

Liquidity Management and Trading in the Interbank Market¹

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Abstract

The assessment of banks' liquidity management and interbank operations is a crucial element in the evaluation of the stability and efficiency of financial systems.

However, there is little economic analysis on this key issue. This paper is an

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attempt to bridge such a gap. In this paper, we develop an analytical framework to understand banks' strategic behavior in managing and trading liquidity. Our main results are (i) that competition does affect interbank liquidity, (ii) that (inefficient) premature liquidation is more likely to happen in highly competitive environments, (iii) both liquidity provision rules and central bank open market operations are required to achieve efficient liquidity management.

1. Introduction

Banks' liquidity management and its relationship with interbank operations are crucial not only for the profitability of banks in normal time and their insolvency in crisis time, but also for the stability and efficiency of financial systems. However, there is little economic analysis on this key issue. This paper is an attempt to bridge such a gap.

In this paper, we develop an analytical framework to address banks' strategic behavior in managing and trading liquidity. We focus on the benefits of an interbank market and model the role of liquidity management in preventing banking crisis. We implicitly assume that banks under prudential liquidity management are solvent and leave aside the issue of a contagious risk emerged from interbank market.

We specifically analyze three cases: liquidity management without the interbank market; liquidity management and trading in the interbank market when banks long in liquidity engage in competitions in supplying liquidity; and liquidity management and trading in the interbank market when banks long in liquidity collude in their supplying liquidity. Our model also allows us to address important policy issues, ranging from optimal liquidity management, to liquidity provision

rules, and to open market operations policy.

With the presence of an interbank market, banks hit by liquidity shocks would have incentives to borrow from others long in liquidity in order to meet early demand withdrawals so that they will need to liquidate less (costly) premature investment. Moreover, anticipating this possibility, banks ex ante can put aside less liquidity in cushion and invest more in illiquid projects. Whether banks long in liquidity engage in competitions or a collusion in supplying liquidity affects liquidity management, liquidity price in the interbank market, and the efficiency of the interbank risk sharing.

Our main results include that competition does affect interbank liquidity, that (inefficient) premature liquidation is more likely to happen in highly competitive environments, and that both liquidity provision rules and central bank open market operations are required to achieve efficient liquidity management.

2. The Model

Our basic model is built to capture the effects of liquidity management and trading in the interbank market.

2.1. Depositors

There are N banks, and ex ante each bank has mass-1 depositors. The model has three date (two periods). At date 0, each bank receives $I = 1/N$ amount of deposit. At $t = 1$, a portion of $(1 - \gamma)N$ banks face early withdrawals from some depositors, as in Diamond and Dybvig. At date 0, neither depositors nor banks know who are the early withdrawals, except the probability distribution.

Banks exist, as in Diamond and Dybvig (1983), because of their role in creating liquidity through using demand deposit contracts, according to which a depositor is allowed to withdraw w_t at time t .²

2.2. Bank

All N banks are members of the interbank market. Each bank invests the deposits it has taken in a partly illiquid project, which generates at date 2 a return of $(1+r)$ for each unit of initial investment. That is, at date 1 the project does not yield any liquidity, except through costly premature liquidation of a portion of its assets, q^i . Doing so can generate additional liquid revenue of $\rho q^i(1+r)$, where $\rho < 1$.

To capture the insight that when more banks chooses to liquidate their investment prematurely, the market for liquidation becomes more depressed so that

²We do not model banks' monitoring function here.

the lower is the value of (prematurely) liquidated asset, an insight suggested by Shleifer and Vishny (1992), and to endogenize this process of liquidation market, we assume that the amount of disinvestment due to many banks liquidation is

$$q^i = (1 + B \sum_{j=1}^n d^j) d^i, \quad (2.1)$$

where d^i is the amount of liquidity generated through prematurely liquidating q^i investment, $n \leq N$ is the total number of banks in premature liquidation, and $B > 0$ is a parameter.

Obviously,

$$d^i = \rho q^i < q^i.$$

In other words,

$$q^i = \frac{1}{\rho} d^i = (1 + B \sum_{j=1}^n d^j) d^i,$$

or,

$$1 + B \sum_{j=1}^n d^j = \frac{1}{\rho} \geq 1.$$

Moreover,

$$\frac{\partial \rho}{\partial (\sum_{j=1}^n q^j)} < 0.$$

That is, when more banks have to liquidate their investment prematurely, the more depressed is the liquidation market, and the lower is the value generated by liquidation for a given portion of asset.

We implicitly assume that banks have positive charter value. More specifically, banks under prudential liquidity management are solvent even if they do not get help from interbank market.

3. Liquidity Management without Interbank Trading

We start our analysis from the benchmark case of liquidity management when there is no interbank market to trade liquidity. In this benchmark case, without prudential liquidity management banking crises would be an equilibrium outcome.

In the benchmark case, if a bank faces no excessive early withdrawals, there would be no liquidity shock and thus no bank run or insolvency. When the bank faces liquidity shocks and it has not putted in place a sound liquidity management policy, however, a bank run or insolvency becomes a possibility, because as described above, any liquidation of the remaining investment at date 1 can only generate $\rho < 1$ for each dollar of investment.

Thus, from a perspective of liquidity management, a bank should put aside sufficient liquidity to meet early withdrawals and prevent banking crisis from

breaking out. This is, in case it face early withdrawals at $t = 1$, it can still avoid to liquidate too much premature assets. Assuming that a bank puts $1/N - \alpha^i \geq 0$ amount of liquidity in cushion. If it faces early withdrawals of $\frac{S}{N} < \frac{1}{N}$ at $t = 1$, it still need d^i amount of new liquidity to meet the early withdrawals. That is,

$$\frac{S}{N} - \left(\frac{1}{N} - \alpha^i\right) = \alpha^i - \frac{1 - S}{N} = d^i.$$

Obviously, the bank has to liquidate q^i amount of investment, and

$$q^i = \left(1 + B \sum_{j=1}^n d^j\right) d^i.$$

In doing so, the amount of initial investment is reduced, and the return from investment becomes only:

$$(\alpha^i - q^i)(1 + r).$$

Bank i 's optimization problem is:

$$\gamma \alpha^i (1 + r) + (1 - \gamma) (\alpha^i - q^i) (1 + r) \tag{3.1}$$

With $n = (1 - \gamma)N$ banks facing early withdrawals,

$$\begin{aligned}
q^i &= (1 + B \sum_{j=1}^n d^j) d^i \\
&= \left\{ 1 + B[(1 - \gamma)N - 1]d + Bd^i \right\} d^i \\
&= \left\{ 1 + B[(1 - \gamma)N - 1]d + B\left(\alpha^i - \frac{1 - S}{N}\right) \right\} \left(\alpha^i - \frac{1 - S}{N}\right).
\end{aligned}$$

Using this condition in (), we have

$$\max_{\alpha^i} \gamma \alpha^i (1+r) + (1-\gamma) \left[\alpha^i - \left\{ 1 + B[(1 - \gamma)N - 1]d + B\left(\alpha^i - \frac{1 - S}{N}\right) \right\} \left(\alpha^i - \frac{1 - S}{N}\right) \right] (1+r).$$

The F.O.C. leads to

$$\begin{aligned}
\alpha^* &= \frac{1 - S}{N} + \frac{\gamma}{B(1 - \gamma)[(1 - \gamma)N + 1]}, \\
d^* &= \alpha^* - \frac{1 - S}{N} = \frac{\gamma}{B(1 - \gamma)[(1 - \gamma)N + 1]}, \\
q^* &= \left[1 + \frac{N^2 \gamma}{(1 - \gamma)N + 1} \right] d^* = \left[1 + \frac{N^2 \gamma}{(1 - \gamma)N + 1} \right] \frac{\gamma}{B(1 - \gamma)[(1 - \gamma)N + 1]}.
\end{aligned}$$

The comparative statics of this result, i.e.,

$$\frac{\partial \alpha^*}{\partial N} < 0, \quad \frac{\partial \alpha^*}{\partial \gamma} > 0,$$

$$\frac{\partial d^*}{\partial N} < 0, \frac{\partial d^*}{\partial \gamma} > 0,$$

$$\frac{\partial q^*}{\partial N} > 0(?), \frac{\partial q^*}{\partial \gamma} > 0,$$

are quite intuitive.

And obviously,

$$q^* = \left[1 + \frac{N^2 \gamma}{(1 - \gamma)N + 1} \right] d^* > d^*.$$

Since we have implicitly assumed that banks under prudential liquidity management are solvent even if they do not get help from interbank market, no bank will default on its depositors. But in this case of no interbank market, there is a waste of liquidity in the banking system. The total amount of extra liquidity wasted in the banking system is

$$\Gamma \equiv \gamma N \left(\frac{1}{N} - \alpha^i \right) = \gamma S - \frac{N \gamma^2}{B(1 - \gamma)[(1 - \gamma)N + 1]} > 0.$$

Moreover,

$$\frac{\partial \Gamma}{\partial N} < 0.$$

That is, the larger is the total number of banks in the banking system, the more is the total liquidity waste in the banking system when there is no interbank market.

Furthermore, from the condition of

$$\alpha^i = \frac{1-S}{N} + \frac{\gamma}{B(1-\gamma)[(1-\gamma)N+1]} < \frac{1}{N},$$

we have

$$N\gamma < BS(1-\gamma)[(1-\gamma)N+1].$$

To summary our results, we present the following proposition.

Proposition 3.1. *In a banking system without interbank market, ex ante each bank puts $\frac{1}{N} - \alpha^* = \frac{S}{N} - \frac{\gamma}{B(1-\gamma)[(1-\gamma)N+1]}$ aside as cushion liquidity, and there is a waste of $\Gamma = \gamma S - \frac{N\gamma^2}{B(1-\gamma)[(1-\gamma)N+1]}$ total amount of liquidity in the banking system.*

4. Liquidity Management and Trading

As we have shown in the above section, much of liquidity is wasted when there is no interbank market. When all the N banks are linked by an interbank market, the interbank market shall provide a mechanism to trade liquidity. As a result, it shall be able to provide some of the needed liquidity, and enable banks to better manage their liquidity. In contrast to the Aghion, Bolton and Dewatripont (2000)

model, we focus on the benefits of an interbank market. More specifically, we focus on the role of liquidity management in preventing banking crisis and thus leave aside the issue of a contagious risk emerged from interbank market.³

With the presence of an interbank market, the banks hit by liquidity shocks would have incentives to borrow from the other banks in order to meet the demand withdrawals at date 1 so that they will need to liquidate less premature investment. Moreover, anticipating this possibility at $t = 1$, banks ex ante can put aside less liquidity in cushion and invest more in illiquid projects. That is, they can save a portion of $1/N - \alpha^*$ and use it to invest in real projects, which can generate additional returns.

Obviously those banks not facing liquidity shocks may or may not have incentives to lend their excessive liquidity to illiquid banks, unless the price is right and illiquid banks are solvent and can provide collateral to insure the liquidity they intend to borrow. Because we focus on liquidity management in preventing banking crisis and thus leave aside the issue of insolvent banks, price for liquidity becomes the only issue to be concerned in the rest of paper.

Again, from a perspective of liquidity management, a bank should put aside

³See Aghion, Bolton and Dewatripont (1999) and Huang and Xu (2000) for contagious risk in a domestic interbank market, and Goodhart and Huang (2000) for contagious risk in an international interbank market.

some liquidity, along with possible borrowings from the interbank market, to meet early withdrawals and prevent bank run. This is, in case it face early withdrawals at $t = 1$, it can still avoid to liquidate too much premature assets. Assuming that a bank puts $1/N - \alpha_I^i \geq 0$ amount of liquidity in cushion. If it faces early withdrawals at $t = 1$, in addition to borrow from the interbank market, it still need d^i amount of new liquidity to meet the early withdrawals. That is,

$$\frac{S}{N} - \left(\frac{1}{N} - \alpha_I^i\right) - d^i = \beta_i.$$

Again, the bank has to liquidate q^i amount of investment, and

$$q^i = \left(1 + B \sum_{j=1}^n d^j\right) d^i.$$

In doing so, the amount of initial investment is reduced, and the return from investment becomes only:

$$(\alpha_I^i - q^i)(1 + r).$$

Ex ante, bank i 's optimization problem is:

$$\gamma \left[\alpha_I^i (1+r) + \left(\frac{1}{N} - \alpha_I^i - \phi^i \right) (1+r_I) \right] + (1-\gamma) \left\{ (\alpha_I^i - q^i) (1+r) + \left[\frac{S}{N} - \left(\frac{1}{N} - \alpha_I^i \right) - d^i \right] (1+r_I) \right\}, \quad (4.1)$$

where the subscript I denotes variables under the interbank market, r_I the interest rate of liquidity traded in the interbank market, $\phi^i \geq 0$ the amount of liquidity a bank not facing liquidity shock wants to withhold.

In the following, we discuss two cases: competitive liquidity supply whereby competitive on the supply-side drives $\phi^i = 0$, and collusion in liquidity supply whereby liquidity suppliers jointly form a monopolist and thus $\phi^i > 0$.

4.1. Competitive liquidity supply

Since there are $n = (1 - \gamma)N$ banks facing early withdrawals,

$$q^i = \left(1 + B \sum_{j=1}^n d^j \right) d^i = \left\{ 1 + B[(1 - \gamma)N - 1]d + Bd^i \right\} d^i.$$

Moreover, the market clearing condition (the total demand equals the total supply) gives:

$$\sum_{i=1}^n \left[\frac{S}{N} - \left(\frac{1}{N} - \alpha_I^i \right) - d^i \right] = \sum_{i=1}^{N-n} \left[\frac{1}{N} - \alpha_I^i \right].$$

That is

$$N(1 - \gamma) \left[\frac{S}{N} - \left(\frac{1}{N} - \alpha_I^i \right) - d^i \right] = N\gamma \left[\frac{1}{N} - \alpha_I^i \right],$$

or

$$\alpha_I^i = 1 - (1 - \gamma)S + N(1 - \gamma)d^i.$$

On the demand side for liquidity, the equilibrium condition, given that an illiquid bank has two instruments (to borrow and to liquidate premature investment), should be that the bank is indifference between using these two instruments. But since the supply is limited,

$$\frac{\partial}{\partial d^i} [q^i(1 + r)] \geq \frac{\partial}{\partial d^i} [d^i(1 + r_I)].$$

Noticing that

$$\frac{\partial}{\partial d^i} [q^i(1 + r)] = \{1 + B[(1 - \gamma)N - 1]d + 2Bd^i\} (1 + r),$$

and

$$\frac{\partial}{\partial d^i} [d^i(1 + r_I)] = 1 + r_I + d^i \frac{\partial r_I}{\partial d^i} = 1 + r_I,$$

because under fixed supply, we expect

$$\frac{\partial r_I}{\partial d^i} = 0.$$

Therefore,

$$1 + r_I \leq \left\{ 1 + B[(1 - \gamma)N - 1]d + 2Bd^i \right\} (1 + r)$$

Using this condition in (), we have

$$\begin{aligned} & \max_{\alpha_I^i} \Pi(\alpha_I^i) \\ \equiv & \gamma \left[\alpha_I^i(1 + r) + \left(\frac{1}{N} - \alpha_I^i \right) (1 + r_I) \right] \\ & + (1 - \gamma) \left\{ \left(\alpha_I^i - \left\{ 1 + B[(1 - \gamma)N - 1]d + B\left(\alpha_I^i - \frac{1-S}{N} \right) \right\} \right) \left(\alpha_I^i - \frac{1-S}{N} \right) (1 + r) \right. \\ & \left. + \left[\frac{S}{N} - \left(\frac{1}{N} - \alpha_I^i \right) - d^i \right] (1 + r_I) \right\}. \end{aligned}$$

Substituting

$$1 + r_I = \left\{ 1 + B[(1 - \gamma)N - 1]d + 2Bd^i \right\} (1 + r),$$

$$\alpha_I^i = 1 - (1 - \gamma)S + N(1 - \gamma)d^i,$$

into the above maximization problem, we have the problem redefined as:

$$\max_{d^i} \Pi(d^i)$$

The F.O.C. leads to

$$\alpha_{IC}^* = ,$$

$$d_{IC}^* = ,$$

$$q_{IC}^* = ,$$

where subscript IC denotes the case that in the interbank market there are competitions among liquidity suppliers.

The comparative statics of this result, i.e.,

$$\frac{\partial \alpha_{IC}^*}{\partial N} < ? , \frac{\partial \alpha_{IC}^*}{\partial \gamma} > ? ,$$

$$\frac{\partial d_{IC}^*}{\partial N} < ? , \frac{\partial d_{IC}^*}{\partial \gamma} > ? ,$$

$$\frac{\partial q_{IC}^*}{\partial N} > ? , \frac{\partial q_{IC}^*}{\partial \gamma} > ? ,$$

are quite intuitive.

And obviously,

$$q_{IC}^* < q^*.$$

With the presence of interbank market, there is no waste of liquidity in the banking system. The total amount of liquidity provided by the interbank is

$$\Sigma \equiv \gamma \left(\frac{1}{N} - \alpha_{IC}^* \right) = .$$

Moreover,

$$\frac{\partial \Omega}{\partial N} > 0(?).$$

That is, the larger is the total number of banks in the banking system, the more (?) is the total liquidity generated by the interbank market.

To summary our results, we present the following proposition.

Proposition 4.1. *In a banking system with interbank market and competitive liquidity supply, ex ante each bank puts $\frac{1}{N} - \alpha_I^*$ aside as cushion liquidity, and there is a net liquidity of $\Sigma =$ provided by the interbank market.*

4.2. Collusion in liquidity supply

With collusion in the supply side, the equilibrium condition on the demand side for liquidity, given that an illiquid bank has two instruments (to borrow and to liquidate premature investment), should be that the bank is indifference between using these two instruments. That is:

$$\frac{\partial}{\partial d^i}[q^i(1+r)] = \frac{\partial}{\partial d^i}[d^i(1+r_I)].$$

Noticing that

$$\frac{\partial}{\partial d^i}[q^i(1+r)] = \{1 + B[(1-\gamma)N - 1]d + 2Bd^i\} (1+r),$$

and

$$\frac{\partial}{\partial d^i}[d^i(1+r_I)] = 1 + r_I + d^i \frac{\partial r_I}{\partial d^i}.$$

Therefore,

$$1 + r_I + d^i \frac{\partial r_I}{\partial d^i} = \{1 + B[(1-\gamma)N - 1]d + 2Bd^i\} (1+r)$$

Moreover, the market clearing condition (the total demand equals the total

supply) gives:

$$\sum_{i=1}^n \left[\frac{S}{N} - \left(\frac{1}{N} - \alpha_I^i \right) - d^i \right] = \sum_{i=1}^{N-n} \left[\frac{1}{N} - \alpha_I^i - \phi^i \right].$$

That is

$$N(1 - \gamma) \left[\frac{S}{N} - \left(\frac{1}{N} - \alpha_I^i \right) - d^i \right] = N\gamma \left[\frac{1}{N} - \alpha_I^i - \phi^i \right],$$

or

$$\phi^i = \frac{1 - (1 - \gamma)S - N\alpha_I^i}{N\gamma} + \frac{1 - \gamma}{\gamma} d^i.$$

The profit maximization of the liquidity supply leads to

$$\frac{\partial}{\partial \phi^i} \left[\left(\frac{1}{N} - \alpha_I^i - \phi^i \right) (1 + r_I) \right] = -(1 + r_I) - \frac{\gamma \phi^i}{1 - \gamma} \frac{\partial r_I}{\partial d^i} = 0.$$

This gives

$$\frac{\partial r_I}{\partial d^i} = -\frac{(1 - \gamma)(1 + r_I)}{\gamma \phi^i}.$$

Using this in () leads to

$$1 + r_I - \frac{(1 - \gamma)(1 + r_I)d^i}{\gamma \left[\frac{1 - (1 - \gamma)S - N\alpha_I^i}{N\gamma} + \frac{1 - \gamma}{\gamma} d^i \right]} = \left\{ 1 + B[(1 - \gamma)N - 1]d + 2Bd^i \right\} (1 + r),$$

or

$$\begin{aligned}
1 + r_I &= \left[1 + \frac{N(1-\gamma)d^i}{1 - (1-\gamma)S - N\alpha_I^i} \right] \{1 + B[(1-\gamma)N - 1]d + 2Bd^i\} (1+r) \\
&> \{1 + B[(1-\gamma)N - 1]d + 2Bd^i\} (1+r) \\
&> 1 + r,
\end{aligned}$$

if

$$1 - (1-\gamma)S - N\alpha_I^i \geq 0.$$

And this gives

$$\begin{aligned}
\frac{\partial r_I}{\partial d^i} &= -\frac{(1-\gamma)(1+r_I)}{\gamma\phi^i} \\
&= -\frac{1-\gamma}{\gamma\phi^i} \left[1 + \frac{N(1-\gamma)d^i}{1 - (1-\gamma)S - N\alpha_I^i} \right] \{1 + B[(1-\gamma)N - 1]d + 2Bd^i\} (1+r) \\
&= -\frac{N(1-\gamma) [1 + B[(1-\gamma)N - 1]d + 2Bd^i]}{1 - (1-\gamma)S - N\alpha_I^i} (1+r)
\end{aligned}$$

Using this condition in (), we have

$$\max_{\alpha_0^i} \gamma \left[\alpha_I^i (1+r) + \left(\frac{1}{N} - \alpha_I^i - \phi^i \right) (1+r_I) \right]$$

$$+(1 - \gamma) \left\{ \begin{aligned} &(\alpha_I^i - \{1 + B[(1 - \gamma)N - 1]d + B(\alpha_I^i - \frac{1-S}{N})\}(\alpha_I^i - \frac{1-S}{N}))(1 + r) \\ &+ [\frac{S}{N} - (\frac{1}{N} - \alpha_I^i) - d^i](1 + r_I) \end{aligned} \right\}.$$

The F.O.C. leads to

$$\alpha_{IM}^* = ,$$

$$\phi_{IM}^* =$$

$$d_{IM}^* = ,$$

$$q_{IM}^* = ,$$

where subscript IM denotes the case that in the interbank market there is a kind of monopolistic pricing on the supply side of liquidity.

The comparative statics of this result, i.e.,

$$\frac{\partial \alpha_{IM}^*}{\partial N} < ?, \frac{\partial \alpha_{IM}^*}{\partial \gamma} >?,$$

$$\frac{\partial d_{IM}^*}{\partial N} < ?, \frac{\partial d_{IM}^*}{\partial \gamma} >?,$$

$$\frac{\partial q_{IM}^*}{\partial N} > ?, \frac{\partial q_{IM}^*}{\partial \gamma} >?,$$

are quite intuitive.

And obviously,

$$q_{IM}^* > q_{IC}^*.$$

With the presence of interbank market but when the suppliers of liquidity engage in a collusion, there is also waste of liquidity in the banking system. The total amount of liquidity provided by the interbank is

$$\Gamma_{IM} \equiv \gamma \left(\frac{1}{N} - \alpha_{IM}^* - \phi_{IM}^* \right) = .$$

Moreover,

$$\frac{\partial \Gamma_{IM}}{\partial N} > 0(?).$$

That is, the larger is the total number of banks in the banking system, the more (?) is the total liquidity generated by the interbank market.

To summary our results, we present the following proposition.

Proposition 4.2. *In a banking system with interbank market and collusion in liquidity supply, ex ante each bank puts $\frac{1}{N} - \alpha_0^*$ = aside as cushion liquidity, there is a net liquidity of Γ_{IM} = provided by the interbank market, and a waste of amount of liquidity due to collusion.*

5. Policy Implications

Incomplete.

6. Conclusions

Incomplete.