Endogenous Bank Runs

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Abstract

Models of banking panics have the unsatisfactory property that banks can not reduce the probability of facing runs by holding more liquid assets. This is inconsistent with empirical evidence that suggests that the probability is endogenously determined and negatively related to the liquidity of bank balance sheets. I develop a model consistent with the evidence.

There are important empirical implications. Liquidity should be measured by the liquidation value of total assets as a fraction of short term liabilities. Econometric results are obtained for the Argentine panic of 1995.

Three government policies are analyzed. First, under a scheme of partial deposit insurance, the probability of panics is increasing in the level of coverage. Second, the government can reduce the probability of panics and increase depositor's welfare for a constant expenditure by imposing liquidity requirements. Third, a more capital intensive financial structure reduces the probability of runs. Capital requirements reduce the probability of panics and the government expenditure when a panic episode occurs.

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1 Introduction

The inherent instability of banking systems has always been a matter of concern for policymakers. Before the great depression, the world witnessed recurrent episodes of banking panics, where depositors suddenly demanded conversion of deposits into cash at all or many banks, forcing banks to suspend convertibility. Since then, the common view is that governments must intervene to prevent or reduce the frequency of panics and a myriad of regulatory schemes have been implemented across countries. In spite of the repeated attempts to stabilize banking systems, most countries experienced episodes of banking crises whose frequency and intensity increased in the last twenty years. The most notorious episodes include the savings and loan crises in the United States, the Scandinavian banking crises, the Asian crises and the Mexican and Argentine crises. Banking crises are usually identified as episodes where either a panic occurs or bank runs lead the government to intervene in one or more financial institutions or provide assistance to a major bank. Even though governments usually intervene to stop the contagious effects of runs before they become systemic, panic episodes still occur. The experiences of Estonia in the early 1990s and Argentina in 1995 show that governments with a strong commitment to defend a currency board can be severely limited in their possibilities to act as a lender of last resort and stop the propagation of runs.

Due to these disrupting events, there is a renewed interest in understanding banking panics. The political debate usually focuses on regulation; however, as noted by Calomiris and Gorton (1991), each regulatory scheme presumes a model of the origin of panics.

The theoretical literature has focused mainly on vulnerability and run preventing policies. Factors and policies that could reduce the probability of panics but not prevent them have not been studied extensively. In particular, models of banking panics have the unsatisfactory property that banks can not reduce their exposure to runs by holding more liquid assets in their portfolios, which is inconsistent with the available empirical evidence[1].

The purpose of this paper is to develop a model where the probability of panics is endogenously determined and portfolio composition affects the probability in a sensible way. The theory is used to analyze the effects of

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[1] See for instance, White (1984), Esbitt (1986), Grossman (1994) and Anastasi et. al. (1998).

capital, partial deposit insurance and liquidity requirements on bank runs. In addition, the theoretical results provide important insights for the empirical study of banking panics.

The liquidity of assets in the banking system plays a key role. Because liquid assets are less risky than loans, the higher the fraction of liquid assets in the portfolio the lower is its variance while the sensitivity of the value of the portfolio to the arrival of news about the quality of loans is also lower.

If depositors take their decisions based at least partially on fundamentals, the more liquid is the portfolio of the banking system, the lower is the probability that bad news are bad enough to reduce the expected value of the portfolio to the level where they consider that to withdraw their deposits is the "safest" choice. Hence, the more liquid is the portfolio of assets in the banking system the lower is the probability that a banking panic occurs.

I emphasize the role that liquid assets play in the coordination problem that depositors face. In the economic environment of the model, a representative depositor would never force banks to liquidate loans. In the abscence of a coordination failure, runs would not occur. The confusion that characterizes actual episodes of banking panics is explicitly considered. To a large extent, the source of confusion is the strategic uncertainty that takes place among a large number of depositors who share identical information. They face a situation similar to a coordination game with two equilibrium points where the payoffs are determined by the liquidity of bank portfolios and the information about the quality of loans. Under one equilibrium all depositors try to withdraw their deposits while in the other all depositors wait.

The solution concept of risk dominance is appropriate to analyze these kind of situations and allows us to predict when one equilibrium or the other will be played. As Harsanyi and Selten (1988) state the spirit of their solution concept, "Risk dominance tries to capture the idea that in this state of confusion the players enter a process of expectation formation that may lead to the conclusion that in some sense one of both equilibrium points is less risky than the other".

Risk dominance is a departure from theories of bank runs that completely ignore the strategic uncertainty that takes place among depositors. In contrast to sunspot equilibrium where depositors are assumed to be able to correlate their actions perfectly, risk dominance assumes that depositors are less powerful as a group in terms of their capabilities to correlate their actions. One of the implications is that the run preventing liquidity level in the banking system is lower under risk dominance.

The paper develops a model that uses the liquidity of the portfolio of the banking system, news about the quality of loans and the criteria of risk dominance as building blocks to characterize the behavior of depositors. Bankers explicitly recognize that they can reduce the probability of facing a run by increasing the liquidity of their portfolios. In the absence of runs, banks provide liquidity insurance to depositors. However, when banks are subject to runs, depositors bear the risk of losing their deposits. By increasing the liquidity of their portfolios and reducing the probability of runs, banks are buying partial insurance against this risk on behalf of their depositors.

Cooper and Ross (1998) study the causality from an exogenous (sunspot) probability of bank runs to the liquidity of bank portfolios. In this paper, the other direction of causality is studied, the effect of liquidity on the probability of bank runs.

The model provides clear and testable implications. A gap has emerged in the literature between empirical and theoretical studies. Empirical studies usually focus on the probability that a bank run occurs, but in the models that are supposed to be tested the probability of a run is not an endogenous variable.

The structure of the model developed here is consistent with the empirical analysis of Calomiris and Gorton (1991) for the U.S. national banking era (1863-1913). They find that "the results are consistent with the view that "bad" economic news combined with the vulnerability of banks to shocks...accounts for panics". Calomiris and Gorton use the loan to reserve ratio (illiquidity of asset portfolio) to measure the exposure to risk and leading indicators of economic recessions such as the stock market price index or the liabilities of commercial failures as indicators of macroeconomic news. The evidence suggests that a sufficient predictor of panics is the simultaneous violation of some thresholds whose values are determined by the liquidity of the banking system. In particular, the more illiquid is the banking system, the lower are the changes in stock prices and commercial failures that are needed to trigger a panic.

To place the paper in the literature of bank runs, the theory developed is a natural extension of the models of Diamond and Dybvig (1983), Jacklin (1987), Wallace (1988), Cooper and Ross (1998), Chang and Velasco (2000) and Allen and Gale (1998). The main differences are that the return of the long term technology is random and runs are triggered by signals that contain fundamental information about the long term returns. The strategic uncertainty that takes place among depositors is explicitly modelled and the multiplicity of equilibria is refined using risk dominance as the equilibrium selection concept. The most important contribution is that runs are endogenous, competitive banks recognize that they can reduce the probability of facing a bank run by holding more liquid assets and the equilibrium probability is determined by the structure of the economy and the optimal behavior of banks and depositors.

Because the panics are triggered by fundamental information, the theory also shares some elements present in the information based literature of bank runs. For instance, in Gorton (1985) all depositors observe a common signal of the future return of bank assets and the violation of thresholds trigger the panics. However, in his model banks have no portfolio choice (all their assets are invested in the long term technology) and hence they can not affect the probability of facing a run. In addition, in his paper no strategic uncertainty takes place among depositors.

The model developed here has a number of applicatons. It is shown that the capital structure is relevant to determine the probability of runs. The effect of partial deposit insurance is also analyzed. It is shown that if the government can not commit to provide full deposit insurance and prevent runs, then the probability of banking panics is increasing in the level of coverage. Competitive banks maximize the expected utility of depositors by holding relatively illiquid portfolios and taking excessive risk which can be interpreted as moral hazard. Finally, the effect of liquidity requirements is analyzed. It is shown that the government can reduce the probability of panics and increase depositor's welfare for a constant expenditure, by imposing liquidity requirements on banks' balance sheets.

The Argentine panic of 1995 is analyzed empirically. According to the econometric results, the risk dominance hypothesis can not be rejected.

The rest of the paper is organized as follows. In Section 2, a model of bank runs that occur with an endogenously determined probability is developed. In Section 3 and 4, the properties of the model are analyzed and a parametric example and numerical results are presented. In this section, the comparative statics and its empirical relevance are also analyzed. The effects of government policies on depositor's welfare and the probability of banking panics are presented in Section 5. The empirical analysis is presented in Section 6. Section 7 concludes and summarizes the main results.

2 The Model

Consider an economy populated by a continuum of individuals. The whole population has measure one. There is only one consumption good and three periods. Individuals are borne at period t=0, consume either at t=1 or at t=2 and die at the end of period 2.

At t=0 all the individuals are identical and each one receives one unit of the good. At t=1 the utility function is revealed to each individual. There are two types of individuals. Type 1 (impatient) individuals consume at t=1 only and have a utility function $V(C_1)$. Type 2 (patient) individuals consume at t=2 only and have a utility function $U(C_2)$ where $V(0) = U(0) = 0; V'(\cdot) > 0; U'(\cdot) > 0; V''(\cdot) < 0; U''(\cdot) < 0$ and C_t denotes consumption in period t. Types are private information. π is the probability of being a type 1 individual and $1 - \pi$ is the probability of being a type 2 individual. Individual type distributions are identical and independent, so π and $1 - \pi$ are the fractions of the population that end up being type 1 and type 2 respectively.

The economies studied in subsections 2.1 and 2.2 are characterized by different productive technologies.

In subsection 2.1, there is a risk-free long term constant return to scale technology that returns $R > 1$ units of goods at t=2 per unit invested at t=0. If the long term investment project is liquidated at t=1, $L < 1$ units of the consumption good are obtained.

In subsection 2.2, there is a risky long term constant return to scale technology that returns $\tilde{R}$ units of goods at t=2 per unit invested at t=0. $\tilde{R}$ is a random variable that takes two possible values R_H and R_L where $R_H > R_L$ and $E(\tilde{R}) > 1$. If the investment is liquidated at t=1, $L < R_L$ units are obtained.

These technological assumptions imply that premature liquidation of long term projects is always costly in both the risk-free and risky cases.

In both cases, there are two liquid/short term constant return to scale technologies that allow individuals to store the consumption good for one period (both at t=0 and t=1).

In this economic environment, as in Diamond and Dybvig (1983), autarkic individuals bear an idiosyncratic and diversifiable risk. Hence, a banking institution that pools the endowments of many individuals can exploit the long term technology more intensively, avoid costly liquidations and improve welfare.

2.1 Risk Free Long Term Technology

Consider a decentralized version of the economy where consumers rely on banking institutions to improve their consumption possibilities. Banks compete in offering demand deposit contracts. Free entry guarantees that profits are zero and those banks that survive offer contracts that maximize the expected utility of individuals/depositors at t=0. Due to productive technologies characterized by constant returns to scale, the number and size of banks are indeterminate.

If banks are not subject to runs, the central planner allocation will be implemented, as shown by Diamond and Dybvig (1983), Jacklin (1987) and Wallace (1988). The central planner's problem for this economy is presented in Appendix 1.

Suppose instead that bank runs can occur. Think, for instance, that the government can not credibly commit to act as a lender of last resort or implement any other policy that fully stabilizes the banking system. Credibility of policy announcements and time consistency are crucial to determine a succesful implementation of such policies. Moreover, because banking crises are relatively infrequent events, it is difficult for a government to build the reputation that would enhance credibility. For simplicity, assume in this section that the government is absent and that banks hold deposits but no capital on the liability side of their balance sheets.

Denote by I the fraction of assets invested in short term projects and by $(C_1; C_2)$ the demand deposit contract. The contract states that the depositor can withdraw either C_1 at t=1 or C_2 at t=2 per unit deposit.

The sequence of events is as follows. At t=0 individuals deposit the endowment in the banking system. Banks take portfolio decisions by operating the investment technologies directly (there are no specialized productive firms) and offer demand deposit contracts to depositors. Bankers recognize that the possibility of bank runs is sensitive to their portfolio decisions, so they take into account the constraint imposed by the future behavior of patient individuals when making their investment decisions.

At t=1 depositors learn their types. Type one individuals withdraw their deposits. Type two individuals play a bank run game. After observing the level of liquid assets in the banking system, they take the action that is consistent with the risk dominant equilibrium of the game.

Denote by T_1, T_2 the sets of impatient and patient individuals respectively. Because type 1 individuals always withdraw at t=1, we focus our attention on the restricted game played by type 2 individuals only. Call G_{RF} the restricted game with a measure of $(1-\pi)$ players. Each depositor can take two possible actions, withdraw the deposit (W) at t=1 or wait (NW) until t=2, so the action space for player i is $S_i = \{W; NW\}$. Denote by $S = \times_i S_i$ the set of action profiles of all players and S_{T1}, S_{T2} the set of action profiles of players type 1 and type 2 respectively.

Denote by $x \in [\pi; 1]$ the fraction of the population withdrawing at t=1 (it includes both type 1 and type 2 individuals). The measure of type 2 individuals withdrawing their deposits at t=1 is $x - \pi$.

Denote by x^* the maximum level of withdrawals that can be serviced by the banking system without liquidating long term projects and by x^{**} the minimum level of withdrawals such that the bank resources are completely depleted. Clearly, $C_1 x^* = I$ and $C_1 x^{**} = I + L(1-I)$:

If the fraction of depositors who withdraw at t=1 x is greater than x^* then the banking system has to liquidate long term investment projects to pay back demand deposits. The banking system runs out of assets when the fraction of withdrawals is x^{**}, so if x is greater than x^{**} then no additional deposit can be paid back.

For a given level of withdrawals denote by $c(x; I; C_1; R; \pi; L)$ the resources per depositor available for payment at t=2.

$$c(x; I; C_1; R; \pi; L) = \begin{cases} \frac{R(1-I)+I-xC_1}{1-x} & x \in [\pi; x^*] \\ \frac{R[1-I+\frac{I}{L}-\frac{xC_1}{L}]}{1-x} & x \in [x^*; x^{**}] \\ 0 & x > x^{**} \end{cases}$$

Denote by $U_i(s_i; x)$ the utility level of player i, who plays a pure strategy $s_i \in S_i$ when the level of withdrawals is x.

$$U_i(s_i; x) = \begin{cases} U(C_1) & s_i = W \quad x \in [\pi; x^{**}] \\ U(c(x; I; C_1; R; \pi; L)) & s_i = NW \quad x \in [\pi; x^{**}] \\ 0 & s_i \in S_i \quad x \in (x^{**}; 1] \end{cases}$$

Definition 1 Restricted Bank Run Game (Risk free long term technology) The restricted bank run game consists of the set of all type 2 players $i \in T_2$, and for each player i, the set of actions $S_i = \{W; NW\}$ and the payoff function $U_i(s_i; x)$. Denote this game by $G_{RF} = \{i \in T_2; S_i = \{W; NW\}; U_i(s_i; x)\}$:

Denote by $s^r \in S_{T2}$ the action profile where $s_i = W$ for all $i \in T_2$ and

s^{nr} 2 S_{T2} the action profile where s_i = NW for all i 2 T_2.

Lemma 2 Multiple Equilibria The game G_{RF} has two pure strategy equilibria, s^r and s^{nr}. There is also a mixed strategy equilibrium where the fraction of the population withdrawing deposits at t=1 is $x^{mix} = \frac{1}{(1 ¡ \frac{L}{R})}[\frac{I+L(1 ¡ I)}{C_1} ¡ \frac{L}{R}]$.

The proofs of lemmas and propositions are presented in Appendix 2.

Lemma 2 states that there is a pure strategy equilibrium where no patient depositor withdraws her deposit from the bank and there is another pure strategy equilibrium where a bank run occurs and all type 2 individuals try to withdraw their deposits.

Observe that the structure of the game is characterized by sequential service. When runs occur at t=1, depositors try to excercise their claims and banks liquidate their assets to pay on a first come-first served basis. In a fractional reserve system, some depositors will be able to withdraw their deposits before the assets are completed depleted and the rest of the depositors will not be paid back.

In the mixed strategy equilibrium, each type 2 individual withdraws with probability x^{mix} ¡ ¼. Because the population is continuous and the mixed strategies are independent accross individuals, a fraction x^{mix} of the total population withdraws at t=1. Observe that the measure of individuals who withdraw is increasing in the level of liquid assets. This is contrary to the conventional wisdom which holds that a negative relation exists between liquidity and withdrawals/runs. On the other hand, in a fractional reserve system with sunspots, the strategies of depositors are perfectly correlated and runs are independent of the level of liquid assets. In contrast, the equilibrium concept defined below and used in the rest of this paper implies that a negative relation exists between liquidity and runs, which has intuitive appeal.

In order to select the equilibrium that will be played, the theory of equilibrium selection developed by Harsanyi and Selten (1988) is applied. According to this theory, the solution concept called risk dominance is used for pairwise comparisons of solution candidates (the pure strategy equilibria) and a unique solution is determined.

Definition 3 Risk dominant equilibrium (Risk-free long term technology) A risk dominant equilibrium of G_{RF} is defined as a profile of actions $s \in S_{T2}$ with the property that for every $i \in T_2$ we have that $U_i(s_i; \bar{x}) \geq U_i(s'_i; \bar{x})$, $s_i \in S_i$, $s'_i \in S_i$ and $\bar{x} = \pi + \frac{1-\pi}{2}$:

Let us use the convention that when the individual is indifferent between W and NW, then she chooses NW, so if $U_i(s_i; \bar{x}) = U_i(s'_i; \bar{x})$ then s^{nr} is the risk dominant equilibrium of G_{RF}.

We make use of the result that states that risk dominance is equivalent to $\frac{1}{2}$ dominance in games where each player can play only two strategies. In symmetric games with two strategies per player, the concept of $\frac{1}{2}$ dominance states that if all players strictly prefer the same action when their predictions are that all the other players play that action with probability $\frac{1}{2}$ then the profile where each player plays that action is the risk dominant equilibrium.

Observe that with a continuum of players it is easy to find the risk dominant equilibrium. We must find the best response of the typical type two individual when $\frac{1}{2}$ of the type 2 population withdraws ($\bar{x} = \frac{1-\pi}{2} + \pi$).

For each player, the utility of withdrawing (W) is $U(C_1)$ and the utility of waiting (NW) is $U(c(\bar{x}; I; C_1; R; \pi; L))$: If $C_1 > c(\bar{x}; I; C_1; R; \pi; L)$ then s^r is the risk dominant equilibrium (there is a bank run). If $C_1 \leq c(\bar{x}; I; C_1; R; \pi; L)$ then s^{nr} is the risk dominant equilibrium (there is no bank run).

Given C_1, define I^* as the run preventing liquidity level that satisfies $C_1 = c(\bar{x}; I^*; C_1; R; \pi; L)$. This equation implicitly defines $I^*(C_1)$: Bankers can fully stabilize the banking system and eliminate the possibility of facing a run by holding a portfolio with liquidity levels greater or equal than $I^*(C_1)$. However, in equilibrium, banks will never hold an overliquid portfolio where $I > I^*(C_1)$.

Lemma 4 $\bar{x} \in [x^*; x^{**}]$; $I^*(C_1) = -\frac{L}{1-L} + \left[\frac{\frac{L}{R} + \bar{x}(1-\frac{L}{R})}{1-L}\right]C_1$:

Proposition 5 For all $C_1 > \frac{L}{\frac{L}{R} + \bar{x}(1-\frac{L}{R}) - \pi(1-L)}$, s^r is the risk dominant equilibrium of G_{RF} iff $I \in [\pi C_1; I^*(C_1))$ and s^{nr} is the risk dominant equilibrium iff $I \geq I^*(C_1)$: For all $C_1 \leq \frac{L}{\frac{L}{R} + \bar{x}(1-\frac{L}{R}) - \pi(1-L)}$, $I \geq \pi C_1$, s^{nr} is the risk dominant equilibrium of G_{RF}.

At t=0, bankers anticipate that their choices of $(I; C_1)$ will determine the outcome of the bank run game that depositors will play in the next period. Moreover, they can forecast with certainty whether a run will occur. Of course, no individual will deposit her endowment in a bank where a run occurs with certainty, so in order to survive in the market, bankers will choose $(I; C_1)$ such that no run occurs.

Lemma 4 guarantees that $c(\bar{x}; I; C_1; R; \pi; L)$ is increasing in I and runs can be aliminated by holding high levels of liquid assets. Observe that if lemma 4 were false, $c(\bar{x}; I; C_1; R; \pi; L)$ would be decreasing in I, the relation between runs and liquid assets would be positive and runs would occur for high levels of liquidity.

Proposition 5 determines the set of contracts such that no run occurs at t=1. In Figure 1, the set of such contracts includes all the points above the line labeled $I^*(C_1)$ where $I \geq \pi C_1$.

Formally, the problem (P_1) that a bank solves is the following:

$$\underset{C_1; C_2; I; l}{\text{Max}} \ \pi V(C_1) + (1 - \pi) U(C_2)$$

s:t:

$$\pi C_1 \leq I + Ll \qquad (1)$$

$$(1 - \pi) C_2 \leq R(1 - I - l) + I + Ll - \pi C_1 \qquad (2)$$

$$I \geq I^*(C_1) \qquad (3)$$

$$C_2 \geq C_1 \qquad (4)$$

$$0 \leq l \leq 1 - I \qquad (5)$$

Constraint (1) requires that total consumption of impatient consumers at t=1 is not greater than the total short term investment plus the value of liquidated long term assets. (2) is the resource constraint at t=2; total consumption can not be greater than the total returns from the non-liquidated long term investments and short term reinvestments. (3) is the risk dominance constraint. (4) is the incentive compatibility constraint for type 2 individuals. In order for the contract to be implementable, individuals must reveal their types (remember that types are private information) and the incentive compatibility constraint guarantees that they have an incentive to reveal them truthfully.

In the optimal solution to (P_1), $l = 0$ and (2) holds as an equality. Because banks never hold an overliquid portfolio, either (1) and/or (3) bind in equilibrium.

Figure 1 depicts constraints (1), (3) and (4) along with the narrow banking constraint in the $(C_1; I)$ plane. According to narrow banking, the level of liquidity has to be high enough to pay all potential withdrawals at t=1. Formally, the narrow banking liquidity level satisfies $C_1 \leqq I + L(1 - I)$ or $I \geq -\frac{L}{1-L} + \frac{C_1}{1-L}$:

Remark 6 the risk dominance run preventing liquidity level is always below the liquidity level of narrow banking. This observation suggests that narrow banking overstates the level of liquidity that is necessary to fully stabilize the system.

Observe that bank profits are equal to zero in (P_1). In order to support the solution to (P_1) as a competitive equilibrium the optimal contract must be invariant with respect to the scale of deposits.

Denote by $\psi(C_1; C_2)$ the expected utility of the typical depositor at t=0. Consider D as the deposits in a bank. $(C_1; C_2)$ and I are payments and investment per unit deposit/asset. Similarly, $c(x; I; C_1; R; \pi; L)$ denotes the resources per depositor per unit of asset to be paid at t=2 if a fraction x of the population withdraws at t=1.

Proposition 7 if $\psi(C_1; C_2)$ is homogenous then the optimal contract that solves (P_1) is invariant with respect to the scale of deposits D.

The previous proposition states a sufficient condition for the existence of a competitive equilibrium where the number of banks and their size are indeterminate.

2.2 Risky Long Term Technology

As described before, in this subsection the productive technology is risky and depositors receive interim information about the quality of investments. The variable θ is the probability that $R = R_h$ at t=2 and is revealed to the whole population at t=1. θ plays the role of a leading indicator of the state of the economy in the last period.

As a metaphor, think of long term investments as agricultural projects and θ as the level of rain. If at t=1 the level of rain tuns out to be high then it is very likely that the long term technology will have high returns. On the other hand, if the level of rain is low it is more likely to observe a poor return at t=2.

At t=0 the distribution of the random variable μ is common knowledge. Denote by $f(\mu)$ and $F(\mu)$ the density and cumulative distribution functions of μ. Obviously, the support of these functions is $[0;1]$.

The sequence of events is as follows. At t=0 individuals receive their endowments and deposit them in the banking system. The banking system takes the investment decisions by choosing the portfolio of short term and long term investments. Banks know that depositors behave according to risk dominance principles and take into account that the portfolio choice affects the probability of facing a run at t=1. Again, competition implies that each bank chooses a balance sheet and offers a deposit contract that maximizes the expected utility of the depositors. The contract offered is denoted by $(C_1;C_{2H};C_{2L})$. According to this contract, the depositor has the option to withdraw C_1 at t=1 or C_{2H} at t=2 when no run occurs and the returns of long term investments are high or C_{2L} at t=2 when no run occurs and the returns are low.

The contract implicitly recognizes that a bank run can occur with positive probability and depositors would be serviced sequentially.

At t=1 individuals know the contracts offered and the portfolio decisions taken by the banking system. They observe their own type and the level of rain. As before, they play a bank run game. Finally, either bank runs occur if the level of rain is low enough or no bank run occurs if the level of rain is high enough.

When runs occur at t=1, depositors are paid sequentially. If there is no run type 1 depositors withdraw and type 2 depositors wait until t=2 to excercise their claims. In the last period, the returns of the long term projects are observed and distributed to patient individuals as stated in the contract.

Denote by $U_i(s_i;x;\mu)$ the interim expected utility of player i, who plays a pure strategy $s_i\ 2\ S_i$ when the level of withdrawals is x and the level of rain is μ. $x^{¤}$ and $x^{¤¤}$ are defined as in the risk free case.

$$U_i(s_i;x;\mu) = \begin{cases} U(C_1) & s_i = W \quad x\ 2\ [\text{¼};x^{¤}] \\ \mu U(c(x;I;C_1;R_H;\text{¼};L)) \\ \quad +(1\ \text{¡}\ \mu)U(c(x;I;C_1;R_L;\text{¼};L)) & s_i = NW \quad x\ 2\ [\text{¼};x^{¤¤}] \\ 0 & s_i\ 2\ S_i \quad x\ 2\ (x^{¤¤};1] \end{cases}$$

The bank run game and the risk dominant equilibrium are defined as in the risk free case where $U_i(s_i;x)$ is replaced by $U_i(s_i;x;\mu)$. Denote by $G_R = f\,i\ 2\ T_2;\ S_i = fW;NWg;\ U_i(s_i;x;\mu)g$ the restricted bank run game (risky long term technology). Furthermore, define $I^{¤}_L$ as the liquidity level that satisfies $C_1 = c(\bar{x};I;C_1;R_L;\text{¼};L)$ and $I^{¤}_H$ as the liquidity level that satisfies $C_1 = c(\bar{x}; I; C_1; R_H; \pi; L)$.

Proposition 8 In the restricted bank run game G_R,

1) for all $C_1 \geq \frac{L}{\frac{L}{R_L}+\bar{x}(1-\frac{L}{R_L})-\pi(1-L)}$, $\theta \in [0;1]$, if $I \geq I_L^*(C_1)$ then s^{nr} is the risk dominant equilibrium;

2) for all $C_1 \geq \frac{L}{\frac{L}{R_L}+\bar{x}(1-\frac{L}{R_L})-\pi(1-L)}$;

if $I_L^*(C_1) > I \geq \max\{\pi C_1; I_H^*(C_1)\}$ then there exists a threshold level $\bar{\theta}$ such that s^{nr} is the risk dominant equilibrium for all $\theta \geq \bar{\theta}$, and s^r is the risk dominant equilibrium for all $\theta < \bar{\theta}$, $\bar{\theta} \in (0;1]$;

if $I_H^*(C_1) \geq I \geq \pi C_1$ then s^r is the risk dominant equilibrium;

3) for all $C_1 < \frac{L}{\frac{L}{R_L}+\bar{x}(1-\frac{L}{R_L})-\pi(1-L)}$, if $I \geq \pi C_1$ then s^{nr} is the risk dominant equilibrium.

Corollary 9 $F(\bar{\theta})$ is the probability that a bank run occurs. The choice of I and C_1 at t=0 determines the probability that a run occurs at t=1. Hence, the probability of a bank run is endogenously determined at t=0.

Proposition 8 determines the set of contracts where a run occurs with positive probability. It states that if a bank chooses a liquidity level that is above the $I_L^*(C_1)$ line in Figure 2, then the probability of a bank run is zero and if it is below $I_H^*(C_1)$ then the probability of facing a run is 1. Observe that if $I = I_L^*(C_1)$ then $\bar{\theta} = 0$, and if $I = I_H^*(C_1)$ then $\bar{\theta} = 1$. Hence, at any point in between those two lines the probability of a run is positive.

Proposition 10 For all $\bar{\theta} \in (0;1)$ the probability that a run occurs $F(\bar{\theta})$ is decreasing in I and increasing in C_1. Moreover, given $(I; C_1)$, the probability that a run occurs $F(\bar{\theta})$ is increasing in π and decreasing in R_H, R_L and L:

The previous proposition states that banks can decrease their exposure to bank runs by holding a more liquid portfolio of assets or by reducing the promised payment to impatient depositors.

Taking as given the bankers' decisions at t=0, the higher are the returns of the long term technology and the lower is the cost of liquidation, the lower is the probability that a run occurs at t=1. The higher is the fraction of impatient individuals in the population, the higher is the probability of runs.

The problem (P_2) that a bank solves is the following:

$$\max_{C_1, C_{2H}, C_{2H}, l, I} (1 - F(\bar{\theta}))\{\pi V(C_1) + (1-\pi)[E_{\theta=\bar{\theta}<\theta<1} U(C_{2H}) + (1 - E_{\theta=\bar{\theta}<\theta<1})(U(C_{2L})]\}$$

$$+F(\bar{\theta})\,[\frac{l + L(1-l)}{C_1}][\pi V(C_1) + (1-\pi)U(C_1)]$$

$s.t.$

$$\pi C_1 \le l + LI \tag{6}$$

$$(1-\pi)C_{2H} \le R_H(1 - l - I) + l + LI - \pi C_1 \tag{7}$$

$$(1-\pi)C_{2L} \le R_L(1 - l - I) + l + LI - \pi C_1 \tag{8}$$

$$\bar{\theta} = \frac{U(C_1) - U(c(\bar{x}; l; C_1; R_L; \pi; L))}{U(c(\bar{x}; l; C_1; R_H; \pi; L)) - U(c(\bar{x}; l; C_1; R_L; \pi; L))} \tag{9}$$

$$\theta U(C_{2H}) + (1-\theta)U(C_{2L}) \ge U(C_1); \theta \ge \bar{\theta} \tag{10}$$

$$0 \le I \le 1 - l \tag{11}$$

The objective function is the expected utility of the depositor conditional on the information available at t=0, when the contract is determined.

The first term is the probability that no bank run occurs at t=1 multiplied by the expected utility obtained if there is no run.

The second term is the probability that a run occurs times the probability of a depositor arriving to the bank on time (before assets are depleted) multiplied by the utility of withdrawing C_1. Remember that in the event of a run all depositors will claim to be type 1 and those type 2 depositors who are able to withdraw will store the good for consumption at t=2. It is assumed that all individuals are equally likely to arrive to the bank before resources are completely depleted. Hence, the probability of arriving to the bank on time and withdrawing C_1 is the liquidation value of assets over total claims $(\frac{l+L(1-l)}{C_1})$. Observe that this ’liquidity ratio” is also a key determinant of $\bar{\theta}$.

Constraint (6) is the same as in (P_1). (7) and (8) are the resource constraints at t=2 for the high and low states respectively. (9) is the risk dominance constraint. (10) is the incentive compatibility constraint for type 2 individuals. Observe that the incentive compatibility constraint states that type 2 individuals do not claim to be type 1 only for those levels of rain where no run occurs ($\mu \geq \bar{\mu}$).

Given the parameter values $R_H; R_L; \pi; L$, the utility function and the distribution of the leading indicator μ, program (P_2) determines endogenously the probability of banking panics $F(\bar{\mu})$, the contract $(C_1; C_{2H}; C_{2L})$ and the asset composition I.

At the optimal solution to (P_2), $I = 0$ and constraints (7) and (8) hold as equalities. Once $(I; C_1)$ are determined, C_{2H} and C_{2L} are obtained from (7) and (8). The solution to $(I; C_1)$ in (P_2) is given by some point in the area ABCD in Figure 2, including all the border points except segment DC where a run occurs for sure. The solution could be at some point in the segment AB where the probability of a run is zero. The solution could also be at some interior point in the segment AD where (6) binds. The solution could also be interior, where the probability of a run is positive and (6) is slack. The solutions located in segment BC will be considered interior because the probability of a run is positive.

In (P_2) a bank faces the following trade-offs. The benefits of increasing liquidity are that the bank reduces the probability of facing a run and increases the probability that the depositor arrives to the bank on time to be paid back her claims when a run occurs. In addition, to hold more liquid assets is a way to insure depositors against period 2 consumption risk. More liquidity reduces the difference between C_{2H} and C_{2L}, which benefits risk averse depositors. On the other hand, holding more liquid assets is costly because the high return long term technology is operated less intensively reducing period 2 payments and the expected utility when no run occurs.

The benefit of increasing C_1 is that the bank increases the expected utility when no run occurs. However, to increase C_1 is also costly for two reasons. First, the probability that a run occurs is increasing in C_1. Second, the fraction of people whose deposits can be paid back when a run occurs is decreasing in C_1.

Intuitively, whether the probability of a run is zero or positive in equilibrium depends on the properties of the distribution of the leading indicator and the long term technology. If the variability of the leading indicator is high relative to long term returns, increasing one unit of liquid assets has a strong negative impact on the probability of runs, so the benefits of increasing liquidity are high relative to the costs and the bank could choose a run preventing liquidity level in segment AB. On the other hand, as long as the returns of the long term technology are high relative to the variability of the leading indicator (if the probability of a run falls smoothly enough as I increases), the bank will choose an interior contract where the equilibrium probability of runs is positive.

Notice that, as in the risk free case, a bank will not offer a contract above the dotted line $I = \bar{x} C_1$. When $I > \bar{x} C_1$ the relation between liquidity and probability of a run is reversed, the probability of a run becomes increasing in I. The basic trade-off is broken, so a reduction of liquidity is always beneficial, the probability of a run falls and the expected utility when no run occurs increases.

As in the risk free case, in order to support the solution to (P_2) as a competitive equilibrium, the solution must be invariant to the scale of deposits. An example of utility functions that satisfy that requirement is $V(C) = U(C) = C^{®}$: In this case, the number of banks and their size are indeterminate in the competitive equilibrium.

3 Particular Case-Uniform Distribution of the Leading Indicator

Consider the case where μ has a uniform distribution, $R_H > R_L > 1 > L$ and $V(C) = U(C)$. Denote the objective function of (P_2) by $^{a} = (1 ¡ \bar{\mu})U^{nr} + \bar{\mu} U^{r}$, where U^{nr} and U^{r} are the expected utilities if no run occurs and if a run occurs respectively.

Let us analyze the properties of an interior solution to (P_2) where neither (6) nor (10) bind and the probability of a run is positive.

First, it is interesting to compare the case where the probability or runs is endogenous (program (P_2)) with the case of a sunspot economy where the probability is constant. The question is how do the solutions differ. Second, the comparative statics is analyzed.

Most of the mathematical details are presented in Appendix 3.The subindex $\bar{\bar{\mu}}$ is used in reference to the sunspot economy to denote that the probability $\bar{\mu}$ is kept constant in the excercise.

3.1 Constant Probability of Runs

The first order conditions that are satisfied in an interior solution are the following:

$$({}^{a}{}_{I})_{\underset{\mu}{=}} = ((1 \text{ ¡ } \bar{\mu})(U_I^{nr})_{\underset{\mu}{=}} + \bar{\mu}\, U_I^{r} = 0$$

$$({}^{a}{}_{C_1})_{\underset{\mu}{=}} = ((1 \text{ ¡ } \bar{\mu})(U_{C_1}^{nr})_{\underset{\mu}{=}} + \bar{\mu}\, U_{C_1}^{r} = 0$$

From these conditions we obtain the solution $I^{sun}(R_H; R_L; ¼; L; \bar{\mu})$ and $C_1^{sun}(R_H; R_L; ¼; L; \bar{\mu})$. The other endogenous variables can be found from the binding constraints. As shown in the appendix, in an interior maximum $(U_I^{nr})_{\underset{\mu}{=}} < 0$ and $U_I^r > 0$, $U_{C_1}^r < 0$ and $(U_{C_1}^{nr})_{\underset{\mu}{=}} > 0$:

Assuming that the sufficient conditions for a maximum are satisfied, $({}^{a}{}_{II})_{\underset{\mu}{=}} < 0$, $({}^{a}{}_{C_1C_1})_{\underset{\mu}{=}} < 0$ and $({}^{a}{}_{II})_{\underset{\mu}{=}}({}^{a}{}_{C_1C_1})_{\underset{\mu}{=}} \text{ ¡ } [({}^{a}{}_{IC_1})_{\underset{\mu}{=}}]^2 > 0$. Optimally, in the sunspot economy the bank holds a higher level of liquid assets (I) and offers lower period 1 payments (C_1) than in an economy where runs are ruled out.

The comparative statics results are ambiguous. However, some conditional statements can be made. As shown in the appendix, $({}^{a}{}_{IC_1})_{\underset{\mu}{=}} < 0$: Due to risk aversion, the higher is C_1 the greater is the expected marginal utility of long term investments and period 2 consumption. Notice that this cross effect depends on the concavity of the utility function, the less concave is the utility function (the lower is $U^{00}(:)$ in absolute value) the weaker is the cross effect. In the limit, if the utility function were linear, $({}^{a}{}_{IC_1})_{\underset{\mu}{=}}$ would be 0.

Effects of changes in R_H and R_L: When R_H increases, the long term technology is more productive, so the bank has an incentive to reduce I and invest more in long term projects. However, there are two other effects that play in the opposite direction. Given I and C_1, an increase in R_H decreases the marginal utility of period 2 consumption when no run occurs, so the bank has an incentive to increase I in order to increase the utility when runs occur. On the other hand, an increase in R_H increases the difference between C_{2H} and C_{2L}, so a risk averse consumer faces a higher period 2 consumption risk that could be reduced by increasing I.

If the level of risk aversion is high enough, the last two effects dominate the first one and I^{sun} is increasing in R_H. If the degree of risk aversion is low, the first effect dominates the other two, C_1^{sun} is increasing and I^{sun} is decreasing in R_H. The analysis is similar for the case of R_L.

Effects of changes in L: an increase in L reduces the costs of liquidating long term investments during panic episodes, so it is reasonable to expect banks to hold more long term assets in their portfolios. However, this is not necessarily the case. As L increases, the marginal utility of holding liquid assets when a run occurs U_I^r increases as well. To restore the equilibrium, I must fall and/or C_1 must increase to increase $U_{C_1}^{nr}$. In principle, I and C_1 could move in the same negative direction, but what makes the result ambiguous is that $(\Psi_{IC_1})_{\theta=\bar{\theta}} < 0$: If the cross effect $(\Psi_{IC_1})_{\theta=\bar{\theta}}$ is weak enough (the level of risk aversion is low) then I falls as L increases and banks invest more in long term projects.

Effects of changes in π: as the fraction of impatient depositors increases, so does the weight of period 1 consumption in the utility function, so we expect banks to offer higher levels of C_1. As C_1 increases, both the marginal utilities of liquid assets when a run occurs and when no run occurs fall, inducing the bank to reduce I.

An additional effect exists. Given I and C_1, as π increases, the difference between C_{2H} and C_{2L} increases as well, so a banker would increase I and reduce period 2 consumption risk in order to attract risk averse depositors. In the limit case, as the utility function approaches linearity, C_1 increases as π increases, the effect on I remains ambiguous.

3.2 Endogenous Probability of Runs

The first order conditions are:

$$\Psi_I = (\Psi_I)_{\theta=\bar{\theta}} + \{(\frac{1-\pi}{2})[U(C_{2H}) - U(C_{2L})](1-\bar{\theta}) - [U^{nr} - U^r]\}\bar{\theta}_I = 0$$

$$\Psi_{C_1} = (\Psi_{C_1})_{\theta=\bar{\theta}} + \{(\frac{1-\pi}{2})[U(C_{2H}) - U(C_{2L})](1-\bar{\theta}) - [U^{nr} - U^r]\}\bar{\theta}_{C_1} = 0$$

In the first condition the second term is positive and in the second condition the second term is negative, implying that at the optimum, $(\Psi_I)_{\theta=\bar{\theta}} < 0$ and $(\Psi_{C_1})_{\theta=\bar{\theta}} > 0$:

Hence, when the probability is endogenous the bank holds more liquid assets I and offers a lower C_1 than in the case where the probability is constant.

The results from the comparison of the solutions to the bank problem when no run is possible, in the sunspot economy and when the probability is endogenous can be summarized as follows.

Consider an economy with parameters $(R_H; R_L; \pi; L)$. Suppose that $I^{cp}; C_1^{cp}$ is a solution to the central planner's problem or to a bank's problem when runs are ruled out. Suppose that $I^{sun}; C_1^{sun}$ is an interior solution to the bank's problem in a sunspot economy where the probability of runs is $\bar{\theta}$. Finally, $I^{end}; C_1^{end}$ is an interior solution to program (P_2) where the probability of runs $\bar{\theta}^{end}$ is endogenously determined.

Proposition 11 $I^{end} > I^{sun} > I^{cp}; C_1^{end} < C_1^{sun} < C_1^{cp}$: Moreover, $\bar{\theta}^{end} < \bar{\theta}$:

Summarizing the results of this section, the previous proposition ranks the solutions to the three problems in terms of asset composition and contracts offered to depositors.

When the probability is endogenous, the results of the comparative static analysis are even more ambiguous than in the sunspot economy. In this case the cross effect Ψ_{IC_1} has an ambiguous sign, so the changes in parameter values can affect the contracts and portfolios in any direction.

4 A Parametric example, Numerical Results and Comparative Statics

In the previous section, we assumed that the leading indicator θ was uniformly distributed. In this section, the effect of the variability of the leading indicator on the contracts and portfolio choices and on the probability that a run occurs is analyzed. In addition, some numerical examples and comparative statics excercises are presented.

Consider the following specifications. The utility functions of type 1 and type 2 individuals are γC^{α} and C^{α} respectively, where $0 < \alpha < 1$ and $\gamma > 1$. $\bar{\theta}$ has the following functional form:

$$\bar{\theta} = -\frac{(R_L)^{\alpha}}{(R_H)^{\alpha} - (R_L)^{\alpha}} + \frac{[(1 - \bar{x})L]^{\alpha}}{[(R_H)^{\alpha} - (R_L)^{\alpha}]} \frac{1}{[\frac{I + L(1 - I)}{C_1} - \bar{x}]^{\alpha}}$$

Furthermore,assume that θ is a truncated normal random variable[2].

Observe that under this specifications, I and C_1 affect the probability of runs only through their effect on the liquidity ratio $\frac{I + L(1-I)}{C_1}$. Moreover,

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[2] θ is distributed as a truncated normal random variable with support [0,1]. The pdf and cdf are the following:

given $(I;C_1)$, as ® increases, the depositors become less risk averse and the probability of a bank run falls (ceteris paribus).

Let us construct a suggestive numerical example that describes how the variables in the model could be matched to observable variables and how the results could be interpreted. Consider six months as the time that elapses between t=0 and t=1, and one year as the time between t=0 and t=2. Let us think of R_H as one plus the real lending rate in the banking system (if the economy is booming there is no default on loans) and set $R_H = 1:15:$ Let us think of $R_L = (1 + R_H)(1$ ¡ default rate) where the default rate is the fraction of loans that are non performing when the economy is in a recession and set a default rate of 0:2, $R_L = 0:92:$

Set ° = 1:3. This number is arbitrary. Alternatively, we could think that there is a discount of consumption at t=2 relative to consumption at t=1 equal to 0:77. However, in the context of this model the meaning of discount is not clear given that consumers have preferences for consumption in only one period.

Consider the general stock market index as a leading indicator which summarizes the available information with respect to risky asset returns. Think of ¾ as the standard deviation of the general stock market index when its detrended mean is normalized to be 0:5. Set ¾ = 0:15, so the six-month standard deviation from the mean is 30%.

Interpret C_1 as one plus the real interest rate offered on a 6-month certificate of deposit. As an approximation, we could interpret the average value between C_{2H} and C_{2L} as one plus the one year real interest rate on a CD deposit. Observe that even when the model is designed to deal with demand deposits, we could think that banks offer CD deposits and individuals have the option to roll over the deposits every six months. Interpreted in this way, a bank run occurs when nobody renews the deposits at the end of a six-month period.

$$f(\text{µ}=\text{¹};\text{¾}^2) = \frac{\text{Á}(\text{µ}=\text{¹};\text{¾}^2)}{\text{©}(1=\text{¹};\text{¾}^2)\ \text{¡}\ \text{©}(0=\text{¹};\text{¾}^2)} \quad 0\ \mathbf{5}\ \text{µ}\ \mathbf{5}\ 1$$

$$F(\text{µ}=\text{¹};\text{¾}^2) = \frac{\text{©}(\text{µ}=\text{¹};\text{¾}^2)\ \text{¡}\ \text{©}(0=\text{¹};\text{¾}^2)}{\text{©}(1=\text{¹};\text{¾}^2)\ \text{¡}\ \text{©}(0=\text{¹};\text{¾}^2)} \quad 0\ \mathbf{5}\ \text{µ}\ \mathbf{5}\ 1$$

where $\text{Á}(:=\text{¹};\text{¾}^2)$ and $\text{©}(:=\text{¹};\text{¾}^2)$ are the pdf and cdf of a normally distributed random variable with support $[\text{¡}\ 1;+\ 1]$ mean ¹ and variance ¾^2.

The expression for the conditional expectation $E_{\text{µ}=\bar{\text{µ}}<\text{µ}<1}$ is the following:

$$E_{\text{µ}=\bar{\text{µ}}<\text{µ}<1} = \text{¹} + \text{¾}\left[\frac{\text{Á}(\frac{\bar{\text{µ}}\ \text{¡}\ \text{¹}}{\text{¾}}=0;1)\ \text{¡}\ \text{Á}(\frac{1\ \text{¡}\ \text{¹}}{\text{¾}}=0;1)}{\text{©}(\frac{1\ \text{¡}\ \text{¹}}{\text{¾}}=0;1)\ \text{¡}\ \text{©}(\frac{\bar{\text{µ}}\ \text{¡}\ \text{¹}}{\text{¾}}=0;1)}\right] \quad 0\ \mathbf{5}\ \bar{\text{µ}}\ \mathbf{5}\ 1$$

In addition, set ® = 0:5. In the absense of better estimates, assume that idiosyncratic liquidation values L observed in the US are also valid for systemic episodes and set L = 0:65[3]. Finally, the government insures deposits partially and if a panic occurs, depositors receive 0:8 per unit deposit. This kind of government intervention will be explored in more detail in Section 5.

The values of the endogenous variables are the following.

Variable	l	C_1	C_{2H}	C_{2L}	$\bar{µ}$	$F(\bar{µ})$	$\frac{l+L(1¡l)}{C_1}$
Value	0:744	0:992	1:085	0:967	0:221	0:0310	0:918

The probability that a banking panic occurs is 3.19% in a six month period. If the probabilities were independent for different periods, on average, we should observe a banking crisis every 16 years. Bank runs would occur whenever the stock market falls almost two standard deviations from its detrended mean. The real interest rates offered are negative 0.8% in six months and 2.6% in one year. The liquidity ratio is 0.918, which means that if a run occurs then the system can pay back the deposits to 91.8% of the depositors and the others are partially reimbursed by the government.

Now, we analyze the sensitivity of the optimal contracts and the equilibrium probabilities of bank runs to changes in parameter values.

The qualitative results are presented in Figures 3-9. Most of the results are robust to even large changes in all the parameter values. The most robust results are those obtained for the liquidity ratio, the probability of banking panics, l and C_1, so the following analysis focuses mainly on those variables.

The effects on $\bar{µ}$ are also discussed. Except for the comparative statics with respect to ¾, $\bar{µ}$ is directly linked to the behavior of $F(\bar{µ})$. In the numerical excercise, the optimal values of C_{2L} were usually below 1, that is why as l increases, C_{2L} increases and C_{2H} falls (ceteris paribus). Observe that the optimal liquidity ratio changes monotonically with respect to the different parameters.

Effects of changes in ¾. For a given liquidity ratio, an increase in ¾ increases the probability of a bank run. Banks adjust to find the new equilibrium contract by increasing the liquidity ratio and reducing the threshold level of the leading indicator. Whether the probability of a run is greater or lower in the new equilibrium depends on whether the effect on $\bar{µ}$ is strong enough to offset the higher dispersion of the leading indicator. Eventually, when ¾ is large enough the banking system chooses a run-preventing liquidity

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[3] Berger et. al (1995).

contract and the probability of a run is zero.

Effects of changes in ¼. For a given liquidity ratio, an increase in the fraction of impatient individuals increases the threshold level $\bar{\mu}$ and the probability that a run occurs. Banks offset this effect partially by increasing their liquidity. Both I and C_1 increase because depositors weight more heavily consumption at $t=1$. In equilibrium, the probability of a bank run is increasing in the fraction of impatient depositors ¼. Observe that depositor's welfare is increasing in ¼. This result is driven by the assumption ° > 1:

Effects of changes in R_H and R_L. For a given liquidity ratio, an increase in R_H or R_L reduces $\bar{\mu}$ and the probability that a run occurs. In order to restore equilibrium, banks take advantage of the higher returns of long term projects by holding more illiquid portfolios. In equilibrium, depositor's welfare is increasing and the probability of a bank run is decreasing in R_H and R_L.

Effects of changes in L. For a given $(I;C_1)$; an increase in L increases the liquidity ratio, reduces $\bar{\mu}$ and the probability of a bank run. However, banks adjust to restore the equilibrium by reducing I and investing more in long term projects whose liquidation value is higher. In the new equilibrium, even when the liquidity ratio is higher, the direct positive effect of L on the numerator of $\bar{\mu}$ is greater than the effect of the liquidity ratio, so the probability of a bank run is increasing in L.

Effects of changes in ®. As ® increases, the concavity of the utility function falls and depositors are willing to accept riskier contracts. The liquidity ratio falls and the fraction of assets invested in long term projects increases.

For a given liquidity ratio, an increase in ® reduces $\bar{\mu}$. The falling liquidity ratio increases the probability of a bank run and the direct effect of ® reduces that probability. The equilibrium probability of panics is increasing for low levels of ® and decreasing for high levels.

5 Some Extensions of the Basic Model

In this section, we extend the basic model in two directions. First, we introduce a government who has different policy instruments to affect the outcomes of the banking system. Second, we analyze the role that equity plays in the bank run game.

5.1 Partial Deposit Insurance

I introduce partial deposit insurance to the model developed in Section 2. The government commits credibly to pay some amount $0 < \tau < C_1$ to any individual who has lost her deposit in a bank run. It is assumed that the government collects the revenue to guarantee the program by taxing some other sectors of the economy that are not considered for simplicity. We analyze the effect of this policy on the probability of runs, on the contract offered by the banking system and on the welfare of depositors. It is shown that the higher is the level of coverage τ the higher is the probability of runs, the more illiquid is the banking system and the higher is the welfare of depositors. Because the liquidity ratio is decreasing in τ the fraction of the population that can be served by the banking system in a panic is also decreasing in τ. This implies that by increasing the coverage the government has to commit funds to pay a larger amount to a larger fraction of depositors. Hence, the government expenditure when a run occurs is increasing in τ.

The objective function of the program that a bank solves at t=0 must be redefined. Denote by Ψ^τ the objective function when τ is positive and by Ψ the objective function when $\tau = 0$ (as in the basic model).

$$\Psi^\tau = \Psi + \beta\theta[1 - \frac{I + L(1-I)}{C_1}]U(\tau)$$

Observe that τ does not affect $\beta\theta$ directly. It affects the probability of runs only indirectly through its effect on bank decisions at t=0 (I and C_1).

The first order conditions for a interior solution where the probability of a run is positive are the following:

$$\Psi^\tau_I = \Psi_I - \beta\theta[\frac{1-L}{C_1}]U(\tau) + \frac{\partial \beta\theta}{\partial I}[1 - \frac{I + L(1-I)}{C_1}]U(\tau) = 0$$

$$\Psi^\tau_{C_1} = \Psi_{C_1} + \beta\theta[\frac{I + L(1-I)}{(C_1)^2}]U(\tau) + \frac{\partial \beta\theta}{\partial C_1}[1 - \frac{I + L(1-I)}{C_1}]U(\tau) = 0$$

Because $\beta\theta$ is increasing in C_1 and decreasing in I, at the optimum $\Psi_I > 0$ and $\Psi_{C_1} < 0$. As long as the sufficient conditions for a maximum are satisfied when $\tau = 0$, the objective function is concave at the interior solution, so as τ increases, I falls and C_1 increases.

From the first order conditions, $\Psi^{\tau}_{I\tau} < 0$ and $\Psi^{\tau}_{C_1\tau} > 0$. Hence, if $\Psi_{IC_1} < 0$ at an interior solution,

$$\frac{\partial I}{\partial \tau} = \frac{\Psi^{\tau}_{IC_1}\Psi^{\tau}_{C_1\tau} - \Psi^{\tau}_{C_1C_1}\Psi^{\tau}_{I\tau}}{\Psi^{\tau}_{C_1C_1}\Psi^{\tau}_{II} - (\Psi^{\tau}_{IC_1})^2} < 0; \frac{\partial C_1}{\partial \tau} = \frac{\Psi^{\tau}_{IC_1}\Psi^{\tau}_{I\tau} - \Psi^{\tau}_{C_1\tau}\Psi^{\tau}_{II}}{\Psi^{\tau}_{C_1C_1}\Psi^{\tau}_{II} - (\Psi^{\tau}_{IC_1})^2} > 0$$

As long as the objective function is concave at the optimum and the cross effect between I and C_1 is negative, an increase in government insurance coverage will induce banks to hold more illiquid portfolios and offer higher period 1 payments to depositors. Hence, an increase in partial coverage reduces the liquidity ratio and increases the probability of bank runs.

5.2 Liquidity Requirements

Suppose that there is a limited amount of funds available to rescue the system in the event that a panic occurs. Consider two policy options.

Under the first policy option, the government sets τ such that its expenditure if a panic occurs is equal to the funds available and banks choose their liquidity ratios freely. An alternative policy is to increase τ and impose a liquidity constraint such that the government expenditure is constant. It is shown that the second alternative can achieve higher welfare levels for depositors. The purpose of this excercise is to show that liquidity requirements can be justified as a regulatory tool when the government has access to a limited amount of funds to insure depositors and the amount is not high enough to provide run-preventing full deposit insurance.

The results are presented in Figure 8. The government has a fixed amount available to spend if a panic occurs equal to 0.06584. With this amount the government can set a level of $\tau = 0.8$ (or lower) and let banks choose the endogenous variables freely, while being able to keep its promises if a panic occurs. Alternatively, the government could set $\tau > 0.8$ and impose a liquidity requirement equal to the liquidity ratio shown in Figure 8.C, to guarantee that the expenditure is not greater than the amount available.

By imposing a liquidity requirement and increasing the coverage τ the government reduces the probability of panics and increases the fraction of deposits that banks can pay back if a panic occurs. Hence, the government pays a higher coverage to a lower fraction of depositors, while keeping the expenditure constant.

As we can see in Figure 8.A, the level of τ that maximizes the expected utility of depositors is 0.804. A liquidity requirement equal to 0.9181 must be imposed. Observe that by imposing this requirement the government can reduce the probability of banking panics from 3.2% to 2.7%.

In Figures 8.I and 8.J the government has access to a greater amount of funds (0.07099) and it can cover up to $\tau = 0.85$ of individual deposits, while letting banks choose the endogenous variables freely. In this excercise, it is observed that as the amount available to insure the system increases, the difference between the optimal τ (with liquidity constraint) and the maximum unconstrained τ increases as well. If the amount available to insure the system is sufficiently low, then no liquidity requirement must be imposed in order to maximize the expected utility of depositors.

5.3 Bank Equity

We analyze the effect of the capital structure on the bank run game played by depositors at t=1. It is shown that the capital structure is relevant because it affects the probability that a run occurs.

Suppose that the liability side in the balance sheets of banking institutions includes both equity (E) and demand deposits (D). Total assets are $A = E + D$. $(C_1; C_{2H}; C_{2L})$ and I are payments and investment per unit deposit. Similarly, $c(x; A; D; I; C_1; R; \pi; L)$ denotes the resources per unit of deposit to be paid at t=2 if a fraction x of the population withdraws at t=1. $x^* = \frac{A}{D}\frac{I}{C_1}$ and $x^{**} = \frac{A}{D}[\frac{I+L(1-I)}{C_1}]$.

$$c(x; A; D; I; C_1; R; \pi; L) = \begin{cases} \frac{\frac{A}{D}[R(1-I)+I] - xC_1}{1-x} & x \in [\pi; x^*] \\ \frac{RC_1}{1-x}[\frac{A}{D}(\frac{I+L(1-I)}{C_1}) - x] & x \in [x^*; x^{**}] \\ 0 & x > x^{**} \end{cases}$$

In the parametric case where $V(C) = U(C) = C^\alpha$, the threshold level of the leading indicator is the following.

$$\bar{\mu} = -\frac{(R_L)^\alpha}{(R_H)^\alpha - (R_L)^\alpha} + \frac{[(1-\bar{x})L]^\alpha}{[(R_H)^\alpha - (R_L)^\alpha]}\frac{1}{[\frac{A}{D}(\frac{I+L(1-I)}{C_1}) - \bar{x}]^\alpha}$$

Observe that the adjusted liquidity ratio is $(1 + \frac{K}{D})(\frac{I+L(1-I)}{C_1})$.

The probability that a bank run occurs is a decreasing function of that ratio. The ratio is also equal to the probability that each depositor arrives to the bank before its assets are depleted (under sequential service) given that a run occurs. It is important to observe that the ratio depends both on the capital structure and the composition of assets and they both affect directly the probability that a run occurs. This implies that the capital structure is relevant. A change in the capital structure affects the probability of a bank run and it changes the value of the banking firm. Hence, the markets would impose an optimal capital structure. Following Berger et al. (1995), we can call it the "market capital requirement". To determine the market capital requirement at t=0, the marginal cost of raising additional equity with the marginal benefits to depositors of a lower exposure to runs. The complexity of the model must be increased to solve for the decisions on capital structure at t=0. The excercise is left for future research. However, the analysis of the effect of equity at the interim period is a useful guide for the empirical analysis that follows.

6 Empirical Analysis

The model developed in this paper has clear empirical implications. The most important testable implication is the risk dominance hypothesis that states the relation between liquidity and the probability of runs. This hypothesis is related to the events that take place at t=1 in the model. The fact that liquidity is modeled as being exogenous when runs occur is a good approximation to actual panic episodes where liquidity is difficult to obtain. Typically, once the panic unfolds, the main symptom of the fierce scramble for liquidity that takes place among banks is a sharp increase in interbank interest rates.

Empirically, the risk dominance hypothesis must be tested during panic episodes where a significant number of runs take place. Unfortunately, the more frequent episodes of banking crises where governments intervene to prevent contagion would not be satisfactory to test the hypothesis.

As pointed out before, several studies of banking panics have estimated equations where the probability of bank failure is a function of some measure of liquidity, usually using logit/probit econometric techniques. However, due to the lack of a theoretical framework, the approach has been heuristic so far. The risk dominance approach developed in this paper provides a well defined functional form, hypothesis about the value of the parameters to be estimated and a complete set of variables that should be included in the regression analysis (subject to the simplifying assumptions of the model). The limits of the model must be carefully considered while testing the risk dominance hypothesis. For instance, some banks could be too big to fail, so the theory does not apply for those banks.

Suppose that during a panic episode, a cross section of banks indexed by i are analyzed. Define $y_i = 1$ if there is a run on bank i and $y_i = 0$ otherwise. Define the dummy variable $d_i = 1$ if bank i is too big to fail and $d_i = 0$ otherwise. Denote $y_i^{¤} = ¡(µ ¡ \bar{µ}_i)$ where $\bar{µ}_i = ¡\frac{R_l}{R_H ¡ R_L} + \bar{R}_i$; $\bar{R}_i = \frac{L_i(1 ¡ \bar{X}_i)}{¢_i ¡ \bar{X}_i}$. Observe that this is the simplest formulation where depositors are assumed to be risk neutral and the variable $\bar{R}_i$ is observed for each bank.

The model developed in previous sections states that

if $y_i^{¤} > 0$ and $d_i = 0$ then $y_i = 1$

if $y_i^{¤} > 0$ and $d_i = 1$ then $y_i = 0$

if $y_i^{¤}$ **5** 0 then $y_i = 0$

To obtain the empirical model, we must add the random disturbance u_i and redefine $y_i^{¤}$ as

$y_i^{¤} = ¡(µ + \frac{R_l}{R_H ¡ R_L}) + \bar{R}_i + °d_i + u_i$

Hence, the model to be estimated is the following:

$y_i^{¤} = ® + ¯\,\bar{R}_i + °d_i + ±^{0}z_i + u_i$

$y_i^{¤}$ is unobservable, but we observe the dummy variable y_i

$y_i = 1$ if $y_i^{¤} > 0$

$y_i = 0$ otherwise.

To test theoretical completeness z_i includes other variables that should be omitted according to the model. For instance, we would like to know whether the size of a bank is empirically relevant to affect the outcome of the coordination game[4].

In order to decide whether the evidence is consistent with the risk dominance hypothesis we must test whether $® < 0$; $¯ = 1$ and $± = 0$. Observe that the constant term ® captures the common shock that affects the risky assets of all the banks included in the sample.

The second implication of the model is the relation between liquidity and the structural parameters of the economy. The model suggests that the parameters of the distributions of risky asset returns as well as proxies for government policies such as partial deposit insurance must be included as regressors to explain liquidity. Empirically, the relation can be estimated during non-panic periods. Observe that this second equation is a good complement to the VAR methodology used by bankers and regulators. The VAR approach requires the estimation of parameters of risky asset returns that

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[4] For a model of bank runs where the size of banks is relevant, see Temzelides (1997).

can be used as regressors in this equation. The equation links a managerial decision (portfolio composition) to the basic inputs of risk analysis. Thus, while the VAR approach measures risk taking, this equation adds a theory of a banking firm's best response to measures of risk.

In the rest of this section I concentrate on the estimation of the risk dominance equation and the analysis of banking panic episodes.

6.1 Measurement or $\bar{R}_i$ and Econometric Results

How do we actually measure $\bar{R}_i$? We must extrapolate from the simple three period, two asset model to actual data that involves many types of assets, liabilities and maturities. What follows is a proposed methodology.

Suppose that we observe J categories of liabilities indexed by j and classified according to their contractual maturities, where j stands for the time that elapses from the date when the contract is signed up until the maturity date (in number of periods). Denote $D_1; D_2; :::; D_J$ the values of the corresponding liabilities. We also observe K categories of assets indexed by k and classified according to their liquidation value per unit of asset (L_k). These values correspond to liquidation within T periods; hence, the dependent variable in the regression is the probability that a run occurs within the next T periods. For instance, if $T = 3$ and each period is one month, then L_k is the value that can be obtained by liquidating one unit of asset k within the next three months. Denote $A_1; A_2; :::; A_K$ the values of the different types of assets whose corresponding liquidation values are $L_1; L_2; :::; L_K$, where A_1 is cash, $L_1 = 1$ and $L_K < L_{K\,¡1} < ::: < L_3 < L_2 < L_1$. We can estimate the components of $\bar{R}_i$ as follows (the index i is omitted).

$$\text{¢} = \frac{\sum_{k=1}^{K} A_k L_k}{\sum_{j\quad T} D_j + \sum_{j>T} (\frac{T}{j}) D_j} \qquad L = \frac{\sum_{k=2}^{K} A_k L_k}{A\ ¡\ A_1} \qquad \bar{X} = \frac{1}{2D}[\sum_{j\quad T} D_j + \sum_{j>T} (\frac{T}{j}) D_j]$$

$$A = \sum_{k=1}^{K} A_k \qquad D = \sum_{j=1}^{J} D_j$$

¢ is estimated as the liquidation value of assets as a fraction of the total liabilities that mature within the next T periods. L is a weighted average of the liquidation values per unit of asset of all the asset categories except cash.

Notice that an important difference between the model and actual data is that banks lose deposits through time in the model even when no run occurs. In reality, banks face withdrawals and receive new deposits in every period. A reasonable benchmark for non-panic periods is that there is no net withdrawals over time. Hence, the application of risk dominance is consistent with 1/2 of the maturing liabilities not being rollovered within the next T periods. $\bar{X}$ measures the fraction of liabilities that are not rollovered according to risk dominance. Because D_j denotes the liabilities that mature in j periods from the contractual date, it is assumed that their maturities are evenly distributed over time, so $\frac{D_j}{j}$ mature in the current period and $\frac{T}{j}D_j$ mature in the next T periods.

The methodology is applied to the Argentine banking panic of 1995. Bad news hit the Argentine financial markets in the second half of December 1994 when Mexico devalued its currency. Argentine asset prices fell dramatically and several banks lost deposits during the first two months of 1995. However, the crises only became systemic in March 1995, when almost all banks lost deposits. The magnitude of the crises is revealed by the fact that the system lost 18% of total deposits from December 1994 to May 1995. For a more detailed description of the panic and the regulations, see Calomiris and Powell (2000).

The variables were constructed using balance sheet information of November 1994, right before the devaluation of the Mexican Peso. The sample includes 168 banks.

Ideally, we would have a detailed description of the maturity structure of liabilities and good estimates of the liquidation values per unit of asset (L_k). However, this information is not publicly available. I assigned reasonable values for the different types of assets and liabilities. The values are presented in Table 1. It must be remarked that the experience of bankers and regulators would be very useful to assign better liquidation values and test the risk dominance hypothesis more precisely.

I assigned the value $y_i = 1$ (failure) to those banks that closed or were acquired during 1995. A total of 49 failures were recorded in the sample. Failure was almost always a direct consequence of a large loss of deposits during the panic episode of March. T was set equal to three months and one period is one month.

Besides the constant term and $\bar{R}_i$, the variable total assets was included to explore whether the model is missing something related to the size of banking institutions. Dummy variables for public, foreign and cooperative banks were also included in the analysis. Finally, a dummy variable for too big to fail banks was constructed assigning the value 1 to those banks with total assets greater than one billion pesos/US dollars (17 banks).

The logit/probit regressions are presented in Table 2.

The evidence is not inconsistent with the main elements of the model. The results indicate that $\bar{R}_i$ is a significant variable to explain the probability of failure and the constant term is significant and negative. Moreover, the estimated coefficient of $\bar{R}_i$ is not significantly different from 1. Furthermore, the dummy variable for too big to fail banks is not significant to explain failure.

On the other hand, total assets is a significant explanatory variable. According to the model the scale of the banking institution should not affect the probability of run once $\bar{R}_i$ is taken into account. However, the regressions indicate that size matters. I interpret these results as an indication that size matters not because of a too big to fail effect but because of a possible effect of size on the coordination game that is not adequately captured in the theoretical model.

The dummy variable for cooperative banks is also significant. The other dummy variables for public and foreign banks were not found significant once $\bar{R}_i$ and total assets were included as regressors.

With respect to the goodness of fit, the probit analysis predicts correctly 81% of the actual cases. The results are presented in Table 3.

7 Summary and Conclusion

A model of bank runs consistent with the main elements of the empirical evidence available is developed. Inefficient bank runs are triggered by leading indicators that contain fundamental information about the quality of loans. By holding more liquid portfolios, banks can reduce the probability of facing a run because liquid assets are less sensitive than loans to the arrival of information. Hence, the probability that a run occurs is endogenously determined.

The solution to the coordination problem and the strategic uncertainty among depositors that characterize bank runs is modeled with the equilibrium selection concept of risk dominance. Depositors take the action that is considered as less risky. The application of this equilibrium concept results in a negative relation between liquid assets and bank run probability, as opposed to mixed strategy and sunspot equilibria. According to risk dominance the run-preventing liquidity levels are lower than those implied by narrow banking.

The theoretical results have clear and testable implications. The risk dominance approach provides a functional form, hypothesis about the value of the parameters to be estimated and a complete set of variables that should be included in the regression analysis. The degree of exposure to panics should be measured by the "liquidity ratio", the liquidation value of total assets as a fraction of demand deposits and short term liabilities. The volatility of leading indicators can be used to measure the quality of the information observed by depositors. The quality of the signal that depositors observe is important because it implies that fluctuations have to be much larger in a very volatile economy than in a more stable one in order to trigger a banking panic. The results also suggest that in those countries where the fraction of deposits that are rolled over every period is high (average maturity is low), the probability of panics should also be high. Other variables such as the level of interest rates, the easiness with which loans can be liquidated and the loan default rates during recessions are important variables. The risk dominance hypothesis is tested for the Argentine banking panic of 1995 and, according to the econometric results, the hypothesis can not be rejected. However, contrary to the implications of the model, the empirical excercise suggests that bank size is a significant variable to explain failure during a panic episode.

With respect to banking policies and regulation, this paper focuses on the probability that runs occur as opposed to vulnerability. The literature has focused on run-preventing policies such as the existence of a lender of last resort, full deposit insurance and suspension of convertibility. The basic framework presented in this paper is flexible enough to tackle a much larger set of policies that could potentially reduce the probability of panics, but not prevent them. Examples are partial deposit insurance, liquidity and capital requirements. It is shown that under a scheme of partial deposit insurance, the probability of panics is increasing in the level of coverage and competitive banks take excessive risk by holding relatively illiquid portfolios. It is also shown that the government can reduce the probability of panics and increase depositor's welfare for a constant expenditure by imposing liquidity requirements on banks' balance sheets. The effect of capital is also studied. For a banking firm, the probability of facing a run depends on the composition of assets and the capital structure. A more capital intensive financial structure reduces the exposure to runs. By imposing binding capital requirements, the government can reduce the probability of panics and the expenditure when a panic episode occurs.

Appendix 1-Central Planner's Problem

$\text{Max}_{l,\, C_1,\, C_2;\, I} \quad \pi V(C_1) + (1 - \pi)U(C_2) \quad \text{s.t.}$

$$\pi C_1 \leq I + Ll \tag{12}$$

$$(1 - \pi)C_2 \leq R(1 - I - l) + I + Ll - \pi C_1 \tag{13}$$

$$C_1 \leq C_2 \tag{14}$$

$$0 \leq l \leq 1 - I \tag{15}$$

Constraint (12) implies that total consumption of impatient consumers at t=1 is not greater than the total short term investment plus the value of liquidated long term assets. (13) is the resource constraint at t=2; total consumption can not be greater than total returns from non-liquidated long term investments and short term reinvestments. (14) is the incentive compatibility constraint for type 2 individuals. Observe that the central planner has to ask the individuals to reveal their types. (15) says that liquidation can not be greater than total long term investments. In the optimal solution there is no liquidation of long term assets, $l^* = 0$. (12) holds as an equality. The central planner holds just enough liquid assets to pay normal and predictable withdrawals. (13) also holds as an equality. The optimal solution (C_1^*, C_2^*) satisfies $R = \frac{V'(C_1^*)}{U'(C_2^*)}$. In the particular case where $U(.) = V(.)$ then $0 < C_1^* < C_2^*$ so (14) holds with strict inequality and type 2 individuals are better off than type 1 individuals (ex-post).

Appendix 2-Proofs

Proof-Lemma 2. Mixed Strategy Equilibrium

Define x^{mix} as the fraction of the population withdrawing deposits at t=1 such that type 2 depositors are indifferent between playing W or NW. x^{mix} satisfies $C_1 = c(x^{mix}, I, C_1, R, \pi, L)$.

$x^{mix} \in [x^*, x^{**}]$. Suppose that $x^{mix} \in [\pi, x^*]$. This implies that x^{mix} is indeterminate and the equality is satisfied only if $C_1 = R(1 - I) + I$. However, because individual types are private information and the return of the long term technology is greater than 1, banks will offer contracts where $C_2 > C_1$, which implies $R(1 - I) + I > C_1$.

$C_1 = \frac{R}{(1 - x^{mix})}[1 - I + \frac{I}{L} - \frac{x^{mix}C_1}{L}]$; which can be written as $(1 - x^{mix})\frac{L}{R} + x^{mix} = \frac{I + I(1 - I)}{C_1} > \frac{L}{R}$, $x^{mix} = \frac{1}{(1 - \frac{L}{R})}[\frac{I + L(1 - I)}{C_1} - \frac{L}{R}]$.

Pure Strategy Equilibria

Because $c(x; I; C_1; R; ¼; L)$ is decreasing in x, for all $x > x^{mix}$, $C_1 > c(x; I; C_1; R; ¼; L)$ so every type 2 depositor plays W and s^r is the pure strategy equilibrium of the game. Otherwise, for $x < x^{mix}$, $C_1 < c(x; I; C_1; R; ¼; L)$, every type 2 depositor plays NW and s^{nr} is the pure strategy equilibrium of the game. ■

Proof-Lemma 4. Suppose that it is false. Suppose that $\frac{I}{C_1} = x^{¤} > \bar{x}$: In Figure 1, the points that satisfy that condition are above the dotted line $I = \bar{x}\, C_1$.

Observe that any contract above the line $C_2 = C_1$ is subject to a run because it is not incentive compatible. Remember that individual types are private information and if $C_2 < C_1$, all type 2 depositors would pretend to be type 1 and would withdraw at t=1. No bank will ever offer such contracts.

It is not optimal for a bank to offer an incentive compatible contract above the dotted line either. For a given C_1, the bank could reduce the level of liquid assets and still avoid runs. Hence, a bank will try to exploit the high return long term technology more intensively offering an incentive compatible contract with the mnimum level of liquid assets such that no run occurs. All those contracts are below the dotted line and satisfy $\frac{I}{C_1} = x^{¤} < \bar{x}$, $\bar{x} 2 [x^{¤}; x^{¤¤}]$: ■

Proof-Proposition 5. Observe in Figure 1 that the line $I^{¤}(C_1)$ intersects the line $I = ¼C_1$ at point A where $C_1 = \frac{L}{\frac{L}{R} + \bar{x}(1 ¡ \frac{L}{R}) ¡ ¼(1 ¡ L)}$. From lemma 4, $c(x; I; C_1; R; ¼; L)$ is increasing in I, $\frac{@c(x;I;C_1;R;¼;L)}{@I} = \frac{¡R[1 ¡ \frac{1}{L}]}{1 ¡ \bar{x}} > 0$. Hence, if $I \;¸\; I^{¤}(C_1)$ then $U_i(W; \bar{x}) = U(C_1)\ \mathbf{5}\ U(c(x; I; C_1; R; ¼; L)) = U_i(NW; \bar{x})$ for all $i\ 2\ T_2$. Hence, each type 2 individual waits and s^{nr} is the risk dominant equilibrium of G_{RF}. If $I < I^{¤}(C_1)$ then $U_i(W; \bar{x}) = U(C_1) > U(c(x; I; C_1; R; ¼; L)) = U_i(NW; \bar{x})$ for all $i\ 2\ T_2$, each type 2 individual withdraws her deposit and s^r is the risk dominant equilibrium of G_{RF}. ■

Proof-Proposition 7. The result is immediate once it is observed that all the redefined constraints are invariant to the scale of deposits D, because all the inequalities are multiplied by D on both sides. If the utility function is homogeneous of degree m then $À(DC_1; DC_2) = D^m À(C_1; C_2)$. Hence, the solution $C_1; C_2; I; I$ that solves (P_1) also solves a similar maximization problem where the objective function is $À(DC_1; DC_2)$. ■

Proof-Proposition 8. (Figure 2) As it is explained in the main text, a bank will not offer a contract where $I > \bar{x}\, C_1$, so both $c(\bar{x}; I; C_1; R_H; ¼; L)$ and $c(\bar{x}; I; C_1; R_L; ¼; L)$ are increasing in I. Obviously, $c(\bar{x}; I; C_1; R_H; ¼; L) >$

$c(\bar{x}; I; C_1; R_L; ¼; L)$:

1) If I ¸ $I^¤_L(C_1)$ then $c(\bar{x}; I; C_1; R_L; ¼; L)$ ¸ C_1. For those liquidity levels, it is always convenient to play NW for any type 2 depositor. Hence, s^{nr} is the risk dominant equilibrium for any level of rain and no run occurs.

2) At any point between the lines $I^¤_L$ and $I^¤_H$ or between $I^¤_L$ and $¼C_1$, $c(\bar{x}; I; C_1; R_H; ¼; L)$ ¸ $C_1 > c(\bar{x}; I; C_1; R_L; ¼; L)$: There exists a $\bar{µ}$ such that $\bar{µ}\, U(c(\bar{x}; I; C_1; R_H; ¼; L)) + (1 ¡ \bar{µ})U(c(\bar{x}; I; C_1; R_L; ¼; L)) = U(C_1)$, $\bar{µ}2\ (0; 1]$: Hence, $\bar{µ} = \frac{U(C_1) ¡ U(c(\bar{x};I;C_1;R_L;¼;L))}{U(c(\bar{x};I;C_1;R_H;¼;L)) ¡ U(c(\bar{x};I;C_1;R_L;¼;L))}$: If $µ$ ¸ $\bar{µ}$ then $U_i(NW; \bar{x}; µ) = µU(c(\bar{x}; I; C_1; R_H; ¼; L)) + (1 ¡ µ)U(c(\bar{x}; I; C_1; R_L; ¼; L))$ ¸ $U(C_1)$ and each type 2 individual plays NW, s^{nr} is the risk dominant equilibrium If $µ < \bar{µ}$ then $U_i(NW; \bar{x}; µ) < U(C_1)$ each player plays W and s^r is the risk dominant equilibrium.

3) For all $C_1 < \frac{L}{\frac{1}{R_L} + \bar{x}(1 ¡ \frac{1}{R_L}) ¡ ¼(1 ¡ L)}$, I ¸ $¼C_1$, it is also true that $I > I^¤_L$ so s^{nr} is the risk dominant equilibrium. ■

Proof-Proposition 10. $\bar{µ}$ satis…es $\bar{µ}\, U(c(\bar{x}; I; C_1; R_H; ¼; L)) + (1 ¡ \bar{µ})U(c(\bar{x}; I; C_1; R_L; ¼; L)) = U(C_1)$

Di¤erentiate and rearrange to obtain

$$\frac{@\bar{µ}}{@I} = ¡ \frac{½ \frac{@c(\bar{x};I;C_1;R_H;¼;L)}{@I}\bar{µ}U^0(c(\bar{x};I;C_1;R_H;¼;L)) + \frac{@c(\bar{x};I;C_1;R_L;¼;L)}{@I}(1 ¡ \bar{µ})U^0(c(\bar{x};I;C_1;R_L;¼;L)) ¾}{U(c(\bar{x};I;C_1;R_H;¼;L)) ¡ U(c(\bar{x};I;C_1;R_L;¼;L))}.$$

From the de…nition of $c(\bar{x}; I; C_1; R; ¼; L)$; $\frac{@c(\bar{x};I;C_1;R;¼;L)}{@I} = R(\frac{1}{L} ¡ 1)(\frac{1}{1 ¡ \bar{x}}) > 0.$

Furthermore, $c(\bar{x}; I; C_1; R_H; ¼; L) > c(\bar{x}; I; C_1; R_L; ¼; L)$. Hence, $\frac{@\bar{µ}}{@I} < 0$:

Similarly, $$\frac{@\bar{µ}}{@C_1} = ¡ \frac{½ \frac{@c(\bar{x};I;C_1;R_H;¼;L)}{@C_1}\bar{µ}U^0(c(\bar{x};I;C_1;R_H;¼;L)) + \frac{@c(\bar{x};I;C_1;R_L;¼;L)}{@C_1}(1 ¡ \bar{µ})U^0(c(\bar{x};I;C_1;R_L;¼;L)) ¾}{U(c(\bar{x};I;C_1;R_H;¼;L)) ¡ U(c(\bar{x};I;C_1;R_L;¼;L))}; \frac{@c(\bar{x}}{}$$

$¡\frac{\bar{x}R}{L(1 ¡ \bar{x})} < 0$. Hence, $\frac{@\bar{µ}}{@C_1} > 0$:

Similarly, with respect to $¼$, R_H, R_L and L

$$\frac{@\bar{µ}}{@¼} = ¡ \frac{½ \frac{@c(\bar{x};I;C_1;R_H;¼;L)}{@\bar{x}}\bar{µ}U^0(c(\bar{x};I;C_1;R_H;¼;L)) + \frac{@c(\bar{x};I;C_1;R_L;¼;L)}{@\bar{x}}(1 ¡ \bar{µ})U^0(c(\bar{x};I;C_1;R_L;¼;L)) ¾ \frac{@\bar{x}}{@¼}}{U(c(\bar{x};I;C_1;R_H;¼;L)) ¡ U(c(\bar{x};I;C_1;R_L;¼;L))}; \frac{@c(\bar{x};I;C_1;R;¼;L}{@\bar{x}}$$

$\frac{C_1R}{L(1 ¡ \bar{x})^2}[\frac{I + L(1 ¡ I)}{C_1} ¡ 1] < 0$, $\frac{@\bar{x}}{@¼} = \frac{1}{2}$: Hence, $\frac{@\bar{µ}}{@¼} > 0$:

$$\frac{@\bar{µ}}{@R_H} = ¡ \frac{\bar{µ}U^0(c(\bar{x};I;C_1;R_H;¼;L))\frac{@c(\bar{x};I;C_1;R_L;¼;L)}{@R_H}}{U(c(\bar{x};I;C_1;R_H;¼;L)) ¡ U(c(\bar{x};I;C_1;R_L;¼;L))}; \frac{@\bar{µ}}{@R_L} = ¡ \frac{\bar{µ}U^0(c(\bar{x};I;C_1;R_L;¼;L))\frac{@c(\bar{x};I;C_1;R_L;¼;L)}{@R_L}}{U(c(\bar{x};I;C_1;R_H;¼;L)) ¡ U(c(\bar{x};I;C_1;R_L;¼;L))}; \frac{@c(\bar{x};I;C}{@}$$

$\frac{1 ¡ I + \frac{I}{C_1} ¡ \frac{\bar{x}C_1}{L}}{1 ¡ \bar{x}} > 0$. Hence, $\frac{@\bar{µ}}{@R_H} < 0$, $\frac{@\bar{µ}}{@R_L} < 0$:

$$\frac{@\bar{µ}}{@L} = ¡ \frac{½ \frac{@c(\bar{x};I;C_1;R_H;¼;L)}{@L}\bar{µ}U^0(c(\bar{x};I;C_1;R_H;¼;L)) + \frac{@c(\bar{x};I;C_1;R_L;¼;L)}{@L}(1 ¡ \bar{µ})U^0(c(\bar{x};I;C_1;R_L;¼;L)) ¾}{U(c(\bar{x};I;C_1;R_H;¼;L)) ¡ U(c(\bar{x};I;C_1;R_L;¼;L))}; \frac{@c(\bar{x};I;C_1;R;¼;L)}{@L} =$$

$-\frac{R(l-\bar{x}C_1)}{(1-\bar{x})L^2} > 0$. Hence, $\frac{\partial\bar{\theta}}{\partial L} < 0.$ ■

Appendix 3-Particular Case-Uniform Distribution of the Leading Indicator

Comparative Statics-Formulas. $U^{nr} = \pi U(C_1)+(1-\pi)(\frac{1+\bar{\theta}}{2})U(C_{2H})+(1-\pi)(\frac{1-\bar{\theta}}{2})U(C_{2L})$;

$U^r = \frac{l+L(1-l)}{C_1}U(C_1)$. The general formula for the comparative statics excercises is the following. The derivative of the endogenous variables l and C_1 with respect to some parameter y, where the objective function is Ω, is given by: $\frac{\partial l}{\partial y} = \frac{\Omega_{lC_1}\Omega_{C_1y}-\Omega_{C_1C_1}\Omega_{ly}}{\Omega_{ll}\Omega_{C_1C_1}-(\Omega_{C_1l})^2}$; $\frac{\partial C_1}{\partial y} = \frac{\Omega_{lC_1}\Omega_{ly}-\Omega_{ll}\Omega_{C_1y}}{\Omega_{ll}\Omega_{C_1C_1}-(\Omega_{C_1l})^2}$

Constant Probability of Run

$(U_l^{nr})_{\bar{\theta}} = -(R_H-1)(\frac{1+\bar{\theta}}{2})U'(C_{2H})-(R_L-1)(\frac{1-\bar{\theta}}{2})U'(C_{2L}) < 0; U_l^r = (\frac{1-L}{C_1})U(C_1) > 0$. From strict concavity of the utility function and $U(0) = 0$ we know that $U'(0)C_1 > U(C_1) > U'(C_1)C_1; U^r_{C_1} = [l+L(1-l)][\frac{U'(C_1)C_1-U(C_1)}{C_1^2}] < 0.$ Hence, from the first order conditions,

$(U^{nr}_{C_1})_{\bar{\theta}} = \pi U'(C_1)-\pi(\frac{1+\bar{\theta}}{2})U'(C_{2H})-\pi(\frac{1-\bar{\theta}}{2})U'(C_{2L}) > 0.$

$(\Omega_{lC_1})_{\bar{\theta}} = (1-\bar{\theta})(U^{nr}_{lC_1})_{\bar{\theta}}+\bar{\theta}U^r_{lC_1}$

$(U^{nr}_{lC_1})_{\bar{\theta}} = (\frac{\pi}{1-\pi})[(R_H-1)(\frac{1+\bar{\theta}}{2})U''(C_{2H})+(R_L-1)(\frac{1-\bar{\theta}}{2})U''(C_{2L})] < 0; (U^r_{lC_1})_{\bar{\theta}} = (1-L)[\frac{U'(C_1)C_1-U(C_1)}{C_1^2}] < 0.$ Hence, $(\Omega_{lC_1})_{\bar{\theta}} < 0.$ Observe that if $U''(.) = 0$ then $(U^{nr}_{lC_1})_{\bar{\theta}} = 0; U^r_{lC_1} = 0; (\Omega_{lC_1})_{\bar{\theta}} = 0.$

Effects of Changes in R_H and R_L

$(\Omega_{lR_H})_{\bar{\theta}} = (1-\bar{\theta})(U^{nr}_{lR_H})_{\bar{\theta}}+\bar{\theta}U^r_{lR_H}$

$(U^{nr}_{lR_H})_{\bar{\theta}} = -(R_H-1)(\frac{1+\bar{\theta}}{2})(\frac{1-l}{1-\pi})U''(C_{2H})-(\frac{1+\bar{\theta}}{2})U'(C_{2H}); U^r_{lR_H} = 0. (\Omega_{lR_H})_{\bar{\theta}}$ could be either positive or negative. If $U''(C_{2H})$ is low (in absolute value) then $(\Omega_{lR_H})_{\bar{\theta}} < 0.$

$(\Omega_{C_1R_H})_{\bar{\theta}} = (1-\bar{\theta})(U^{nr}_{C_1R_H})_{\bar{\theta}}+\bar{\theta}U^r_{C_1R_H}$

$(U^{nr}_{C_1R_H})_{\bar{\theta}} = -(\frac{1+\bar{\theta}}{2})\pi U''(C_{2H})(\frac{1-l}{1-\pi}) > 0; U^r_{C_1R_H} = 0$ Hence, $(\Omega_{C_1R_H})_{\bar{\theta}} > 0.$ Observe that if $U''(.) = 0$ then $(U^{nr}_{lR_H})_{\bar{\theta}} < 0; (\Omega_{lR_H})_{\bar{\theta}} < 0; (U^{nr}_{C_1R_H})_{\bar{\theta}} = (\Omega_{C_1R_H})_{\bar{\theta}} = 0.$ Similarly for R_L.

Effects of Changes in L

$(\Omega_{lL})_{\bar{\theta}} = (1-\bar{\theta})(U^{nr}_{lL})_{\bar{\theta}}+\bar{\theta}U^r_{lL}$

$(U^{nr}_{IL})_{\bar{\theta}} = 0, U^{r}_{IL} = -\frac{U(C_1)}{C_1} < 0.$ Hence, $(\Upsilon_{IL})_{\bar{\theta}} < 0.$

$(\Upsilon_{C_1L})_{\bar{\theta}} = (1-\bar{\theta})(U^{nr}_{C_1L})_{\bar{\theta}} + \bar{\theta}\, U^{r}_{C_1L}$

$(U^{nr}_{C_1L})_{\bar{\theta}} = 0, U^{r}_{C_1L} = (1-I)[\frac{U'(C_1)C_1 - U(C_1)}{C_1}] < 0.$ Hence, $(\Upsilon_{C_1L})_{\bar{\theta}} < 0.$ Observe that if $U''(.) = 0$ then $(\Upsilon_{IL})_{\bar{\theta}} < 0, (\Upsilon_{C_1L})_{\bar{\theta}} = 0.$

Effects of Changes in π

$(\Upsilon_{I\pi})_{\bar{\theta}} = (1-\bar{\theta})(U^{nr}_{I\pi})_{\bar{\theta}} + \bar{\theta}\, U^{r}_{I\pi}$

$(U^{nr}_{I\pi})_{\bar{\theta}} = -(R_H - 1)(\frac{1+\bar{\theta}}{2})U''(C_{2H})\frac{\partial C_{2H}}{\partial \pi} - (R_L - 1)(\frac{1+\bar{\theta}}{2})U''(C_{2L})\frac{\partial C_{2L}}{\partial \pi}, U^{r}_{I\pi} = 0. \frac{\partial C_{2H}}{\partial \pi} > 0$ always. However, $\frac{\partial C_{2L}}{\partial \pi}$ could be either positive or negative. If both C_{2H} and C_{2L} are increasing in π then $(\Upsilon_{I\pi})_{\bar{\theta}} > 0.$

$(\Upsilon_{C_1\pi})_{\bar{\theta}} = (1-\bar{\theta})(U^{nr}_{C_1\pi})_{\bar{\theta}} + \bar{\theta}\, U^{r}_{C_1\pi}$

$(U^{nr}_{C_1\pi})_{\bar{\theta}} = U'(C_1) - (\frac{1+\bar{\theta}}{2})U'(C_{2H}) - (\frac{1-\bar{\theta}}{2})U'(C_{2L}) - [(\frac{1+\bar{\theta}}{2})U''(C_{2H})\frac{\partial C_{2H}}{\partial \pi} + (\frac{1-\bar{\theta}}{2})U''(C_{2L})\frac{\partial C_{2L}}{\partial \pi}].$ Again, if both C_{2H} and C_{2L} are increasing in π then $(\Upsilon_{C_1\pi})_{\bar{\theta}} > 0.$ Observe that if $U''(.) = 0$ then $(U^{nr}_{I\pi})_{\bar{\theta}} = 0$ and $(\Upsilon_{C_1\pi})_{\bar{\theta}} > 0.$ ■

Proof-Proposition 11. The result is obtained from the first order conditions presented in subsections 3.1 and 3.2 and from the assumption that the objective functions are concave at the interior solutions. ■

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Figure 1
Risk Free Long Term Technology and Risk Dominance

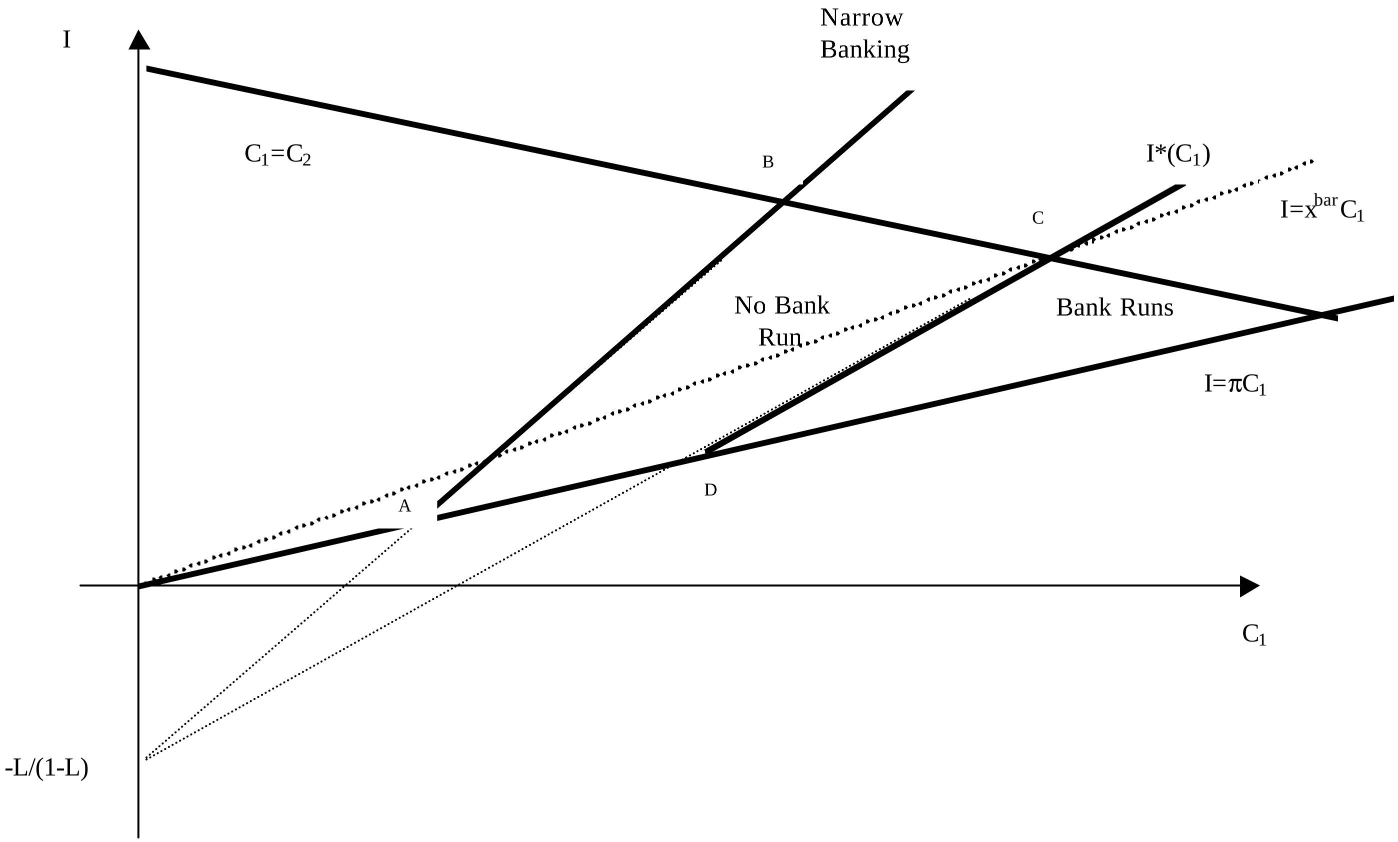


Figure 2
Risky Long Term Technology and Risk Dominance

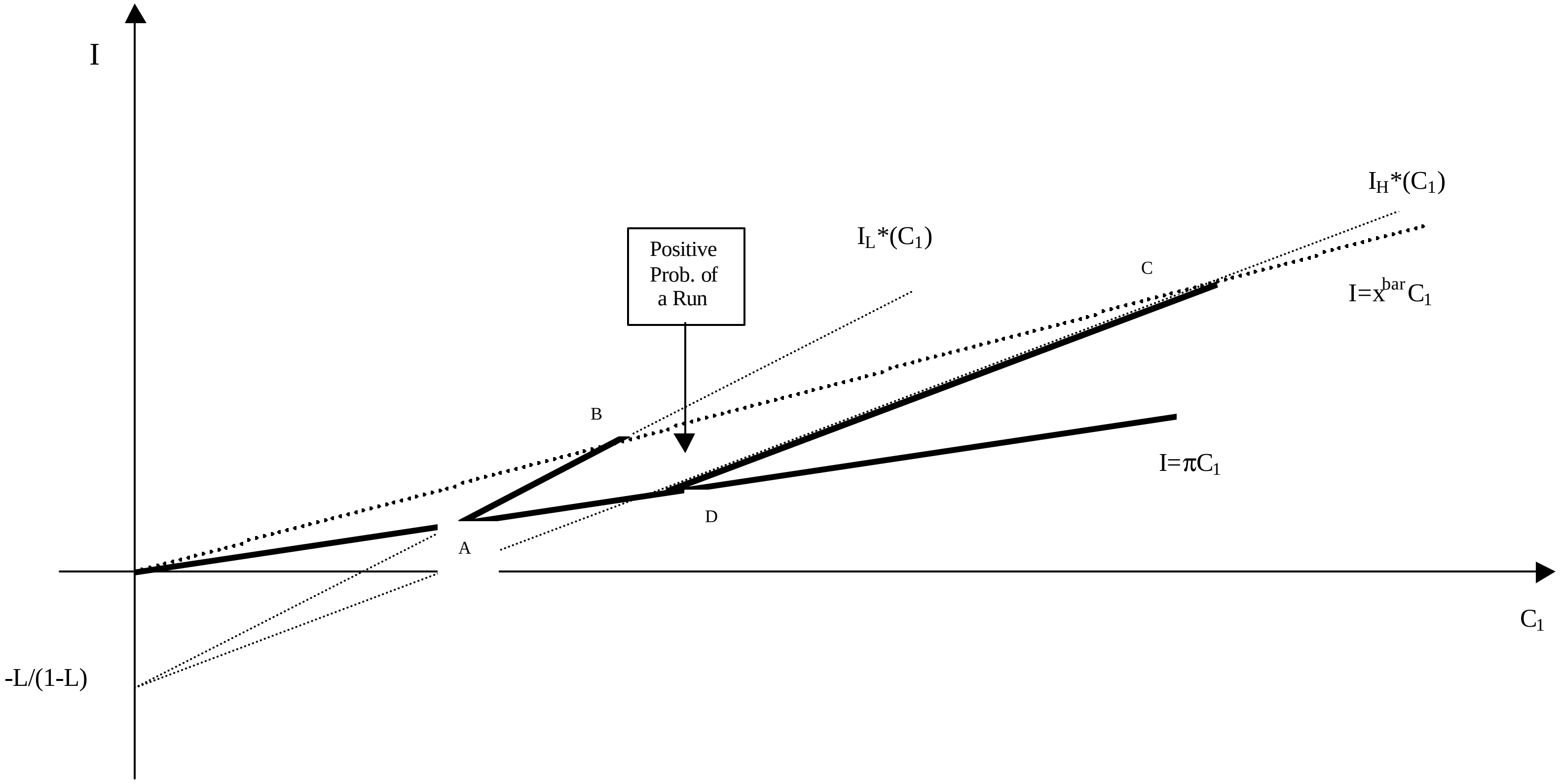
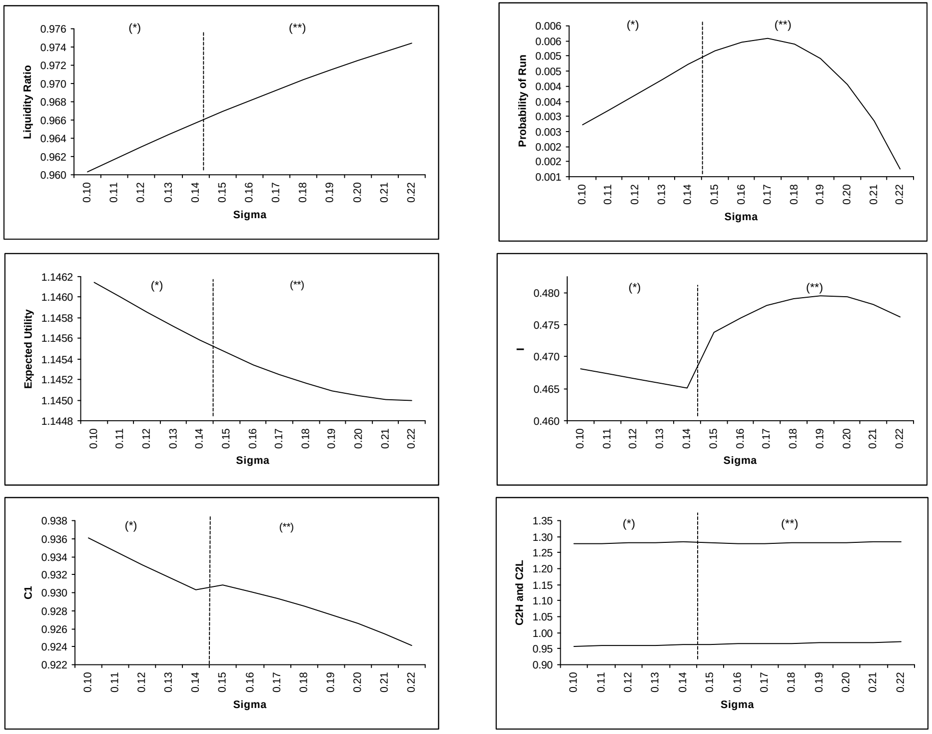


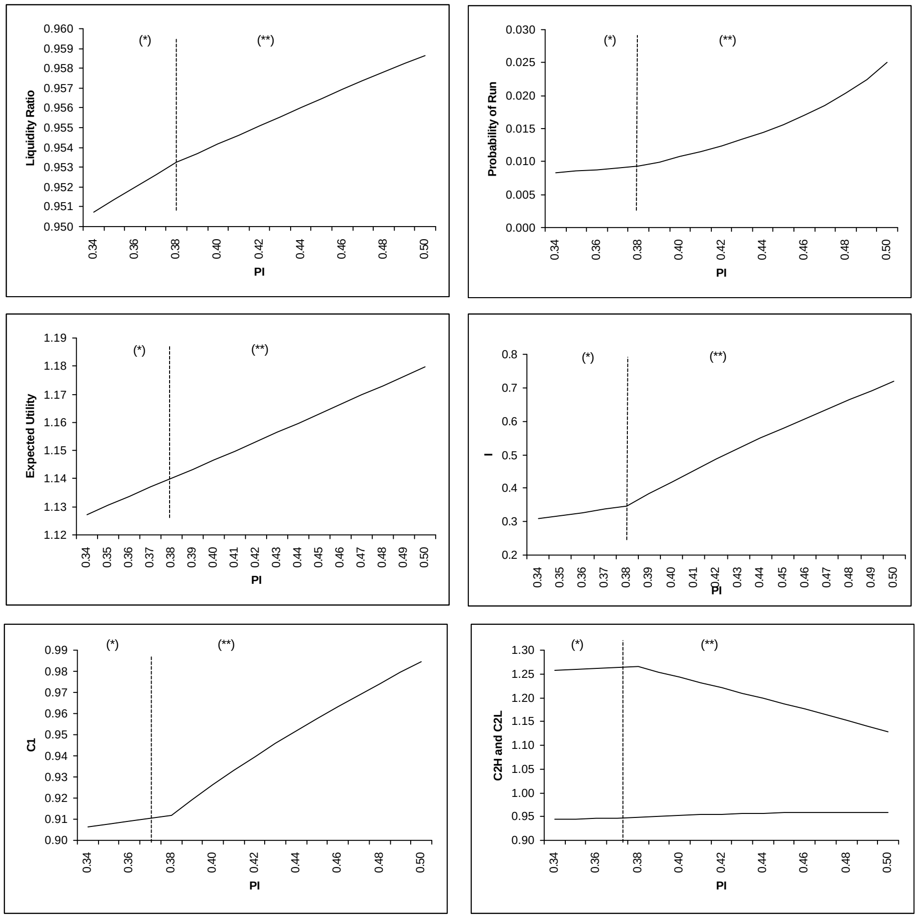
Figure 3-Comparative Statics-Effects of an Increase in s



Parameter Values:
RH=1.2 RL=0.9 L=0.81 α=0.5 γ=1.28 π=0.5 τ=0

(*) The short term resource constraint is binding.
(**) Neither the short term nor the incentive compatibility constraint is binding.

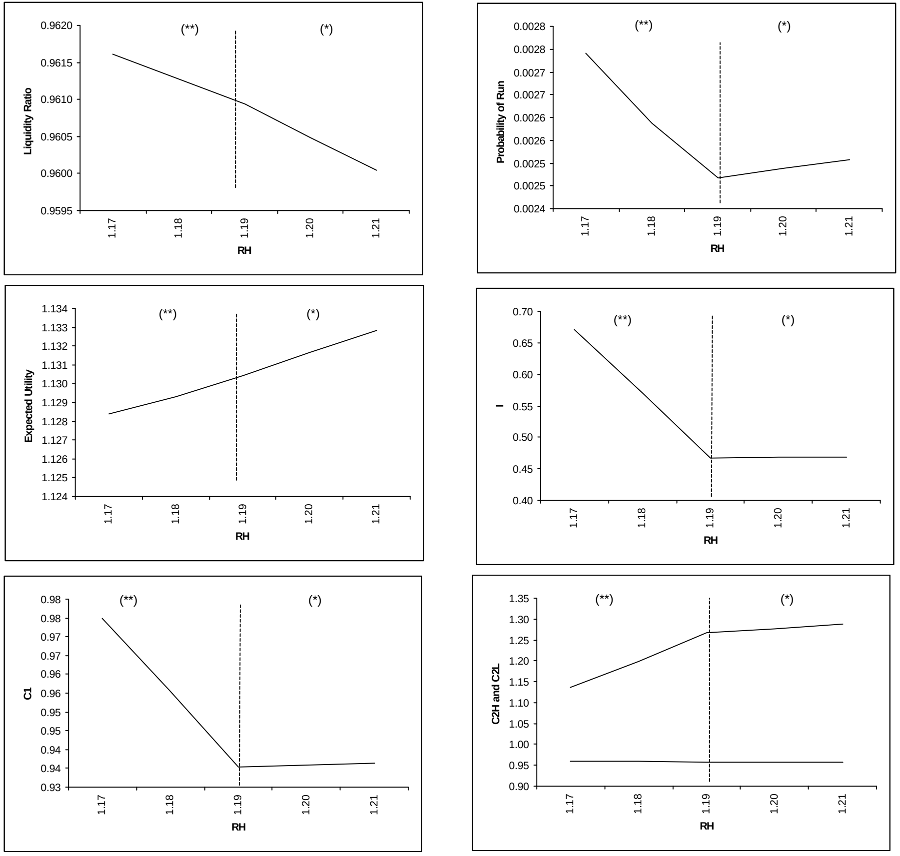
Figure 4-Comparative Statics-Effects of an Increase in p



Parameter Values:
RH=1.2 RL=0.9 L=0.8 α=0.5 γ=1.35 σ=0.15 τ=0.5

(*) The short term resource constraint is binding.
(**) Neither the short term nor the incentive compatibility constraint is binding.

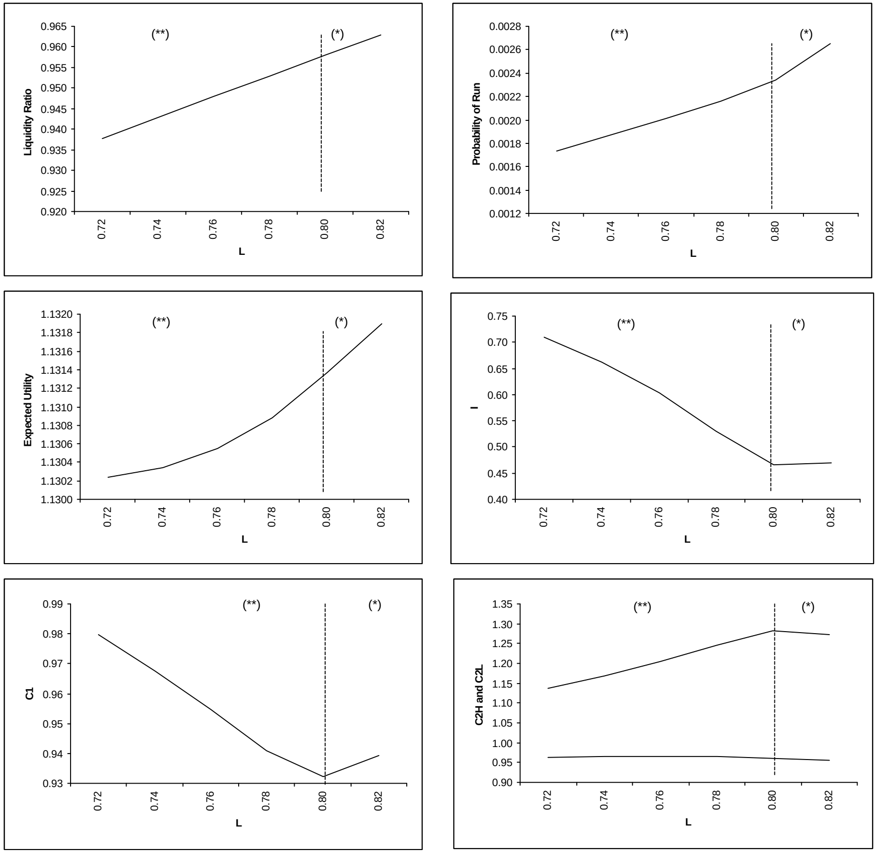
Figure 5-Comparative Statics-Effects of an Increase in RH



Parameter Values:
RL=0.9 π=0.5 α=0.5 L=0.81 γ=1.25 σ=0.1 τ=0

(*) The short term resource constraint is binding.
(**) Neither the short term nor the incentive compatibility constraint is binding.

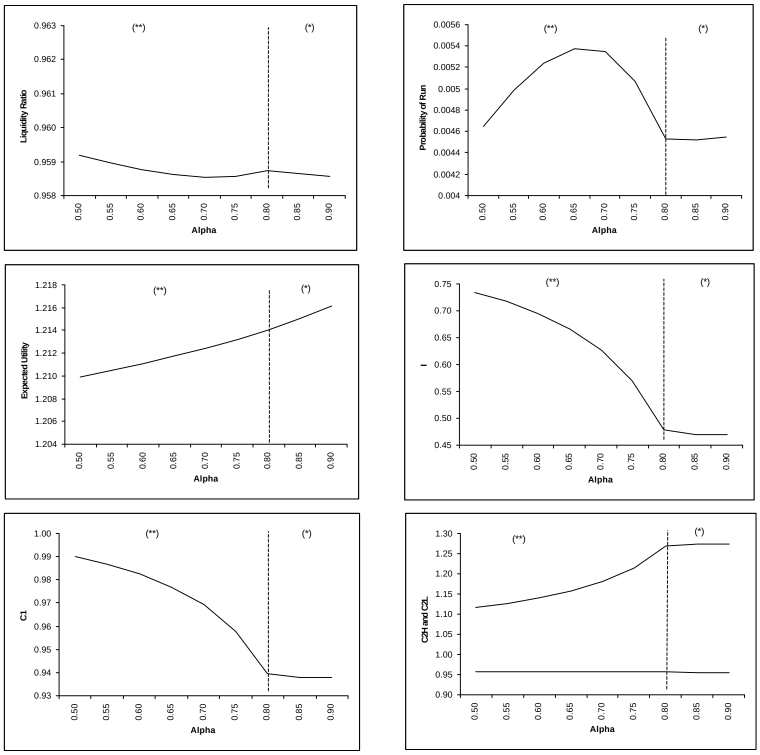
Figure 6-Comparative Statics-Effects of an Increase in L



Parameter Values:
RH=1.2 RL=0.9 π=0.5 α=0.5 γ=1.25 σ=0.1 τ=0

(*) The short term resource constraint is binding.
(**) Neither the short term nor the incentive compatibility constraint is binding.

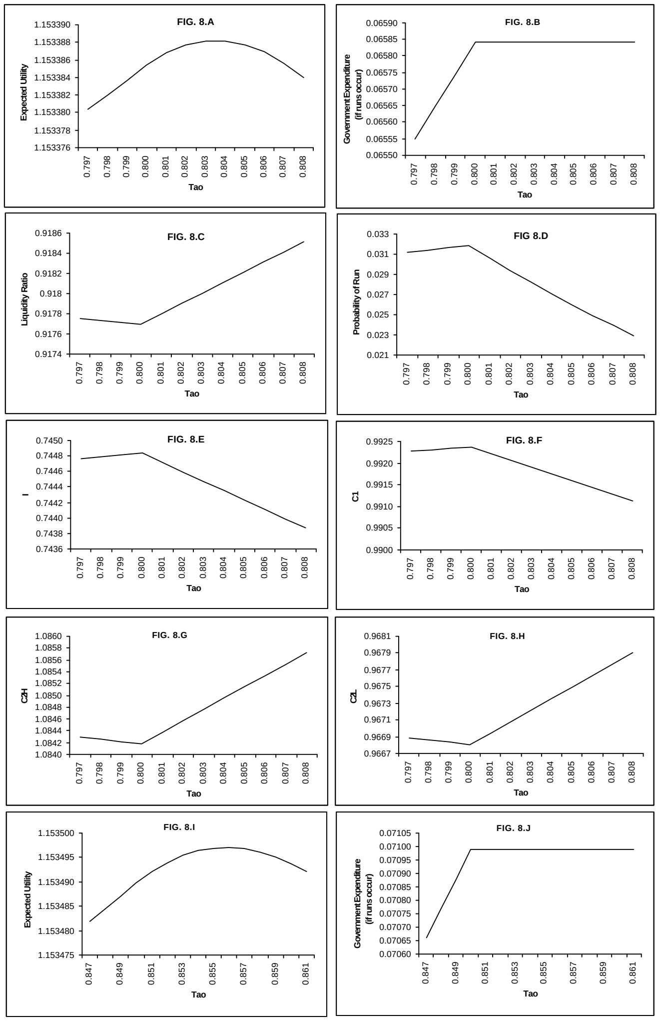
Figure 7-Comparative Statics-Effects of an Increase in a



Parameter Values:
RH=1.2 RL=0.9 π=0.5 L=0.81 γ=1.41 σ=0.1 τ=0

(*) The short term resource constraint is binding.
(**) Neither the short term nor the incentive compatibility constraint is binding.

Figure 8-Liquidity Requirements



Parameter Values: Rh=1.5 RL=0.92 π=0.5 α=0.5 L=0.65 γ=1.3 σ=0.15

Table 1
Liquidation Values per Unit of Asset (L_k) and Liability Contractual Maturity (j)

L_k (3 month period)	Asset	j (months)	Liability
1	-Cash	1	-Demand (checking) Deposits -Saving Deposits
0.9	-Bonds	3	-Certificates of Deposit
0.7	-Mortgages and Other Loans with Collateral	4	-Other Deposits
0.6	-Loans (excluded Mortgages and Other Loans with Collateral) -Other Types of Credit	12	-Other Debts
0.3	-Fixed and Intangible Assets	24	-Subordinated Debt

Source: Banco Central de la Republica Argentina.

Notes: ASSETS: Cash=Disponibilidades; Bonds=Titulos Publicos; Loans=Prestamos; Mortgages and Other Loans with Collateral=Prestamos Hipotecarios y Prendarios; Other Types of Credit=Otros Creditos por Intermediacion Financiera, Bienes en Locacion Financiera, Participaciones en Otras Sociedades y Creditos Diversos; Fixed and Intangible Assets=Bienes de Uso, Bienes Diversos, Bienes Intangibles y Partidas Pendientes de Imputacion. LIABILITIES: Demand Deposits= Depositos en Cuenta Corriente; Saving Deposits=Depositos en Caja de Ahorro; Certificates of Deposit=Depositos a Plazo Fijo; Other Deposits=Depositos en Dinero-Tasa Fija-, Depositos en Dinero-Tasa Variable-, Depositos en Titulos Publicos; Depositos en Titulos Privados, Otros Depositos, Intereses Devengados y Diferencias de Cotizacion a Pagar; Other Debts=Otras Obligaciones por Intermediacion Financiera, Obligaciones Diversas, Previsiones, Partidas Pendientes de Imputacion; Subordinated Debt=Obligaciones Subordinadas.

Table 2
Regression Analysis-PROBIT-Dependent Variable y

Variable	Regression 1	Regression 2	Regression 3	Regression 4
Constant	-1.405518 (0.338628) (**)	-2.026559 (0.382949) (**)	-1.299439 (0.375672) (**)	-1.475535 (0.335743) (**)
R	1.174485 (0.536578) (*)	1.999314 (0.596736) (**)	1.644164 (0.555676) (**)	1.926640 (0.513938) (**)
Assets	-2.90E-07 (1.29E-07) (*)		-3.84E-07 (1.37E-07) (**)	-3.61E-07 (1.38E-07) (**)
TBTF		-7.192629 (1765216)		
Cooperative	1.554442 (0.291148) (**)	1.491054 (0.289431) (**)		
Foreign			-0.334706 (0.354177)	
Public				-0.352087 (0.306561)
Log likelihood	-69.33435	-67.53042	-82.00630	-82.14339
Restr. Log likelihood	-101.4111	-101.4111	-101.4111	-101.4111
Likelihood Ratio Index	0.316304	0.334092	0.191348	0.189996
Likelihood Ratio Statistic (3 df)	64.15341	67.76127	38.80951	38.53534
Probability	7.62E-14	1.29E-14	1.90E-08	2.18E-08
Total Observations	168	168	168	168
Observations y=0	119	119	119	119
Observations y=1	49	49	49	49

Note: (**) Significant at 1% significance level (2-tail test); (*) Significant at 5% significance level (2-tail test). The other variables are not significant at 10% significance level (2-tail test).

Table 3
Prediction-Probit Model (Regression 1)

		Predicted		
		$y_i=0$	**$y_i=1$**	**Total**
Actual	**$y_i=0$**	103	16	119
	$y_i=1$	16	33	49
	Total	119	49	168
Correct Predictions/Total Observations 0.81				

Note: Predicted values are obtained as follows. If the predicted probability is greater or equal than 0.29 then the predicted y is equal to 1, otherwise it is 0. The threshold value 0.29 was chosen so that the frequencies of both types of error are equal.