

Legislatures vs. Political Parties: Endogenous Policy with Strategic Voters

Luis Fernando Medina*

Department of Economics, Stanford University

email: `lmedina@leland.stanford.edu`

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Abstract

A model of elections and legislation is analyzed under two types of constitutional framework: with or without legislative initiative. Political parties have policy preferences but need to field candidates who maximize their probability of victory (subject to the constraint of securing the party's endorsement). At the same time, voters use strategic reasoning in assessing their candidates in a multi-district election. The model predicts that, in spite of the candidates' purely electoral motives, voting stances and party will be correlated in the legislature. With legislative initiative, the policy outcomes will gravitate toward the median district, while without it, the parties can fully implement their (divergent) platforms. In this second case, the ability of parties to implement non-centrist policies decreases as the *intradistrict* homogeneity of the electorate increases. This has implications for the study of endogenous economic policy, in particular the way federalism affects the redistributive outcomes of electoral competition.

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1 Introduction

Democratic polities differ in the degree of legislative initiative allowed by their constitutional arrangements. While in some regimes (henceforth called open-rule regimes), the legislature is

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free to amend any legislative proposal submitted to its consideration, in others (closed-rule regimes), its agenda is limited so that, by and large, it can only vote on the platforms proposed by the political parties. The United States, Great Britain in the period prior to the First Reform Act (1832), and the Fourth Republic in France are well-known cases of open-rule regimes whereas contemporary Great Britain, the Fifth French Republic, Japan and some “strong” presidential regimes, especially in Latin America, constitute examples of democracies akin to the closed-rule model. Intuitively, these two types of system imply different ways of determining the “popular will.” Open-rule regimes rest on the assumption that the legislature is the arena where the preferences of the citizenry are aggregated. Local constituencies elect representatives expected to voice their voters’ grievances in the forum provided by the legislature. In contrast, in closed-rule regimes such aggregation of individual preferences takes place at the general election, mediated by the political parties. Once the masses have spoken, giving the majority to a party or set of parties, it is the legislature’s task to implement their will in as faithful and diligent a manner as possible—hence the constraints placed over its agenda. In this case, were the legislature to introduce amendments of its own over the agenda proposed by the parties it would somehow distort the mandate received.

Given this characterization of the two models of democracy, we should expect them to lead to different allocations of political power and, hence to different policy outcomes. After all, in most democracies, the major parties are nationally-based coalitions of citizens (with regional parties playing a subsidiary role). Therefore, whether the legislature responds to constituencies or to parties is far from an immaterial consideration. The goal of this paper is to give a rigorous treatment to this insight.

To that end, I will develop a model of two abstract polities, identical in all respects except in the degree to which they allow legislative initiative, and then characterize when and how their policy outcomes will differ. A thorough comparison of the two systems requires an analysis of elections (contested by parties) *and* legislation. For the most part, the existing literature on endogenous economic policy has focused either on electoral competition among parties or on legislative decision-making but not on a unified model of both. Examples of party-based analyses are Alesina [1], on macroeconomic policy, Alesina and Tabellini [2] and Persson and Svensson [19] on public debt, Persson and Tabellini [20] on fiscal policy in a common market and Roemer [22, 23] on the democratic class struggle. Legislature-based models are used in Baron and Ferejohn [4] and Weingast, Shepsle and Johnsen [27] to analyze the geographic distribution of costs and benefits of public projects. A skeptical view of party-based models (at least of their relevance for the US) is proposed by Krehbiel [11, 12].

There are other examples of related literature as well, although there are some important ways in which this paper departs from them. A formal comparative perspective on the role of legislatures and parties can be found in Persson, Roland and Tabellini [18]. Diermeier and Feddersen [9] study also legislative institutions and how they affect partisan behavior in the legislature. However, their focus concentrates on one particular institutional setting (the confidence vote in parliamentary regimes). None of these two papers considers the electoral

origins of the legislature.

It may be argued that a polity's constitutional framework, far from being a *datum* that affects the choices of the political parties, is in fact the endogenous result of the interaction of those same parties. But in order to understand the endogenous formation of constitutional arrangements, there is prior step that needs to be taken: examining the impact of such arrangements on the opportunities available to the political parties. This is the task undertaken here.

An important feature of the present model is its extensive use of sophisticated voting. The voters use their knowledge about the constitutional setting to assess the real impact of different candidates on final policy outcomes. The idea of combining strategic voting with explicit institutional settings is also present in recent work by Austen-Smith [3]. There are, however, a few differences. First, the present model introduces electoral uncertainty, which will prove to have important implications. Second, Austen-Smith assigns a crucial role to *party constitutions*, that is, social choice rules that generate the parties' platforms from their candidates' strategies, and which will dictate the policy of the elected legislature. As he rightly states, this framework is more reasonable the more the parties can be thought of as controlling the legislature. In contrast, a major goal of this paper is to show how *legislative institutions* can shape policy outcomes, even keeping constant the internal structure of the parties. A central argument of the paper is that this interaction between the political institutions and the electorate's strategic behavior can explain some of the most crucial patterns of legislative behavior.

First, the model predicts that the voting positions observed in the legislature will be correlated with the party labels. In other words, the legislators casting the most 'rightist' votes will systematically belong to the 'Rightist' party while the 'leftist' legislators will belong to the 'Leftist' party. This is such a widely observed pattern in so many different legislatures that we tend to take it for granted. However, it is surprising that many standard models of open-rule regimes that rely on the sincere voting assumption, consistently *fail* to predict this same pattern. In fact, according to these models, extremist legislators should have *equal* chances of belonging to any party.

The present model also takes a step further, generating testable theories as to how such correlation will vary as the distribution of voters' preferences change in a given polity. In this sense, the model is relevant to the literature on quantitative studies of legislative behavior. (McCarty, Poole and Rosenthal [16], Collie and Mason [6] and Smith [25] are examples from this literature which focus on the US Congress.)

In this paper, parties control resources vital for the electoral viability of the candidates. This would suggest that they can use such resources to influence the legislators' behavior regardless of the constitutional setting. However, the second result of this paper shows that decision-making rules that allow for legislative initiative deprive political parties of most (though not all) of their influence over policy outcomes. In this case, the policy will be determined by the median *district* since it is the district that will return the pivotal legislator. In turn, the third result proves that without legislative initiative, the parties propose divergent platforms

and that the victorious party is fully able of implementing its platform as policy. Intuitively, legislative initiative allows for the individual members of the legislature to undo the partisan arrangements that may have been made during the electoral phase of the political process.

For decades, comparative political economy has highlighted the extent to which the expansion of franchise in many countries led to the formation of national, class-based parties (e.g. Labor in Britain).¹ However, the US, allegedly the first country to attain virtually universal franchise, followed a very different path. The results of this model suggest the following hypothesis: while Britain, at the time of expanding the franchise, had already completed the transition to a closed-rule regime, the US remained (as it still is today) an open-rule polity. In the light of the findings of this paper, cross-district, class-based coalitions in the US were bound to be much less successful than what they eventually became in Britain; under an open rule the political parties, for all their control over their candidates, cannot prevent the policy results from gravitating toward the median districts' preferred outcomes.

A fourth result from this model says that the degree to which the constitutional framework affects policy outcomes depends upon the level of intra-district homogeneity of the voters' preferences. In a closed-rule polity, the more homogeneous the preferences *within* the districts, the more the parties' platforms will converge to the median voter of the median district, and hence, the more the policy outcome will resemble the one that would obtain under an open-rule regime. This raises a host of relevant questions about policies that modify the distribution and sorting of citizens across districts. Districts' boundaries and sizes may be changed by legislative fiat, but there are also other, deeper economic forces at work in this regard, like migration. Once the model is developed and analyzed, I will argue in the concluding section that this fact has implications for the way we think of endogenous economic policies in federalized systems.

A widely held view attributes the different legislative patterns observed here to different degrees of "party strength," where the latter is interpreted as the ability of parties to coerce their members' vote in the legislature. Here this is not the case. In this paper, the organizational prerogatives of the parties vis-a-vis their caucuses remain identical across models. Furthermore, the legislature votes following strict majority rule. I believe this approach, on top of being parsimonious, has the advantage of making explicit what is meant by party strength. In fact, the present model suggests that we ought to distinguish between "system-wide" sources of party strength, emanating from the constitutional allocation of decision-making power and "party-specific" sources, that depend on the internal organization of each party.

The paper is organized as follows: Section 2 presents the basic setup and notation of the model. Section 3 calculates the equilibrium of the model under an open-rule legislative institution, while Section 4 does the same for the closed-rule case. Section 5 studies the impact of intra-district homogeneity on the policy outcomes of the model. Finally, section 6 summarizes the main results, offers some concluding remarks and discusses possible extensions. Lengthy proofs are relegated to the Appendix.

¹A list of studies of this nature would constitute a paper in itself. Lipset and Rokkan [13] is one of the classical references. A more recent example is Luebbert [14].

2 Basic Setting, Definitions and Notation

2.1 Institutions

A set of citizens is partitioned into N districts, where N is an odd number. The subindex A will be used to denote a generic district while the subindex M will refer to a special district (the “median district”) to be defined below; I will make explicit use of the district subindex only when needed to avoid confusion. There is one policy issue common to all districts so that the policy space is represented by $X \subseteq \mathfrak{R}$. Each district elects a *legislator*. All the legislators convene in a single body (the *legislature*) and they choose the policy to be implemented by *majority rule*. I will consider two alternative agenda-setting rules later.

2.2 Agents

There are three groups of agents: citizens, (local) candidates, and (national) parties. These have single-peaked preferences over policy outcomes. For convenience, these will be represented by a Euclidean utility function though none of the essential arguments depend on this specific functional form. The ideal policy point of a citizen will then be a sufficient description of his type. To simplify notation, if a citizen’s ideal point is x_i , his type is will be written as i . The policy preferences of a type i citizen are described by the function u_i where:

$$u_i(x) = -(x_i - x)^2.$$

The same is true of the parties. In particular, there are two parties: Left and Right, and τ_L and τ_R denote their respective ideal points. The Left’s policy preferences will be labeled u_L while u_R will correspond to the Right’s policy preferences. Summarizing:

$$u_L(x) = -(\tau_L - x)^2,$$

$$u_R(x) = -(\tau_R - x)^2.$$

Without loss of generality, and to ease visualization, $\tau_L < \tau_R$, that is, the Left party is actually “to the left of” the Right party.

The candidates’ objective is to maximize their probability of victory. It is premature to explain how this probability is calculated since we need some other ingredients of the model. In every district there are two candidates, one for each national party. Thus, l and r will represent the Left’s and Right’s candidates respectively (with their respective district subindex when needed).

2.3 Stages of the Game and Strategies

The game consists of the following stages:

Convention Stage: Parties L and R each choose a point in the policy space, respectively called x_L and x_R , which will be their *national platforms*.

Campaign Stage: The $2N$ candidates choose points in \mathfrak{R} which will represent their *local platforms*. The local platform of candidates l, r in district A will be denoted x_{lA} and x_{rA} respectively. These local platforms become common knowledge for the national parties but are not yet disclosed to the voters.

Endorsement Stage: The parties decide, for each district, whether or not to field their respective candidate. If party L nominates a candidate in district A , I will write $e_{LA} = 1$; otherwise, $e_{LA} = 0$. Analogous definitions hold for party R .

Electoral Stage: The local platforms of those candidates actually fielded are revealed to the citizens. A state ω is chosen by Nature. Within each district, a group of voters (whose distribution is governed by ω) is selected from the pool of citizens and elections take place. So, the electoral outcome is uncertain.

Legislative Stage: The victorious candidates become legislators and convene to decide the policy. Each legislator votes according to Euclidean preferences whose ideal point is the *local platform* she announced in the campaign stage. From now on, x_A will denote the platform of district A 's *legislator*, i.e. the platform of the victorious candidate ($x_A \in \{x_{lA}, x_{rA}\}$). The set of ideal points of a legislature will be $\mathbf{x} = \{x_1, \dots, x_N\}$. The final policy outcome, x^* , depends on *all* the strategies chosen in the other stages.

Strictly speaking:

$$x^* = x^*(x_L, x_R, \{x_{lA}\}_{A \in \{1, \dots, N\}}, \{x_{rA}\}_{A \in \{1, \dots, N\}}, \{e_{lA}\}_{A \in \{1, \dots, N\}}, \{e_{rA}\}_{A \in \{1, \dots, N\}}).$$

To simplify notation, when referring to x^* , I will drop the arguments not directly relevant to the claim being made.

2.4 Legislative Rules

It is important to keep in mind that the legislature in this model always uses majority rule to arrive to any decision. What changes between one institutional setting and the other is the way the agenda is shaped. In one case, the *open-rule* system, the legislature is allowed to introduce any kind of amendment to the legislative proposals it receives. In contrast, under the *closed-rule* regime, the agenda is composed of only two possible alternatives: the national platforms x_L, x_R proposed by the parties. No further amendment is possible. Formally, let \mathbf{a} denote the agenda- that is, the set of policy alternatives admissible to be considered by the legislature. The two possible settings are then:

1. Open-rule legislature: $\mathbf{a}_O = \mathfrak{R}$.
2. Closed-rule legislature: $\mathbf{a}_C = \{x_L, x_R\}$.

2.5 Auxiliary Assumptions

The basic setup just described needs to be complemented by some extra assumptions, specifically with regard to *a*) the distribution of voters within districts, *b*) how is electoral uncertainty introduced, *c*) what the role of endorsements is, and *d*) how the probability of victory is calculated within the model.

Electoral Districts There is a continuum of citizens in every district. Further, across districts the distribution of citizens' preferences varies. Thus, letting G_A be the cumulative distribution of citizens' ideal points in district A , with cumulative distribution G_B for district B , then in general, $G_A(x) \neq G_B(x)$ for any x . Define the location of the median citizen in district A by μ_A (i.e. $G_A(\mu_A) = 1/2$). Without loss of generality, label districts so that $\mu_1 < \dots < \mu_M < \dots < \mu_N$, where $M \equiv \frac{N+1}{2}$; therefore, district M is the *median district*. Furthermore, the parties' ideal points will be assumed to bound the ideal point of the median citizen in the median district: $\tau_L < \mu_M < \tau_R$. This is a plausible assumption. Without it, the "left" party would be to the right of the majority of districts (or the "right" party would be to the left of them).

Electoral Uncertainty The approach used here to introduce electoral uncertainty is the one common in models of probabilistic voting (see, for instance, the work of Calvert [5], Coughlin [7], Wittman [28] and Roemer [22], among others). Let $\omega \in [0, 1]$ be a state chosen by Nature at election day with a cumulative distribution $F(\omega)$. After that draw, in every district a sample of *voters* is selected from the set of citizens. The actual location of the *median voter* in each district depends upon the state ω selected. Formally, denoting by i_{mA} the median voter of district A , then $i_{mA}(\omega)$ is a strictly increasing function of ω . Thus, for any location of the median voter, we can retrieve the value of the state with the inverse function i_m^{-1} . Furthermore, I will make the following assumption about the behavior of i_m : In every district, for any two intervals of types $[i_0, i_1], [i'_0, i'_1]$,

$$G(i_1) - G(i_0) \geq G(i'_1) - G(i'_0) \iff F(i_m^{-1}(i_1)) - F(i_m^{-1}(i_0)) \geq F(i_m^{-1}(i'_1)) - F(i_m^{-1}(i'_0)).$$

This assumption means that the mapping from states to locations of the median voter is such that as more citizens are located in certain interval, the more likely it is that the median voter belongs to that same interval. It is a plausible condition on the responsiveness of the process generating the median voter location to the actual distribution of preferences in the electorate.

From this assumption, we obtain a simple but useful lemma. Its proof is straightforward and is peripheral to the main point of the paper, so I omit it:

Lemma 1 *For every district A , the location of its median voter in the median state ω_M , defined by $F(\omega_M) = 1/2$, is equal to the location of its median citizen, that is: $\forall A, F(i_m^{-1}(\mu_A)) = 1/2$.*

I will also assume that in every district, $G(\tau_L) > 0$ and $G(\tau_R) < 1$. This assumption implies that both parties have some support (no matter how small) in every district.

Since there is electoral uncertainty, the parties cannot know what the exact effect of supporting their candidates will be. The policy outcome is decided by a legislature whose composition is unknown at the endorsement stage. From now on, I will denote by $p(\mathbf{x})$ the probability that legislature \mathbf{x} is elected. So, both parties and voters evaluate a candidate by the expected utility they obtain from her being elected. For example, for party L , the expected utility derived from electing candidate l in district A is given by

$$E(u_L(x_{lA})) = \sum_{\mathbf{x}_{-A}} u_L(x^*(\mathbf{x}_{-A}, x_{lA}))p(\mathbf{x}_{-A}),$$

where a similar definition holds for party R and for any given citizen of type i .

Endorsements In this model, parties command resources essential for the electoral viability of the candidates. The following assumptions capture the basic elements of the relationship between parties and candidates:

- If a candidate is not endorsed by her party, her probability of victory is 0.
- If, at the end of the endorsement stage, a candidate turns out to run unopposed, the platform she will disclose is the ideal point of her party. That is: $e_{LA} = 1, e_{RA} = 0 \Rightarrow x_{lA} = \tau_L; e_{LA} = 0, e_{RA} = 1 \Rightarrow x_{rA} = \tau_R$.
- If both parties refuse to field a candidate in a district, this district's legislator will be selected at random from the citizens so that the distribution of the would-be legislator's preferences is the same as the distribution of citizens' preferences.

The first assumption implicitly denies the possibility of independent candidates, that is, candidates that run without the support of one of the national parties. The second assumption specifies how the candidate breaks ties between strategies if her victory is assured. The third assumption amounts to claim that the only case in which independent candidates have any viability occurs when the two major parties fail to field a candidate of their own. This is consistent with the basic outlook adopted in the first assumption.

Probability of Victory When both candidates in a district are endorsed, their probability of victory is a function of the platforms proposed. From now on, $\pi_A(x_l, x_r)$ will denote the probability of victory of candidate l in district A , given that the platforms are x_l, x_r . By the same token, $1 - \pi_A(x_l, x_r)$ will denote r 's probability of victory.

Legislators vote according to their platform regardless of party labels. If both candidates in a district choose the same platform, their votes in the legislature will be exactly the same so their impact on the policy outcome chosen will also be the same. Hence, voters will be indifferent between them and their vote will be determined by the toss of a fair coin. In any other case, we need to know the location of the type of voter indifferent between the two candidates. This location depends upon the legislative rule employed, so, for the time being, we derive the general form of the probability of victory, filling the details once we enter the discussion of each institutional setting.

Under sincere voting, single-peakedness has the convenient implication of partitioning the electorate into two *convex* sets of types: one supporting the Left candidate and the other supporting the Right. The following lemma ensures that the same is true under strategic voting.

Lemma 2 (Single-Crossing Property of Voter's Preferences) *Denote by i^* the voter indifferent between two candidates in any district. If $x^*(\mathbf{x}_{-A}, x_A)$ is monotonic non-decreasing in x_A , then $x_{lA} < x_{rA}$ implies that:*

- $\forall i < i^*, x_{lA} \succ_i x_{rA}$ and
- $\forall i > i^*, x_{lA} \prec_i x_{rA}$

Proof: See Appendix A.1.

Without loss of generality, we describe how the probability of victory is determined for the case in which the types of voters who vote for l are lower (to the left of) those who vote for r . In this case, l will win her district's election if the type of the median voter is lower than i^* , or

$$\Pr(i_m < i^*) = \Pr(\omega : i_m(\omega) < i^*) \equiv F(i_m^{-1}(i^*)).$$

Putting all these results together, we obtain:

$$\pi(x_l, x_r) = \begin{cases} F(i_m^{-1}(i^*)) & \text{if } x_l < x_r \\ 1/2 & \text{if } x_l = x_r \\ 1 - F(i_m^{-1}(i^*)) & \text{if } x_l > x_r \end{cases}$$

Notice that the data of a political system are given by the makeup of the electorate (as defined by the distributions G_A and the median-voter generating processes i_{mA}), the legislative institution and the ideal points of the parties. We can now bring together all these elements of the model in the following definition:

Definition 1 A *polity* is a collection $\mathcal{P} = \langle \{G_A\}_{A \in \{1, \dots, N\}}, \{i_{mA}\}_{A \in \{1, \dots, N\}}, \mathbf{a}, \tau_L, \tau_R \rangle$.

Now we need to specify the solution concept to be used. Verbally, it is simply a restatement of the usual conditions for a Nash equilibrium of a game, that is, that all the strategies are chosen optimally taking as given all the remaining strategies of the other players.

Definition 2 Given a polity \mathcal{P} , a *political (Nash) equilibrium* is a collection of strategies $\langle x_L^*, x_R^*, \{x_{lA}^*\}_{A \in \{1, \dots, N\}}, \{x_{rA}^*\}_{A \in \{1, \dots, N\}}, \{e_{lA}^*\}_{A \in \{1, \dots, N\}}, \{e_{rA}^*\}_{A \in \{1, \dots, N\}} \rangle$ such that:

- $x_L^* = \arg \max_{x_L \in \mathfrak{R}} E(u_L(x^*(x_L, x_R^*)))$ with a perfectly analogous condition for x_R^* .
- For each A , $x_{lA}^* = \arg \max_{x_{lA} \in \mathfrak{R}} \pi(x_{lA}, x_{rA}^*)$, $x_{rA}^* = \arg \min_{x_{rA} \in \mathfrak{R}} \pi(x_{lA}^*, x_{rA})$
- $e_{lA}^* = \arg \max_{e_{lA} \in \{0,1\}} E(u_L(x^*(x_L^*, x_R^*, \{e_{lB}^*\}_{B \neq A}, \{e_{rA}^*\}_{A \in \{1, \dots, N\}})))$ with an analogous condition for e_{rA}^* .

Under this definition, the parties choose their national platforms x_L^* and x_R^* so as to maximize the expected utility derived from the (uncertain) electoral outcome. The same is true of their endorsement strategies $\{e_{lA}^* - A \in \{1, \dots, N\}\}$ and $\{e_{rA}^* - A \in \{1, \dots, N\}\}$. Notice that the endorsement strategy of a party in district A depends not only on the other party's endorsement strategy in that same district but also on the strategies chosen by *both* parties in *all* the remaining districts. In turn, the candidates choose local platforms that maximize their probability of victory in their respective district.

Our next task is to solve for the political equilibria of the two types of political systems considered here: those with open rules and closed rules. That is the goal of the next sections. To distinguish between the two types of model, I will call \mathcal{P}_O an open-rule polity and \mathcal{P}_C a closed-rule polity.

The following result, pertaining to the endorsement strategies, is common to both models. It says that in any district, in equilibrium both candidates are endorsed and the parties are not indifferent between the platform of their candidate and yielding that district to the other candidate running on the ideal point of her party:

Lemma 3 For any polity \mathcal{P} , in a political equilibrium the endorsement strategies and the local platforms are such that, for all districts A

1. $e_{lA} = e_{rA} = 1$
2. $E(u_L(x^*(\mathbf{x}_{-A}, x_{lA}))) > E(u_L(x^*(\mathbf{x}_{-A}, \tau_R)))$ and
3. $E(u_R(x^*(\mathbf{x}_{-A}, x_{rA}))) > E(u_R(x^*(\mathbf{x}_{-A}, \tau_L)))$

Proof: If inequalities 2 and 3 do not hold, then the parties are indifferent between endorsing a candidate or just yielding that district to the other party. So, any randomization between, say, $e_{LA} = 1$ and $e_{LA} = 0$ is consistent with sub-game perfection. From the point of view of the candidates, securing endorsement is a dominant strategy. Therefore, if (say) x_{lA} is such that 2 does not hold, party L has the credible threat of choosing $e_{LA} = 0$ so that x_{lA} is not optimal for the candidate. If 2 and 3 hold, then it is easy to verify that $e_{LA} = 0, e_{RA} = 1$ is not an equilibrium, because party L could increase its pay-off by choosing $e_{LA} = 1$. But, neither is $e_{LA} = e_{RA} = 0$ an equilibrium. In that case, A 's representative will be chosen from G_A so that there is a positive probability that it will be of type τ_R or τ_L . So, L can secure that she is *not* of type τ_R by simply deviating to $e_{LA} = 1$.

3 The Open-Rule Model

Given that this is a multi-stage game, the natural solution concept is that of backward induction. In this section, I will solve the game for the case of open-rule institutions using the customary procedure of solving first for the last stages and working up the decision tree until reaching the first stage.

Legislative Stage As has been stressed repeatedly throughout the paper, this is the only stage in which there is some structural difference between both types of political systems. As said before, under an open rule, the set of alternatives to be voted upon is not constrained in any way. Any member of the legislature can introduce, if she so desires, a new point in the agenda (an *amendment*) to be considered against whatever may be the *status-quo* at the time.

Ideally, a full description of the legislative institution used in each case would include the extensive form of the game underlying the policy-making process. However, there is no need for such detail: the reduced-form spelled out here is enough. It is a well-known fact that under the conditions specified in this definition, the policy outcome is the ideal point of the median legislator. To make this precise, the operator $m(\cdot)$ will denote the median of a set of ideal points. Hence, under an open rule, $x^* = m(\mathbf{x})$.

Electoral Stage The median operator is monotonic non-decreasing with respect to the legislators' platforms. Therefore, Lemma 2 ensures that under strategic voting every district's electorate is partitioned into two convex sets of types, each supporting a different candidate. Thus, if $x_{lA} < x_{rA}$, there will exist a type i^* such that all voters left of i^* will vote for l and, likewise, all voters to the right of i^* will vote for r .

Endorsement Stage As described in Lemma 3, all candidates secure endorsement by choosing platforms such that their parties are not indifferent between fielding them and yielding that district to the other party.

Campaign Stage There is a continuum of equilibria for the local platforms. However, they all share some very important properties. The following theorem describes this set of equilibria. Due to its length, its proof can be found in Appendix A.2.

Theorem 1 *Let $\mathcal{P} = \langle \{G_A\}_{A \in \{1, \dots, N\}}, \{i_{mA}\}_{A \in \{1, \dots, N\}}, \mathbf{a}_O, \tau_L, \tau_R \rangle$ be an open-rule polity and ϵ an arbitrarily small constant such that $0 < \epsilon < \min[\mu_{M+1} - \mu_M, \mu_M - \mu_{M-1}]$. Then, in any equilibrium the local platforms of the candidates are such that:*

- $x_{lA} < \mu_M - \epsilon, x_{rA} = \mu_M + \epsilon$ if $A < M$,
- $x_{lM} = \mu_M = x_{rM}$
- $x_{lA} = \mu_M - \epsilon, x_{rA} > \mu_M + \epsilon$ if $A > M$

Intuitively, the endorsement constraints force the candidates to take stances that, if elected, will lead to a different median legislator at least for some possible electoral outcomes. However, since this differentiation is costly for the candidates in terms of their probability of victory, they try to minimize it by remaining as close as possible to their districts' medians.

The following result is valuable for what follows. Its proof is trivial and will be omitted:

Corollary 1 *For all possible legislatures, the location of the median legislator $m(\mathbf{x}) \in \{\mu_M - \epsilon, \mu_M, \mu_M + \epsilon\}$.*

The second corollary of this theorem, although straightforward to prove, is very important: it establishes that the political equilibria of this model generate a correlation between voting stances and party labels in the possible legislatures. In other words,

Corollary 2 *For an open-rule polity \mathcal{P}_O , given that a legislator's platform is to the left of that of the median legislator, her probability of belonging to the Left party is greater than her probability of belonging to the Right party: that is*

$$\Pr(x_A = x_{lA} \mid x_A < m(\mathbf{x})) > \Pr(x_A = x_{rA} \mid x_A < m(\mathbf{x}))$$

and, likewise,

$$\Pr(x_A = x_{lA} \mid x_A > m(\mathbf{x})) < \Pr(x_A = x_{rA} \mid x_A > m(\mathbf{x}))$$

Proof: I will consider only the first statement, the second being proven by identical arguments. If we can prove that $\Pr(x_A = x_{lA}, x_A > m(\mathbf{x})) > \Pr(x_A = x_{rA}, x_A > m(\mathbf{x}))$, the result follows. Now:

$$\begin{aligned}
& \Pr(x_A = x_{lA}, x_A > m(\mathbf{x})) \\
&= \Pr(x_A = x_{lA}, x_A < \mu_M - \epsilon) + \Pr(x_A = x_{lA}, \mu_M - \epsilon \leq x_A < \mu_M, m(\mathbf{x}) \geq \mu_M) + \\
&\quad \Pr(x_A = x_{lA}, \mu_M \leq x_A < \mu_M + \epsilon, m(\mathbf{x}) = \mu_M + \epsilon) \\
&\leq \Pr(x_{M-1} = x_{lM-1}) + \Pr(x_N = x_{lN}, m(\mathbf{x}) \geq \mu_M) + \Pr(x_M = x_{lM}, m(\mathbf{x}) = \mu_M + \epsilon)
\end{aligned}$$

where the last inequality comes from the fact that the only l candidates that adopt platforms $< \mu_M - \epsilon$ are those running in districts $1, \dots, M - 1$ while the l candidates with platforms $> \mu_M - \epsilon$ run in districts $M + 1, \dots, N$. Applying a similar reasoning to the other expression (that for the Right) we conclude that:

$$\begin{aligned}
& \Pr(x_A = x_{rA}, x_A > m(\mathbf{x})) \\
&= \Pr(x_A = x_{rA}, x_A < \mu_M - \epsilon) + \Pr(x_A = x_{rA}, \mu_M - \epsilon \leq x_A < \mu_M, m(\mathbf{x}) \geq \mu_M) + \\
&\quad \Pr(x_A = x_{rA}, \mu_M \leq x_A < \mu_M + \epsilon, m(\mathbf{x}) = \mu_M + \epsilon) \\
&= \Pr(x_M = x_{rM}, m(\mathbf{x}) = \mu_M + \epsilon)
\end{aligned}$$

because M is the only district where an r candidate chooses a platform $< \mu_M + \epsilon$. Since in district M , both candidates have a probability of victory of $1/2$, the claim follows.

The reason this corollary is so important is as follows: a legislature generated by this model will be such that the legislators to the left of the median member are more likely to be from the L party while those to the right are more likely to be from the R party. The most ‘rightist’ legislator of the L party will be close to the center and the same is true for the most ‘leftist’ legislator of the R party. In other words, there will be a correlation between voting stances and party membership a pattern displayed by many real-life legislatures.

Space considerations rule out a detailed discussion, but it is important to realize how this result would break down if we dropped any of the crucial assumptions.

Case 1. No endorsement constraints, sincere voting: Here both candidates in every district will choose the platform that maximizes their probability of victory, not unlike the standard Downsian model. In that case, the probability of victory for all candidates is $1/2$. This implies that the most leftist legislator is equally likely to be from the Left or from the Right. Notice that this overtly wrong prediction comes from the *standard* median-voter model when applied to the multi-district case.

Case 2. Endorsement constraints, sincere voting: Under these assumptions, the candidates still need to differentiate themselves from their rivals so as to secure endorsement. But, sincere

voting means that voters fail to see through the legislative decision-making process. Therefore, the “majority” candidates (that is, the Left candidates in districts $A \leq M - 1$ and the Right candidates in districts $A \geq M + 1$) benefit from choosing platforms as close as possible to those of their rivals as far as is consistent with securing endorsement, *even if this does not affect* the possible median locations in the legislature. In that case, all the candidates will choose platforms infinitesimally close to μ_M . Needless to say, this is another implausible prediction.

Convention Stage As seen in Corollary 1, in equilibrium under an open rule the median legislator, and hence the policy outcome, is infinitesimally close to μ_M and, furthermore, her location is independent of the party platforms.² Therefore, the parties’ pay-offs, are also independent of their national platforms. The optimal national platforms are then arbitrary. All the elements of this model can be put together in the following theorem, whose proof is already contained in the preceding arguments:

Theorem 2 (Legislative Government under Open Rule) *Let \mathcal{P} be a polity with the legislative institution of open rule prevailing and ϵ a vanishingly small constant $0 < \epsilon < \min[\mu_{M+1} - \mu_M, \mu_M - \mu_{M-1}]$. Then, in all the political Nash equilibria:*

- $e_{L,A}^* = e_{R,A}^* = 1 \quad \forall A \in \{1, \dots, N\}$
- *The local platforms are as described in Theorem 1*

and the policy outcome x^ is independent of x_L, x_R and is such that $x^* \in \{\mu_M - \epsilon, \mu_M, \mu_M + \epsilon\}$.*

Remark: Notice that under an open rule, although national platforms are irrelevant, parties themselves are not. In fact, it is thanks to their endorsement prerogatives that the policy outcome is not fixed at μ_M . Were we to use a sincere voting model with no endorsement constraints, the policy outcome would be entirely fixed at μ_M . In that case, it would be hard to explain why political parties come to exist in the first place. But this model shows that, although limited, there is a role for parties: they have the possibility of fielding minority candidates that, if elected (no matter how unlikely this may be) will actually modify to some extent the legislature’s choice.

4 The Closed-Rule Model

Legislative Stage As said in the introduction, the main difference between open and closed rules as interpreted here is the fact that in the latter the agenda is entirely dictated by the parties’ platforms. Therefore, the policy outcome is the party platform which obtains the simple majority in the legislature:

²Alternatively, if we think of endorsements as costly for the parties, then the value of ϵ would be the one necessary for them to recover such cost.

$$x^* = \begin{cases} x_L & \text{if } \#\{A : u_A(x_L) > u_A(x_R)\} > \#\{A : u_A(x_L) < u_A(x_R)\} \\ x_R & \text{if } \#\{A : u_A(x_L) > u_A(x_R)\} < \#\{A : u_A(x_L) < u_A(x_R)\} \end{cases}$$

where, u_A stands for the utility function of district A 's legislator which is, as said before, the utility function whose ideal point is the platform she announced as a candidate.

Electoral Stage Under a closed rule, the only relevant feature of the candidates the voters care about is the agenda point they will support (x_L or x_R) if elected. The candidates' specific platforms become *irrelevant*. Unlike in standard spatial models, here $u_i(x_l) > u_i(x_r)$ is not a sufficient condition for voter i to vote for l . In fact, in spite of this difference in utility, if both candidates would support the same platform in the legislature, if elected, all the voters will be indifferent between them and the probability of victory for both candidates will be $1/2$. However, the inequality is still a necessary condition. Therefore, if the candidates vote differently in the legislature (e.g. $u_l(x_L) > u_l(x_R), u_r(x_L) < u_r(x_R)$), the indifferent type in every district is: $i^* = \frac{x_L + x_R}{2}$. So:

$$\pi_A(x_l, x_r) = \begin{cases} F(i_{mA}^{-1}(\frac{x_L + x_R}{2})) & \text{if } x_l < x_r \\ 1/2 & \text{if } x_l = x_r \\ 1 - F(i_{mA}^{-1}(\frac{x_L + x_R}{2})) & \text{if } x_l > x_r \end{cases}$$

Endorsement Stage Once again, each candidate is endorsed and the platform she chooses is such that the parties are not indifferent between fielding her or allowing the other party's candidate to win her district.

Campaign Stage Once again, securing endorsement is a dominant strategy for the candidates. Therefore, from the analysis of the endorsement stage, we conclude that for the candidates to achieve this, they need to propose local platforms that support their respective parties. This can be stated as the following lemma:

Lemma 4 For a polity $\mathcal{P} = \langle \{G_A\}_{A \in \{1, \dots, N\}}, \{i_{m,A}\}_{A \in \{1, \dots, N\}}, \mathbf{a}_C, \tau_L, \tau_R \rangle$, in any equilibrium, the local platforms of the candidates are such that

$$|x_{lA} - x_L^*| < |x_{lA} - x_R^*|, |x_{rA} - x_L^*| > |x_{rA} - x_R^*| \quad \forall A$$

Convention Stage Lemma 4 is crucial to the analysis of the convention stage under a closed rule. The first thing to notice is that in this model, unlike what happens in the open rule model, there is one sense in which a local candidate is really a *party's* candidate: if elected she will support her party's national platform and that is the end of it. In the closed rule model, a party's victory implies more than simply obtaining a majority of dubious relevance in the legislature. Here victory means actually getting to implement its national platform.

There is a second implication of this result. From the point of view of a national party, the general election resembles strikingly a *single-district* election under majority rule, where each district represents an individual vote. Call $\Pi(x_L, x_R)$ the probability that party L wins a majority of seats in the legislature, i.e. $\Pi(x_L, x_R) = \Pr(\#\{A : u_A(x_L) > u_A(x_R)\} > \{A : u_A(x_R) > u_A(x_L)\})$. Therefore, each party's pay-off becomes:

$$E(u_L(x_L)) = u_L(x_L)\Pi(x_L, x_R) + u_L(x_R)(1 - \Pi(x_L, x_R))$$

$$E(u_R(x_R)) = u_R(x_L)\Pi(x_L, x_R) + u_R(x_R)(1 - \Pi(x_L, x_R))$$

Notice that this is exactly the same pay-off function usually assumed for parties with policy preferences in one-district elections. This enables us to use a well-known result of spatial competition which will form part of the following theorem characterizing the equilibrium under closed rules.

Theorem 3 (Party Government under Closed Rules) *Let \mathcal{P} be a polity with the legislative institution of closed rule. Then, in all the political Nash equilibria*

- $e_{LA}^* = e_{RA}^* = 1 \quad \forall A \in \{1, \dots, N\}$
- $x_{lA}^* < \frac{x_L^* + x_R^*}{2}, x_{rA}^* > \frac{x_L^* + x_R^*}{2} \quad \forall A \in \{1, \dots, N\}$
- $x_L^* < x_R^*$

and the policy outcome x^ is the national platform that obtains the majority of the legislature: $x^* \in \{x_L^*, x_R^*\}$.*

Proof: The first two claims of the theorem have already been proven. The third claim requires a somewhat lengthy proof, to be found in Appendix A.3

Remark: According to this result, the candidates will virtually become “employees” of their parties, with the task of supporting it in the legislature. This coincides with the way individual legislators are normally thought of in closed-rule regimes.

5 Legislative Outcomes and the Electorate's Structure: Comparative Analysis

The two main theorems show how the legislative institutions contribute to shape the policy outcomes in a democracy. However, their relative importance actually depends on the makeup of the electorate. Is there any special feature of the citizenry that will dictate the actual performance of the legislative institutions in shaping the policy outcomes? The main result of this section is that there is one: the degree of *intradistrict heterogeneity*.

Before going into the main results, it is worth making precise what it will mean formally for a polity to experience an increase in intra-district homogeneity. To that end, I will generate a sequence of polities for which the electorate becomes increasingly homogeneous at the district level. The first step is, for every district A , choose a sequence of intervals $\{\underline{\mu}, \bar{\mu}\}_{A,n}$ of citizens' types with the following properties:

- $\underline{\mu}_{A,n} < \mu_A < \bar{\mu}_{A,n} \quad \forall n$ i.e. each of the n intervals bounds the median citizen's type in district A .
- $\forall m > n, \bar{\mu}_{A,m} - \underline{\mu}_{A,m} \leq \bar{\mu}_{A,n} - \underline{\mu}_{A,n}$ i.e. as we go further in the sequence, the intervals narrow.
- $\lim_{n \rightarrow \infty} \bar{\mu}_{A,n} - \underline{\mu}_{A,n} = 0$ i.e. in the limit, the intervals become arbitrarily small.

For every district A , consider a sequence of probability measures $\{G_A\}_n$ such that:

- $G_{A,n+1}(\mu_A) = G_{A,n}(\mu_A) = 1/2, \quad \forall n, A$.
- $G_{A,n+1}$ assigns at least as much probability as $G_{A,n}$ to every subinterval in $[\underline{\mu}, \bar{\mu}]_{A,n}$.
- $G_{A,n+1}$ assigns no more probability than $G_{A,n}$ to every subinterval in $(-\infty, \underline{\mu}]_{A,n}$.
- $G_{A,n+1}$ assigns no more probability than $G_{A,n}$ to every subinterval in $[\bar{\mu}, \infty)_{A,n}$.

Therefore, $G_{A,n+1}$ is a *median-preserving risk reduction* (henceforth m.p.r.r) of $G_{A,n}$. (This definition is analogous to the definition of mean-preserving risk reduction given in Machina and Pratt [15].) A sequence of measures thus constructed will be said to describe a sequence of polities with ***increasing intradistrict homogeneity***.

What are the effects of a change of intradistrict homogeneity in the two cases? The easiest case to analyze is that of the open rule. Since the only determinant of the policy outcome is the location of the median citizen in the median district, we know that the legislation implemented will not change. However, the correlation between party and ideology will increase as the districts become more homogeneous. This is a testable implication of the model, related to the current studies on legislative polarization in countries like the US. This is the claim of the following lemma:

Lemma 5 *Let $\mathcal{P}_{O,n}$ be a sequence of polities under an open rule with increasing intradistrict homogeneity. Then, the probability of a legislator to the left of the median belonging to the Left party goes to one. The same is true about the probability of a legislator to the right of the median belonging to the Right party. Formally, $\lim_{n \rightarrow \infty} \Pr_n(x_A = x_{lA} \mid x_A < m(\mathbf{x})) = 1$. Analogously, $\lim_{n \rightarrow \infty} \Pr_n(x_A = x_{rA} \mid x_A > m(\mathbf{x})) = 1$.*

Proof: There are only two types of candidates running on local platforms in the interval $[\mu_M - \epsilon, \mu_M + \epsilon]$: l candidates running in districts $A > M$ and r candidates running in districts $A < M$. Focusing only on the first group, their probability of victory is $\pi_A(x_l, x_r) \approx F(i_{mA}^{-1}(\mu_M))$.

Since the sequence of polities is such that intradistrict homogeneity is increasing, then $\lim_{n \rightarrow \infty} \pi_A(x_l, x_r) = 0$. Therefore, $\lim_{n \rightarrow \infty} \Pr_n(\mu_M - \epsilon < x_A < \mu_M + \epsilon) = 0$, and therefore:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \Pr_n(x_A = x_{lA} \mid x_A < m(\mathbf{x})) \\ &= \lim_{n \rightarrow \infty} \frac{\Pr_n(x_A = x_{lA}, x_A < \mu_M - \epsilon) + \Pr_n(x_A = x_{lA}, \mu_M - \epsilon < x_A < \mu_M + \epsilon)}{\Pr_n(x_A < \mu_M - \epsilon) + \Pr_n(\mu_M - \epsilon < x_A < m(\mathbf{x}))} \\ &= \lim_{n \rightarrow \infty} \frac{\Pr_n(x_A = x_{lA}, x_A < \mu_M - \epsilon)}{\Pr_n(x_A < \mu_M - \epsilon)} \\ &= 1 \end{aligned}$$

A decrease in interdistrict homogeneity would have the same effect of raising the correlation between ideology and party in the legislature. This can be seen by fixing μ_M and allowing all the values $\mu_A < \mu_M$ to shift leftwards and all the values $\mu_B > \mu_M$ to shift rightwards as described in the next lemma:

Lemma 6 *Let $\mathcal{P}_O, \mathcal{P}'_O$ be two open rule polities identical in all respect except for:*

- $\mu'_A < \mu_A$ for all $A < M$
- $\mu'_M = \mu_M$
- $\mu'_A > \mu_A$ for all $A > M$.

Then, $\Pr(x_A = x_{lA} \mid x_A < m(\mathbf{x})) < \Pr(x'_A = x'_{lA} \mid x'_A > m(\mathbf{x}'))$.

Proof: The l candidates in districts $A < M$ in polity \mathcal{P}'_O have a higher probability of victory than in polity \mathcal{P}_O . This is due to the fact that the indifferent type i^* remains at the same location in both polities ($\approx \mu_M$) but the distributions G'_A for $A < M$ assign more mass to the interval $(-\infty, \mu_M]$ than the distributions G_A . An analogous argument holds for the probability

of victory of the r candidates in districts $A > M$. Since all these are the only candidates that chose platforms out of the interval $[\mu_M - \epsilon, \mu_M + \epsilon]$, the result follows.

Remark: This result is in keeping with findings in American politics about the rise of “partisanship” in Congress, as related to changes in the electorate.³ As districts become more different from each other, the legislatures become more polarized. Notice, however, that this does not imply necessarily more extreme policy outcomes. In the example described in the lemma, the median legislator remains in the same location throughout the changes in the electorate, so that the policy choices do not change.

Closed-rule polities respond quite differently to intradistrict homogeneity. In this case, the policy outcome actually changes as a function of changes in the districts’ preference distribution. Moreover, as the districts become more homogeneous, the policy outcomes approach what would obtain under an open-rule institution. In other words, the degree to which the constitutional setting affects the policies of a society depend on the way voters are sorted into districts. That is the result of the next theorem. To grasp the intuition behind it, consider what would happen if intradistrict heterogeneity were entirely suppressed, that is, if in each district all the citizens were of the same type and, therefore, the location of the median voter became deterministic. In that case, from the point of view of the parties, their probability of winning any specific seat in the legislature would be either 0 or 1, regardless of the pair of platforms chosen. But that means that for the election as a whole, the probability of victory would also be either 0, 1 or 1/2 (the last of these, only if both propose the same platform). It is a well-known fact that without electoral uncertainty even parties with policy preferences will converge in their platforms. Now, where will this convergence occur? For all intents and purposes, each district becomes, from the point of view of the parties, like a single voter. Therefore, it is not surprising that for each party, the maximin strategy is to propose as a policy the ideal point of *the* voter of the median district. Since this is a zero-sum game, the standard arguments of Downsian convergence apply and hence in equilibrium both parties propose that same platform.

Theorem 4 *Let $\{\mathcal{P}_{C,n}\}$ be a sequence of closed-rule polities with increasing intradistrict homogeneity. Then, the sequence of equilibrium policy outcomes $(x_{L,n}^*, x_{R,n}^*)$ is such that $x_{L,n}^* \rightarrow \mu_M, x_{R,n}^* \rightarrow \mu_M$.*

Remark: Technically, this theorem is the multi-district version of a result obtained previously by Roemer [22]. However, the interpretation given here is different since in this model there is an explicit reason for the reduction of the electoral uncertainty: changes in the population’s distribution.

Proof: See Appendix A.4

³See, for example, Smith [25].

Table 6: Determination of Policy under each Legislative Rule

		Type of Regime	
		Open-Rule	Closed-Rule
<i>Intradistrict</i> Distribution of Preferences	Heterogeneous	Median District	(National) Parties
	Homogeneous	Median District	Median District

6 Concluding Remarks

Before going into a broader discussion of the results of this paper, I will summarize them. The paper argues that constitutional rules, coupled with strategic voting, dictate the balance of power between the political parties and the legislature. The main theorems of the paper are consequences of this basic insight. First, strategic voting generates legislatures in which their members’ voting stances are correlated with their party labels. (That is, “leftist” legislators belong to the “Left” party.) In open-rule legislatures, this correlation increases as *intradistrict* homogeneity increases and *interdistrict* homogeneity decreases. Second, closed-rule legislatures allow for a determinant role of parties and, moreover, for policy differentiation between them. In contrast, under an open rule, for all their prominence as endorsers of local candidates, the parties cannot prevent the policy outcomes from gravitating toward the median citizen of the median district. Furthermore, under closed-rule legislation, the parties’ decisiveness in the policy-making process is undermined as the electorate is sorted into increasingly homogeneous districts. In the limit, if all the districts are populated by voters with identical preferences, then both parties propose the same platform, which coincides, once again, with the median citizen of the median district. In other words, the closed-rule model produces the same policy outcome as the open-rule model. Table 6 summarizes this result.

This comparative analysis is relevant for the political economy of federalism. Models of competition between local communities in the provision of public goods (in the tradition begun by Tiebout [26]) generally conclude that fiscal federalism leads to a stratification of agents whereby the population of each community becomes increasingly homogeneous in terms of preferences and income (see for example Epple and Romer [10]). A tacit assumption in the analysis of endogenous economic policies under federalism is that there is some sort of separability between the local and the national components of taxation. According to this view, competition between localities may affect (in fact, is intended to affect) the level of public goods provision and hence the level of taxation within each community, while leaving intact national taxation. This separability has powerful political implications: if it holds, it implies that whatever the distributional consequences of fiscal federalism, governments will still have the possibility of undoing them through national, redistributive taxation. Under this assumption, fiscal federalism should be able to command a wide consensus: individuals of opposing views on income

distribution could support it, “saving their energies” for the discussion of distributive taxation.

The model proposed here challenges this separability assumption. It argues that the endogenous choice of national policies depends on the specific way in which voters are sorted into districts. Therefore, it predicts that, in a closed-rule regime, fiscal federalism will have unintended consequences for the endogenous determination of economic policy. As local jurisdictions are allowed to choose different tax schedules and different levels of public goods provision, this generates incentives for the citizens to move to their preferred localities. In other words, the jurisdictions become populated by agents with similar tastes and income. By virtue of Theorem 4, such increase in intradistrict homogeneity brought about by stratification raises the cost the parties would incur were they to propose extremist platforms. As extremist districts become “safe seats” for the parties, the latter have more incentives to win the centrist districts by converging toward their ideal points. If the relevant policy dimension is economic redistribution (e.g. through taxes) as occurs so often in most modern democracies, this convergence will shape the spectrum of politically viable distributive proposals. As citizens sort themselves into homogeneous districts, a massive process of “spontaneous gerrymandering” is set in motion, enhancing the strategic importance of the median citizens (arguably, the middle class in industrialized democracies), that now live and vote in homogeneous and pivotal districts. Parties cannot afford to ignore these voters.⁴

The characterization of legislative institutions offered here matches observations made in countries that have undergone significant constitutional changes affecting the degree of legislative initiative. The upsurge of party government in England, ushered in by the First Reform Act of 1832, brought an end to what has been called the “Golden Age of the MP.” It has been argued (see Cox [8]) that this shift in the balance of power between the parties and Parliament is responsible for the modern pattern of partisan voting both among the electorate and the MP’s, just as predicted by this model.

The prediction of correlation between ideology and party in the legislature provides support for the assumption of strategic voting. Usual models of constituency elections, when solved under the assumption of sincere voting, predict that the local candidates will converge to their district’s median point so that they should have equal chances of winning. Were this to be true, all the legislators across the ideological spectrum would have a 50/50 chance of belonging to any of the two parties, a prediction consistently falsified by representative bodies in many different contexts.

The results about intradistrict heterogeneity have implications for our understanding of the political effects of structural changes among the electorate and their interaction with constitu-

⁴It is important to emphasize that the model’s assumption about single-peakedness in voters’ preferences is supple enough to accommodate different economic decision problems. Even when single-peakedness does not obtain, problems of distributive taxation can be formulated so that, under some general properties, there is a “natural” ordering of citizens. Thus, the conventional arguments about the decisiveness of the median voter (which is the crucial point here) still hold (see for instance Roberts [21], Meltzer and Richard [17], and Roemer [22]).

tional rules. A first area where these results could be put to use is in the current research that looks for connections between the electorate's structure and the parties' presence in the American Congress. Given that the U.S. largely resembles an open-rule regime (as defined in this paper), the hypothesis that increased intradistrict homogeneity and interdistrict heterogeneity increase the party vote in open-rule legislatures is, in principle, testable.

Another place where this type of modeling could be employed is in the study of electoral districting and the historic process of franchise expansion. In fact, in any country, these two legal procedures are the most immediate mechanism to alter the composition of the electorate. The conclusions derived from strategic voting could shed light on the impact of such measures on the policy outcomes chosen by democracies under different constitutional frameworks.

Finally, another direction in which this line of argument can be extended lies in the analysis of other types of electoral systems. This paper has focused entirely on single-member district elections. But a fuller understanding of the interaction between legislative institutions and characteristics of the electorate requires the analysis of proportional representation. This I regard as a necessary step in future research.

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A Proofs of Results

A.1 Proof of Lemma 2

Under strategic voting, the voters evaluate their candidates not by their declared local platform, but by the impact such platform will have on the final policy outcome. However, since there is electoral uncertainty, the voters need to evaluate their candidates' impact over all the possible legislatures. Therefore, we will need some notation for the probability of different legislatures being elected. Hence, $p(\mathbf{x})$ is the probability that legislature $\mathbf{x} = (x_1, \dots, x_N)$ is elected:

$$p(\mathbf{x}) = \prod_{A=1}^N p_A(x_A), \quad p_A(x_A) = \begin{cases} \pi_A(x_l, x_r) & \text{if } x_A = x_{lA} \\ 1 - \pi_A(x_l, x_r) & \text{if } x_A = x_{rA} \end{cases}$$

Since $x^*(\mathbf{x}_{-A}, x_A)$ is monotonic non-decreasing in x_A , then for all possible legislatures, $x^*(\mathbf{x}_{-A}, x_{lA}) \leq x^*(\mathbf{x}_{-A}, x_{rA})$. (For some legislatures, $x^*(\mathbf{x}_{-A}, x_{lA}) = x^*(\mathbf{x}_{-A}, x_{rA})$.)

Suppose, for the sake of clarity, that there are only two possible legislatures $\mathbf{x}'_{-A}, \mathbf{x}''_{-A}$ such that $x^*(\mathbf{x}'_{-A}, x_{lA}) < x^*(\mathbf{x}'_{-A}, x_{rA})$ and $x^*(\mathbf{x}''_{-A}, x_{lA}) < x^*(\mathbf{x}''_{-A}, x_{rA})$. (The extension to the general case poses no major difficulty.) For each of them, there is an indifferent voter i', i'' respectively:

$$i' = \frac{x^*(\mathbf{x}'_{-A}, x_{lA}) + x^*(\mathbf{x}'_{-A}, x_{rA})}{2},$$

$$i'' = \frac{x^*(\mathbf{x}''_{-A}, x_{lA}) + x^*(\mathbf{x}''_{-A}, x_{rA})}{2},$$

Without loss of generality, suppose that $i' < i''$. Hence, for any type $i < i'$, $E(u_i(x_{lA})) > E(u_i(x_{rA}))$ since, regardless of the actual legislature, those types belong to the set of supporters of l . Likewise, for any $i > i''$, $E(u_i(x_{lA})) < E(u_i(x_{rA}))$. For all i such that $i' < i < i''$:

$$u_i(\mathbf{x}'_{-A}, x_{lA}) < u_i(\mathbf{x}'_{-A}, x_{rA}),$$

$$u_i(\mathbf{x}''_{-A}, x_{lA}) > u_i(\mathbf{x}''_{-A}, x_{rA}),$$

Therefore, for every such i there exists a unique probability $p(\mathbf{x}'_{-A})$ for which:

$$\begin{aligned} E(u_i(x_{lA})) &= p(\mathbf{x}'_{-A})u_i(\mathbf{x}'_{-A}, x_{lA}) + (1 - p(\mathbf{x}'_{-A}))u_i(\mathbf{x}''_{-A}, x_{lA}) \\ &= p(\mathbf{x}'_{-A})u_i(\mathbf{x}'_{-A}, x_{rA}) + (1 - p(\mathbf{x}'_{-A}))u_i(\mathbf{x}''_{-A}, x_{rA}) \\ &= E(u_i(x_{rA})) \end{aligned}$$

By continuity of u , as i increases between i' and i'' , $u_i(\mathbf{x}'_{-A}, x_{rA}) - u_i(\mathbf{x}'_{-A}, x_{lA})$ increases while $u_i(\mathbf{x}''_{-A}, x_{lA}) - u_i(\mathbf{x}''_{-A}, x_{rA})$ decreases. Therefore, the value $p(\mathbf{x}'_{-A})$ that renders i indifferent increases. The fact that this value p is unique for each i and increasing in i implies that for each $p(\mathbf{x}'_{-A})$ there is also a *unique* type i that is indifferent between the two candidates. This proves the lemma's claim.

A.2 Proof of Theorem 1

Since the policy outcome is determined by the median point of the elected legislature, the strategies chosen by candidates in one district depend upon the local platforms chosen in all the other districts. With $2N$ candidates and 2^N possible legislatures, a direct calculation of the equilibria is close to impossible. Therefore, I shall start by ruling out possible strategies.

Step 1: Non-deterministic policy outcomes First, I will prove that in equilibrium the location of the median legislator is uncertain. This result is important in itself because it rules out full convergence of the candidates to the median of their districts, a commonly assumed pattern in models of constituency elections.

Lemma 7 *Let \mathcal{P}_O be an open-rule polity. In equilibrium, there is no policy outcome $x \in \mathfrak{R}$ such that $\Pr(m(\mathbf{x}) = x) = 1$.*

Proof: In such an equilibrium, L would be indifferent between endorsing and not endorsing all its candidates that propose $x_l > x$. If they are not endorsed, their district's platform will become τ_R but this will not affect the median. But, instead of randomizing its endorsement

strategies in those districts, party L can in fact choose $e_L = 0$. That way, those candidates will be forced to propose platforms $x_l < x$ which would, if elected, increase L 's pay-off. A similar reasoning would hold for party R .

There is one special case that needs to be considered: $x_{lA} = x_{rA} = x$ for all districts. In that case, no L candidate proposes a platform greater than x so that the preceding argument does not hold. However, this set of strategies cannot be an equilibrium. Suppose, without loss of generality, that $x > \mu_M$ (if $x \leq \mu_M$, the following argument goes through for the R candidates). Here, if endorsed, all the candidates have a probability of victory of $1/2$. But, the L candidates of districts $A < M$ can improve (weakly) their pay-off by choosing platforms $x_{lA} = x - \epsilon$, for an $\epsilon > 0$ small enough. In fact, this would mean that, if elected, these candidates would yield a lower median which, in its turn, implies that the voters in their districts will no longer be indifferent and, then in those districts the pivotal voter will be in the interval $[x - \epsilon, x]$. With this pivotal location, the probability of victory of these candidates will be in the interval $[i_{mA}^{-1}(x - \epsilon), i_{mA}^{-1}(x)]$ and all these values are $> 1/2$.

Step 2: Range of possible medians In this step I will constrain the range in which the possible median legislators can be located.

Lemma 8 *Let $\underline{m}, \overline{m}$ denote the lowest and highest possible location of the median legislator in a political equilibrium of \mathcal{P}_O . Then:*

$$x_{lA} < \overline{m}, x_{rA} > \underline{m} \quad \forall A$$

Proof: If an L candidate chooses $x_{lA} > \overline{m}$, party L will be indifferent between fielding her and allowing her district to return to the legislature a representative with platform τ_R . Therefore, she will be denied endorsement. The same holds for an R candidate that chooses $x_{rA} < \underline{m}$.

An implication of this result is that \underline{m} is the median of a legislature formed only by L candidates and, likewise, \overline{m} is the median when only R candidates are elected.

Lemma 9 *The range of possible median legislators is such that:*

$$\mu_{M-1} < \underline{m} < \mu_M < \overline{m} < \mu_{M+1}$$

Proof: If $\underline{m} > \mu_M$, this means that the indifferent voter in districts $A < M$ is: $i_A^* > \mu_M$ and, since $\mu_A < \mu_M$, this means that $i_{mA}^{-1}(\mu_M) > 1/2$. However, if an L candidate from those districts chooses a platform $\underline{m} < x_{lA} < \overline{m}$, her rival can choose $x_{rA} = x_{lA}$ ensuring a probability of victory of $1/2$. Notice that in so doing, the R candidate will retain endorsement because if that district elects a legislator with platform τ_L (which is what would happen if party R denies

endorsement) this will shift the \underline{m} to the left. Therefore, all L candidates from these districts choose a platform $x_{lA} < \underline{m}$. But this implies that the median of a legislature formed entirely by L candidates cannot be \underline{m} . This contradicts the condition on \underline{m} derived in the last result. A similar contradiction, applied to R candidates will prove that $\overline{m} > \mu_M$.

The other inequalities can be proven by similar arguments: $\underline{m} < \mu_{M-1}$ requires that all the L candidates of districts 1 to $M - 1$ propose platforms $x_l < \mu_{M-1}$. But this would imply that the L candidate in district $M - 1$ can shift to the right the location of her district's pivotal voter (and hence, increase her probability of victory) by choosing a platform $x_{lM-1} > \underline{m}$. By the same token, we obtain that $\overline{m} < \mu_{M+1}$.

Step 3: Minimal Differentiation Here I will prove that, as the candidates attempt to reduce differentiation, in order to maximize their probability of victory, this narrows the interval of possible median locations.

Lemma 10 *In district M , the candidates choose platforms $x_{lM} = x_{rM} = \mu_M$.*

Proof: The logic used to prove Lemma 9 also shows that the candidates in district M do not benefit from choosing platforms out of $[\underline{m}, \overline{m}]$. So, this is the only district such that both candidates platforms' are contained in said interval. The crucial implication of this is that both candidates can secure endorsement even if their platforms converge within this range. In fact, district M will return the median legislator in at least one possible legislature: the one formed by L candidates from districts 1 to $M - 1$ and R candidates from districts $M + 1$ to N . So, if a party denies endorsement in district M this will shift the median against its preferences. This means that the usual argument of Downsian convergence applies here and $x_{lM} = x_{rM} = \mu_M$.

Up to this point, we have determined that, in equilibrium, the following pattern holds:

1. All the L candidates from districts 1 to $M - 1$ propose platforms $x_l \leq \mu_{M-1}$.
2. All the R candidates from districts $M + 1$ to N propose platforms $x_r \geq \mu_{M+1}$.
3. $x_{lM} = x_{rM} = \mu_M$.
4. All the L candidates from districts $M + 1$ to N and all the R candidates from districts 1 to $M - 1$ propose platforms in the interval (μ_{M-1}, μ_{M+1}) .

This means that the candidates in the interval $[\underline{m}, \overline{m}]$ are *minority* candidates and, therefore, they maximize their probability of victory by approaching their rivals' strategy as much as is compatible with preserving the endorsement. In the case of the L candidates, this is accomplished by choosing $x_l = \mu_M - \epsilon$ while the R candidates choose $x_r = \mu_M + \epsilon$.

A.3 Proof of Theorem 3

The only part of the theorem that has not been proven yet is the conclusion $x_L^* \neq x_R^*$. In fact it includes two claims: one about existence of a Nash equilibrium in the convention stage and one about the actual nature of such Nash equilibrium, in particular, about policy divergence. I will discuss both of them in that order.

Existence: As Roemer [22] has pointed out, the pay-off function of the parties in this game is in general not quasi-concave. That means that a direct appeal to Kakutani's fixed point theorem is not possible because the best-response correspondences may fail to be convex valued. However, it is possible to restore the properties required for the application of the theorem. The first thing to notice is that the pay-off function of, say, party L can be rewritten as:

$$E(u_L(x_L)) = (u_L(x_L) - u_L(x_R))\Pi(x_L, x_R) + u_L(x_R)$$

Therefore, for any given strategy x_R , we know that L will never choose x_L such that $u_L(x_L) < u_L(x_R)$. Were it to do so, the first term would become negative and therefore, the pay-off would be inferior to the one obtained by choosing $x_L = x_R$. Since the policy preferences of the parties are represented by concave functions, then, the set $\tilde{x}_L(x_R) = \{x_L \mid u_L(x_L) \geq u_L(x_R)\}$ is a convex set. More precisely, $\tilde{x}_L(x_R) = [2\tau_L - x_R, x_R]$ (or $[x_R, 2\tau_L - x_R]$ if $x_R < \tau_L$).

On the other hand, $\Pi(x_L, x_R)$ is monotonic in x_L over the sets $x_L < x_R$ and $x_L > x_R$. To see why, let's first write down $\Pi(x_L, x_R)$ as a function of a specific $\pi_A(x_L, x_R)$:

$$\begin{aligned} \Pi(x_L, x_R) &= \Pr(\#\{B : u_B(x_L) > u_B(x_R)\} \geq \frac{N+1}{2}) \\ &= \Pr(\#\{B : u_B(x_L) > u_B(x_R), B \neq A\} \geq \frac{N+1}{2}) + \\ &\quad \pi_A(x_L, x_R) \Pr(\#\{B : u_B(x_L) > u_B(x_R), B \neq A\} = \frac{N-1}{2}) \end{aligned}$$

In words, $\Pi(x_L, x_R)$, as a function of $\pi_A(x_L, x_R)$ is equal to the probability of L obtaining a majority in all the districts, excluding A plus the probability of L obtaining a tie in all those districts *and* breaking that tie in its favor by winning the elections in A .

Therefore, it is clear that:

$$\frac{\partial \Pi}{\partial \pi_A} = \Pr(\#\{B : u_B(x_L) > u_B(x_R), B \neq A\} = \frac{N-1}{2}) > 0$$

On the other hand, for $x_L < x_R$, we have that:

$$\frac{\partial \pi_A(x_L, x_R)}{\partial x_L} = \frac{\partial^{i-1}(\frac{x_L+x_R}{2})}{\partial x_L} > 0$$

where the inequality follows from the fact that $i_{m_A}^{-1}$ is the inverse of a monotonically increasing function.

Likewise, we can conclude that $\frac{\partial \pi_A(x_L, x_R)}{\partial x_L} < 0$ if $x_L > x_R$. If we put these two expressions together, we obtain the derivative of the probability of victory with respect to x_L :

$$\frac{\partial \Pi(x_L, x_R)}{\partial x_L} = \sum_{A=1}^N \frac{\partial \Pi(x_L, x_R)}{\partial \pi_A(x_L, x_R)} \frac{\partial \pi_A(x_L, x_R)}{\partial x_L}$$

This derivative will be positive if $x_L < x_R$ and negative if $x_L > x_R$. Anyway, we know that in $\tilde{x}_L(x_R)$, x_L is never $> x_R$ and $< x_R$ at the same time, therefore, when constrained to $\tilde{x}_L(x_R)$, the function $\Pi(x_L, x_R)$ is monotonic in x_L which means that it is also *quasi-concave*.

From this we can conclude that the pay-off function $E(u_L(x_L))$, when constrained to $\tilde{x}_L(x_R)$, is quasi-concave. In fact, remember that $E(u_L(x_L)) = (u_L(x_L) - u_L(x_R))\Pi(x_L, x_R) + u_L(x_R)$ and that $(u_L(x_L) - u_L(x_R))$ is positive in this subset. Therefore:

$$\frac{\partial E(u_L(x_L))}{\partial x_L} = (u_L(x_L) - u_L(x_R)) \frac{\partial \Pi(x_L, x_R)}{\partial x_L} + u'_L(x_L) \Pi(x_L, x_R)$$

since $\Pi(x_L, x_R)$ is a monotonic function, its derivative dictates the sign of the first term. On the other hand, the concavity of u_L ensures that u'_L is monotonically decreasing. So, it is the case that we can always partition $\tilde{x}_L(x_R)$ into two convex subsets: one where $E(u_L(x_L))$ is increasing and one where it is decreasing. This ensures the quasi-concavity of the pay-off function when constrained to $\tilde{x}_L(x_R)$.

It is also true that the correspondence $\tilde{x}_L(x_R)$ is upper hemi-continuous. This can be seen by writing down its formula:

$$\tilde{x}_L(x_R) = \begin{cases} [x_R, 2\tau_L - x_R] & \text{if } x_R < \tau_L \\ x_R & \text{if } x_R = \tau_L \\ [2\tau_L - x_R, x_R] & \text{if } x_R > \tau_L \end{cases}$$

So, it is immediate to verify that for any sequences $x_{R,n}, x_{L,n}$, if $x_{L,n} \in \tilde{x}_L(x_{R,n}) \forall n$ and $x_{R,n} \rightarrow x_R$, then $x_L \in \tilde{x}_L(x_R)$ which is the definition of upper hemi-continuity.

Essentially the same arguments can be applied to the set of strategies for R so that it is possible to obtain a correspondence $\tilde{x}_R(x_L)$ which is upper hemi-continuous and convex valued and also with a pay-off function $E(u(x_R))$ quasi-concave over $\tilde{x}_R(x_L)$. Therefore, all the conditions for Kakutani's theorem are fulfilled and it is possible to claim that a Nash equilibrium (x_L^*, x_R^*) exists.

Policy Divergence Now, suppose that $x_L^* = x_R^*$ so that $\Pi(x_L^*, x_R^*) = 1/2$. It can be proven that in this case there will exist a profitable unilateral deviation for at least one of the two parties so that this cannot be a Nash equilibrium.

In particular, under total policy convergence, L 's pay-off is $u_L(x_L^*)$. Now, consider an alternative policy $x'_L = x_L^* - \epsilon$ for some $\epsilon > 0$ small enough. Then, $u_L(x_L^*) < u_L(x'_L)$. On the other hand:

$$E(u_L(x^*(x'_L, x_R^*))) = u_L(x'_L)\Pi(x'_L, x_R^*) + u_L(x_L^*)\Pi(x_L^*, x_R^*)$$

Thus, x'_L constitutes a profitable deviation if $\Pi(x'_L, x_R^*) > 0$. This will be true unless $i_{mA}^{-1}\left(\frac{x'_L + x_R^*}{2}\right) = 0 \quad \forall A$. So, if this condition does not hold, L can deviate from x_L^* .

Finally, suppose that such condition actually holds. Then, by a similar argument, it is easy to prove that R can profitably deviate to $x'_R = x_R^* + \epsilon$ for some $\epsilon > 0$ arbitrarily small. In fact, $u_R(x'_R) > u_R(x_R^*)$ so that R 's pay-off increases if $1 - \Pi(x_L^*, x'_R) > 0$. Since, by assumption, $i_{mA}^{-1}\left(\frac{x'_L + x_R^*}{2}\right) = 0 \quad \forall A$, for any $\epsilon > 0$, the only possibility of $1 - \Pi(x_L^*, x'_R) = 0$ is if $i_{mA}^{-1}\left(\frac{x_L^* + x'_R}{2}\right) = 0 \quad \forall A$ for any ϵ . But this would imply that the location of the median voter is a degenerate random variable in *all* the districts something that violates the assumption of electoral uncertainty. Therefore, R has a profitable deviation and (x_L^*, x_R^*) , where $x_L^* = x_R^*$, is not a political Nash equilibrium.

A.4 Proof of Theorem 4

The proof will proceed in two major steps. First, I will prove that the multi-district case is, in a crucial sense, analogous to the single-district case. Thus, the arguments used by Roemer [22] to prove the single-district case can be used here. In the second step I will show how such line of reasoning goes.

Step 1. Intradistrict homogeneity in the multi-district case. The main goal of this step is to prove the following lemma:

Lemma 11 *Let $\mathcal{P}_{C,n}$ be a sequence of closed-rule polities with increasing intradistrict homogeneity. Then, for all x_L, x_R for which $\frac{x_L + x_R}{2} \neq \mu_M$:*

$$\lim_{n \rightarrow \infty} \Pi_n(x_L, x_R) \in \{0, 1\}$$

Remark: In words, this lemma claims that as intradistrict homogeneity decreases, the sequence of polities converges to one without electoral uncertainty.

Proof: This follows simply from the fact that $\mathcal{P}_{C,n+1}$ is obtained by a m.p.r.r. of the distributions G_n of $\mathcal{P}_{C,n}$.

Without loss of generality, assume that $\mu_{M+q} > \frac{x_L + x_R}{2} > \mu_M$, for some integer $q \geq 1$. Then, $\pi_{1,n}(x_L, x_R) \geq \dots \geq \pi_{M,n}(x_L, x_R) \geq \pi_{M+q,n}(x_L, x_R) \geq \dots \geq \pi_{N,n}(x_L, x_R)$ for all n .

Since $G_{A,n+1}$ is a m.p.r.r. of $G_{A,n}$ for all A , then,

$$\lim_{n \rightarrow \infty} \pi_{A,n}(x_L, x_R) = \begin{cases} 1 & \text{if } A < M + q \\ 0 & \text{if } A \geq M + q \end{cases}$$

The probability of victory $\Pi_n(x_L, x_R)$ is obtained by a summation of terms of the form

$$\prod_{A \subseteq N} \pi_{A,n}(x_L, x_R) \prod_{B=N \setminus A} (1 - \pi_{B,n}(x_L, x_R))$$

where A contains all the possible combinations of a majority of districts ($\#A \geq \frac{N+1}{2}$). Therefore, the only term that does not converge to 0 is:

$$\lim_{n \rightarrow \infty} \Pi_n(x_L, x_R) = \lim_{n \rightarrow \infty} \prod_{A=1}^{M+q-1} \pi_{A,n}(x_L, x_R) \prod_{B=M+q}^N (1 - \pi_{B,n}(x_L, x_R)) \rightarrow 1$$

A similar argument can be made to prove that if $\frac{x_L + x_R}{2} < \mu_M$ then $\Pi_n(x_L, x_R) \rightarrow 0$.

Step 2a: Convergence of the pivotal location to μ_M . An important consequence of Step 1 is the following lemma:

Lemma 12 *Let $(x_L^*, x_R^*)_n$ be the equilibrium of polity $\mathcal{P}_{C,n}$ in the sequence. Then:*

$$\lim_{n \rightarrow \infty} \frac{x_{Ln}^* + x_{Rn}^*}{2} = \mu_M.$$

Proof: Without loss of generality, let's assume, to the contrary, that there exists a subsequence $(x_L^*, x_R^*)_{n_1}$ such that $\lim_{n_1 \rightarrow \infty} \frac{x_{L,n_1}^* + x_{R,n_1}^*}{2} > \mu_M$. Let $\delta > 0$ be a constant and x'_{L,n_1} a sequence of platforms such that $\lim_{n_1 \rightarrow \infty} x'_{L,n_1} = x'_L$, $x'_{L,n_1} < x_{L,n_1}^* \forall n_1$ and $\lim_{n_1 \rightarrow \infty} \frac{x'_{L,n_1} + x_{R,n_1}^*}{2} > \mu_M$ with $u_L(x'_{L,n_1}) - u_L(x_{L,n_1}^*) = \delta > 0$. I claim that for all large n_1 , $E(u_L(x^*(x'_{L,n_1}, x_{R,n_1}^*))) > E(u_L(x^*(x_{L,n_1}^*, x_{R,n_1}^*)))$:

$$\begin{aligned} & E(u_L(x^*(x'_{L,n_1}, x_{R,n_1}^*))) - E(u_L(x^*(x_{L,n_1}^*, x_{R,n_1}^*))) \\ &= (u_L(x'_{L,n_1}) - u_L(x_{L,n_1}^*)) \Pi_{n_1}(x'_{L,n_1}, x_{R,n_1}^*) - (u_L(x_{L,n_1}^*) - u_L(x_{R,n_1}^*)) \Pi_{n_1}(x_{L,n_1}^*, x_{R,n_1}^*) \\ &= \delta \Pi_{n_1}(x'_{L,n_1}, x_{R,n_1}^*) + (u_L(x_{L,n_1}^*) - u_L(x_{R,n_1}^*)) (\Pi_{n_1}(x'_{L,n_1}, x_{R,n_1}^*) - \Pi_{n_1}(x_{L,n_1}^*, x_{R,n_1}^*)) \end{aligned}$$

Since the limit of the pivotal location is $> \mu_M$ for both sequences, we know from Lemma 11 that $\lim_{n_1 \rightarrow \infty} \Pi_{n_1}(x_{L,n_1}^*, x_{R,n_1}^*) = \lim_{n_1 \rightarrow \infty} \Pi_{n_1}(x'_{L,n_1}, x_{R,n_1}^*) = 1$. In turn, this implies that: $\lim_{n_1 \rightarrow \infty} E(u_L(x^*(x'_{L,n_1}, x_{R,n_1}^*))) - E(u_L(x^*(x_{L,n_1}^*, x_{R,n_1}^*))) = \delta > 0$.

This establishes the claim and, therefore, the sequence x_{L,n_1}^*, x_{R,n_1}^* is not a sequence of equilibria.

Step 2b: Convergence of equilibrium platforms. Lemma 12 proves that along the sequence of equilibria, for n large enough, $x_{L,n}^* = \mu_M - a_n$, $x_{R,n}^* = \mu_M + b_n$, for $a_n, b_n > 0$ and that $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n$. Now we need to prove that the limit of both sequences is 0.

Assume, to the contrary, that $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n = c > 0$. Let there be a sequence of strategies $x'_{L,n}$ with $\lim_{n \rightarrow \infty} x'_{L,n} = x'_L$, such that $x_{R,n}^* > x'_{L,n} > x_{L,n}^*$ and $u_L(x'_{L,n}) - u_L(x_{R,n}^*) > 1/2(u_L(x_{L,n}^*) - u_L(x_{R,n}^*)) > 0, \forall n$. Then, I will prove that for n large enough, $E(u_L(x^*(x'_{L,n}, x_{R,n}^*))) > E(u_L(x^*(x_{L,n}^*, x_{R,n}^*)))$, $\forall n$.

We know that:

$$\begin{aligned} E(u_L(x^*(x'_{L,n}, x_{R,n}^*))) - E(u_L(x^*(x_{L,n}^*, x_{R,n}^*))) = \\ (u_L(x'_{L,n}) - u_L(x_{R,n}^*))\Pi_n(x'_{L,n}, x_{R,n}^*) - (u_L(x_{L,n}^*) - u_L(x_{R,n}^*))\Pi_n(x_{L,n}^*, x_{R,n}^*) \end{aligned}$$

On the other hand, $\lim_{n \rightarrow \infty} \frac{x'_{L,n} + x_{R,n}^*}{2} > \mu_M$ so that

$$1 = \lim_{n \rightarrow \infty} \Pi_n(x'_{L,n}, x_{R,n}^*) > \lim_{n \rightarrow \infty} \Pi_n(x_{L,n}^*, x_{R,n}^*) = 1/2$$

(The last inequality follows from the fact that μ_M is the median district.) Therefore:

$$\begin{aligned} \lim_{n \rightarrow \infty} E(u_L(x^*(x'_{L,n}, x_{R,n}^*))) - E(u_L(x^*(x_{L,n}^*, x_{R,n}^*))) \\ = (u_L(x'_L) - u_L(x_{R,n}^*)) - 1/2(u_L(x_{L,n}^*) - u_L(x_{R,n}^*)) \\ > 0 \end{aligned}$$

This inequality establishes that $x_{L,n}^*$ is not a sequence of best responses to $x_{R,n}^*$ and this contradiction proves the theorem.