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Demand Estimation in the Context of Bundled Goods: An Application to the Costa Rican Tourism Sector

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Abstract

A wide variety of goods and services purchased in the market today are offered in packages or bundles, each containing several types of goods. In general, the cost of the package is known to the researcher, but the cost corresponding to each particular good in the package is unknown. This is especially true when the costs of the packages are non-additive. This means that the cost of a package containing one unit of good A and one unit of good B is not equal to the cost of a package containing 1 unit of good A only plus the cost of another containing 1 unit of good B.

Using traditional methods, the estimation of the demand for goods sold in this fashion is difficult because any way of distributing the total fixed cost among the goods contained in the package would be arbitrary. Parametric demand models require having the price of the good (independent of other goods purchased) as one of the explanatory variables. Discrete choice models, as traditionally applied, not only require knowing the price of the good that the individual chooses, but also require that the individual only buy one of the goods available.

We use a multinomial logit model treating the bundle as the object of choice, and taking the quantities and main features of each of the goods comprised in the bundle as attributes of this alternative. Hence, packages containing different goods or different quantities of the same goods are taken as different alternatives. This eliminates the need to allocate the joint costs over multiple goods.

The potentially intractable size of the choice set is reduced by using stated preference data from a survey in which each individual was given to choose from a small number of alternative packages. Each individual's choice set was randomly drawn from a universal set containing a large number of possible alternatives based on the characteristics (type and variance) of bundles offered in the current market.

We apply this framework to the case of demand for recreational sites. The goods to be included in the package are recreational sites, and an entire trip constitutes what we have called a bundle. The quantity of each good is the length of time spent at each site. Each package has a fixed cost component which is the access (or travel) cost associated to the trip. This cost is likely to be subadditive especially if sites share a common route. We classify the sites in seven main types: volcanoes, beaches, forests, rivers, golf resorts, islands, and fishing sites.

Using this framework, we focus on three questions: 1) How do constraints on trip length affect the demand and willingness to pay for different site types? 2) How do changes in costs associated to visiting a site type affect the demand for other types of sites? Which site types are closer substitutes for each other? 3) Is there a preference for variety in the type of sites visited over a trip?

We find that beaches and forests are close substitutes for each other, and so are rivers and volcanoes too. As a result of the non-separability and the decreasing marginal utility of time spent at each site type, we also find that there is a high WTP for site variety over a single trip.

1.0 Introduction

A wide variety of goods and services purchased in the market today are offered in packages or bundles, each containing several types of goods. In general, the cost of the package is known to the researcher, but the cost corresponding to each particular good in the package is unknown. This is especially true when the costs of the packages are non-additive. This means that the cost of a package containing one unit of good A and one unit of good B is not equal to the cost of a package containing 1 unit of good A only plus the cost of another containing 1 unit of good B. Bundles of goods usually have a fixed cost component associated to the combination of goods included, and a variable cost component associated to the number of units of each good. It is in the fixed component that non-additivity typically takes place. Examples of such bundled goods include telephone services, vacation-tour packages, and banking and satellite-media services.

Using traditional methods, the estimation of the demand for goods sold in this fashion is difficult because any way of distributing the total fixed cost among the goods contained in the package would be arbitrary. Parametric demand models require having the price of the good (independent of other goods purchased) as one of the explanatory variables, together with the prices of substitutes and complements. Discrete choice models, as traditionally applied, not only require knowing the price of the good that the individual chooses, but also require that the individual only buy one of the goods available. This is obviously a very strong assumption, especially in the case of goods that are offered in packages.

In this paper, we develop and apply a practical method for estimating the demand for bundles of goods and for particular goods contained within a bundle. Our modeling approach allows us to describe the way in which the utility individuals derive from a good varies, with the type and quantities of other goods consumed in the bundle. We illustrate our modeling approach using data on demand for tour packages that contain multiple sites in Costa Rica in 1998.

We use a multinomial logit model treating the bundle as the object of choice, and taking the quantities of each of the goods comprised in the bundle, as well as the main features of these goods as attributes of this alternative. Hence, packages containing different goods or different quantities of the same goods are taken as different alternatives. This eliminates the need to allocate the joint costs over multiple goods. It also allows us to employ a likelihood function that has a multi-dimensional probability integral with a closed form.

Defining each alternative in this way raises difficulties in terms of the large number of packages that the individual can choose from, each one corresponding to a different combination of goods included and quantities of each good. We reduce the potentially intractable size of the choice set by using stated preference data from a survey in which each individual was given to choose from a small number of alternative packages. Each individual's choice set was randomly drawn from a universal set containing a large number of possible alternatives. Choice sets vary from individual to individual and the alternatives differ from each other in the sites included, their price, and the length of stay at each site. We construct the universal choice set based on the characteristics (type and variance) of bundles offered in the current market equilibrium.

This ensures that the choice set is known, feasible and designed as statistically efficiently as possible.

Since the prices of the alternatives in the choice set are randomly determined, the stated preference data set solves a second difficulty that arises in the context of demand estimation in general: the endogeneity of the price variable. Real prices may be correlated with unobservable features of each site. The stated preference survey allows us to use a set of prices that is independent of site unobservables.

In the empirical section, we apply this framework to the case of demand for recreational sites. The goods to be included in the package are recreational sites, and an entire trip constitutes what we have called a bundle. The quantity of each good is the length of time spent at each site. Each package has a fixed cost component which is the access (or travel) cost associated to the trip. This cost jumps every time a new site is added in the trip and is likely to be subadditive if sites share a common route, for example. The lodging costs constitute a variable cost associated to the length of stay at each site. Each site is characterized by one main feature that determines the site type. A package is characterized by the number of days spent at each site type, and by the associated cost. In this application, we have seven site types: volcanoes, beaches, forests, rivers, golf resorts, fishing sites, and islands.

Using this framework, we focus on three questions: 1) How do constraints on trip length affect the demand and willingness to pay for different site types? 2) How do changes in costs associated to visiting a site type affect the demand for other types of sites? Which site types are closer substitutes for each other? 3) Is there a preference for variety in the type of sites visited over a trip?

The answers to these questions have important policy implications for recreational site managers, tour package wholesalers and retailers, and a country's resource managers. The answer to the first and second question provides relevant information for recreational site managers. The first one provides information on the proportion of tourists on long trips that they might be receiving at a recreational site. This is important because the on-site services demanded by tourists are likely to depend on how long they stay at the site. The second one helps managers predict the effect on their business of changes in the prices of other recreational sites in the country.

Tour operators and country resource managers are likely to benefit from the answers to the first and third questions since they give information on the relative value of different trip characteristics to different types of visitors. In particular, the optimal combination of sites for a tour operator to offer in a package is likely to be different in short and longer trips. Resource managers will find the willingness to pay for each site type useful in identifying socially profitable investments in public recreational sites. Finally, the relative preference for multiple-site trips can guide strategies for promoting the country as a whole as a tourist destination. A strong preference for visits to multiple types of sites implies that a large variety of site types and relatively small transportation costs are important assets for the country as a recreational destination.

We answer the first question by simulating an individual's choices with and without a constraint on trip length. We find that the relative demand and willingness to pay for different site types is strongly dependent on trip length constraints. In particular, beaches and forests are the favorite sites for individuals with no binding constraints on trip length. In contrast, when constrained to a five day trip, rivers and volcanoes become the most popular sites.

To answer the seond question, we begin by testing whether the utility function is separable in the consumption of different site types. This gives us an idea of the degree to which the utility from consuming one of the goods depends on the other goods consumed even if the travel costs were additive. Next, we simulate a 1% increase in the lodging prices at all sites of a type and look at the effect that this has on the probability of visitation and expected length of stay at other sites.

We find that the utility function is not separable. Changes in the lodging price of sites of a given type affect both, the expected length of stay and the probability of visitation of other types of sites. The effect on the demand for different site types varies considerably, meaning that some sites are closer substitutes than others. Assuming no constraints on trip length, beaches and forests appear as strong substitutes for one another, as is the case too for rivers and volcanoes.

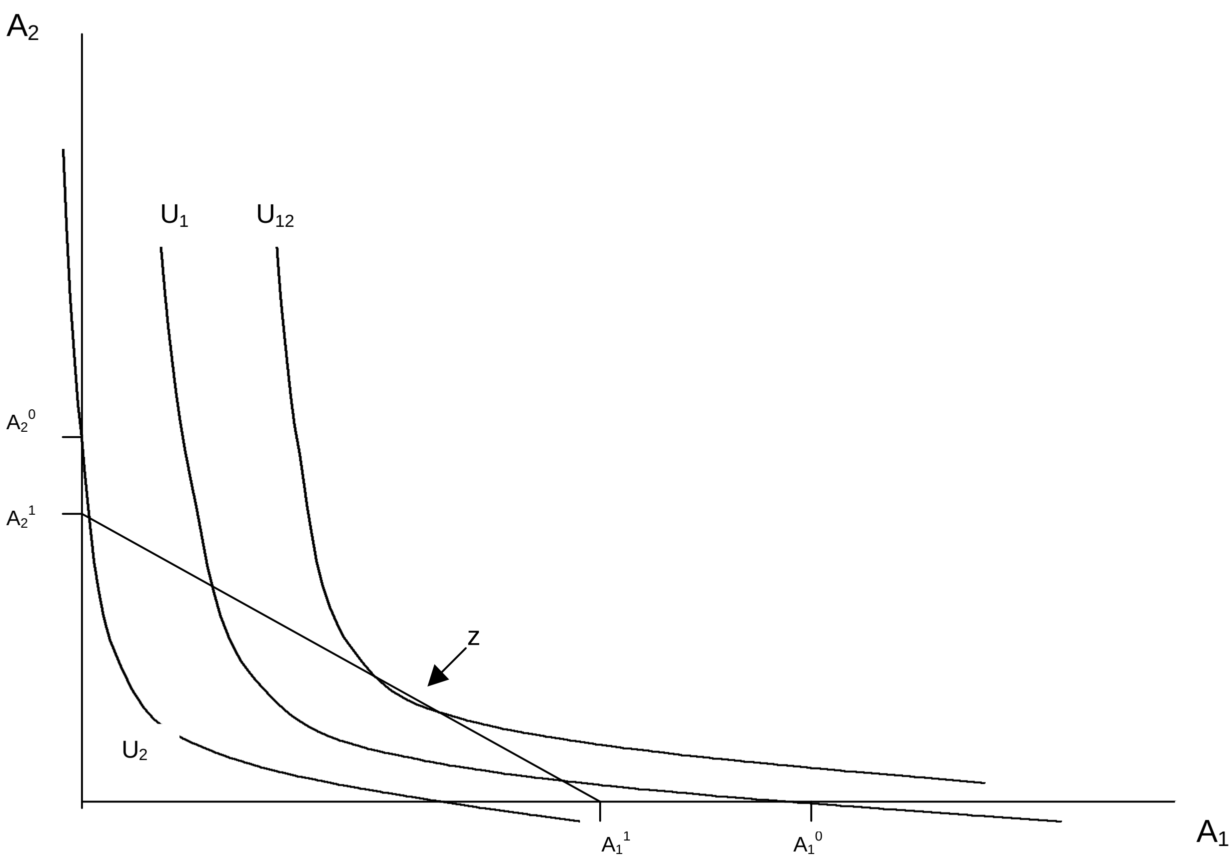
We answer the third question by calculating the WTP for an increase in the number of site types visited on a trip. We find that there is a high WTP for access to multiple sites on a single trip. For example, we estimate that an individual constrained to 5 day trips is willing to pay more than $2600 for increasing the site types visited from 1 to 2 and $908 for increasing them from 2 to 3.

The paper is organized in the following way. Section 2 presents a review of the literature. Section 3 presents the consumer's problem. We show how non-separability in the utility function and sub-additive cost functions make the WTP and quantity demanded of each good dependent on the other goods present in the chosen package. We also refer to the connection between preference convexity and the WTP for visiting multiple destinations. Section 4 presents the econometric model. Section 5 describes our data and results and Section 6 presents the conclusions.

2.0 Related Literature

We can relate the distinctive features of this choice problem to the literature with a simple graph. Consider an individual who is choosing between two products A_1 and A_2. Most scholars employ discrete choice models that assume individuals have convex preferences but that these alternatives are *mutually exclusive in consumption*. The individual must decide which alternative to choose and how much of it to consume given their budget constraint and prices. Figure 1 depicts the case of an individual with a given budget to buy units of these two goods. The assumptions result in a corner solution in which the individual consumes a positive amount of only one alternative which, in the case depicted would be A_1^0 units of A_1 at utility level U_1 (this yields a higher utility than buying units of A_2 only). Much of the frontier in discrete choice modeling is focused modeling demand under these conditions. For discrete-continuous models see Wales and Woodland, 1983; Hanemann, 1984; Lee and Pitt,1987; Ransom, 1987; Srinivasan and Winer, 1994; and Phanuef, Kling and Herriges, 1997. For discrete-discrete model applications to this problem see Train, McFadden and Ben-Akiva, 1997 and DeShazo and Saenz, 2000.

Figure 1



We allow for a more general choice framework in which individuals may consume a positive quantity of both goods on a choice occasion. In the figure we depict the case in which consuming a positive amount of both goods implies an addition fixed cost to the consumer, so that the maximum feasible quantity of each good declines from $A_1^0 \rightarrow A_1^1$ and $A_2^0 \rightarrow A_2^1$. However, we see in Figure 1 that consuming a positive quantity of both goods leads to a higher level of utility (U^{12}) because of the convexity of individuals' preferences.[1] Hence, as long as the extra costs incurred from consuming a positive quantity of both goods are not too high, the assumption that each individual consumes only one of them is too strong. In theory a hedonic model accommodates the consumption of multiple alternative-quantities in presence of complex costs and utility interactions (citations). However, in practice, scholars have struggled to estimate the changes in consumer surplus associated with changes in the quantity of an alternative consumed and to predict the choice of alternatives.

3.0 Theoretical Framework

We solve the individual's maximization problem in two steps because the cost function is discontinuous each time the inivdidual adds a site to the package. First, the individual chooses the optimal combination of sites. He or she then chooses the optimal time to spend at each site, conditional on a combination of goods chosen.

To formalize this probem we denote the vector of lengths of stay at each site in a combination by t^j. We represent the associated cost function with $c\left(t_1,...,t_n\right)$ which is discontinuous at t_i=0 for all i. The per day varaible prices for each site in a combination are $p_1,..., p_n$ for n possible sites. Z is an outside good with its price normalized to one. *I* denotes the individual's income while *T* represents the maximum trip length that he or she may choose. The indivdual solves the following:

$$\underset{j}{Max}\left\{\underset{t^j,z}{Max}\left\{U\left(t_1,...,t_n,z\right)\right\}\right\}$$

$$\text{s.t.} \qquad I = c\left(t_1,...,t_n\right)+\sum_{i=1}^{n} p_i t_i + z \qquad (1.1)$$

$$\sum_{i=1}^{n} t_i = T$$

where

$$c\left(t_1,...,t_n\right)=\begin{cases} 0, \text{ if } t_1 = t_2 = ... = t_n = 0 \\ c_1, \text{ if } t_1 > 0,\ t_2 = t_3 = ... = t_n = 0 \\ c_2, \text{ if } t_2 > 0,\ t_1 = t_3 = ... = t_n = 0 \\ c_{12}, \text{ if } t_1 > 0,\ t_2 > 0,\ t_3 = ... = t_n = 0 \\ \quad ... \\ c_{1...n}, \text{ if } t_1 > 0, ..., t_n > 0 \end{cases} \qquad (1.2)$$

[1] We do not make any assumptions about the sub- or super-additive properties the fixed costs functions. Notice, however, that if A_1 and A_2 were strong enough complements, individuals *might* choose to consume a positive amount of both even if the fixed costs of doing so were super-additive.

The Lagrangian function that corresponds to the maximization (with respect to length of stay) conditional on a given set of sites visited is:

$$L = U\left(t_1,...,t_n,z\right) + l\left(I - c\left(t_1,...,t_n\right) - \sum_{i=1}^{n} p_i t_i - z\right) + f\left(T - \sum_{i=1}^{n} t_i\right)$$

Let Ω^j denote the set of sites included in combination j , with j=1,…,J; $J = 2^n$. The First Order Conditions for a local maximum, conditional on site combination j being visited, are:[2]

$$\frac{\partial L}{\partial t_i} = \frac{\partial U}{\partial t_i} - l\, p_i - f = 0; \text{ for all i} \in \Omega^j. \tag{1.3}$$

$$\frac{\partial L}{\partial z} = \frac{\partial U}{\partial z} - l = 0 \tag{1.4}$$

$$\frac{\partial L}{\partial l} = I - c\left(t_1,...,t_n\right) - \sum_{i=1}^{n} p_i t_i - y = 0 \tag{1.5}$$

$$\frac{\partial L}{\partial f} = T - \sum_{i=1}^{n} t_i \geq 0; \quad f \geq 0; \quad f\frac{\partial L}{\partial f} = 0 \tag{1.6}$$

Henceforth, we assume that the utility function is linear in the quantity of the outside good.[3] When U is a quasi-linear utility function (linear in the numaraire good "Z"), then the FOC above can be rewritten as:

$$\frac{\partial L}{\partial t_i} = \frac{\partial U}{\partial t_i} - a\, p_i - f = 0, \text{ for all i in site combination j.} \tag{1.7}$$

$$\frac{\partial L}{\partial y} = a - l = 0 \tag{1.8}$$

$$\frac{\partial L}{\partial l} = I - c\left(t_1,...,t_n\right) - \sum_{i=1}^{n} p_i t_i - y = 0 \tag{1.9}$$

$$\frac{\partial L}{\partial f} = T - \sum_{i=1}^{n} t_i \geq 0; \quad f \geq 0; \quad f\frac{\partial L}{\partial f} = 0 \tag{1.10}$$

We assume that income is high enough for z>0 to hold at the critical point. Once the individual determines optimal length of stay for each site, conditional on a set of sites visited, he or she obtains the optimal site combination to visit by comparing the value of the indirect utility function that results from solving equations (1.3) through (1.6). The individual then chooses the site combination with the highest associated indirect utility.

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[2] We assume no satiation in the utility function and no corner solutions.

[3] This simplifies the theoretical analysis and is compatiable with the empirical section where we employ a utility specification that is linear in income.

The Effect of changes in Varaible Costs and Site Availabilty on the Quantity Choosen.

We first consider the problem when the time constraint is not binding and then when it is. When the time constraint is not binding, $f = 0$ in the FOC's above, and (1.7) becomes:

$$\frac{\partial L}{\partial t_i} = \frac{\partial U}{\partial t_i} - a\, p_i = 0, \text{ for all i in site combination j.}$$

What is the effect of a change in the variable costs (e.g.the price of lodging) at one of the sites? Without an assumption of additive separability, the optimal length of stay at site *i* depends on the lodging prices of other sites. Therefore, marginal utility from time spent at site *i* may depend on the time spent at site *j*, and changes in the lodging price at site *j* affect the length of stay at *i*. If they are substitutes, the cross price elasticity should be positive. The own price elasticity should be negative as long as the marginal utility of time at each site is decreasing.

Changes in the set of available sites can also affect the optimal length of time when the utility function is not additively separable. If site *j* complements site *i* (in the sense that the marginal utility of time spent at *i* increases with time at site *j*), then when site *j* is omitted from the choice set, time spent at site *i* is likely to decrease. If they are substitutes the inverse holds.

If the individual has a binding time constraint, then $f > 0$ in (1.7) through (1.10). This implies that even if the utility function were additively separable, the optimal time at site j would depend on the lodging price at other sites visited. An increase in the lodging price of a site will shorten the stay at this site while increasing the stay at all other sites in the combination visited (this is formally shown in the appendix).

With a non-separable utility function, the length of stay at a site again depends on the lodging and access prices of potentially all other sites. However, in contrast with the case of additively separable utilities, the direction in which the length of stay changes is uncertain: the marginal utility of time spent at a site may depend on the length of stay at other sites. Therefore, we might observe a reduction in the length of stay at a site when lodging price or the access cost to another site increases.

The elimination of a site from the choice set (and from the visited combination) can also affect the optimal time at other sites. With a binding time constraint, this will happen even with an additively separable utility function. This is because the time spent at the eliminated site is now zero, and time at other sites will be substituted for it.

Optimal Site-Combinations and Willingness to Pay for a Site

We assume that the individual determines the optimal site-combination by comparing the utility associated with all combinations and then choosing the one that yields the highest utility. To express this theoretically, we denote c^j as the set of all sites included in combination *j* and $\underset{\{-k\}}{\Omega}$ as the set of possible site combinations when all sites are considered except for site *k*. We define willingness to pay (WTP) for site *k* as:

$$WTP(k) = \frac{1}{a}\left[\underset{j \in \Omega}{Max}\left\{ V_j\left(c^j, p_1,..., p_n, M\right)\right\} - \underset{j \in \underset{\{-k\}}{\Omega}}{Max}\left\{ V_j\left(c^j, p_1,..., p_n, M\right)\right\} \right] \quad (1.11)$$

We next consider how an indivdual's probability of choosing a site and WTP for a site varies with the varaible costs and on the presence of other sites in the package.

Interdependencies across sites may arise because of interactions in the individual's utility function, the subadditivity of the cost function or a presence of binding time constraint. First, significant interactions across sites within the utility function, such as site complementarity or substitution, will affect the the probability of choosing a site, the quantity choosen and their WTP for a site. For example, if sites *i* and *j* are close substitutes (complements) for the individual, one would expect the increase in utility associated to visiting *i* to be smaller (larger) when *j* is present than when only *i* is present. Therefore, an increase in the varaible costs of *j* would cause the individual to shorten his or her stay at *j* or even choose package without *j*. This cost increase in *j* would also increase the indivdual's WTP for site *i*.

Second, the presence of the a subadditive cost function may affect the consumer surplus their derive from a site or package. When two sites are located along a common route, the travel cost incurred in visiting both of them in one trip is lower than the cost incurred if both sites are visited on separate trips. In this case, even if the utility function were separable in these two sites, the consumer surplus derived from site *j* increases when site *i* is visited too because the additional travel cost imposed by *i*'s visit is smaller when *j* is already included in the trip. (Manrique, you need to be clear about the difference between WTP and CS in this discussion. I have change it accordingly but you may wish to fine tune this.)

Third, even if the utlity function were seperable and cost function strictively additive, the presence of a binding time constraint would induce an interdependency across sites. The descion to consume a postive amount of more than one site implies trading off utility at different sites. Therefore, when evaluating the addition of new site to the package, the researcher must consider the difference utility the indivduals derives from that quantity of consumption. [4]

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[4] Consider, for example an individual with two sites in his choice set: *i* and *j*. Assume that the utility and cost functions are additively separable. Assume also that his trip length is restricted to a maximum of T days and that this constraint is binding. The utility associated with choosing both *i* and *j* for is:

$$U_{ij} = v_j\left(T - t_i^*\right) - a \cdot \left[p_j \cdot \left(T - t_i^*\right) + c_j \right] + v_i\left(t_i^*\right) - a \cdot \left[p_i t_i^* + c_i \right]$$

The utility associated with choosing site j only is:

$$U_j = v_j\left(T\right) - a \cdot \left(p_j T + c_j\right) + v_j\left(0\right)$$

As long as $U_{ij}>U_i$ and $U_{ij}>U_j$, so that a convex combination of *i* and *j* is preferred over a visit to only one of them, the WTP for *i* is:

$$WTP(i) = \frac{1}{a}\left[v_i\left(t_i^*\right) - \left[v_j\left(T\right) - v_j\left(T - t_i^*\right)\right] - a \cdot \left[c_i + \left(p_i - p_j\right) t_i^* \right] - v_i\left(0\right)\right]$$

If $p_j = p_i$ then:

$$WTP(i) = \frac{1}{a}\left[v_i\left(t_i^*\right) - \left[v_j\left(T\right) - v_j\left(T - t_i^*\right)\right] - a\, c_i - v_i\left(0\right)\right]$$

A Measure of Willingness to Pay for Variety

Preference for site variety, requires that sites be imperfect substitutes and that preferences be convex in the recreational sites available. In this case, assuming that access costs are small enough, an individual who is not time constrained will choose to visit multiple sites instead of sticking to one site only. Even under a time constraint, individuals will choose to visit multiple sites as long as the implied increase in access cost is low enough and as long as there is no one site with a marginal utility so high that it is preferred over the rest even after spending most of the time available at it.

We can measure the WTP for variety, by comparing the maximum utility achieved by an individual who is constrained to visit 1 site only v.s. one that is unconstrained in the number of sites to visit. Formally, denote by Ω_r the set of elements in Ω that consist of combinations of r sites or less. We define the WTP for an increase in the number of sites from r to n as:

$$WTP_{r,n} = \underset{j \in \Omega_n}{Max}\left\{V_j\left(c^j, p_1,..., p_n, M\right)\right\} - \underset{j \in \Omega_r}{Max}\left\{V_j\left(c^j, p_1,..., p_n, M\right)\right\}$$

Note that the discussion in the previous section about how the WTP for a site depends on the presence and lodging price of other sites in the choice set reflects the fact that part of the WTP for a site stems from its contribution to variety over the trip.

4.0 Econometric Model

We assume that the individual chooses between different types of trips or tour-packages and use a multinomial logit to model this choice. Each element in an individual's choice set is a complete trip, characterized by one or more sites being visited (for different time lengths) and an associated cost. We use a random utility function that depends mainly on the total number of days spent at each of seven different types of sites: beaches, forests, volcanoes, rivers, golf resorts, fishing sites and islands. As might be obvious, each of these classifications is based on the main feature of the sites in it.

Assuming that utility is linear in income, a desirable specification for the utility function is:

This illustrates that the WTP for *i* depends on price for consumption of *j* in an indirect way because the optimal t_i depends on the price of *j*, and directly because the sacrificed time at *j* (due to time spent at *i*) is accounted for as well.

$$U_j = f(t_{1j},...,t_{nj}) + d\left(I - P_j\right)$$

where

$$f(t_{1j},...,t_{nj}) = \sum_{m=1}^{M} l_m D_{mj} + \sum_{m=1}^{M-1}\sum_{k=m+1}^{M} j_{mk} D_{mj} D_{kj} + \sum_{m=1}^{M}\left[a_m\left(\sum_{i=1}^{n} t_{ij}\right) + b_m\left(\sum_{i=1}^{n} t_{ij}\right)^2\right]$$
$$+\sum_{m=1}^{M-1}\sum_{p=m+1}^{M} b_{mp}\left(\sum_{i=1}^{n} t_{ij}\right)\left(\sum_{i=1}^{n} t_{ij}\right)$$

where

n = total number of sites. (1.12)

M = number of site types

D_{mj} = dummy for having site type i in package j.

t_{ij} = number of days at site type i in package j.

P_j = cost of trip j

I = income

This function has both a time-independent and a time-dependent component. The time-independent component captures any instantaneous satisfaction that an individual may obtain from visiting each site type; it is allowed to be different for each possible combination of sites. The time-dependent component is quadratic in the time spent at each site and allows the marginal utility of time on site to depend on the total time spent at the site. The interaction terms allow this marginal utility to depend on the time spent at other sites also.

Attribute specific dummies and their interaction capture the effect of each site type combination. Each of these dummies turns on when a positive amount of the site type in question is consumed. This may have important implications in terms of possible complementarity or substitution between site types.

4.0 Research Design and Data

We take the package to be the object of choice and the alternative-quantities to be attributes of this object. This accommodates the joint and conditional nature of the fixed costs and the utility derived from the quantity of an alternative. However, this approach results in a choice set that is extremely large and complex, varying along two continuous dimensions and encompassing all permutations of sites, on-site time and trip length. Such a choice may be intractable in empirical estimation. Yet even with a choice set this large and complex, if scholars use revealed preference data to estimate demand for site portfolios they are *still likely to omit* measures of site, lodging, and transportation quality which would be correlated with on-site time, trip length and price.

We address these potential issues by using stated preference demand techniques in which we observe an individual's choice from the choice set that we constructed. This ensures that 1) we know the extent of the choice set, 2) it is based on market-feasible portfolios (available at the current market equilibrium), and 3) it is exogenous to the individual. In this way, we are able to reduce the dimensionality of the choice set and ensure that essential variables are not subject to endogeniety.

We begin by collecting and transforming market data on tour packages, before we design the stated preference choice sets. We constructed the *feasible market choice* set by collecting the 239 tour package itineraries available in the Costa Rican market in 1998. We identified six dominant package "themes" plus four types of one day trips from this market equilibrium. The package themes included: beaches only, beach & forest, rainforest only, a sample of destinations, outdoor adventure, bird & wildlife viewing.

We offered respondents all six package-themes, seven one-day trips, plus the option to purchase none of them. We designed each theme package in following way. First, from the market data we identified all possible sites that were incorporated into existing packages, the length of stay at these sites and the package prices. This allowed us to characterized the feasible market choice set which we did by setting bounds on the minimum and maximum 1) number sites in that package theme, 2) length of stay per site, and 3) price per package theme using market data. Second, for each theme, we then randomly drew sites, days at these sites and prices from these market feasible sets to construct a packages of sites, days and a price that was random and orthogonal. (See Appendix C for a sample choice set.[5]) . The set of one-day trips (which contains seven trips) was formed by randomly drawing two volcanoes, two beaches and two forests, plus a trip to either a forest, a river, a golf resort, a fishing spot, or an island. Our analysis evaluates the demand for 27 sites that individuals consumed for between zero and three days, over a total trip length that ranges from zero to twelve days.

Respondents were given, first, a choice set with seven one-day trips to choosse from plus a "None of the Above" option. Then, they were given a choice set of six multiple-day packages to choose from. Finally, they were asked to choose between their preferred options in the one-day and multiple-day trip choice sets.

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[5] The survey preamble to this choice exercise is presented in Appendix D. Through this preamble, we endeavor to hold the quality of complements constant such as lodging, food, and transportation.

The variables used in the econometric estimation are the following:

Description of Empirical Variables

Variable	Description
Dvolcano	Dummy for one or more volcanoes in package
Dbeach	Dummy for one or more beaches in package
Dforest	Dummy for one or more forests in package
Driver	Dummy for one or more rivers in package
Dgolf	Dummy for one or more golf resorts in package
Dfish	Dummy for one or more fishing trips in package
Disland	Dummy for one or more islands in package
Dbeach_forest	=Bea*Fore
Dbeach_volcano	=Bea*Volc
Dforest_volcano	=Fore*Volc
Dvolcano_river	=Volc*River
DAYS volcano	Days at the volcano
DAYS beach	Days at the beach
DAYS forest	Days at the forest
DAYS river	Days at the river
DAYS volano2	=Dayvol^2
DAYS beach2	=Daybea^2
DAYS forest2	=Dayfore^2
DAYS river2	=Dayriver^2
DAYS beach_forest	=Daybea*Dayfore
DAYS beach_volcano	=Daybea*Dayvol
DAYS forest_volcano	=Dayfore*Dayvol
DAYS forest_river	=Dayfore*Dayriver
DAYS volcano_river	=Dayvol*Dayriver
Package price/$1000	=Price of the package in thousands of dollars

5.0 Description of Results

In this section, we focus on the relative value to the consumer of different types of site that can be visited over a trip. We answer the following questions: a) What is the probability of visitation associated to each site type? b) What is the optimal length of stay? c) Which site tyes are complements and which are substitutes for each other? d) How much more are individuals willing to pay for a trip to multiple sites than for a single-site trip? e) How does the answer to these questions change when the individual faces a constraint on trip length? In particular, we are interested in the behavior of individuals constrained to a relatively short trip (five days for example).

The answers to these questions have important policy implications for the resource managers who determine type and number of sites avaible in a country. Such information will also help them provide the optimal type and quantity of on-site services. Tour package wholesalers and retailers will also find this information useful for designing the optimal pakcages given the portfolios offered by a region or country. Finally, these results are likely to be relevant when building a marketing strategy for the country as a tourist destination since this requires knowing the trip characteristics that are most appealing to the potential visitors.

5.1 Description of Coefficients

We present the estimated coefficents in Table 1 which are largely significant at the five percent level and consistently of the expected signs. The time-depedent coefficents (DAYS volcano, DAYS beach, DAYS forest, DAYS river, DAYS volcano2, DAYS beach2, DAYS forest2, DAYS river2) suggest a positive, diminishing marginal utlity of time at each site type when visited on a single-site trip[6]. The coefficients on the time-quadratic terms are significantly negative for volcanoes and rivers, and their point estimate is significantly more negative than for beaches and forests. That is, the marginal utility decreases at a higher rate at volcanoes and rivers and this will reflect in a shorter optimal length of stay at rivers and volcanoes in our simulations described on the next section.

The coefficients on the interaction terms suggest that the utlity function is not additively seperable. The instantaneous interaction terms (Dbeach_forest, Dbeach_volcano, Dforest_volcano, Dvolcano_river) are all postive and generally signficant. Interestingly, the time-dependent interaction terms are negative and generally sigfinicant. This implies that the marginal utility of time spent at a site type decreases with the time spent at another site type. This is consistent with our expectation that marginal utility of "recreation" as an aggregate good should decline as the quantity consumed increases.

[6] If more than one of these site types is visited on a trip then the marginal utility of time at each of them will also depend on the time-interaction terms (DAYS beach_forest, DAYS beach_volcano, DAYS forest_volcano, DAYS forest_river, DAYS volcano_river).

5.2 Simulations

In order to answer the questions stated above, we take as a starting point the choice predictions for an individual who faces a choice set similar to the one available in reality (as opposed to the fictitious one faced in the stated preference survey). We construct this "real" choice set by including the same 27 sites used for the construction of the SP experiment. Using these sites we form a set of approximately 9000 possible trips available to the individual. These trips are formed by making combinations of the 27 sites and different possible lengths of stay at each of them. For each trip, we calculated the travel and lodging costs based on the distance that must be traveled on a round trip to visit the sites (departing from the airport) and the low-end lodging prices at each site.

This set is constructed with the following restrictions: Each trip can include a maximum of four types of site, and three sites within each type. Hence the maximum number of sites visited on a trip is twelve. The maximum length of stay at each site is three days, and the maximum length for a complete trip is sixteen days. We obtain all the possible site and length-of-stay combinations given the 27 sites used and the constraints just described.

5.2.1 Site Types: Optimal Length of Stay, Probability of Visitation and Willingness to Pay

Table 2 shows the probability of choice associated to each type of site, the average optimal length of stay at each site type conditional on it being visited, and the "unconditional" expected length of stay. The probability of choice associated to each type is the sum of the probabilities associated to all trips in the choice set that contain this site type. The "conditional" expected length of stay is the weighted average of the length of stay across all trips in which this site type is visited. The weight of each trip is given by the probability of choice conditional on the trip containing this site type. Hence, it tells us, given that an individual visits a certain site type, how long we should expect her to say. The unconditional expected length of stay at a site type is the product of the conditional length of stay times the probability of choice associated to the site type. It can be interpreted as the number of days that each visitor entering the country is expected to spend at the corresponding site type (i.e. before we know that she will visit that site type for sure).

The visit of one site type does not exclude other types from being visited and therefore the probabilities of visitation sum up to more than one. As shown in Table 2, all of these probabilities are higher than 85%, which reflects a preference for visiting multiple site types. The beach is the site type with the highest probability of visitation (96%) followed by forests, volcanoes, and finally rivers.

As suggested by the coefficients on the quadratic terms of the utility function, the optimal length of stay is shorter at volcanoes and rivers than at beaches and forests. While visits to volcanoes and rivers are expected to be 2 days long or less, forests and beaches are expected to be visited for more than 3.8 days. The unconditional expected length of stay is very similar to the conditional one given the high probabilities of choice associated to each site type. Note, however, that the gap between expected time at volcanoes and rivers, and expected time at beaches and forests is slightly higher.

Table 3 shows the length of stay and visitation probabilities when the trip length is restricted to a five day maximum. The constraint on trip length produces a reversal in the probabilities of visitation: volcanoes are now the most popular sites, followed by rivers, beaches and forests (in that order). The conditional length of stay is now shorter for all sites. The site types that are more sensitive to this constraint on trip length are beaches and forests where the length of stay reduces by more than 50%, while the length of stay at rivers and volcanoes reduces by less than 15%. Hence, it seems like the shorter stay at beaches and forests hurts the popularity of these site types significantly for shorter trips.

5.2.2 Substitution and Complementarity between Site Types

Changes in the cost of visiting a site type may not only affect the expected visitation to it but also other site types that might be substitutes or complements for it. This information is relevant for recreational site administrators in order to forsee the effect on the demand for their product when other products' prices change. For this reason, we estimate the elasticity of the choice probability and optimal length of stay with respect to changes in the lodging price of different site types. This is done by simulating a 1% increase in the lodging price at all sites of a certain type and obtaining the percentage change in visitation probability and expected length of stay at all site types.

In principle, we could also simulate a change in the lodging price at a single site (instead of at all sites of a type). However, our model only contains a few site characteristics and the only variable that distinguishes different sites of the same type is the cost of the trip that visits them. Hence, we feel more comfortable using our model to predict demand for site types (groups of sites of the same type) than to predict demand for single sites. This still allows us to make the point that there may be important differences in the degree to which different sites substitute each other in multiple-site trips.

Table 4a shows the elasticity of the conditional expected time on site given no binding constraints on trip length. The decreasing marginal utility of time on site at any of the site types implies a negative own price elasticity in our model. Beaches and forests have the highest own price elasticities. The crossed price elasticities of conditional time one site are all positive which is consistent with the fact that all the time interaction terms have negative coefficients. This means that a decrease in the time on site at one site type increases the marginal utility of time at the other sites.

Table 4b shows the elasticity of the probability of visitation to each site type with respect to lodging prices. An increase in the lodging prices of a given site type reduces the consumer surplus associated with it and therefore reduces the probability of visitation to it. This explains the negative own price elasticities. On the other hand, an increase in the lodging prices at a site type affects the probability of visitation to other sites through two mechanisms. First, if the site type that becomes more expensive is still visited by the tourist, the number of days at it are likely to decrease. The negative day-interaction coefficients imply that the marginal utility of time at the other site types will increase, and with them the consumer surplus associated. Hence this mechanism increases the probability of visiting other sites.

Second, if the raise in price causes the tourist not to visit this site at all, then the effect on the consumer surplus associated with other sites depends on the coefficients of the dummy interaction terms and the time-interaction terms. For example, when only site types m and k are being visited, the net effect of the site interaction terms is given by:

$$j_{mk} + b_{mk} t_m t_k \, .$$

This expression is the partial derivative of the utility function in (1.12) with respect to the dummy product D_m D_k. Figures 1 through 5 show the value that this expression takes for different pairs m and k, as a function of the value taken by the time interaction $t_m t_k$. Each graph displays the upper and lower limits of the 95% confidence intervals for the expression above as well as the point estimate. For time interaction values below five, these interaction terms take positive and statistically significant values for the site pairs volcano-beach, volcano-forest, volcano-river, and beach-forest. Hence, in these cases, leaving one of these sites out of the trip may reduce the probability of visiting others too.

As shown by table 4b, the first effect seems to be the stronger of the two, producing positive crossed elasticities of visitation probabilities for all site types. On table 4c we present the elasticity of the Unconditional Time on Site, which is just the sum of the elasticities in tables 4a and 4b. Using this table, we can see that there are important differences in the crossed prices elasticities (i.e. the degree to which different sites are substitutes for each other). Rivers have the highest elasticity with respect to volcano prices and viceversa, while forests are the sites that are more elastic to changes in the prices of beaches and viceversa. We take this as evidence that rivers and volcanoes are close substitutes for each other, and beaches and forests are good substitutes for each other too.

Tables 5a – 5c show the elasticities obtained when the individual is constrained to a maximum trip length of 5 days. In general, the elasticities in this case are higher in absolute value than when no constraint on trip length was imposed. The higher value in the crossed price elasticities makes sense: with a binding time constraint, the numeraire good can no longer be used as a substitute for the recreational site that becomes more expensive. Instead, the consumer uses other recreational sites as substitutes and this produces higher crossed price elasticities between site types.

5.2.3 Willingness to Pay for Site Types and Willingness to Pay for Site Variety

The WTP for a site type is calculated as the change in expected utility produced by the elimination from the choice set of all trips that visit sites of the corresponding type. This change is divided by the marginal utility of money to express it in dollars. The utility associated to each trip in the choice set already takes in account basic travel and lodging costs. Hence the WTP that we obtain is already net of these costs. The reason why we do it this way is that, to an important extent, these costs are determined by the location of the sites and the time necessary to consume the different attributes that it offers. For example, when we consider volcanoes and beaches, we should take in account that most volcanoes are a lot closer to the capital in Costa Rica than any of the beaches available. When we compare the WTP for these different types of sites, we should control for these “unavoidable” costs.

Table 6 shows the willingness to pay (WTP) for different site types. The first column displays the results assuming two possible scenarios: first, the tourist has no constraint on trip length and second, the tourist is constrained to trips 5 days long or less. Observe that if we rank the different site types by WTP, the ranking depends on whether trip length is constrained. In particular, with no constraint on trip length, the WTP for beaches and forests is considerably higher than that for rivers and volcanoes. Instead, with a 5 day constraint on trip length, the WTP for volcanoes is higher than for any other site, followed by rivers, beaches, and finally forests.

Part of the value of each site type comes from the consumer’s preference for varietey over a trip. Convexity in preferences make multiple-site trips in general more attractive than single site trips, and the negative coefficients on the quadratic time-on-site variables is strong evidence that of convexity in our estimated preferences.

Table 7 gives us an idea of how important this preference for variety is. For both constrained and unconstrained trip lengths, it shows the willingness to pay for increasing by one the number of site types visited over a trip. Without trip length constraints, the representative tourist is willing to pay $4380 for increasing the number of sites visited form one to two. The WTP for additional site types decreases as the number of types visited increases. When the trip length is constrained to 5 days or less, the WTP for additional site types decreases. This is to be expected because visiting an additional site type implies sacrifying time that would otherwise be spent at other recreational sites. The WTP decreases rapidly once two or three sites are already being visited in the five day trip.

This information is useful both for tour operators and for individuals building strategies to promote the country as a tourist destination. Given the strong preference for site variety, the possibility of visiting several types of site with a relatively small travel cost is likely to be one of the appealing characteristics of countries like Costa Rica.

6.0 Conclusions

Models of discrete choice can be adapted to estimate demand for bundled goods whose cost functions are typically non-linear. The object of choice is the bundle of goods, and it can be characterized by the attributes of the goods contained in it. Using a quadratic utility function, these models can allow for different degrees of substitution or complementarity between goods by including interactions between the attributes of different goods. These interaction terms together with decreasing marginal utilities can also explain a preference for variety in goods purchased.

We used this model to estimate the demand for recreational sites by foreign tourists who typically visit several sites on a single trip (i.e. in a bundle). Through simulations using the estimated coefficients we learned several lessons on recreational demand. First, constraints on trip length can strongly affect the willingness to pay and probability of visiting different types of sites. In particular, a constraint on trip length strongly hurts the willingness to pay for beaches and forests. Hence, for short trips, volcanoes and rivers are preferred over beaches and forests even though the inverse is true for longer trips.

Second, different site types are substitutes for each other but in very different degrees. When trip length is unconstrained, forests are a closer substitute for beaches than any of the other site types, as evidenced by the crossed price elasticities with respect to lodging prices. In the same way, volcanoes and rivers are close substitutes for one another too. Third, the provision of site variety in a touristic region has a huge impact on the demand for recreational sites. Convexity in preferences, together with the sub-additive nature of travel costs make of trips with multiple destinations a highly valued activity. This is true for both short and long trips.

Due to the growing popularity of goods sold in bundles, research on estimation methods for this kind of situations is very important. We have proposed using a discrete choice framework that, together with a stated preference survey, is able to deal with the non-continuous pricing of bundles at the same time that the size of the choice sets used for estimation is kept small. Additional research is necessary in order to incorporate non-exclusive characteristics for each of the goods, and in order to allow for complementarities between attributes within and across goods.

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Table 1
Coefficient Estimates

Variable	Coefficient	z
Dvolcano	-1.5290	-3.44
Dbeach	-1.6513	-8.53
Dforest	-1.2898	-7.54
Driver	-2.2684	-3.35
Dgolf	-1.9634	-3.85
Dfish	-0.8243	-2.58
Disland	-1.4434	-3.69
Dbeach_forest	1.9100	8.53
Dbeach_volcano	1.1572	3.85
Dforest_volcano	0.2656	1.01
Dvolcano_river	1.4994	4.13
DAYS volcano	1.2652	2.28
DAYS beach	0.2844	2.71
DAYS forest	0.3144	2.86
DAYS river	2.6855	3.42
DAYS volano2	-0.2450	-1.86
DAYS beach2	-0.0129	-1.06
DAYS forest2	-0.0173	-1.44
DAYS river2	-0.5275	-2.91
DAYS beach_forest	-0.0245	-1.93
DAYS beach_volcano	-0.0654	-1.92
DAYS forest_volcano	-0.0198	-0.70
DAYS forest_river	-0.0366	-1.02
DAYS volcano_river	-0.2013	-2.78
Package price/$1000	-0.7126	-3.08
Log-L	-2998.622	
n	1209	

Table 2
Visitation Probability and Expected Length of Stay by Site Type
(no constraint on trip length)

	Conditional Length of Stay	Probability of Visitation	Unconditional Length of Stay
Beach	3.83	0.96	3.70
Forest	3.86	0.95	3.68
River	2.00	0.86	1.73
Volcano	1.89	0.89	1.69

Table 3
Visitation Probability and Expected Length of Stay by Site Type
(5 day maximum trip length)

	Conditional Length of Stay	Probability of Visitation	Unconditional Length of Stay
Beach	1.71	0.60	1.04
Forest	1.77	0.56	0.99
River	1.76	0.69	1.21
Volcano	1.68	0.73	1.22

Table 4a
Elasticity of Expected Time at Each Type (conditional on being visited)
with Respect to Changes in Lodging Prices
(No restrictions on Trip Length)

	Pvolc	Pbeach	Pfore	Priv
Volc	-0.0040	0.0027	0.0005	0.0013
Beach	0.0007	-0.0174	0.0030	0.0003
Fore	0.0005	0.0044	-0.0109	0.0010
Riv	0.0006	0.0002	0.0009	-0.0035

Table 4b
Elasticity of Probability of Visitation by Site Type
with Respect to Changes in Lodging Prices
(No restrictions on Trip Length)

	Pvolc	Pbeach	Pfore	Priv
Volc	-0.0018	0.0012	0.0005	0.0002
Beach	0.0000	-0.0023	0.0002	0.0001
Fore	0.0001	0.0006	-0.0021	0.0002
Riv	0.0006	0.0007	0.0012	-0.0038

Table 4c
Elasticity of (Unconditional) Expected Time at Each Type
with Respect to Changes in Lodging Prices
(No restrictions on Trip Length)

	Pvolc	Pbeach	Pfore	Priv
Volc	-0.0059	0.0039	0.0010	0.0015
Beach	0.0008	-0.0197	0.0032	0.0003
Fore	0.0006	0.0050	-0.0130	0.0012
Riv	0.0012	0.0009	0.0021	-0.0073

Table 5a
Elasticity of Expected Time at Each Type (conditional on being visited)
with Respect to Changes in Lodging Prices
(Trip Length Restricted to 5 Days)

	Pvolc	Pbeach	Pfore	Priv
Volc	-0.0035	0.0021	0.0007	0.0008
Beach	0.0008	-0.0075	0.0016	0.0018
Fore	0.0010	0.0025	-0.0049	0.0018
Riv	0.0007	0.0015	0.0014	-0.0030

Table 5b
Elasticity of Probability of Visitation by Site Type
with Respect to Changes in Lodging Prices
(Trip Length Restricted to 5 Days)

	Pvolc	Pbeach	Pfore	Priv
Volc	-0.0031	0.0017	0.0016	-0.0007
Beach	0.0010	-0.0098	0.0002	0.0030
Fore	0.0017	0.0011	-0.0074	0.0038
Riv	0.0009	0.0025	0.0026	-0.0073

Table 5c
Elasticity of (Unconditional) Expected Time at Each Type
with Respect to Changes in Lodging Prices
(Trip Length Restricted to 5 Days)

	Pvolc	Pbeach	Pfore	Priv
Volc	-0.0066	0.0038	0.0023	0.0001
Beach	0.0018	-0.0173	0.0019	0.0048
Fore	0.0027	0.0036	-0.0123	0.0056
Riv	0.0017	0.0040	0.0040	-0.0103

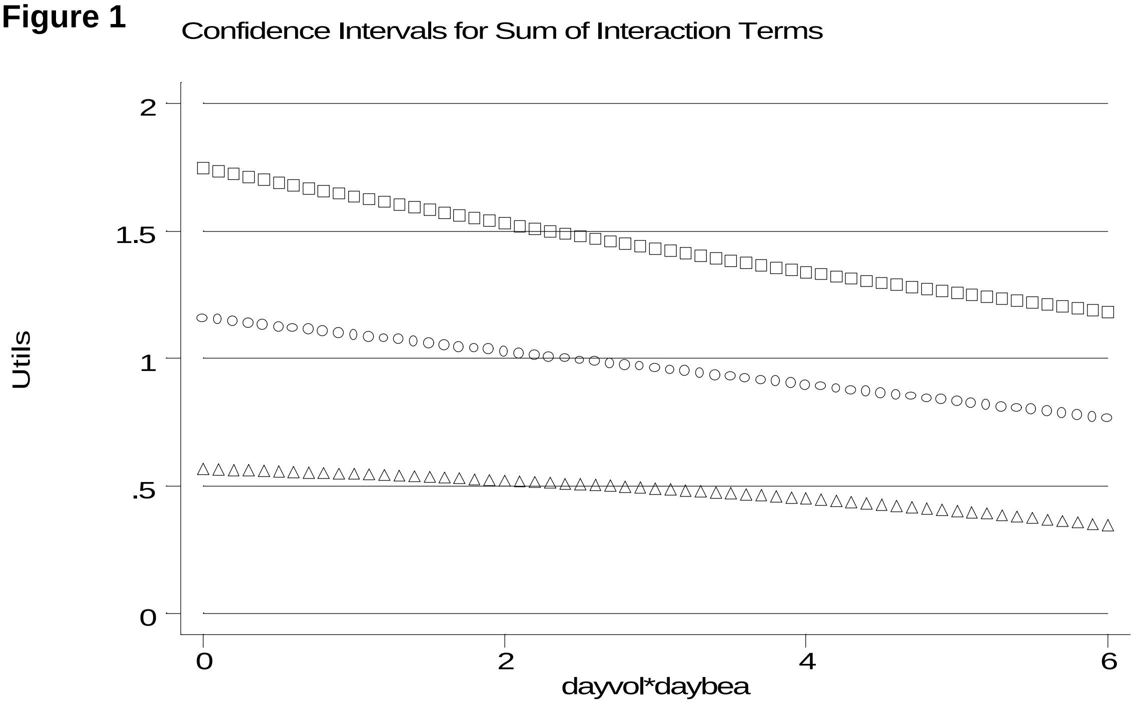
Table 6
Consumer Surplus from Site Types
With and Without Constraint on Trip Length

	Maximum Trip Length	
	Unconstrained	5 Days
Beach	4665.0	1308.1
Forest	4275.0	1153.3
River	2811.1	1645.2
Volcano	3154.2	1842.2

Table 7
Willingness to Pay for Including an Additional Site Type
Maximum Trip Length

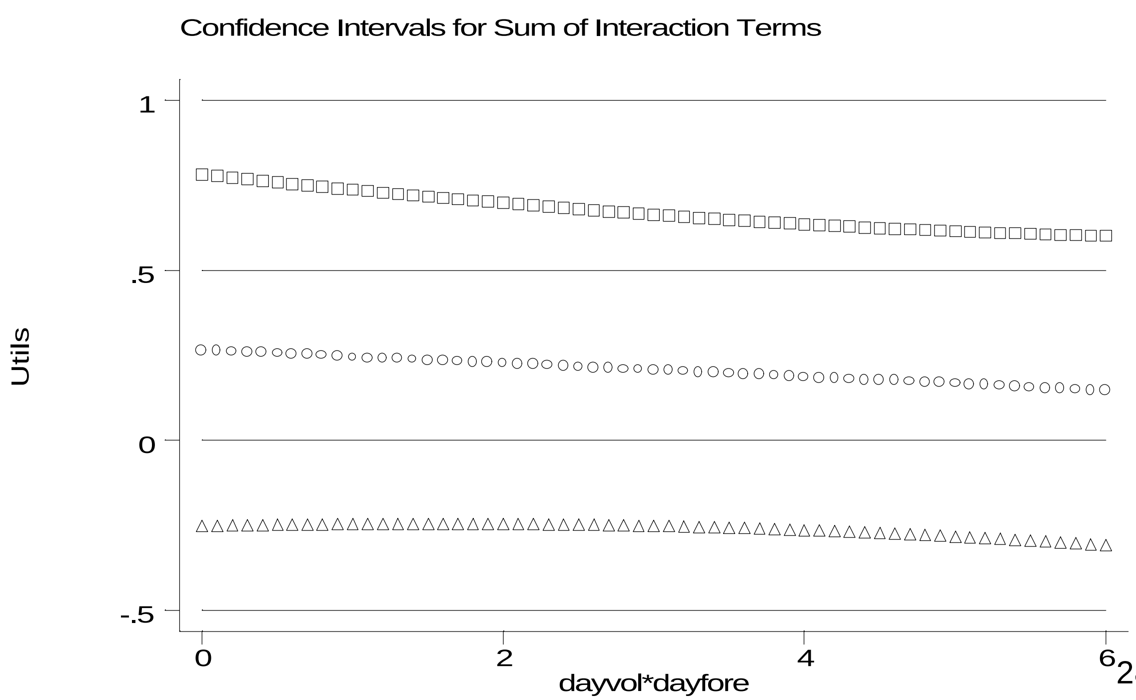
Current Constraint	Maximum Trip Length	
on # of Types	None	5 Days
Increase from 1 to 2 types	4380.9	2676.9
Increase from 2 to 3 types	3174.8	908.1
Increase from 3 to 4 types	1721.4	176.0

Figure 1



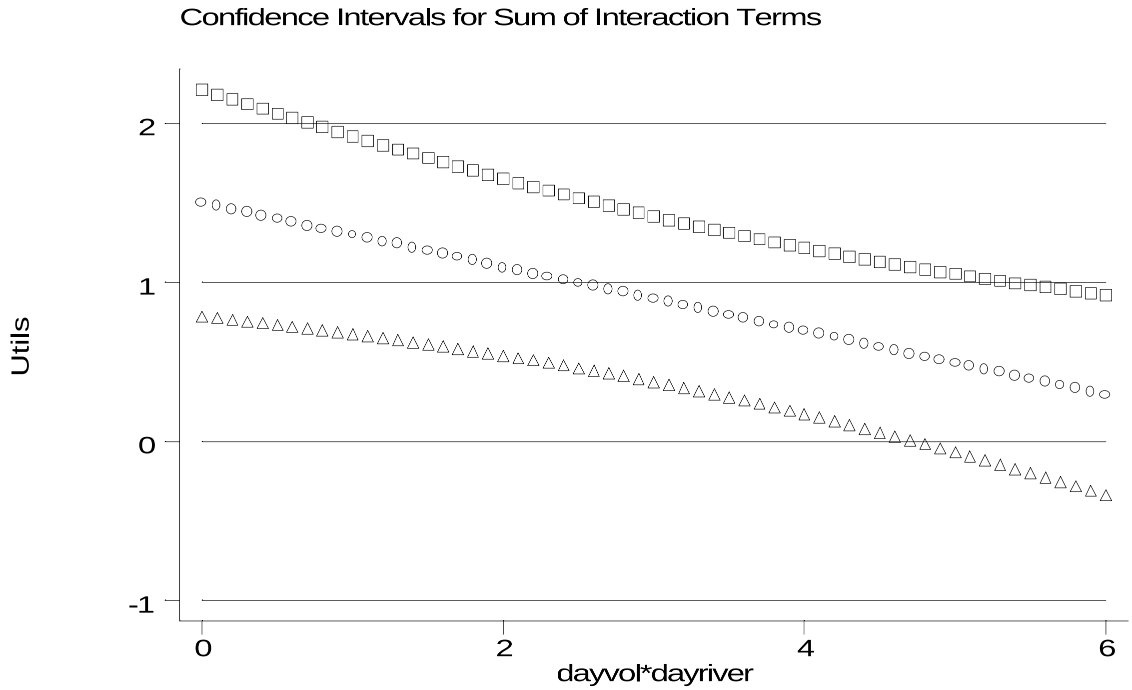
Volcano and Beach Interaction

Figure 2



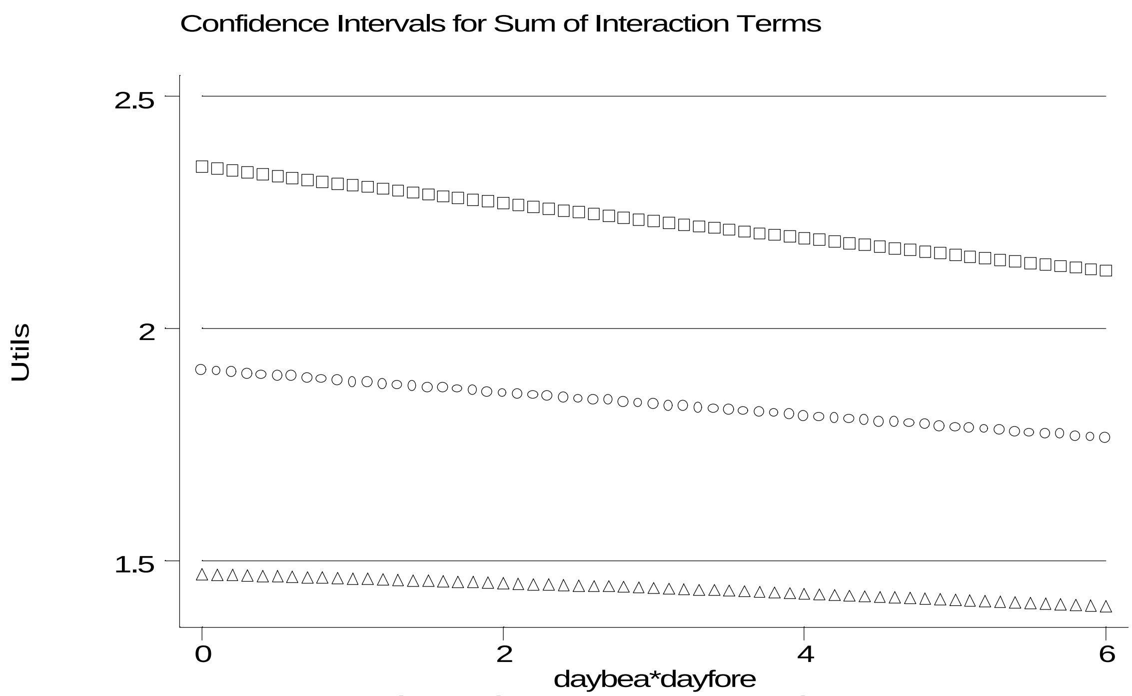
Volcano and Forest Interaction

Figure 3



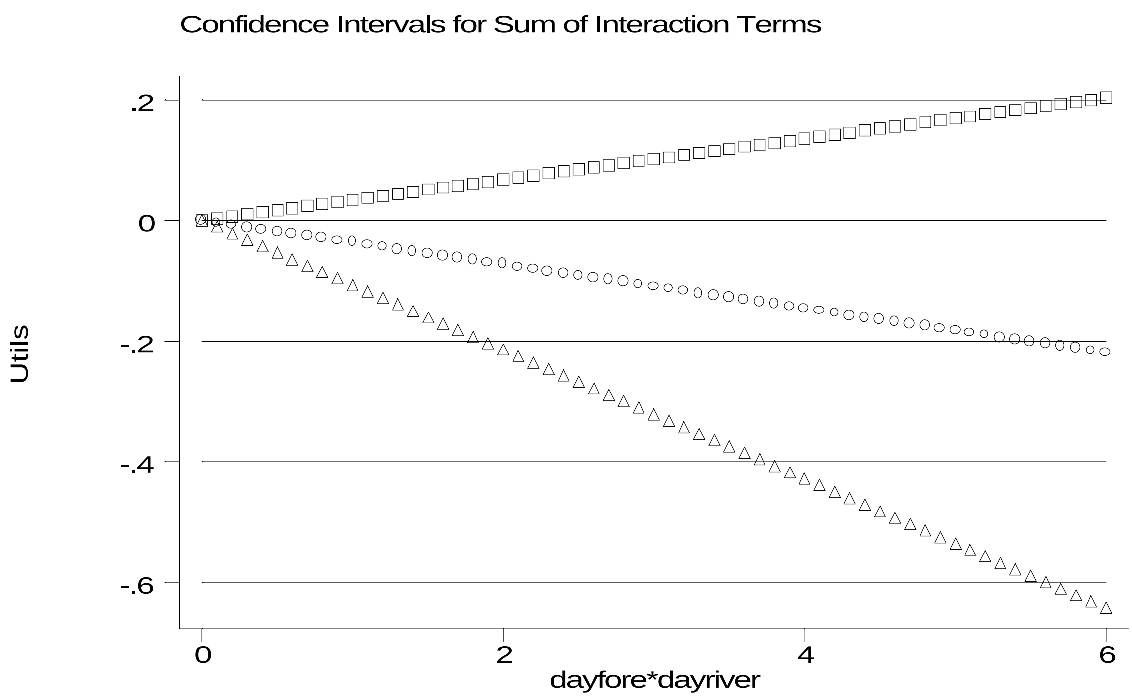
Volcano and River Interaction

Figure 4



Beach and Forest Interaction

Figure 5



Forest and River Interaction

Appendix A

Effect of a Change in Lodging Prices on Time on Site with a Binding Time Constraint and Separable Utility Function

Conditional on a chosen combination of sites, and assuming a utility function that is additively separable, quasilinear, and with nonincreasing marginal utility of time at any site, the FOC's for maximization are:

$$\frac{\partial L}{\partial t_i} = \frac{\partial U}{\partial t_i} - a\, p_i - f = 0, \text{ for all i in the site combination visited.} \qquad \text{(A.1)}$$

$$\frac{\partial L}{\partial y} = a - l = 0 \qquad \text{(A.2)}$$

$$\frac{\partial L}{\partial l} = I - c(t_1, \ldots, t_n) - \sum_{i=1}^{n} p_i t_i - y = 0 \qquad \text{(A.3)}$$

$$\frac{\partial L}{\partial f} = T - \sum_{i=1}^{n} t_i \geq 0; \quad f \geq 0; \quad f \frac{\partial L}{\partial f} = 0 \qquad \text{(A.4)}$$

If time constraints are non-binding, then $f = 0$. Differentiating **(A.1)** with respect to p_j yields:

$$\frac{\partial^2 U}{\partial t_i^{\,2}} \cdot \frac{dt_i}{dp_j} = 0, \text{ if } i \neq j \qquad \text{(A.5)}$$

$$\frac{\partial^2 U}{\partial t_j^{\,2}} \cdot \frac{dt_j}{dp_j} - a = 0 \qquad \text{(A.6)}$$

Since we have assumed that the second derivative of the utility function with respect to time on site is negative for all sites, **(A.5)** implies that $\frac{dt_i}{dp_j} = 0$ and **(A.6)** implies that $\frac{dt_j}{dp_j} < 0$. This means that the own price elasticity of time on site with respect to lodging price is negative conditional on that site being visited, while the cross price elasticity is zero.

In the case of a binding time constraint, $f > 0$ and $\sum_{i=1}^{n} t_i = T$. Differentiating **(A.1)** and **(A.4)**, we obtain:

$$\frac{\partial^2 U}{\partial t_i^{\,2}} \cdot \frac{dt_i}{dp_j} = \frac{df}{dp_j}, \text{ if } i \neq j$$
$$\frac{\partial^2 U}{\partial t_j^{\,2}} \cdot \frac{dt_j}{dp_j} - a = \frac{df}{dp_j} \tag{A.7}$$

$$\sum_{i=1}^{n} \frac{dt_i}{dp_j} = 0 \tag{A.8}$$

Substituting **(A.7)** into **(A.8)**, we obtain the following values for own and cross price elasticities of time on site:

$$\frac{dt_i}{dp_j} = -\frac{a}{u_{ii} \cdot u_{jj}} \left[\frac{1}{\sum_{k=1}^{n} \left(\frac{1}{u_{kk}} \right)} \right] > 0 \tag{A.9}$$

$$\begin{aligned} \frac{dt_j}{dp_j} &= a \left[1 - \frac{1}{u_{jj}} \left(\frac{1}{\sum_{k=1}^{n} \left(\frac{1}{u_{kk}} \right)} \right) \right] \frac{1}{u_{jj}} \\ &= a \left[1 - \frac{\prod_{k \neq j} u_{kk}}{\prod_{k \neq 1} u_{kk} + \prod_{k \neq 2} u_{kk} + \ldots + \prod_{k \neq n} u_{kk}} \right] \frac{1}{u_{jj}} < 0 \end{aligned} \tag{A.10}$$

Hence, if the utility function is separable and quasilinear, all cross price elasticities will be positive, and the own price elasticity will be negative. The shadow price of time decreases the higher the lodging prices.

Appendix B

Willingness to Pay for a Site under No Trip Length Constraints

Assume that an individual can choose out of 3 possible sites: s1, s2, s3. Denote by v_{ij} the utility net of cost associated to a trip visiting sites i and j only. Assuming no constraints on trip length or number of sites, the WTP for s3 is given by:

$$WTP_{s_3} = \frac{1}{a}\left(\ln\left\{1+e^{v_1}+e^{v_2}+e^{v_3}+e^{v_{12}}+e^{v_{13}}+e^{v_{23}}+e^{v_{123}}\right\}-\ln\left\{1+e^{v_1}+e^{v_2}+e^{v_{12}}\right\}\right)$$

$$WTP_{s_3} = \frac{1}{a}\left(\ln\left\{1+e^{v_1}+e^{v_2}+e^{v_{12}}+e^{v_3}\left(1+e^{v_{13}-v_3}+e^{v_{23}-v_3}+e^{v_{123}-v_3}\right)\right\}-\ln\left\{1+e^{v_1}+e^{v_2}+e^{v_{12}}\right\}\right)$$

$$WTP_{s_3} = \frac{1}{a}\left(\ln\left\{1+e^{v_1}+e^{v_2}+e^{v_{12}}+K_{3,12}e^{v_3}\left(1+e^{v_1}+e^{v_2}+e^{v_{12}}\right)\right\}-\ln\left\{1+e^{v_1}+e^{v_2}+e^{v_{12}}\right\}\right)$$

$$WTP_{s_3} = \frac{1}{a}\left(\ln\left\{1+K_{3,12}e^{v_3}\right\}\right)$$

where $K_{3,12} = \dfrac{\left(1+e^{v_{13}-v_3}+e^{v_{23}-v_3}+e^{v_{123}-v_3}\right)}{\left(1+e^{v_1}+e^{v_2}+e^{v_{12}}\right)}$

If the utility function were additively separable and the cost function were additive, then $K_{3,12}$ would be equal to 1 and the willingness to pay for site 3 would depend on the utility associated to this site only and not on the utility associated to other sites, the costs associated to them or the composition of the set of other sites in the choice set.

Probability of Visiting a Site in the Multi-Site Trip Framework

Assume an individual with 3 sites available in his choice set. With no binding constraints on trip length or number of sites, the probability associated to visiting s3 is given by:

$$P(s_3) = \frac{e^{v_3}+e^{v_{13}}+e^{v_{23}}+e^{v_{123}}}{1+e^{v_1}+e^{v_2}+e^{v_3}+e^{v_{12}}+e^{v_{13}}+e^{v_{23}}+e^{v_{123}}}$$

$$\Rightarrow P(s_3) = \frac{K_{3,12}e^{v_3}\left[1+e^{v_1}+e^{v_2}+e^{v_{12}}\right]}{\left[1+K_{3,12}e^{v_3}\right]\left[1+e^{v_1}+e^{v_2}+e^{v_{12}}\right]} = \frac{K_{3,12}e^{v_3}}{1+K_{3,12}e^{v_3}}$$

where $K_{3,12}$ is as defined above.

If the utility and cost functions are additively separable, the $K_{3,12}$ is equal to 1 and the probability of visiting site 3 is independent of the set of other sites that compose the choice set. This probability is exactly the same one as we would have in a logit model where visiting site 3 or choosing an outside option were the only two alternatives in the choice set.