Using Balance Sheet Data to Identify Sovereign Default and Devaluation Risk.

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The purpose of this paper is to study the implications of currency denomination mismatch in the balance sheets of banks on the relationship between currency risk and sovereign default risk. This issue is a key ingredient in the discussion of the pros and cons of dollarizing economies in which the monetary authority lacks credibility.

Most of the arguments in favor of dollarization depend on the alleged high relationship between devaluation risk and default risk. In fact, if a mayor determinant of default risk is the risk of devaluation, then, by taking away the technology to devalue from the domestic Central Bank, dollarization can substantially reduce the default risk with all the implied advantages that a reduction on the cost of capital conveys. Thus, one may argue, this relationship between devaluation risk and default risk is at the heart of the debate.

The obvious question to raise at this time is how are we to think conceptually about this relationship between devaluation risk and default risk. One of the main, although not the only one, proposed channels for this relationship is the existence of explicit or implicit bail outs that the Central Bank is forced to do to avoid generalized bankruptcies in the banking system. Thus, the argument may go, if the banks have a share of assets in domestic currency that is larger than the share of liabilities, a devaluation may bankrupt otherwise solvent banks. The fiscal needs generated by this bail-out may force a default on government bonds.

The purpose of this paper is to take this story seriously and study its implications in terms of interpreting the equilibrium interest rates on the different bonds issued by governments that lack credibility. If the story is correct, interest rate data should reveal a high correlation between expected default on sovereign bonds and expected devaluations. This is an important issue, since we would like to have a tractable conceptual framework with which to analyze the available evidence.

In order to interpret the data, we start by considering the different ways governments have to change the value of their existing liabilities, namely, to default on domestic currency bonds, to default on foreign currency bonds, and to devalue. As these variables are unobservable we need to use economic theory to uncover them.

A common approach to this problem, which is our departure point, is to look at the correlation between $i_t - i_t^*$ and $i_t^* - r_t$, where i_t is the interest rate on domestic currency sovereign bonds, i_t^* is the interest rate on foreign currency sovereign bonds, and r_t is the risk-free rate on foreign currency bonds. This correlation is interpreted as the correlation between default risk and devaluation risk. The problem with this approach is that the two arbitrage relations among these bonds are not enough to identify the correlations between the three variables of interest. We illustrate how different assumptions regarding the behavior of the government imply very different interpretations of the data.

We propose to solve this identification problem by studying a simple dynamic Ramsey problem in which the government is allowed to default on its debt and devalue at the start time of its plan, but must guarantee a minimum level of profits to banks. The analysis shows how existing balance sheets determine the government's optimal choices. Balance sheet structures affect the choice of the government between the alternative default instruments. Our simple model solves the identification problem mentioned above.

Several caveats are in order. First, our model does not take the story mentioned above literally, in which *given* a devaluation, a bail-out and a default on bonds, that otherwise would not happen, follow. In the model we assume, as we believe is sensible, that the government chooses jointly to devalue, to bail-out and to default on bonds. Second, to model default in equilibrium is a tricky business. We take a simple approach. Since we are interested in studying how governments optimally behave when, with small probability and by some reason we do not explain, they opt for breaking promises, we use a model in which a government can default only once. The simplest way to do this is to solve a Ramsey problem, assume commitment, and let the government do the - first and - last break in its promises. This model provides a theory on how governments behave when they can default only once, which, hopefully, has implications similar to the ones of a model in which governments do default in equilibrium with small probability, a theory we cannot write.

1 The Identification Problem without an Optimizing Government.

The standard approach to look at the correlation between sovereign default risk and devaluation risk rests on the arbitrage restrictions,

$$\begin{aligned} (1+i_t)\, E\left(\frac{1-\delta_{t+1}}{1+\varepsilon_{t+1}}\right) &= (1+i_t^*)\, E\left(1-\delta_{t+1}^*\right) \\ (1+i_t^*)\, E\left(1-\delta_{t+1}^*\right) &= (1+r_t) \end{aligned}$$

where δ^* and δ are the repudiation rates of dollar and peso denominated sovereign debt and ϵ is the devaluation rate. For simplicity, we assume that agents are risk-neutral.

The identification problem arises because these two equations and the interest rate data are insufficient to uncover the unobserved variables of interest. Substituting for $1-\delta^*_{t+1}$we obtain $(1+i_t)\,E\left(\frac{1-\delta_{t+1}}{1+\varepsilon_{t+1}}\right) = 1+r_t$: an equation with two unknowns. It is clear that there is a missing equation.

The consequences of this identification problem are serious. Different assumptions about the missing equation imply radically different interpretations of the data. To see this, using the approximation

$$\frac{1-\delta_{t+1}}{1+\varepsilon_{t+1}} \cong 1-\delta_{t+1}-\varepsilon_{t+1},$$

and ignoring products of small numbers, re-write these equations as

$$\begin{aligned} i_t - i^*_t &= E\delta_{t+1} + E\varepsilon_{t+1} - E\delta^*_{t+1} \\ i^*_t - r_t &= E\delta^*_{t+1} \end{aligned}$$

The covariance between $i_t - i^*_t$ and $i^*_t - r_t$ then is

$$\begin{aligned} cov\,(i_t - i^*_t, i^*_t - r_t) &= cov\left(E\delta_{t+1} + E\varepsilon_{t+1} - E\delta^*_{t+1}, E\delta^*_{t+1}\right) \\ &= cov\left(E\varepsilon_{t+1}, E\delta^*_{t+1}\right) + \\ &\quad \left[cov\left(E\delta_{t+1}, E\delta^*_{t+1}\right) - var\left(E\delta^*_{t+1}\right)\right] \end{aligned}$$

Alternative assumptions on the government's default policy have very different implications for the interpretation of the data.

Consider first the case in which the government always defaults on domestic and foreign currency debt simultaneously: $\delta_t = \delta^*_t$ for all t. Under this assumption,

$$\begin{aligned} i_t - i^*_t &= E\varepsilon_{t+1} \\ i^*_t - r_t &= E\delta^*_{t+1} \\ cov\left(E\varepsilon_{t+1}, E\delta^*_{t+1}\right) &= cov\,(i_t - i^*_t, i^*_t - r_t)\,, \end{aligned}$$

so the covariance between the currency spread and the sovereign spread reveals the covariance between the devaluation risk and the default risk on foreign currency sovereign bonds.

Alternatively, consider the case in which the government never defaults on domestic currency sovereign debt explicitly, but it does so implicitly by devaluing. In this case, $\delta_t = 0$ for all t,

$$\begin{aligned} i_t - r_t &= E\varepsilon_{t+1} \\ i^*_t - r_t &= E\delta^*_{t+1} \\ cov\left(E\varepsilon_{t+1}, E\delta^*_{t+1}\right) &= cov\,(i_t - r_t, i^*_t - r_t)\,. \end{aligned}$$

The usual measure of the correlation between default and devaluation risk is $cov\,(i_t - i^*_t, i^*_t - r_t) = cov\left(E\varepsilon_{t+1}, E\delta^*_{t+1}\right) - var\left(E\delta^*_{t+1}\right).$

In the first case, $cov\left(i_t - i_t^*, i_t^* - r_t\right) = 0$ implies that the case for dollarization is weak because in the data there is no relation between currency and sovereign default risk on foreign currency debt. In the second case, the same ovservation implies that $cov\left(E\varepsilon_{t+1}, E\delta_{t+1}^*\right) = var\left(E\delta_{t+1}^*\right) > 0$, so the case for dollarization is consistent with the data.

The next section addresses this identification problem by studying a simple dynamic Ramsey problem for a government that chooses devaluation rates, default rates, and guarantees a minimum profit to banks. The theory will tell us when is the government going to default on its domestic currency debt explicitly through repudiation or implicitly through devaluation.

2 Sovereign Default and Devaluation in a Ramsey Problem with Bank Bailouts.

We study the decision problem faced by the government of a small open economy that, at time $t = 0$, must finance a given stream of government expenditures choosing a sequences of income tax rates, exchange rates and default rates on debts it inherits from the past. We assume that there is a fixed exchange rate regime in which government commits to exchange any amount of domestic and foreign currency at pre-announced exchange rates. A second key assumption of our model is that the government guarantees a minimum level of profits to the banking system.

It is assumed throughout the exercise that this government can perfectly commit to future policies.

2.1 Economic environment and definition of equilibrium.

There is no uncertainty and agents have perfect foresight.

There are two goods produced with the linear **technology**,

$$\begin{aligned} y_{1t} &= n_{1t} \\ y_{2t} &= n_{2t} \end{aligned}$$

where y_{it}, n_{it} denote output and labor for goods $i = 1, 2$. Good 1 is not traded internationally and good 2 is. Purchasing power parity holds for the traded good

Firms choose labor inputs to maximize

$$p_{1t} y_{1t} + e_t p_{2t}^* y_{2t} - w_t \left(n_{1t} + n_{2t}\right). \tag{1}$$

subject to $n_{1t}, n_{2t} \geq 0$. e_t, w_t, p_{1t} are the domestic currency prices of foreign currency, labor, the non-traded good. p_{2t}^* is the foreign currency price of the traded good. This problem has an interior solution only if

$$p_{1t} = e_t p_{2t}^* = w_t \quad \text{for all } t.$$

Other possibilities will not be considered[1].

[1] This without loss of generality because there are no equilibria with corner solutions.

Household preferences over these two goods and work effort, n_t, are described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_{1t}\right) + u\left(c_{2t}\right) - \alpha n_t \right], \tag{2}$$

where $0 < \beta < 1$, $\alpha > 0$, $u : R \rightarrow R$ is monotonically increasing, concave, and satisfies $\lim_{x \rightarrow 0} u'(x) = \infty$ and $\lim_{x \rightarrow \infty} u(x) = 0$.

Purchases of the non-tradable good c_{1t} have to be paid with domestic cash, while those of c_{2t} can be paid with credit. The cash in advance constraint for good 1 is

$$p_{1t} c_{1t} \square M_t \qquad \text{for all } t. \tag{3}$$

Household's budget constraints are given by

$$\begin{aligned}
\left(B_t^H + M_t + D_t - L_t\right) + e_t \left(B_t^{H*} + D_t^* - L_t^*\right) \square \\
M_{t-1} + \left(1 + i_{t-1}\right) \left(\left(1 - \delta_t\right) B_{t-1}^H + D_{t-1} - L_{t-1}\right) \\
+ e_t \left(1 + i_{t-1}^*\right) \left(\left(1 - \delta_t\right) B_{t-1}^{H*} + D_{t-1}^* - L_{t-1}^*\right) \\
\Pi_t + p_{1t} g + w_t \left(1 - \tau_t\right) n_t - p_{1t} c_{1t} - p_{2t}^* e_t c_{2t}
\end{aligned}$$

for $t = 0, 1, 2, ...$where B_t^H, B_t^{H*} are one period bonds held by households from t to $t+1$ in domestic and foreign currency, respectively, M_t are end of period money balances, D_t, D_t^*, L_t, L_t^* are domestic and foreign currency denominated end of period bank deposits and loans, g are government transfers, τ_t are income tax rates, and Π_t are bank profits. $0 \square \delta_0, \delta_0^* \square 1$ represent default rates on domestic and foreign bonds at time 0.

Implicit in these budget constraints is the assumption that there are no arbitrage opportunities: $1 + i_t = \left(1 + i_t^*\right) e_{t+1}/e_t$ for all $t \geq 0$. Note also that, as we assume the government can commit, we can ignore future default rates[2]. Thus, we impose $\delta_t = \delta_t^* = 0$ for $t > 0$ without loss of generality.

In addition to the flow budget constraint above, households are restricted by the no Ponzi game condition

$$\lim_{t \rightarrow \infty} \beta^{t+1} \left(\frac{B_{t+1}^H + M_{t+1} + D_{t+1} - L_{t+1}}{e_{t+1} p_{2,t+1}^*} + \frac{\left(B_{t+1}^{H*} + D_{t+1}^* - L_{t+1}^*\right)}{p_{2,t+1}^*} \right) \geq 0$$

Financial intermediaries costlessly receive deposits and lend money to the government and to the private sector. End of period balance sheets are described by

$$L_t + e_t L_t^* + B_t^b + e_t B_t^{b*} = D_t + e_t D_t^* \quad \text{for } t = -1, 0, 1, \ldots$$

[2] For $t > 0$ assume $0 \square \delta_t, \delta_t^* < 1$ for all t. No arbitrage implies that the return on bonds and other assets is the same. If i_t^b is the interest rate on bonds, no arbitrage implies $1 + i_t = \left(1 - \delta_t\right)\left(1 + i_t^b\right)$ for $t > 0$.

and bank profits are

$$\begin{aligned}\Pi_0 &= (1+i_{-1})\left((1-\delta_0)B^b_{-1}+L_{-1}-D_{t-1}\right)\\ &\quad +e_0\left(1+i^*_{-1}\right)\left((1-\delta^*_0)B^{b*}_{-1}+L^*_{-1}-D^*_{t-1}\right)+T_0\end{aligned}$$

$$\begin{aligned}\Pi_t &= (1+i_{t-1})\left(B^b_{t-1}+L_{t-1}-D_{t-1}\right)\\ &\quad +e_t\left(1+i^*_{t-1}\right)\left(B^{b*}_{t-1}+L^*_{t-1}-D^*_{t-1}\right)+T_t \quad \text{for } t=1,2,\ldots\end{aligned}$$

T_t is a transfer scheme that guarantees banks a minimum level of profits. In our case this level of profits is zero, but we can easily accommodate any constant. The transfer scheme is

$$T_t = \max\{-\Pi_t + T_t, 0\}$$

A government insurance protects the banking system from aggregate shocks such as devaluations and sovereign defaults. The rule is meant to capture in a simple way the contingent debt nature of banks negative profits for the government.

Perfect foresight and no arbitrage conditions imply that bank profits and bailouts are zero for all $t \geq 1$. Bank bailouts at $t=0$ are

$$\begin{aligned}T_0(e_0,\delta_0,\delta^*_0) &= \max\left\{(1+i_{-1})\left(D_{-1}-(1-\delta_0)B^b_{-1}-L_{-1}\right)+\right.\\ &\quad \left. e_0\left(1+i^*_{-1}\right)\left[D^*_{-1}-(1-\delta^*_0)B^{b*}_{-1}-L^*_{-1}\right],0\right\},\end{aligned}$$

Observe that defaults on government bonds may cause transfers to banks. Also, liability dollarization in the banking system makes the transfers that occur when banks have negative profits an increasing function of the exchange rate.

The government has to service its debt and pay for transfers to banks and households by levying income taxes, issuing money and by choosing repudiation rates on its liabilities. Let B^{g*}_t, B^g_t be government bond holdings. Government's budget constraints are

$$\begin{aligned}e_0B^{g*}_0+B^g_0-M_0 &= (1+i_{-1})(1-\delta_0)B^g_{-1}-M_{-1}\\ &\quad +e_0\left(1+i^*_{-1}\right)(1-\delta^*_0)B^{g*}_{-1}+\tau_0\omega_0 n_0-p_{10}g_0-T_0\end{aligned}$$

$$\begin{aligned}e_tB^{g*}_t+B^g_t-M_t &= (1+i_{t-1})B^g_{t-1}-M_{t-1}\\ &\quad +e_t\left(1+i^*_{t-1}\right)B^{g*}_{t-1}+\tau_t\omega_t n_t-p_{1t}g \ \text{ for } t=1,2,\ldots\end{aligned}$$

and the no Ponzi game condition

$$\lim_{t\to\infty}\beta^{t+1}\frac{e_{t+1}B^{g*}_{t+1}+B^g_{t+1}-M_{t+1}}{e_{t+1}p^*_{2,t+1}}\geq 0.$$

For simplicity, assume the foreign government follows the **Friedman rule** for monetary policy—i.e.

$$\begin{aligned} i_t^* &= 0 \text{ for } t = -1, 0, 1, \ldots \\ p_{2t}^* &= \beta^t \text{ for } t = 0, 1, \ldots \end{aligned}$$

for $t = 0, 1, 2, \ldots$..To normalize, we also assume that $e_{-1} = 1, i_{-1} = 0$.

A consequence of this assumption is that an equilibrium exists only if exchange rates satisfy

$$e_{t+1} \geq e_t \text{ for all } t. \tag{4}$$

Otherwise, interest on domestic bonds will be negative, creating an arbitrage opportunity.

Combining household's and government's flow budget constraints, the no Ponzi game condition, expressions for bank profits, and the Friedman rule assumption for foreign monetary policy, we obtain

$$\sum_{t=0}^{t} \beta^t \left[c_{1t} + c_{2t} + \left(1 - \frac{e_t}{e_{t+1}}\right) \frac{M_t}{p_{1t}} - (1 - \tau_t)\, n_t - g \right] = \tag{5}$$
$$\frac{M_{-1} + (1 - \delta_0)\left(B_{-1}^H + B_{-1}^B\right)}{e_0} + (1 - \delta_0^*)\left(B_{-1}^{H*} + B_{-1}^{B*}\right) + \frac{T_0\left(e_0, \delta_0, \delta_0^*\right)}{e_0}$$

for households, and

$$\sum_{t=0}^{\infty} \beta^t \left[g - \left(1 - \frac{e_t}{e_{t+1}}\right) \frac{M_t}{p_{1t}} - \tau_t n_t \right] =$$
$$(1 - \delta_0) \frac{B_{-1}^g - M_{-1}}{e_0} + (1 - \delta_0^*)\, B_{-1}^{g*} - \frac{T_0\left(e_0, \delta_0, \delta_0^*\right)}{e_0}$$

for the government,

Adding the government's and the household's budget constraint we obtain the country's budget constraint,

$$\sum_{t=0}^{\infty} \beta^t \left((c_{1t} - n_{1t}) + (c_{2t} - n_{2t})\right) = (1 - \delta_0) \frac{B_{-1}^H + B_{-1}^B + B_{-1}^g}{e_0} \tag{6}$$
$$+ (1 - \delta_0^*)\left(B_{-1}^{H*} + B_{-1}^{B*} + B_{-1}^{g*}\right)$$

Market clearing for the non–internationally-traded good, labor and bonds requires

$$n_{1t} = c_{1t} \quad \text{for all } t \tag{7a}$$
$$n_{1t} + n_{2t} = n_t \quad \text{for all } t \tag{7b}$$
$$B_{-1}^H + B_{-1}^B + B_{-1}^g = 0 \quad \text{for all } t \tag{7c}$$
$$B_{-1}^{H*} + B_{-1}^{B*} + B_{-1}^{g*} + B_{-1}^{F*} = 0 \quad \text{for all } t \tag{7d}$$

We assume that all domestic currency bonds are held domestically. Thus, when markets clear the government budget constraint becomes

$$\sum_{t=0}^{\infty} \beta^t \left(c_{2t} - y_{2t}\right) \square \left(1 - \delta_0^*\right) \left(B_{-1}^{H*} + B_{-1}^{B*} + B_{-1}^{g*}\right) \tag{8}$$

This equation states that the present value of the country's trade deficits equal the country's initial non-defaulted net foreign assets.

Denote initial portfolios as

$$I_0 = \left\{M_{-1}, B_{-1}^H, B_{-1}^B, B_{-1}^G, D_{-1}, L_{-1}, B_{-1}^{*H}, B_{-1}^{*B}, B_{-1}^{*G}, D_{-1}^*, L_{-1}^*\right\}.$$

For given initial portfolios I_0, an allocation $\{c_{1t}, c_{2t}, n_{1t}, n_{2t}, M_t\}_{t=0}^{\infty}$ is a **competitive equilibrium with bank bailouts, taxes and default** if, and only if,

(i) consumers maximize utility (2) subject to budget (5) and cash-in-advance (3) constraints,

(ii) government policies $\{\delta_0, \delta_0^*\}$ and $\{g, , \tau_t, e_t\}_{t=0}^{\infty}$ are consistent with the government's budget constraint (**??**) and the non-negativity of domestic currency interest rates (4),

(iii) firms solve (1),

(iv) the market clearing conditions (7) are satisfied.

Household's problem. The first order conditions of the household's problem for $t = 0, 1, 2, ...$ are

$$\begin{aligned} \frac{u'\left(c_{1t}\right)}{u'\left(c_{2t}\right)} &= \left(1 + \left(1 - \frac{e_t}{e_{t+1}}\right)\right) \\ \frac{\alpha}{u'\left(c_{2t}\right)} &= \left(1 - \tau_t\right) \end{aligned} \tag{9}$$

and

$$M_t = p_{1t} c_{1t} \tag{10a}$$

$$\begin{aligned} c_2 &= (1-\beta) a_0 + g \\ &\quad + (1-\beta) \sum_{t=0}^{t} \beta^t \square \left(1-\tau_t\right) n_t - c_{1t} - \left(1 - \frac{e_t}{e_{t+1}}\right) \frac{M_t}{e_t p_{2t}^*} \Bigg] \end{aligned} \tag{10b}$$

where

$$a_0 = \frac{M_{-1} + \left(1 - \delta_0\right)\left(B_{-1}^H + B_{-1}^B\right)}{e_0} + \left(1 - \delta_0^*\right)\left(B_{-1}^{H*} + B_{-1}^{B*}\right) + \frac{T\left(\delta_0, \delta_0^*, e_0\right)}{e_0}$$

The household's optimality conditions, the non-negativity of interest rates (4), and the budget constraint (5) yield the implementability conditions

$$u'(c_2)\left[(1-\beta)\,a_0^H + g\right] + (1-\beta)\sum_{t=0}^{t} \beta^t\left[\alpha n_t - u'(c_{1t})\,c_{1t} - u'(c_2)\,c_2\right] = 0$$
$$u'(c_{1t}) \geq u'(c_2)\,. \tag{11}$$

Using the implementability condition we obtain a simpler definition of equilibrium. For initial portfolios I_0 given, an allocation $\{c_{1t}, c_{2t}, n_{1t}, n_{2t}\}_{t=0}^{\infty}$ is a **competitive equilibrium with bank bailouts, taxes and default** if, and only if, it satisfies the conditions (6), (7) and (11).

3 Solution of The Ramsey Problem

Governments problem is to choose $\{e_0, \delta_0, \delta_0^*\}$ and $\{c_{1t}, c_{2t}, n_{1t}, n_{2t}, M_t\}_{t=0}^{\infty}$ in order to maximize (2) subject to conditions (6), (7) and (11), with initial portfolios I_0 given. Conditions (6), (7) and (11) insure that the chosen allocation is a competitive equilibrium. The taxes and exchange rates that implement each allocation are given by (9).

We will focus on the case where all bonds at $t = -1$ are issued by the government and all private agents have initial positive bond holdings—i.e.

$$B_{-1}^H, B_{-1}^B, -B_{-1}^G, B_{-1}^{*H}, B_{-1}^{*B}, B_{-1}^{*F}, -B_{-1}^{*G} > 0.$$

Note that consumption of the credit good must be constant over time, so the government can only choose the level of the tax, but must keep it constant[3]. On the other hand, the government can choose different values for consumption of the cash good over time by changing the devaluation rate constrained to satisfy the non-negativity of nominal interest rates. Finally, the government also chooses the initial nominal exchange rates and default rates. Government chooses $\{c_{1t}, n_{1t}, n_{2t}\}_{t=0}^{\infty}$ and $\{c_{2,}e_0, \delta_0, \delta_0^*\}$ to maximize the Lagrangean

$$\begin{aligned}
\mathcal{L} \;=\; & \sum \beta^t\left[U(c_{1t}) + U(c_2) - \alpha(n_{1t}+n_{2t})\right] \\
& -\lambda\left[u'(c_2)\left[(1-\beta)\,a_0^H + g\right] + (1-\beta)\sum_{t=0}^{t}\beta^t\left[\alpha n_t - u'(c_{1t})\,c_{1t} - u'(c_2)\,c_2\right]\right] \\
& -\sum \beta^t\gamma_t[c_{1t} - n_{1t}] - \omega[\sum \beta^t(c_{2t} - n_{2t}) - (1-\delta_0^*)B_{-1}^*] \\
& -\mu_0[0-\delta_0] - \mu_1[\delta_0 - 1] - \mu_0^*[0-\delta_0^*] - \mu_1^*[\delta_0^* - 1]
\end{aligned}$$

[3] This is due to the fact that this is a small open economy so the real interet rate is constant and because leisure entres linearly in the utility function. Thus, if taxes were not the same in two consecutive periods, the domestic real interest rate would be different from the foreign and capital inflows would be unbounded.

We ignore the non-negativity constraints on nominal interest rates and later verify that they will be satisfied.

First, we discuss the necessary conditions for an interior optimum with respect to consumption and labor. To simplify the discussion and focus on the optimal choices of default rates and devaluation rates, consider the case in which $U(c) = c^{1-\sigma}/(1-\sigma)$. These first order conditions can be combined to yield

$$U'(c_{1t}) = \frac{[1+\lambda(1-\beta)]}{[1+\lambda(1-\beta)(1-\sigma)]} \tag{12}$$

and

$$U'(c_2) - \lambda U''(c_2)(1-\beta)\left[(1-\beta)\,a_0^H + g\right] = \frac{[1+\lambda(1-\beta)]}{[1+\lambda(1-\beta)(1-\sigma)]} \tag{13}$$

First, note that (12) implies that the optimal nominal interest rate is constant over time, so in the solution $c_{1t} = c_1$ for all t. Also, combining (12) and (13) we obtain

$$U'(c_2) - \frac{\lambda U''(c_2)(1-\beta)\left[(1-\beta)\,a_0^H + g\right]}{[1+\lambda(1-\beta)(1-\sigma)]} = U'(c_1)$$

Note also that the multiplier λ is positive, since it measures the marginal cost of increasing the transfers g. Thus, $U'(c_2) < U'(c_1)$, which means that $\left(1 + \left(1 - \frac{e_t}{e_{t+1}}\right)\right) > 1$, or $1 > \frac{e_t}{e_{t+1}}$. Thus, the optimal policy is characterized by a positive and constant devaluation rate[4]. These results are standard in the literature on dynamic Ramsey problems.

We now focus on the optimal choice of devaluation and default rates at time zero in en economy with bank bailouts.

Using bank's balance sheets at $t = -1$, and assuming $i_{-1} = i^*_{-1} = 0$, $e_{-1} = 1$,

[4] In this economy, Friedman rule fails to be optimal, since the value of the goverenmnt liabilities depends on the value of consumption of the credit good at time one - note that this is the source of time inconsistency in Lucas and Stokey (1983). If this were a close economy, this effect would change the relative price betwen the cash and the credit good only at time zero, and Friedman rule would be optimal form time one on. However, in this open economy model the value of credit good consumption must be constant over time, thus, this effect distorts the relative price between cash and credit gods at every period.

bank bailouts can be written as[5]

$$\begin{aligned}\frac{T(\delta_0,\delta_0^*,e_0)}{e_0} &= \max\left\{\left(1-\frac{1}{e_0}\right)\left(B_{-1}^{B}+L_{-1}-D_{-1}\right)+\delta_0\frac{B_{-1}^{B}}{e_0}+\delta_0^* B_{-1}^{B*},0\right\}\\ &= \max\left\{\left(1-\frac{1}{e_0}\right)\left(D_{-1}^{*}-B_{-1}^{B*}-L_{-1}^{*}\right)+\delta_0\frac{B_{-1}^{B}}{e_0}+\delta_0^* B_{-1}^{B*},0\right\}.\end{aligned}$$

Changes in exchange rates and default on bonds held by banks may trigger bailouts. The relation between bailouts and exchange rates depends on the currency exposure of banks, as shown by the expression

$$\frac{\partial\frac{T(\delta_0,\delta_0^*,e_0)}{e_0}}{\partial e_0}=\frac{(1-\delta_0)\,B_{-1}^{B}+L_{-1}-D_{-1}}{e_0^2}$$

The size of bank bailouts is an increasing function of devaluations when banks have positive net assets denominated in domestic currency. If banks have net liabilities in domestic currency devaluations contribute to ex-post bank profits and reduce the size of bailouts. If there is no default on domestic currency bonds, bank's domestic currency exposure is equal to $D_{-1}^{*}-B_{-1}^{B*}-L_{-1}^{*}$. Bailouts are an increasing function of the exchange rate when there is liability dollarization—$D_{-1}^{*}>B_{-1}^{B*}+L_{-1}^{*}$. When there is a currency mismatch in banks portfolios, we will say that there is liability dollarization when $(1-\delta_0)\,B_{-1}^{B}+L_{-1}>D_{-1}$.

Depending on whether there is liability dollarization or not, bail outs are positive or negative depending on whether $e_0 \gtreqless \bar{e}_0$.

$$\begin{aligned}\text{If }\ (1-\delta_0)\,B_{-1}^{B}+L_{-1}-D_{-1} &> 0: T(\delta_0,\delta_0^*,e_0)>0\Leftrightarrow e_0>\bar{e}_0\\ \text{If }\ (1-\delta_0)\,B_{-1}^{B}+L_{-1}-D_{-1} &< 0: T(\delta_0,\delta_0^*,e_0)>0\Leftrightarrow e_0<\bar{e}_0\end{aligned}$$

where

$$\bar{e}_0=\frac{(1-\delta_0)\,B_{-1}^{B}+L_{-1}-D_{-1}}{\left(B_{-1}^{B}+L_{-1}-D_{-1}\right)+\delta_0^* B_{-1}^{B*}}.$$

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[5]

$$\begin{aligned}T_0 &= \frac{D_{-1}-(1-\delta_0)\,B_{-1}^{B}-L_{-1}}{e_0}+\left(D_{-1}^{*}-(1-\delta_0^*)\,B_{-1}^{B*}-L_{-1}^{*}\right)\\ &= \frac{D_{-1}-B_{-1}^{B}-L_{-1}}{e_{-1}}\left(1-\left(1-\frac{e_{-1}}{e_0}\right)\right)+\left(D_{-1}^{*}-B_{-1}^{B*}-L_{-1}^{*}\right)\\ &\quad+\delta_0^* B_{-1}^{B*}+\delta_0\frac{B_{-1}^{B}}{e_0}\\ &= \frac{B_{-1}^{B}+L_{-1}-D_{-1}}{e_{-1}}\left(1-\frac{e_{-1}}{e_0}\right)+\delta_0^* B_{-1}^{B*}+\delta_0\frac{B_{-1}^{B}}{e_0}\end{aligned}$$

It will be useful to have expressions for private assets with and without bailouts. If there is no bailout private assets are

$$a_0 = \frac{M_{-1} + (1-\delta_0)\left(B_{-1}^{H} + B_{-1}^{B}\right)}{e_0} + (1-\delta_0^*)\left(B_{-1}^{H*} + B_{-1}^{B*}\right),$$

while in the case where there is a bailout they are

$$\begin{aligned} a_0 &= \frac{M_{-1} + (1-\delta_0) B_{-1}^{H} + D_{-1} - L_{-1}}{e_0} + (1-\delta_0^*) B_{-1}^{H*} \\ &\quad + B_{-1}^{B*} + B_{-1}^{B} + L_{-1} - D_{-1}. \end{aligned}$$

Consider now the fist order conditions of the Ramsey problem with respect to δ_0^*

$$-\lambda U'(c_2)(1-\beta)\frac{\partial a_0^H}{\partial \delta_0^*} - \omega B_{-1}^* = \mu_1^* - \mu_0^* \tag{14}$$

and

$$\begin{aligned} \frac{\partial a_0^H}{\partial \delta_0^*} &= -\left(B_{-1}^{H*} + B_{-1}^{B*}\right) \text{ with no bailout} \\ \frac{\partial a_0^H}{\partial \delta_0^*} &= -B_{-1}^{H*} \quad \text{with bailout} \end{aligned}$$

Thus, under the assumption $B_{-1}^{H*}, B_{-1}^{B*} > 0$ in either case the derivative is negative. As, we also assume that initially the country is a net debtor (i.e., $B_{-1}^{F*} > 0$) the left-hand-side of (14) is positive and, since the multipliers ought to be nonnegative, then $\mu_1^* > \mu_0^* = 0$, which means that $\delta_0^* = 1$ is optimal. The intuition is very simple. Even though it is true that by defaulting on foreign currency denominated bonds the government can increase the contingent liabilities since banks can be holding some of those bonds, the net effect is positive, to the extent that households and foreigners hold positive amounts of foreign currency denominated debt.

Now, let us focus on the joint choices of e_0 and δ_0. Note that the derivative of the Lagrangean with respect to δ_0 is given by

$$-\lambda U'(c_2)(1-\beta)\frac{\partial a_0^H}{\partial \delta_0} = \mu_1 - \mu_0$$

where

$$\begin{aligned} \frac{\partial a_0^H}{\partial \delta_0} &= -\frac{B_{-1}^{H} + B_{-1}^{B}}{e_0} \text{ with no bailout} \\ \frac{\partial a_0^H}{\partial \delta_0} &= -\frac{B_{-1}^{H}}{e_0} \text{ with bailout.} \end{aligned}$$

Under our assumptions on initial portfolios, as long as e_0 is bounded in both cases $\mu_1 > \mu_0 = 0$ and it is optimal to set $\delta_0 = 1$. Note, however, that in the case in which e_0 is arbitrarily large, the value of δ_0 is inessential. As there is no benefit from defaulting we will assume that government chooses $\delta_0 = 0$ when e_0 is arbitrarily large.

The first order condition with respect to e_0 is

$$-\lambda U'(c_2)(1-\beta)\frac{\partial a_0^H}{\partial e_0}$$

where

$$\begin{aligned} \frac{\partial a_0^H}{\partial e_0} &= -\frac{1}{e_0^2}\left[M_{-1} + (1-\delta_0)\left(B_{-1}^H + B_{-1}^B\right)\right] \quad \text{with no bailout} \\ \frac{\partial a_0^H}{\partial e_0} &= -\frac{1}{e_0^2}\left[M_{-1} + D_{-1} + (1-\delta_0)\, B_{-1}^H - L_{-1}\right] \quad \text{with bailout} \end{aligned}$$

The optimal choice of the initial exchange rate depends crucially on the interaction between private balance sheet positions and bailouts.

If $(1-\delta_0)\, B_{-1}^B + L_{-1} < D_{-1}$ bailouts are a decreasing function of the exchange rate and become zero for $e_0 > \bar{e}_0$. In this case, for a large enough exchange rate there is no bailout because banks are net debtors in domestic currency and for a large enough e_0 ex-post bank profits will be non-negative. As $M_{-1} + (1-\delta_0)\left(B_{-1}^H + B_{-1}^B\right) > 0$ it is optimal to set e_0 as large as possible. The devaluation removes the incentives to set $\delta_0 = 1$ since it makes the real value of the domestic currency government debt equal to zero. $\delta_0 = 0$.

If there is liability dollarization in the banking system. $(1-\delta_0)\, B_{-1}^B + L_{-1} > D_{-1}$, bailouts are an increasing function of the exchange rate and are positive for $e_0 > \bar{e}_0$. The optimal choice of e_0 in this case depends on whether $M_{-1} + (1-\delta_0)B_{-1}^H \lesseqgtr L_{-1} - D_{-1}$.

If $M_{-1} + (1-\delta_0)B_{-1}^H > L_{-1} - D_{-1}$, it is optimal to increase the nominal exchange rate without bound. Note that this expression will be positive when the gains from devaluing, given by $M_{-1} + (1-\delta_0)B_{-1}^H$, exceeds the bail-out required to keep the banking sector from going bankrupt, given by the net domestic currency denominated assets in the banking sector, $L_{-1} - D_{-1}$. In this case, the net fiscal effect of a devaluation is positive. As we mentioned before, in this case there is no point in using the instrument δ_0. $\delta_0 = 0$.

If $M_{-1} + (1-\delta_0)B_{-1}^H < L_{-1} - D_{-1}$ and there is liability dollarization the optimal exchange rate is $e_0 = \bar{e}_0$. For $e_0 < \bar{e}_0$ there is no bailout so $\partial a_0^H/\partial e_0 > 0$. For $e_0 > \bar{e}_0$ there is a bailout and $\partial a_0^H/\partial e_0 < 0$. The intuition for this result is that if there is no bailout the government levies a lump-sum tax by devaluing. If there is a bailout, the net fiscal effect of a devaluation in this case is negative. Furthermore, since the optimal e_0 is finite government will set $\delta_0 = 1$. As the government lowers e_0 (revalues) it improves bank profits, reducing transfers to banks. It will do so until the value of the transfers is zero. The gain from reducing bailouts exceeds the cost that arises from the increase in the real value of non-defaulted nominal government liabilities, M_{-1}. The value of $\bar{e}_0$ when $\delta_0 = \delta_0^* = 1$ is lower than one. $\bar{e}_0 < 1$.

The following table summarizes all the possible cases for the optimal choice of e_0, δ_0, and δ_0^*.

	δ_0^*	δ_0	e_0
$(1-\delta_0) B_{-1}^B + L_{-1} < D_{-1}$	1	0	∞
$(1-\delta_0) B_{-1}^B + L_{-1} > D_{-1}$ and			
$M_{-1} + (1-\delta_0) B_{-1}^H > L_{-1} - D_{-1}$	1	0	∞
$M_{-1} + (1-\delta_0) B_{-1}^H < L_{-1} - D_{-1}$	1	1	$\bar{e}_0$

.

These conditions imply the following optimal choice of e_0, δ_0, and δ_0^* for each initial balance sheets structure.

	δ_0^*	δ_0	e_0
$B_{-1}^B + L_{-1} < D_{-1}$	1	0	∞
$B_{-1}^B + L_{-1} > D_{-1}$ and			
$M_{-1} + B_{-1}^H > L_{-1} - D_{-1}$	1	0	∞
$M_{-1} < L_{-1} - D_{-1}$	1	1	$\bar{e}_0$

When there is no liability dollarization if the government is to renege on prior commitments at time $t = 0$, it is optimal for it to default on foreign currency denominated debt and implement a large devaluation. Defaulting on foreign currency denominated debt and devaluing will also be the optimal choice if there is liability dollarization, but $M_{-1} + B_{-1}^H > L_{-1} - D_{-1}$. Finally, the government will revalue if there is liability dollarization and $M_{-1} < L_{-1} - D_{-1}$.

4 Concluding Remarks

A key issue in the discussion of the benefits of dollarization is the hypothesis that in economies where there is liability dollarization in the banking sector a devaluation will cause a default on sovereign debt. If this alleged causal relationship actually exists removing the technology to devalue will reduce sovereign risk will all the benefits that this entails.

Testing whether this positive correlation between default risk and devaluation risk exists in the data is crucial when evaluating the benefits of dollarizing an economy. Uncovering this correlation in the data is difficult because expected defaults and expected devaluations are unobserved variables.

We propose a simple dynamic model of a small open economy with fixed exchange rates where the government bails out banks with negative profits and chooses devaluation and default rates on domestic and foreign currency to interpret the data.

The model implies that if $B_{-1}^B + L_{-1} < D_{-1}$ or if $B_{-1}^B + L_{-1} > D_{-1}$ and $M_{-1} + B_{-1}^H > L_{-1} - D_{-1}$ it will be optimal for the government to default on its foreign currency debt, set $\delta_0^* = 1$, and to devalue. As the devaluation takes care of the default on the domestic currency debt, government never defaults on domestic currency debt setting $\delta_0 = 0$. This configuration of balance sheets thus implies that expected devaluation rates and expect default on foreign currency debt are

$$\begin{aligned} E\varepsilon_{t+1} &= i_t - r_t \\ E\delta^*_{t+1} &= i^*_t - r_t \end{aligned}$$

In the remaining case, where balance sheets satisfy the conditions $B^B_{-1} + L_{-1} > D_{-1}$ and $M_{-1} < L_{-1} - D_{-1}$, it is optimal for the government to revalue to reduce bank bailouts and default on domestic and foreign currency debt. In this case $\delta_0 = \delta^*_0$. This configuration of balance sheets implies that expected devaluation rates and expect default on foreign currency debt are

$$\begin{aligned} E\varepsilon_{t+1} &= i_t - i^*_t \\ E\delta^*_{t+1} &= i^*_t - r_t. \end{aligned}$$