The Dynamics of Sovereign Debt: Smoothing and Impatience

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Abstract

We consider the allocation of resources between a borrower and lender in the absence of third-party enforcement of loan contracts. The two parties differ in two respects: (1) The borrower is risk averse and subject to fluctuations in endowments while the lender is risk neutral. (2) The borrower discounts the future at a higher rate than the lender. The first difference provides a consumption-smoothing motive for borrowing and the second a long-term impatience motive for borrowing. In contrast to a situation in which only the first effect operates, sovereignty constraints preclude the unconstrained Pareto optimal allocation. However, the relationship provides sufficient surplus to each party to sustain some lending. The constrained allocation can imply a negative correlation between change in indebtedness and consumption. The equilibrium is supported by the renegotiation-proof and coalition-proof punishments demonstrated by Kletzer and Wright (2000). The model can be used to replicate observed patterns of capital flows to developing country debtors in simulations.

1. Introduction

The phenomenon of sovereign borrowing has raised the question of how a loan market can function in the absence of a third party to enforce loan contracts. One argument has been that loan contracts can be self-enforcing in that a borrower might be willing to make a net resource transfer to a creditor in order to maintain access to international financial markets in the future. Models of sovereign borrowing in which maintaining a "reputation" for repayment provides an incentive to repay have been developed, for example, by Eaton and Gersovitz (1981), Manuelli (1984), Craig (1988), and Kletzer and Wright (1991).

Several recent papers have attacked the notion that maintaining a reputation for repayment suffices to sustain a loan market in the absence of any direct sanctions that creditors can impose on borrowers in default. Bulow and Rogoff (1989), for example, show that, as long as the borrower can continue to invest abroad at the world interest rate, the threatened loss of the ability to borrow again is an insufficient incentive to repay any amount of debt. In their analysis, however, the borrower itself can automatically enforce its own loan contracts.

Rosenthal (1991) and Cohen (1991) go on to claim that, even if default results in the borrower's inability to invest abroad subsequently, the loss of access to financial markets for both borrowing and investment purposes will not provide much (or any) incentive to repay. Rosenthal (1991) shows, in a deterministic Cass-Koopmans dynamic framework with capital accumulation and smooth technology, that even if a debtor economy's optimal trajectory with full enforcement would eventually lead it to become a net creditor, loss of this future status does not provide an incentive to repay debt.

He acknowledges that his model does not incorporate income fluctuations, so that fluctuation-smoothing cannot provide an incentive to repay in his analysis. He justifies this omission empirically:

“The magnitudes of the numbers and the realities of the LDC economies make it seem likely that it is growth, not smoothing, that forms the primary motivation for borrowing by LDC’s, however."

Cohen (1991) makes an even stronger claim, that, even if income fluctuates, “The threat of financial autarky is never sufficient to keep a country from defaulting” (p. 94).

In this paper we model the relationship overtime of two parties, neither of whom has access to an external mechanism to enforce contracts. We show, contrary to the claims in these other papers, that maintaining its relationship with the other can by itself provide each party an incentive to make net resource transfers under particular contingencies. These transfers can be interpreted as those that would emerge from the operation of international loan contracts. Moreover, under reasonable assumptions about the magnitudes of exogenous parameters, the extent of these transfers predicted by the theory are commensurate with that we observe in the markets themselves.

Our analysis resembles that in Kletzer and Wright (1990): One party, the “borrower," has a fluctuating endowment and is risk-averse. The other, the “lender," is risk neutral and seeks to maximize the expected net resource transfer from the borrower. Both can observe the borrower’s endowment upon realization. There is no storage.

We depart from this earlier paper in considering, as does Cohen (1991), a borrower with a higher rate of discount than the lender. This change introduces a motive for borrowing beyond smoothing: Even in the absence of fluctuations, with full enforcement, the borrower would borrow from the lender in early periods to repay in later periods. Such borrowing can be reinterpreted as borrowing to finance growth, as in Rosenthal’s (1991) analysis. We find, however, contrary to what Rosenthal’s remarks suggest, that the desire to smooth fluctuations can provide an incentive to service debt that was incurred by “growth" considerations. Our numerical calculations suggest that the numbers involved are in fact consistent with a smoothing explanation for the levels of sovereign indebtedness that we in fact observe.

Our difference with Cohen (1991) is the consequence of an assumption implicit in his demonstration of his proposition on the insufficiency of the threat of financial autarky to provide an incentive to repay. His proof assumes that once the threat becomes binding on the borrower it remains so thereafter. Hence, once the borrower is indifferent between maintaining access to international capital markets and financial autarky, he remains indifferent. But this is true only if the borrower never expects a net transfer from the creditor again. In fact, it is the expectation of receiving such transfers in the future (when income is low) that provides the incentive to repay. As our analysis shows, under very general assumptions, the borrower can expect to receive transfers in the future, at which point he will strictly prefer his lot with the creditor to that under autarky. This expectation can provide an incentive to service debt currently to the point at which he is indifferent between maintaining his relationship with the creditor and defaulting, suffering autarky thereafter.

2. The Environment

We begin by considering the interaction of two parties, a “borrower” and a “lender.” (To facilitate identification of pronouns with their antecedents, we consider the case of a female lender and a male borrower.) The environment is much like that considered in Kletzer and Wright (1990).

Endowments:

Each period t, $t = 0, ..., \infty$, the borrower receives an endowment of a nonstorable commodity in amount y_t which is governed by a discrete-valued Markov process that can assume one of N values. Hence, in any period t, for any history of endowments $\omega_t = \{y_1, ..., y_t\}$, $y_{t+1} = y^j(y_t)$ with probability $p^j(y_t)$, $j = 1, ..., N$. We order these possible realizations $y^1 < y^2 < ... < y^N$ and restrict $y^1 >> 0$ and

$y^N < \infty$.

Preferences: The borrower's utility in period t is a strictly increasing concave, differentiable function u of his consumption c_t that period, with $u(c) \downarrow -\infty$ and $u'(c) \uparrow \infty$ as $c \downarrow 0$. Feasible consumption is bounded from below by zero. He discounts utility in period t by a factor β^t relative to borrower's objective is to maximize expected discounted utility thenceforth, given by:

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} E[u(c_\tau)|\omega_t].$$

Given a period t endowment of y_t, the borrower's expected discounted utility under autarky is:

$$U_t^A(y^t) = u(y_t) +_\tau \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E_t[u(y_\tau|y_t)].$$

The lender is risk neutral. Her objective at the beginning of period t is to maximize the expected discounted value of subsequent resource transfers from the borrower:

$$\Pi(\omega_t) = y_t - c_t +_\tau \sum_{\tau=t+1}^{\infty} \gamma^{\tau-t} E_t[(y_\tau - c_\tau)|\omega_t] = y_t - c_t + \gamma \sum_{j=1}^{N} p_{t+1}^j(y_t)\Pi(\omega_{t+1}^j),$$

where γ is the (constant) discount factor that she applies to future transfers and $\omega_{t+1}^j = \{\omega_t, y_{t+1}^j\}$. Nonnegativity of consumption, along with the restrictions that $\gamma < 1$ and the finiteness of y ensure that Π is bounded from above.

We find it useful to consider the borrower's objective in terms of the gains that he achieves from his relationship with the lender relative to how he would fare under autarky. At any period t, for given Π_t, the borrower's maximum gain from then onward is the value of the dynamic program:

$$V_t(\Pi_t, \omega_t) = \max_{c_t, \{\Pi_{t+1}^j\}} u(c_t) - u(y_t) + \beta \sum_{j=1}^{N} p^j(y_t) V_{t+1}(\Pi_{t+1}^j, \omega_{t+1}^j) \quad (1)$$

solved subject to the constraint that:

$$y_t - c_t + \gamma \sum_{j=1}^{N} p^j(y_t)\Pi_{t+1}^j \geq \Pi_t, \tag{2}$$

where

$$\Pi_{t+1}^j(\omega_{t+1}^j) = c_{t+1}^j - y_{t+1}^j + \gamma_j \sum_{j'=1}^{N} p^{j'}(y_{t+1}^j)\Pi_{t+1}^{j'}(\omega_{t+1}^j, y_{t+2}^{j'}).$$

3. Pareto-Optimality and Competitive Full-Enforcement Equilibrium

We first characterize the unconstrained Pareto optimal allocation, and discuss loan contracts that can support it. Our focus will ultimately be on competitive equilibria in which competition among potential lenders in period 0 eliminates any profit that the creditor derives from her relationship with the borrower. For this reason we set $\Pi_0 = 0$.

In any period t the Pareto optimal allocation can be described as a choice of c_t and $\Pi_{t+1}^j, j = 1, ..., N$, that solves (1) subject to (2). To characterize the solution we write $V_t(\Pi_t, \omega_t)$ as a Lagrangian:

$$V_t(\Pi_t, \omega_t) = u(c_t) - u(y_t) + \beta \sum_{j=1}^{N} p^j V_{t+1}(\Pi_{t+1}^j, \omega_{t+1}^j) + \lambda_t[y_t - c_t + \gamma \sum_{j=1}^{N} p^j \Pi_{t+1}^j - \Pi_t]$$

where λ_t is the shadow price associated with (2). The borrower's nonsatiation ensures that (2) holds with equality in any period t.

First-order conditions for a maximum are:

$$u'(c_t) = \lambda_t \tag{3}$$

$$\beta V_{\Pi,t+1}^j = \lambda_t \gamma, j = 1, ..., N \tag{4}$$

where $V^j_{\Pi,t+1} = \delta V_{t+1}(\Pi^j_{t+1}, \omega^j_{t+1})/\delta\Pi^j_{t+1}$. Differentiating V_t with respect to Π_t gives:

$$V_{\Pi,t} = \lambda_t. \tag{5}$$

where $V_{\Pi,t} = V_t(\Pi_t, \omega_t)/\delta\Pi_t$

Combining these expressions for any two periods t and, for each j, $t+1$ implies that under a Pareto-optimal allocation the borrower's consumption obeys the equation of motion:

$$u'(c_t) = (\beta/\gamma)u'(c^j_{t+1}), j = 1, ..., N \tag{6}$$

But since no term in (6) depends on j, $c^j_{t+1} = c_{t+1}$ for all j : The borrower's consumption is not event contingent, and declines deterministically to allow his marginal utility of consumption to rise at a rate $(\gamma/\beta) - 1$ independent of realizations of y_t, $t > 0$. Not surprisingly, since the lender is risk-neutral while the borrower is risk averse, any Pareto-optimal allocation leaves the lender bearing all the uncertainty in the borrower's endowment. Moreover, if the lender discounts the future less than the borrower $(\gamma > \beta)$ then in any Pareto-optimal allocation the borrower's consumption declines deterministically and monotonically.

The zero-profit condition $\Pi_0 = 0$ implies that:

$$c_0 = y_0 + \sum_{t=1}^{\infty} \gamma^t [E_0(y_t|y_0) - c_t] \tag{7}$$

Together, (6) and (7) fully characterize the zero-profit Pareto-optimal allocation of resources between the two parties.

A Simple Example: Say that $u(c_t)$ displays isoelastic marginal utility of income, so that $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$, where σ is the elasticity of marginal utility. Equation (6) then implies that c_t evolves according to:

$$c_{t+1} = (\beta/\gamma)^{1/\sigma} c_t.$$

Say, in addition, that $N = 2$, and that y_t is identically and independently across time, equaling y^2 with probability p and y^1 with remaining probability. In this case, for $\beta < \gamma$:

$$c_t = (\beta/\gamma)^{t/\sigma}[1 - (\beta/\gamma)^{1/\sigma}]\{y_0 + \gamma[py^2 + (1-p)y^1]/(1-\gamma)\}.$$

As $t \to \infty$, $c_t \to 0$. Hence, for $y^1 >> 0$, after some period $t * c_t < y^1$ for all $t > t*$. After some point in their relationship, whatever the realization of y, the borrower makes a net resource transfer to the lender.

Market Implementation: Sale of Equity and Simple Long-Term Loans

One mechanism for implementing the Pareto optimal allocation is for the borrower to sell the lender his permanent right to his endowment, for which the lender is willing to pay:

$$\sum_{t=0}^{\infty} \gamma^t E\{y_t|y_0\}.$$

The borrower can then use the proceeds to consume c_0 currently and to make t loans to the lender at a noncontingent interest rate $\gamma - 1$, where loan t is an t period loan in amount $c_t\gamma^t$, where c_t solves (6) and (7). Loan repayment at maturity finances the optimal consumption stream.

Market Implementation: One-Period Loans with Event-Contingent Repayments

Alternatively, the optimal consumption stream can be financed by the following series of break-even one-period loans:

In period 0 the borrower borrows $l_0 = c_0 - y_0$ in exchange for a promise to pay Π_1^j in period 1 contingent on y_1^j. The value of this loan to the lender is $y_0 - c_0 + \gamma E[\Pi_1|y_0] = \Pi_0 = 0$.

In period t, contingent on y_t^j, the borrower borrows $l_t = \Pi_t^j + c_t - y_t$ in exchange for a promise to pay $\Pi_{t+1}^{j'}$ in period $t+1$ contingent on $y_{t+1}^{j'}$. The loan is used to repay the current outstanding debt obligation Π_t^j and the excess of current consumption over the endowment $c_t - y_t$. The value of this loan to the lender is $y_t^j - c_t - \Pi_t^j + \gamma E(\Pi_{t+1}|y_t^j)$, which, from (2), is zero.

4. Constrained Pareto-Optimality and Self-Enforced Competitive Equilibrium

For $\gamma > \beta$, the Pareto-Optimal allocation derived in section 3 requires that the borrower's consumption level decrease monotonically. If the process governing his endowment is stationary or tends to increase over time, then in early periods the borrower will on average be receiving net resource transfers from the lender, and in later periods on average be making net resource to her. Hence the benefit he derives from his relationship with her diminishes over time.

If ever $V(\Pi_t, \omega_t) < 0$ then at that point the borrower, given the option, would prefer to end his relationship with the lender and consume his autarky endowment thereafter. Equivalently, if ever $\Pi_t < 0$ then at that point the lender's total return from maintaining the relationship is negative. In either event, any market arrangement that supports the Pareto-optimal allocation must then rely on an exogenous enforcement mechanism which ensures that the parties adhere to their commitments under the relationship even when it would be in at least one of their interests to terminate it.

The parties' sovereignty may preclude any such mechanism. For example, implementing the Pareto-optimal allocation by transferring the borrower's endowment stream to the lender in exchange for a series of multiperiod loans may fail because the borrower may not be able to commit to relinquishing his claim over his endowment (the problem of expropriation), while implementation via a series of one-period loans may fail because he may not be able to commit to meeting his debt-service obligations (the problem of sovereign default). We now consider what allocation can be sustained in the absence of any external enforcement mechanism. We allow either party to opt for permanent autarky at any point at which it would be advantageous to do so. That is, the borrower cannot commit to any contract that would require him to accept $V(\Pi_t, \omega_t) < 0$ under any contingency at any future period t while the lender cannot commit to any contract that would require her to accept $\Pi_t < 0$ under any contingency at any future period t.

These two requirements add the constraints $V_{t+1}(\Pi_{t+1}^j, \omega_{t+1}^j) \geq 0$ and $\Pi_{t+1}^j(\omega_{t+1}^j) \geq 0$, $j = 1, ..., N$, to the dynamic program characterizing the Pareto-optimal allocation. We call them the sovereignty constraints. Incorporating them into the dynamic program we obtain:

$$V_t(\Pi_t, \omega_t) = u(c_t) - u(y_t) + \beta \sum_{j=1}^{N} p^j [1 + \varphi(\omega^j_{t+1})] V_{t+1}(\Pi^j_{t+1}, \omega^j_{t+1}) + \tag{8}$$

$$\lambda_t [y_t - c_t + \gamma \sum_{j=1}^{N} p^j \Pi^j_{t+1} - \Pi_t] + \gamma \sum_{j=1}^{N} p^j \Psi(\omega^j_{t+1}) \Pi^j_{t+1},$$

where $\beta\varphi(\omega^j_{t+1})$ and $\gamma\Psi(\omega^j_{t+1})$ are the respective multipliers for the contraints $V_{t+1}(\Pi^j_{t+1}, \omega^j_{t+1}) \geq 0$ and $\Pi^j_{t+1}(\omega^j_{t+1}) \geq 0$.

First-order conditions for a maximum are now (3) and (4'), given by

$$\beta[1 + \varphi(\omega^j_{t+1})] V^j_{\Pi,t+1} = \gamma[\lambda_t + \Psi(\omega^j_{t+1})], j = 1, ..., N \tag{4'}$$

Combining (3), (4'), and (5), again at t and, for each j, at $t + 1$, gives as an equation of motion for c_t :

$$u'(c_t) = (\beta/\gamma) u'(c^j_{t+1})[1 + \varphi(\omega^j_{t+1})] - \Psi(\omega^j_{t+1}), j = 1, ..., N \tag{6'}$$

Note that the presence of the (potentially) state-contingent multipliers $\varphi(\omega^j_{t+1})$ and $\Psi(\omega^j_{t+1})$ in (6') means that, given c_t, the (constrained) Pareto-optimal consumption in period $t + 1$ may depend on the realization of y^j_{t+1}. Note also, however, that (6') implies that c^j_{t+1} depends on ω_t only through c_t and the multipliers $\varphi(\omega^j_{t+1})$ and $\Psi(\omega^j_{t+1})$. From this follows:

Lemma 1: The allocation subsequent to any period t depends on ω_t only through c_t. The value functions $\Pi_{t+1}(\omega^j_{t+1})$ and $V_{t+1}(\Pi_{t+1}, \omega^j_{t+1})$ and the multipliers $\varphi(\omega^j_{t+1})$, and $\Psi(\omega^j_{t+1})$ can be redefined respectively as $\Pi_{t+1}(c_t, y^j_{t+1})$, $V_{t+1}(c_t, y^j_{t+1})$, $\varphi(c_t, y^j_{t+1})$, and $\Psi(c_t, y^j_{t+1})$.

Proof: Define $\widetilde{\omega}_{t+i} = \{y_{t+1}, ..., y_{t+i}\}$. The Markov property of $\{y_t\}$ implies that the only element of ω_t affecting the distribution of $\widetilde{\omega}_{t+i}$ is y_t. From (6') we can write $c^j_{t+1} = \sigma_{t+1}[c_t, \varphi(\omega^j_{t+1}), \Psi(\omega^j_{t+1})]$. Iterating forward, we can write $c(\omega_{t+i}) = \sigma_{t+i}[c_t, \widetilde{\omega}_{t+i}, \varphi(\omega^j_{t+1}), \Psi(\omega^j_{t+1})]$. Since

$$\Pi_t = \sum_{i=0}^{\infty} \gamma^i E[y_{t+i} - c_{t+i}],$$

we can write $\Pi^j_{t+1}(\omega^j_{t+1}) = \Pi[c_t, y^j_{t+1}, \varphi(\omega^j_{t+1}), \Psi(\omega^j_{t+1})]$. Since no other element in expression (8) depends on ω_t except through c_t, we can write Π, V, φ, and ψ as stated in the lemma. ∎

Hence, even though the sovereignty constraints may force the borrower's realized consumption in period $t+1$ to depend on the realization of y_{t+1}, his consumption the previous period contains all the information from ω_t needed to implement the constrained Pareto-optimal allocation.

In characterizing the constrained Pareto-optimal allocation, we find it useful to define the following two functions:

$$Z^B(c, y^j) = u(c) - u(y^j) + \beta_j \sum_{j'=1}^{\infty} p^{j'}(y_j) V(c, y^{j'})$$

$$Z^L(c, y^j) = y^j - c + \gamma_j \sum_{j'=1}^{\infty} p^{j'}(y_j) \Pi(c, y^{j'}) = 0.$$

Z^B is the value to the borrower of consuming c when income is y^j given that the constrained Pareto optimum is pursued thereafter. Similarly, Z^L is the value to the lender of the borrower's consuming c when income is y^j given that the constrained Pareto optimum is pursued thereafter.

Lemma 1 allows us to define the following for each possible income level y^j :

The lowest incentive-compatible consumption level $\underline{c}^j$ is defined by the condition $Z^B(\underline{c}^j, y^j) = 0$. The highest incentive-compatible consumption level $\bar{c}^j$ is defined by the condition $Z^L(\bar{c}^j, y_j) = 0$. Since $V \geq 0$ and u is monotonically increasing, $\underline{c}^j \leq y^j$. Since $\Pi \geq 0$, $\bar{c}^j \geq y^j$. Hence $\bar{c}^j \geq y^j \geq \underline{c}^j$.[1]

Given c_t, the unconstrained optimal consumption level $\widetilde{c}_{t+1}$ satisfies:

$$u'(c_t) = (\beta/\gamma) u'(\widetilde{c}_{t+1})$$

Lemma 2: c_{t+1} is nondecreasing in c_t.

Proof: If $\varphi = \psi = 0$ then $c_{t+1} = c_{t+1} \leq c_t$. If $\varphi > 0$ then $V = 0$, so that $c_{t+1} = \underline{c}^j$ independent of c_t. If $\psi > 0$ then $\Pi = 0$ so that $c_{t+1} = \overline{c}^j$ independent of c_t. ∎

An immediate corollary is that $\Pi(c_t, y^j_{t+1})$ and $Z^L(c_t, y^j_t)$ are nonincreasing in c_t while $V(c_t, y^j_{t+1})$ and $Z^B(c_t, y^j_t)$ are nondecreasing in c_t.

The following characterizes the constrained Pareto-optimal allocation:

Theorem 1: Given c_t and y^j_{t+1}, the constrained Pareto-optimum calls for:

$$c_{t+1} = \underline{c}^j \text{ if } \widetilde{c}_{t+1} \leq \underline{c}^j,$$

$$c_{t+1} = \widetilde{c}_{t+1} \text{ if } \widetilde{c}_{t+1} \in [\underline{c}^j, \overline{c}^j]$$

and

$$c_{t+1} = \overline{c}^j \text{ if } \widetilde{c}_{t+1} \geq \overline{c}^j.$$

Proof: The monotonicity of $Z^L(c, y)$ and $Z^B(c, y)$ in c implies that if $\widetilde{c}_{t+1} \in [\underline{c}^j, \overline{c}^j]$ then $\varphi = \psi = 0$. If $\widetilde{c}_{t+1} < \underline{c}^j$ then $\varphi > 0$ and $\psi = 0$, so that $c^j_{t+1} = \underline{c}^j$. If $\widetilde{c}_{t+1} > \overline{c}^j$ then $\varphi = 0$ and $\psi > 0$, so that $c^j_{t+1} = \overline{c}^j$. ∎

Theorem 2: If $\beta < \gamma$ then the distribution of $\{c_t\}$ converges to a steady state that is bounded from above by c^{Max}, where $c^{Max} = \max\{\underline{c}^1, ..., \underline{c}^N\}$.

Proof: Theorem 1 implies that if $\gamma > \beta$ then $\{c_t\}$ is monotonically decreasing except when $\varphi(c_{t-1}, y^j_t) > 0$, in which case $c^j_t = \underline{c}^j$. Strict concavity of $u(c)$ implies that $c_t > \widetilde{c}_{t+1}$ and the sequence $\{\tilde{c}_{t+i}\}$ defined by $u'(\tilde{c}_{t+i}) = (\beta/\gamma)u'(\tilde{c}_{t+i+1})$ for $i > 1$ converges to zero. The condition that $u'(c) \uparrow \infty$ as $c \downarrow 0$ while $y^1 >> 0$ ensures that $\underline{c}^j$ is strictly positive for each j. Hence at some finite time, t^*, $\varphi(c_{t*-1}, y^j_{t*}) > 0$ so that $c_{t*} = c^j_{t*} \leq c^{Max}$. The next time $y = y^{Max}$ (corresponding to c^{Max}), $c = c^{Max}$. Subsequently, $\{c_t\}$ is a Markov chain that renews at c^{Max} whenever $y = y^{Max}$. ∎

In contrast to the unconstrained Pareto optimum, in which the borrower's consumption falls toward zero as a deterministic function of time, the sovereignty-constrained Pareto optimum provides the borrower a consumption level that converges to a Markov chain bounded from above by c^{Max} and from below by $c^{Min}(= \min\{c^1, ..., c^N\}) \gg 0$.

A further restriction on the process governing the borrower's endowment allows a fuller characterization of the constrained optimal allocation:

Theorem 3: Say that $\{y_t\}$ displays first-order stochastic dominance (meaning that, for any $\hat{y}$, the probability that $y_{t+1} > \hat{y}$ is nondecreasing in y_t). In this case $y^1 = \underline{c}^1 < \underline{c}^2 < ... < \underline{c}^N \leq y^N$ and $y^1 \leq \bar{c}^1 < \bar{c}^2 < ... < \bar{c}^N$.

Proof: We first show that $c^{Min} \equiv \min\{\underline{c}^1, \underline{c}^2, ..., \underline{c}^N\} = \underline{c}^1 = y^1$. Say that $c^{Min} = \underline{c}^j$. Then $u(c^{Min}) - u(y^j) + \beta EV(c^{Min}, y_{t+1}) = 0$ by definition. Since the Euler condition implies that $c^*_{t+1} < c^{Min} \leq \underline{c}^i$ for all i, $V(c^{Min}, y_{t+1}) = 0$ for any y_{t+1}. Hence $c^{Min} = y^j$. Say that $j \neq 1$. Then $\underline{c}^1 \geq \underline{c}^j = y^j > y^1$. Since $V(\underline{c}^1, y_{t+1}) \geq 0$ for each y_{t+1}, $u(\underline{c}^1) - u(y^1) + \beta EV(\underline{c}^1, y_{t+1}) > 0$, contradicting the definition of $\underline{c}^1$ so that $\underline{c}^1 < \underline{c}^j$ for all $j \neq 1$.

Suppose that $\underline{c}^1 < \underline{c}^2 < ... < \underline{c}^i$, for some $i < N$. We now show that $\underline{c}^j = \min\{\underline{c}^{i+1}, ..., \underline{c}^N\} = \underline{c}^{i+1}$. Assume not. By definition, $u(\underline{c}^j) - u(y^j) + \beta EV(\underline{c}^j, y_{t+1}) = 0$. First (i) $u(\underline{c}^{i+1}) - u(y^{i+1}) > u(\underline{c}^j) - u(y^j)$. The solution for the constrained equilibrium implies that $V(\underline{c}^{i+1}, y_{t+1}) \geq V(\underline{c}^j, y_{t+1})$ for each y_{t+1}, since $\underline{c}^{i+1} > \underline{c}^j$, and that $V(\underline{c}^j, y_{t+1}) = 0$ for $y_{t+1} = y^k$ for each $k > i$. Since the endowment process displays first-order stochastic dominance, (ii) $EV(\underline{c}^{i+1}, y_{t+1}) \geq EV(\underline{c}^{j}, y_{t+1})$. These two inequalities imply that $u(\underline{c}^{i+1}) - u(y^{i+1}) + \beta EV(\underline{c}^{i+1}, y_{t+1}) > 0$, contradicting the definition of $\underline{c}^{i+1}$.

The corresponding inequalities for $\overline{c}$ are proved in an analogous fashion.

Consider a period t in which $y_t = y^1$ and $c_t = \underline{c}^1$, so that $V(c_{t-1}, y_t) = 0$. It must be that $c_t = \underline{c}_1 \leq c_{t-1}$ and that $y_{t+1} \geq y_t$. Since V is increasing in c_{t-1} and decreasing in y_t, for any j, $V(c_t, y^j_{t+1}) \leq V(c_{t-1}, y_t) = 0$. But $V \geq 0$. Hence, for any j. $V(c_t, y^j_{t+1}) = 0$. Hence $V(c_{t-1}, y^1_t) = 0$ implies that $c_t = \underline{c}^1 = y^1$. ∎

The Simple Example Again

Let's return to the two-state $i.i.d.$ example introduced in section 2, in which $y_t = y^2$ with probability p and y^1 with probability $1-p$. Consider a case in which it will turn out that $\bar{c}^1 \leq \underline{c}^2$ and in steady state there are four possible $c_t's$ as follows:

$$c_t = \underline{c}^2 \text{ if } \omega_t = \{\omega_{t-1}, y^2\}$$

$$c_t = \bar{c}^1 \text{ if } \omega_t = \{\omega_{t-2}, y^2, y^1\}$$

$$c_t = \hat{c}^1 \text{ if } \omega_t = \{\omega_{t-3}, y^2, y^1, y^1\}$$

$$c_t = \underline{c}^1 \text{ if } \omega_t = \{\omega_{t-3}, y^1, y^1, y^1\}$$

It's useful to define the net resource transfers between the lender and borrower in each case: $\underline{\tau}^2 = y^2 - \underline{c}^2$, $\bar{\tau}^1 = \bar{c}^1 - y^1, \hat{\tau}^1 = \hat{c}^1 - y^1$, and $\underline{\tau}^1 = \underline{c}^1 - y^1$. Theorem 3 tells us that $\underline{\tau}^1 = 0$, and we have defined the remaining $\tau's$ to be, as it turns out, positive. The lender's problem tells us that:

$$\underline{\Pi}^2 = \Pi(\omega_{t-1}, y^2) = \underline{\tau}^2/(1-\gamma p)$$

$$\hat{\Pi}^1 = \Pi(\omega_{t-3}, y^2, y^1, y^1) = -\hat{\tau}^1 + \gamma p \Pi(\omega_{t-1}, y^2) + \gamma(1-p)\Pi(\omega_{t-3}, y^1, y^1, y^1)$$

$$\underline{\Pi}^1 = \Pi(\omega_{t-3}, y^1, y^1, y^1) = \gamma p \Pi(w_{t-1}, y^2)$$

Using these relationships, $\underline{\tau}^2$, $\bar{\tau}^1$, and $\hat{\tau}^1$ are determined by the three expressions:

$$\gamma u'(y^1 + \bar{\tau}^1) = \beta u'(y^1 + \hat{\tau}^1)$$

$$u(y^2) - u(y^2 - \underline{\tau}^2) = \beta(1-p)\{[u(y^1 + \bar{\tau}^1) - u(y^1)] + \beta(1-p)[u(y^1 + \hat{\tau}^1) - u(y^1)]\}$$
$$\frac{\gamma p \underline{\tau}^2}{[1 - \gamma(1-p)](1 - \gamma p)} = \bar{\tau}^1 + \gamma(1-p)\hat{\tau}^1.$$

The first is simply the first-order condition that applies if $\varphi = \psi = 0$, which is the case whenever $\omega_t = \{\omega_{t-3}, y^2, y^1, y^1\}$. The second comes from the condition that $V(\omega_{t-1}, y^2) = 0$. The third is from the condition that $\bar{\Pi}^1 = \Pi(\omega_{t-2}, y^2, y^1) = 0$. We also have that:

$$\hat{\Pi}^1 = \Pi(\omega_{t-3}, y^2, y^1, y^1) = \bar{\tau}^1 + \gamma(1-p)\hat{\tau}^1 - \hat{\tau}^1$$

$$\underline{\Pi}^1 = \Pi(\omega_{t-3}, y^1, y^1, y^1) = \bar{\tau}^1 + \gamma(1-p)\hat{\tau}^1$$

so that $\underline{\Pi}^2 = \frac{\gamma p + (1-\gamma)}{\gamma p}\underline{\Pi}^1 > \underline{\Pi}^1 > \hat{\Pi}^1 = \underline{\Pi}^1 - \hat{\tau}^1 > \bar{\Pi}^1 = 0$.

Market Implementation and Implications for Debt Dynamics

The sovereignty-constrained Pareto optimum can be supported by one-period loan contracts with event contingent repayments in much the same way as the unconstrained optimum: Each period t the lender makes a loan $l_t = \Pi_t(\omega_t) + c_t - y_t$ in exchange for a promise to pay $\Pi(\omega_t, y^j)$ in the event $y_{t+1} = y_j$. These loans have an expected return of zero. The term Π can be interpreted as the secondary market value of outstanding debt, or of the seniority rights associated with outstanding debt, $i.e.$, the value of the lender's exclusive relationship with the borrower. As our example shows, this has some surprising implications for debt dynamics.

First, note that, as y repeats, the value of Π stays the same or, if $y = y^1$, it can actually rise. The value of debt falls (to zero, in our example), when income falls, but afterward the value actually goes up.

Second, a falling value of consumption is consistent with an increasing value of outstanding indebtedness, as when $\omega_t = \{\omega_{t-3}, y^2, y^1, y^1\}$, in which case $c_{t-1} > c_t$ but $\Pi_{t-1} < \Pi_t$. Such a relationship has been observed, for example, by Gersovitz (1985) and has been interpreted as a rejection of the consumption-smoothing explanation for borrowing. Our example shows that the observation is consistent with this explanation in the presence of sovereignty constraints.

A Numerical Illustration

The two-state example solved above is simulated for $u(c) = \ln(c)$, $p = 1/2$ and alternative values of the lender's discount factor, γ, and the borrower's discount factor, β. The expected value of the endowment is set at 100, with y^1 and y^2 chosen to be either 80 and 120 or 70 and 130, respectively, in different simulations. The lender's discount factor, γ, is either .9, .95 or .99, while the borrower's, β, is .8, .85, or .9, but less than the lender's. (Higher values of β than those reported implied that $(\beta/\gamma)(y^1 + \hat{\tau}) > y^1$, contradicting the assumption that consumption takes on only four possible values. For this range of discount factors, an endowment fluctuating between 90 and 110 was also inconsistent with our four-value assumption.) The simulations solve for the values of $\bar{\tau}^1$, $\hat{\tau}^1$, $\underline{\tau}^2$, $\hat{\Pi}^1$, $\underline{\Pi}^1$, and $\underline{\Pi}^2$.

The main observation is that sovereignty constraints are consistent with transfers of a substantial portion of income. In high endowment states, the borrower is willing to transfer from 4 to up to 10 per cent of his endowment to the lender to maintain his relationship with her. In return, in a period in which his endowment has dropped from the previous period, she is willing to lend from around 8 to 25 per cent of the endowment in addition. The value of the debt (which is zero after a transition from a high to low endowment) is from 10 to 20 per cent of average endowment in a high endowment state ($\underline{\Pi}^2$), and from 60 to 80 per cent of that in low endowment states after the transition ($\hat{\Pi}^1$ and $\underline{\Pi}^1$). All of these magnitudes are commensurate with those observed during the last two decades. [2]

5. Conclusions and Extensions

We have characterized the Pareto-optimal allocation of resources between a borrower and lender subject to extreme sovereignty contraints: Neither party has any incentive to make transfers to the other other than the expectation of receiving such transfers at a future time. This is also the finding in Kletzer and Wright (1990), but the introduction of different rates of discount introduces a fundamental difference. Kletzer and Wright (1990) show, as the “Folk Theorem” implies, with a common discount factor sufficiently close to one, the two parties can sustain the unconstrained Pareto-optimal allocation, which is a constant consumption level for the borrower. With different discount factors this is not the case: The unconstrained optimum calls upon a smoothly declining consumption level for the borrower, which implies that eventually the borrower will always be making net resource transfers to the creditor. At this point, autarky is a preferable alternative, since it provides a dominating consumption path. As Rosenthal (1991) demonstrates, this outcome is not consistent with sovereignty constraints.

However, the only viable alternative is not autarky. Consumption smoothing can provide an incentive to service some debt. The sovereignty constraint allows the borrower to increase initial consumption to some extent, and then to experience smoothly declining consumption with transfers from the lender up to the point at which he would be unwilling to service more debt to smooth consumption. From this point on debt the debtor will make positive transfers to the lender when his endowment is high and receive positive or zero transfers from the lender when his endowment is low.

This prediction of the model is roughly consistent with the experience of sovereign lending during the last two decades. During the 1970s, the first decade in which the market operated to a significant extent in the postwar era, the major sovereign debtors were in most years net recipients of transfers from creditors. During the 1980s, however, they fluctuated frequently between net recipient and net payer status. (See the Appendix.)

While we have considered the interaction of the lender and borrower in a very limited context, the basic message of our analysis would survive a number of changes in assumption. We consider some major ones:

Investment We assume an endowment economy. We could, instead, allow for productive capital domestically . Indeed, fluctuations in investment opportunities rather than in endowments might by the source of an incentive to maintain access to capital markets. A productive role for capital would destroy the fluctuation-smoothing motive for repayment if it could provide a rate of return that strictly dominates the world interest rate in any period. But an economy with access to an investment opportunity of this type would have few economic problems anyway.

Punishments We have treated the cost of reneging on a debt contract as perpetual autarky. The equilibrium that this threat sustains is vulnerable to the criticism that it is not "renegotiation proof" is that, once the punishment was invoked, it would be to the mutual benefit of the two parties to recontract, and revert to the previous equilibrium. Kletzer and Wright (1990), however, derive punishments that are not susceptible of Pareto-improving renegotiation that, for sufficiently high discount factors, can sustain the same allocation as the threat of permanent autarky. We suspect that similar punishments apply when different discount factors differ.

Other Lenders Finally, we have assumed that there is only one lender. At certain points in their relationship the lender derives positive surplus from her relationship with the borrower. Would other potential lenders then step in to skimming off this surplus by offering the borrower better terms? Seniority provisions disallow this, but is legal force required? Kletzer and Wright (1990) show that, in fact, the threat to skim off anyone who tries to skim off can by itself enforce seniority. No external enforcement mechanism is needed.

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Notes

[1]That $\underline{c}^j$ exists is ensured by the fact that, since $\Pi \geq 0$, c, and hence V, are bounded, that $u(c) \downarrow -\infty$ as $c \to 0$, and that $u(y^j)$ is finite for any realization y^j. That $\bar{c}^j$ exists is ensured by the boundedness of Π and y.

[2]Cohen's (1991) claim that "The threat of financial autarky is never sufficient to keep a country from defaulting" follows from his assumption that once the credit ceiling is "tight" (in terms of our notation, that $V_t = 0$) it is "tight" subsequently (in our notation, that $V_{t+1} = 0$). To be sure, if the constraint is surely going to be tight next period any current net repayment would pull the borrower below his autarky welfare level. But in very general situations tighteness now need not imply tightness the next period.