\delta) d\delta - \int_a^s F(\delta) d\delta\right| = \int_s^r F(\delta) d\delta \leq \int_a^r 1 d\delta$ (the last inequality by the fact that $F(\delta) \leq 1 \forall \delta$). Therefore $\left|\int_a^r F(\delta) d\delta - \int_a^s F(\delta) d\delta\right| \leq r - s < \varepsilon$. Proceding similarly for the case in which $s > r$, $|r - s| < \varepsilon$ implies $\left|\int_a^r F(\delta) d\delta - \int_a^s F(\delta) d\delta\right| < \varepsilon$ and $\int_a^r F(\delta) d\delta$ is continuous.

still be possible to satisfy the incentive compatibility constraint by reducing the present incentives to deviate. For this, the present profits should be lowered so that firms can make sure that no one has an incentive to deviate. In this case, an increase in the discount factor results in an increase on the optimal tacit collusion profits and, hence, in prices.

Theorem 2 $\frac{d\pi^*(\delta)}{d\delta} \geq 0$ *and* $\frac{dp^*(\delta)}{d\delta} \geq 0$. *In addition, if* $\frac{N-1}{N} \leq \overline{\delta} < 1 - \frac{a}{N-1}$ *and* $a \leq \delta < c$, $\frac{d\pi^*(\delta)}{d\delta} = \frac{\pi^m}{c}$.

Proof. By Proposition 1, in cases 1) and 3) changes in δ produce no changes in profits and prices, while $\frac{d\pi^*(\delta)}{d\delta} = \frac{\pi^m}{c} > 0$ in case 2). In addition, given that $\frac{d\phi(\pi)}{d\pi} > 0$, $\frac{dp^*(\delta)}{d\delta} = \frac{d\phi(\pi^*)}{d\pi^*}\frac{d\pi^*(\delta)}{d\delta} > 0$, in that case. ■

3.1 The effects of changes on f(δ)

The next result shows how c changes with changes in the distribution of the discount factor.

Lemma 3 *Consider two cumulative distributions functions* F *and* G *such that* $\frac{N-1}{N} < \overline{\delta}_{F,G} < 1 - \frac{a}{N-1}$ *and* F *second-order stochastically dominates*[4] G, *then* $c_F \leq c_G$.

Proof. From the definition of c_F: $H_F(c_F) = (N-1)\left(1 - \overline{\delta}_F\right) + \int_a^{c_F} F(\delta)d\delta - c_F = 0$. By second-order stochastic dominance $\int_a^{c_F} F(\delta)d\delta \leq \int_a^{c_F} G(\delta)d\delta$ and $\overline{\delta}_F \geq \overline{\delta}_G$. Therefore, $H_G(c_F) = (N-1)\left(1 - \overline{\delta}_G\right) + \int_a^{c_F} G(\delta)d\delta - c_F \geq 0$ and, given that $H_G(.)$ is strictly decreasing and the value of $\overline{\delta}_G$, there exists $c_G \in (a,b)$ such that $H_G(c_G) = 0$ and $c_F \leq c_G$. ■

[4]For two cumulative distributions functions $F(\delta)$ and $G(\delta)$, F second-order stochastic dominates G if for any r, $a \leq r \leq b$, $\int_a^r F(\delta)d\delta \leq \int_a^r G(\delta)d\delta$, and the inequality is strict on some range. In that case, it can be proven that $\overline{\delta}_F \geq \overline{\delta}_G$ and $\int_a^b u(\delta)f(\delta)d\delta \geq \int_a^b u(\delta)g(\delta)d\delta$, for any increasing concave twice-piecewise-differentiable function $u(\delta)$. See Hirshleifer and Riley [3].

Denote $\pi_F^*(\delta)$, $E\pi_F^*$ and $p_F^*(\delta)$ as the optimal tacit collusion function, it's expected value and optimal collusion prices under F, respectively.

Theorem 4 *Consider two cumulative distribution functions, F and G, such that F second-order stochastically dominates G, then $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ and $p_F^*(\delta) \geq p_G^*(\delta)$ for every δ. In addition, $E\pi_F^* \geq E\pi_G^*$.*

Proof. By second-order stochastic dominance $\overline{\delta}_F \geq \overline{\delta}_G$. So, from proposition 1, we can see that if the solution under F belongs to case 1), the solution under G can belong to any of the three cases. If the solution under F belongs to case 2), the solution under G can belong to cases 2) or 3). And if the solution under F belongs to case 3), the solution under G must belong to the same case. For most of this combinations it is straight forward to see that $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ for every δ. The situation in which both the solution under F as under G belong to case 2) needs more analysis. From Lemma 1, F second-order stochastically dominates G, then $c_F \leq c_G$. Then, $\pi_F^*(\delta) = \frac{\delta}{c_F}\pi^m \geq \pi_G^*(\delta) = \frac{\delta}{c_G}\pi^m$ for when the IC constraint is binding in both cases. Therefore, $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ for every δ.

The result on prices follows directly from the positive relationship between profits and prices.

By second-order stochastic dominance, $\pi_F^*(\delta)$ being concave and $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ for every δ we have that $E\pi_F^* = \int_a^b \pi_F^*(\delta) f(\delta) d\delta \geq \int_a^b \pi_F^*(\delta) g(\delta) d\delta \geq \int_a^b \pi_G^*(\delta) g(\delta) d\delta = E\pi_G^*$. ■

If we consider the particular case in which $\overline{\delta}_F = \overline{\delta}_G$, G must be a mean preserving spread of F and the next result follows.

Corollary 5 *If G is a mean preserving spread of F, then $\pi_F^*(\delta) \geq \pi_G^*(\delta)$ and $p_F^*(\delta) \geq p_G^*(\delta)$ for every δ. In addition, $E\pi_F^* \geq E\pi_G^*$.*

Therefore, ceteris paribus, the volatility of the discount factor is inversely related to the firms' expected profits.

3.2 The effects of changes on the number of firms

In this section I study the effect of an increase in the number of firms on expected industry profits and prices under tacit collusion. For this, it is useful to first characterize the total industry optimal tacit collusion profits for a given number of firms.

Proposition 6 *The optimal tacit collusion industry profit function when there are N firms, $\Pi_N^*(\delta)$, depends on $f(\delta)$ and N in the following way:*

1) if $\overline{\delta} \geq 1 - \frac{a}{N-1}$, $\Pi_N^*(\delta) = \Pi^m$;

2) if $\frac{N-1}{N} \leq \overline{\delta} < 1 - \frac{a}{N-1}$, $\Pi_N^*(\delta) = \Pi^m$ for $\delta \geq c$ and $\Pi_N^*(\delta) = \frac{\delta}{c}\Pi^m$ for $\delta < c$, for a number $c \in (a, b]$ that solves the following equation: $c = (N-1)(1-\overline{\delta}) + \int_a^c F(\delta)d\delta$;

3) if $\overline{\delta} < \frac{N-1}{N}$, $\Pi_N^*(\delta) = 0$.

Proof. This result follows by multiplying the solution in Proposition 1 by N and noting that $\Pi^m = N\pi^m$. ■

I next show how a change in the number of firms affects the range of perfect collusion in case 2) of Proposition 7. Second, I analyze the effect of an increase in the number of firms on expected total industry profits. Third, I examine the effect of a change on the number of firms on expected profits per firm and prices.

Lemma 7 *Consider two different number of firms, N and M, such that $N < M$ and $\frac{N-1}{N} < \frac{M-1}{M} < \overline{\delta} < 1 - \frac{a}{N-1} < 1 - \frac{a}{M-1}$, then $c_N < c_M$.*

Proof. From the definition of c_N: $H_N(c_N) = (N-1)\left(1-\overline{\delta}\right) + \int_a^{c_N} F(\delta)d\delta - c_N = 0$. Given that $N < M$, $H_M(c_N) = (M-1)\left(1-\overline{\delta}\right) + \int_a^{c_N} F(\delta)d\delta - c_N > 0$ and, given that $H_M(.)$ is strictly decreasing and the value of $\overline{\delta}$, there exists $c_M \in (a, b)$ such that $H_M(c_M) = 0$ and $c_M > c_N$. ■

Denote the expected level of total profits under tacit collusion with N firms as $E\Pi_N^*$.

Theorem 8 *Consider two different number of firms N and M, $N < M$, then $\Pi_N^*(\delta) \geq \Pi_M^*(\delta)$ and $p_N^*(\delta) \geq p_M^*(\delta)$ for every δ and $E\Pi_N^* \geq E\Pi_M^*$ and $Ep_N^* \geq Ep_M^*$. In addition,* *for any $f(\delta)$ exists a number of firms K, such that for any number of firms $N \geq K$, $\Pi_N^*(\delta) = 0$ and $p_N^*(\delta) = \varsigma$.*

Proof. If $\overline{\delta} \geq 1 - \frac{a}{N-1}$, three things can happen: 1) $\overline{\delta} \geq 1 - \frac{a}{M-1}$, and then $\Pi_N^*(\delta) = E\Pi_M^*(\delta) = \Pi^m$; 2) $\frac{M-1}{M} \leq \overline{\delta} < 1 - \frac{a}{M-1}$, and then with probability $F(c_M)$ the profit will be below the monopoly profit and then, $\Pi_M^*(\delta) \leq \Pi_N^*(\delta) = \Pi^m$ for every δ; 3) $\overline{\delta} < \frac{M-1}{M}$ and then, $\Pi_M^*(\delta) = 0 < \Pi_N^*(\delta) = \Pi^m$ for every δ. If $\frac{N-1}{N} \leq \overline{\delta} < 1 - \frac{a}{N-1}$, two things can happen: 1) $\overline{\delta} < \frac{M-1}{M}$ and then, $\Pi_M^*(\delta) = 0 \leq \Pi_N^*(\delta)$ for every δ; 2) $\frac{M-1}{M} \leq \overline{\delta} < 1 - \frac{a}{M-1}$, and then, by lemma 3, $c_M > c_N$ and, by Proposition 7, $\Pi_M^*(\delta) \leq \Pi_N^*(\delta)$ for every δ. If $\overline{\delta} < \frac{N-1}{N}$, given that $M > N$, $\overline{\delta} < \frac{M-1}{M}$ and $\Pi_M^*(\delta) = \Pi_N^*(\delta) = 0$.

The results on prices and in expected values follows directly from the positive relation of industry profits and prices and taking the expected value, respectively.

For the last result, define K as the lowest integer that makes $\frac{K-1}{K} > \overline{\delta}$. From proposition 7 and the fact that $\frac{N-1}{N} > \frac{K-1}{K}$ for any $N > K$, we have that $\Pi_N^*(\delta) = 0$ and $p_N^*(\delta) = \varsigma$ for any $N > K$. ■

Since the level of firm profits can be calculated by dividing the level of industry profits by the number of firms, this result implies that an increase in the number of firms results in a decrease on expected profits per firm.

Corollary 9 *Consider two different number of firms N and M, $N < M$, then $\pi_N^*(\delta) > \pi_M^*(\delta)$ for every δ and $E\pi_N^* > E\pi_M^*$.*

This section shows that an increase in the number of firms reduces the scope of collusion. The higher the number of firms the higher is the present profit from deviation (if there are N firms, a single firm may steal $\frac{N-1}{N}$ percent of the market by deviating) for a given profit, hence, to eliminate incentives to deviate, collusive profits must be lowered

It is important to note that this result does not depend on fixing the size of the market while there is an increase in the number of firms. If both the size of the market and the number of firms increase in the same proportion[5], a similar result holds. Since

[5]That would consists on multiplying the demand function $D(p)$ and the number of firms N by a positive integer.

an increase in the number of firms and size of the market leaves monopoly profits per firm unchanged but increases the incentives to deviate, the scope of collusion diminishes.

3.3 Example with uniform distributions

In this section I consider the special case in which δ is distributed uniformly between a and b, $0 \leq a < b \leq 1$, which provide examples of the results presented in Section 3.

In the uniform case, taking into consideration that $\overline{\delta} = \frac{a+b}{2}$, I can restate Proposition 1 in the following way:

Proposition 10 *If $\delta \sim U(a,b)$, the function $\pi^*(\delta)$ depends on $U(a,b)$ and N in the following way:*

1) if $b \geq 2 - a\frac{N+1}{N-1}$, $\pi^*(\delta) = \pi^m$;

2) if $\frac{2(N-1)}{N} - a \leq b < 2 - a\frac{N+1}{N-1}$, $\pi^*(\delta) = \pi^m$ for $\delta \geq c$ and $\pi^*(\delta) = \frac{\delta}{c}\pi^m$ for $\delta < c$, with $c = b - \sqrt{N(b^2 - a^2) - 2(b-a)(N-1)}$;

3) if $b < \frac{2(N-1)}{N} - a$, $\pi^*(\delta) = 0$.

Therefore, in the case of uniform distribution of the discount factor, the level of profits for each discount factor depends on the magnitudes of a and b. If $b \geq 2 - a\frac{N+1}{N-1}$ perfect collusion can be supported for any realization of the discount factor. If $\frac{2(N-1)}{N} - a \leq b < 2 - a\frac{N+1}{N-1}$, perfect collusion can be supported only for high discount factors and only lower levels of profits can be supported for lower discount factors. Finally, if $b < \frac{2(N-1)}{N} - a$ no collusion can be supported.

Figure 2 shows the different ranges of a and b for the three cases of tacit collusion, for $N = 2$, $N = 4$, $N = 8$ and $N = 16$.

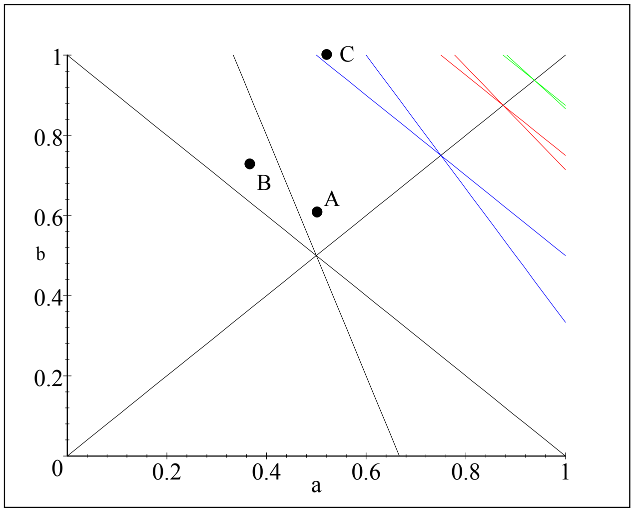


Figure 2: Ranges of tacit collusion

Since $b > a$, the relevant portion of the figure is above the 45 degree line. That part of the graph shows the ranges of a and b that result in different kinds of tacit collusion. For example, to the northeast of the solid black line are the combinations of a and b that results in perfect collusion when there are two firms in the market. Between the solid and dashed black lines we see the combinations that result perfect collusion for high discount factors and imperfect collusion for low discount factors, and below the dashed black line are the combinations that can not support any collusion. The same holds for the colored lines with $N = 4$, $N = 8$ and $N = 16$.[6]

Consider the distributions of δ represented in Figure 2 by the points A and B: the discount factor is distributed $U(0.5, 0.6)$ and $U(0.35, 0.75)$ respectively. Therefore, in both cases the expected value of the discount factor is 0.55 but at point B the variance is higher. We can deduce from Figure 2 that if there are only two firms in the market, perfect collusion can be supported at point A, while perfect collusion can only be supported for a range of high discount factors for point B. Figure 3 shows the tacit collusion profit functions for these two cases assuming that $\Pi^m = 18$. Accordingly with Corollary

[6]Note that for a given N the lines that delimit the different ranges intersect at the 45 degree line, that is, when there is no uncertainty about the discount factor. At this intersection $a = b = \frac{N-1}{N}$, that is exactly the minimum value of the discount factor that allows for tacit collusion when there are N firms and a fixed discount factor.

6, Figure 3 shows that a mean preserving spread in the distribution of the discount factor reduces the expected value of profits. In addition, Figure 3 shows that for some ranges of the discount factor it is possible to have a positive relationship between the discount factor and profits (and therefore, prices).

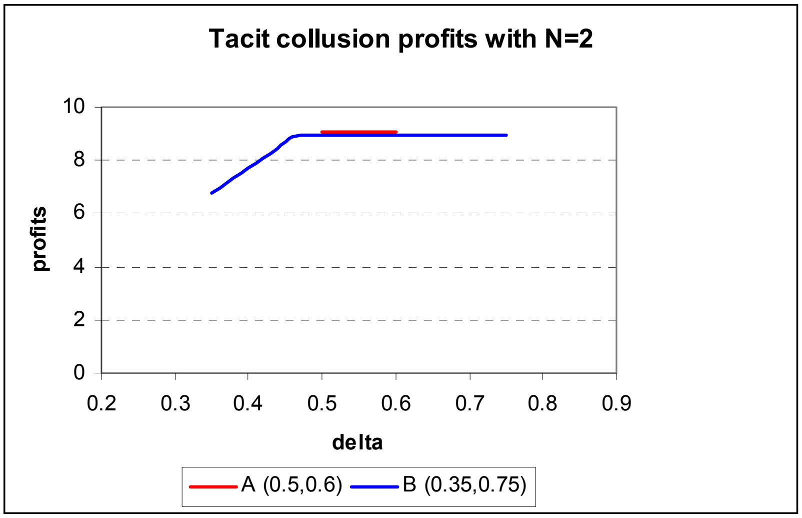


Figure 3

Consider now the distribution of δ represented in Figure 2 by point C: the discount factor is distributed $U(0.52, 1)$. Following Figure 2, we see that, for example, perfect collusion can be supported if $N = 2$, while perfect collusion can only be supported for a range of high discount factors if $N = 4$, and can not be supported at all for $N = 8$. Figure 4 shows the tacit collusion profit functions for these three cases. Consistent with Theorem 9, Figure 4 shows that the expected profit per firm decreases with an increase in the number of firms.

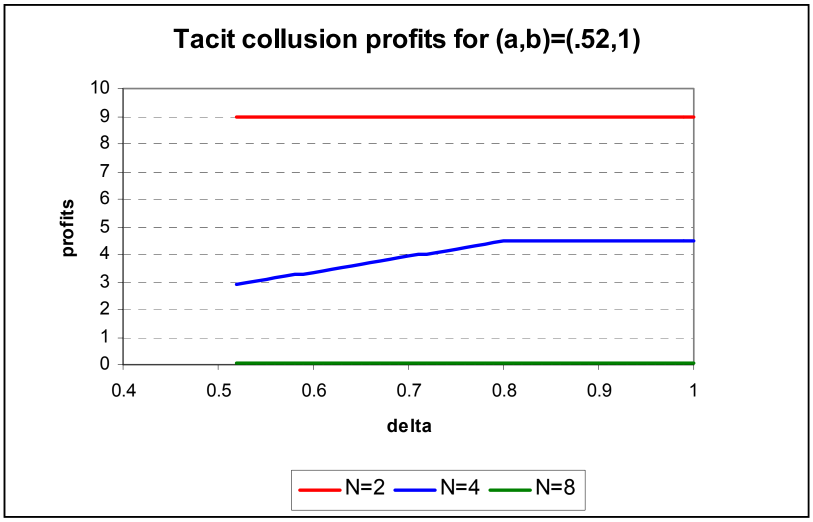


Figure 4

4 Conclusions

In this paper I characterize the optimal symmetric tacit collusion equilibrium in an environment of identical firms, price competition and discount factor fluctuations, and show that it displays the following properties. First, the higher the discount factor, the higher the equilibrium prices and profits. This follows from the fact that the higher the discount factor, the more firms care about the future and the lower their incentives to deviate. Hence the higher the discount factor, the higher profits can be without firms deviating. Second, the higher the volatility of the discount factor, the lower the expected equilibrium prices and profits. This holds because combining the feasibility constraint with the incentive compatibility constraint results in a concave profit function (as a function of the discount factor) and, hence, an increase in volatility leads to a decrease in expected profits. In turn this decrease in expected profits results in a decrease in the range of perfect collusion, further reducing the expected value of profits. Third, an increase in the number of firms reduces the scope of collusion by increasing the incentives to deviate. To counter balance the increase in incentives to deviate, equilibrium profits decrease.

References

[1] Ellison, G. (1994). "Theories of Cartel Stability and the Joint Executive Committee", *Rand Journal of Economics*, 25, 1, 37-57.

[2] Friedman, J.W. (1971) "A Non-Cooperative Equilibrium for Supergames", *Review of Economic Studies*, 38, 1-12.

[3] Hirshleifer, J. and Riley, J.G. (1992). *The Analytics of Uncertainty and Information.* Cambridge University Press.

[4] Porter, R.H. (1983). "A Study of Cartel Stability: the Joint Executive Committee, 1880-1886", *Bell Journal of Economics*, 14, 301-314.

[5] Rotemberg, J.J. and Saloner, G. (1986). "A Supergame-Theoretic Model of Price Wars During Booms," *American Economic Review*, 76, 3, 390-407.