Rule-of-Thumb Behavior and Monetary Policy

Jeffery D. Amato
Bank for International Settlements
Thomas Laubach
Board of Governors of the Federal Reserve System*

First version: April 2000
Revised: August 2000

Abstract

We investigate the implications of rule-of-thumb behavior on the part of consumers or price setters for optimal monetary policy. The existence of such behavior leads to endogenous persistence in output and inflation. By varying the fraction of rule-of-thumb agents, we are able to characterize their impact on optimal monetary policy and the equilibrium outcomes for inflation, output and interest rates. We show that the presence of rule-of-thumb agents alters the welfare objective that policymakers minimize. In particular, we provide a welfare-theoretic motivation for including *output* - not just the output *gap* - in the policymaker's objective. One main result is that highly inertial policy, found to be desirable in a purely forward-looking model, remains optimal even when most agents follow a rule of thumb.

---

*Amato: 4051 Basel, Switzerland, jeffery.amato@bis.org; Laubach: Washington, D.C. 20551, tlaubach@frb.gov. We gratefully acknowledge many helpful discussions with Michael Woodford. All remaining errors are ours. The views expressed herein are those of the authors and do not necessarily reflect those of the Bank for International Settlements, the Board of Governors of the Federal Reserve System, or the staff of either institution.

1 Introduction

The characterization of desirable monetary policy has been the subject of a large body of recent research. The question of what constitutes "optimal" monetary policy within structural models derived from optimizing behavior of households and firms has been a particularly lively area. A number of these studies are based on models in which the non-neutrality of monetary policy is derived from assuming frictions to price adjustment on the part of imperfectly competitive firms (e.g. Ireland 1997, Rotemberg and Woodford 1997, Clarida et al. 1999, Woodford 1999b). In these models, the price decisions of firms that are optimal given the assumed frictions to price adjustment[1] lead to a relation linking current inflation to a measure of the current output gap, or current marginal cost, and expected future inflation, which Roberts (1995) has called the "new-Keynesian Phillips curve." This desription of the supply side of the economy is usually complemented on the demand side by a standard Euler equation characterizing households' optimal consumption choice.

A notable feature of such models is the absence of lagged variables in the structural equations. The dynamics of output and inflation depend entirely on expectations of future values of these variables as well as future monetary policy actions. From an empirical perspective, this class of models has been criticized as being unable to replicate the high serial correlation found in both output and inflation data of many industrialized economies, unless one is willing to assume a substantial degree of serial correlation in the structural disturbances of the model (Fuhrer 1997a,b).

The failure of the consumption Euler equation to capture the dynamics of aggregate nondurable consumption, let alone those of aggregate output, has been debated for a long time (Mankiw et al. 1985, Deaton 1992, and many others). More recently, attention has focused on the question whether the new-Keynesian Phillips curve is able to explain the high serial correlation in inflation in the United States (Fuhrer and Moore 1995a) as well as other industrialized countries (Coenen and Wieland 2000). Fuhrer and Moore argue that the price setting problem underlying the new-Keynesian Phillips curve is misspecified, and propose a contracting specification in which price setters are concerned about their relative *real* contract price. However, their specification seems at odds with optimizing behavior, since profit maximization motivates only a concern for relative *nominal* contract prices or wages. On the other hand, Sbordone (1998) and Gali and Gertler (1999)

[1]The frictions to price adjustment referred to are alternatively Taylor's (1979) overlapping contracts, Rotemberg's (1982) convex costs of price adjustment, or Calvo's (1983) constant hazard model of readjustment of prices.

provide evidence that the source of inflation inertia arises from the sluggish response of firms' real marginal cost to fluctuations in output.

An alternative approach to explaining the apparent dependence of current values of output and inflation on *past* as well as expected future conditions is to allow the choices of some agents to deviate from optimal behavior due to, e.g., limits on their capacity to form fully rational expectations. Campbell and Mankiw (1989) argue that a model in which roughly half the population chooses current consumption based on an Euler equation, while the other half follows a "rule of thumb" by consuming current income, fits aggregate consumption data remarkably well. Similarly, Roberts (1997) and Gali and Gertler (1999) consider deviations from optimal expectations formation on the part of price setters. In particular, Gali and Gertler derive a structural relationship explaining current inflation as depending on *lagged* inflation as well as current marginal cost and expected future inflation by assuming that a fraction of firms set prices by following a rule of thumb, while the remaining firms set prices in the optimal, forward-looking manner. This approach to introducing dependence of current endogenous variables on lagged values — namely, within a utility-based framework — is appealing since it is compatible with evaluation of the effects of monetary policy based on the maximization of households' welfare.

This article studies the implications for optimal monetary policy of rule-of-thumb behavior. We do not attempt to provide a specific explanation for why a fraction of consumers or firms uses a rule of thumb. Independent of the reasons why agents might engage in such behavior, it is natural to ask whether there are any important consequences for the conduct of monetary policy *if* they do so.

We obtain four significant results. First, we show that rule-of-thumb behavior, by consumers or price setters, affects the objective that monetary policy should pursue in order to maximize the representative household's welfare. In addition to the objectives of inflation and output gap stabilization that the existing literature either assumes, or derives from optimizing behavior of households, our results show that the monetary authority should also seek to stabilize the first difference of inflation (in the case of rule-of-thumb price setters), or the first difference of output (in the case of rule-of-thumb consumers). Importantly, this latter result provides justification for the common practice of including a term in the policymaker's objective involving the variability of *output*. However, our derivation also makes clear that terms for both the output gap and output should be present, and that the weight on the latter should vary with the proportion of forward-looking behavior in the economy.

Second, our results affirm the finding of Rotemberg and Woodford (1999) and Woodford (1999b) that a highly inertial interest rate policy is desirable, as characterized by a feedback coefficient on the lagged interest rate exceeding one in simple interest rate rules. This result is robust even when an overwhelming proportion of price setters or consumers are following the rule of thumb. This stands in direct contrast to the suggestion (e.g. Taylor 1999) that such an inertial interest rate policy ceases to be optimal, or even feasible, once backward-looking behavior is incorporated into the structure of the model.

Third, even though endogenous inflation persistence increases as the fraction of rule-of-thumb price setters increases, the variance of inflation *declines* under optimal monetary policy. By itself, an increase in persistence makes shocks to inflation more long-lived, and inflation more variable. But a larger degree of rule-of-thumb behavior also dampens the effects of shocks on inflation. Overall, this latter effect dominates the increase in persistence. At the same time, the variability of the output gap increases. Since monetary policy affects inflation by influencing expected future output gaps, to mitigate the drawn-out effects of shocks on inflation, it becomes necessary to induce larger fluctuations in the output gap. In our model, this policy strategy is optimal because inflation fluctuations are relatively costly from a welfare perspective.

Fourth, as the fraction of rule-of-thumb consumers increases, the average level of welfare increases, even though endogenous output persistence increases at the same time. Increasing the degree of rule-of-thumb consumption reduces the *impact* of shocks on output and inflation in our model. As it turns out, the muted impact of shocks more than offsets their pronounced dynamic effects resulting from greater persistence.

The remainder of the article is structured as follows. The next section introduces rule-of-thumb consumers and rule-of-thumb price setters into an otherwise standard sticky price optimizing model, and derives a representation of households' welfare which monetary policy seeks to maximize. Section 3 analyzes the implications of rule-of-thumb behavior for optimal monetary policy, while section 4 analyzes an extension of the model. Section 5 offers conclusions. The derivation of several results is taken up in Appendix A and some calibration issues are discussed in Appendix B.

2 A Structural Model with Rule-of-Thumb Behavior

To analyze the consequences for optimal monetary policy of rule-of-thumb behavior by a fraction of agents, we use a small structural model derived from optimizing behavior of households and imperfectly competitive suppliers. Except for the presence of rule-of-thumb behavior, our model is identical to that of Woodford (1999b). Specifically, we assume that the economy consists of a continuum of households, each of which is the monopolistic supplier of one differentiated product. Because households derive utility from consuming an aggregate of the differentiated products, suppliers face a downward-sloping demand schedule for their product. To keep the model as simple as possible, the economy is assumed closed, and there is no capital accumulation, so that goods market clearing requires that all output is being consumed each period.

In this section, we first derive the implications of the presence of rule-of-thumb consumers for the relationship between expected future real interest rates and aggregate demand for output. Second, we analyze the implications of rule-of-thumb price setting for the relationship between output and inflation. Finally, we derive an approximation to the representative household's welfare in the presence of both rule-of-thumb consumers and rule-of-thumb price setters.

2.1 Rule-of-Thumb Consumers and Aggregate Demand

The population of households consists of forward-looking and rule-of-thumb consumers. Each forward-looking household i maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t^i; \xi_t) - v(y_t(i); \xi_t) \right] \tag{1}$$

C_t^i is household i's consumption of the usual Dixit-Stiglitz aggregate of individual products, with the elasticity of substitution between products described by θ. The household maximizes (1) subject to its budget constraint and the constraint that its supply of its product equals demand,

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t \tag{2}$$

Here $p_t(i)$ denotes the price that household i charges per unit of its product, and P_t and Y_t denote the utility-based price index and the aggregate of individual households' output respectively, corresponding to the aggregate C_t. The vector-valued random disturbance ξ in (1) represents taste shifters affecting the utility of consumption $u(\cdot; \xi)$ and the disutility of supply $v(\cdot; \xi)$ respectively.

The effects of ξ on u and v are not required to be mutually orthogonal. We assume that financial markets are complete, and that households insure themselves against all idiosyncratic risk, such that their wealth is identical *ex ante*. Therefore all forward-looking households choose the same consumption path, which we denote by $\{C_t^f\}$.

Rule-of-thumb behavior is introduced by assuming that only a fraction ψ of households is able to choose consumption according to the path that maximizes (1). In particular, optimal consumption choice depends on expectations of real interest rates into the indefinite future. Suppose, for example, that only a fraction ψ of households is able to form these expectations. The remaining fraction $1-\psi$ chooses its consumption in period t, C_t^b, following the simple rule of thumb

$$C_t^b = C_{t-1} \tag{3}$$

where $C_t \equiv \psi C_t^f + (1-\psi)C_t^b$ denotes per capita consumption in period t. The rule (3) has the important feature that rule-of-thumb consumers learn from forward-looking households with one period delay. Hence, although consumption choice according to (3) is obviously not optimal, differences between C_t^b and C_t^f will always remain bounded.[2]

Goods market clearing requires that $C_t = Y_t\ \forall t$. Hence (3) implies that $C_t^b = Y_{t-1}$, and therefore

$$C_t^f = \psi^{-1}(Y_t - (1-\psi)Y_{t-1}). \tag{4}$$

As shown in appendix A, taking a log-linear approximation of the forward-looking households' first-order condition characterizing the optimal choice of C_t^f and substituting from (4) leads to an intertemporal IS equation of the form

$$y_t - \tilde{g}_t = (1-\delta)y_{t-1} + E_t[\delta y_{t+1} - \tilde{g}_{t+1}] - \tilde{\sigma}^{-1}[r_t - E_t\pi_{t+1}] \tag{5}$$

Here, y_t denotes the percent deviation of aggregate output from its steady-state (or long run trend) level, π_t is the growth rate of the aggregate price index, i.e. $\pi_t \equiv \log(P_t/P_{t-1})$, and r_t is the nominal interest rate on a one-period riskless bond.[3] The parameter ψ enters (5) through

[2] We assume that tranfers between forward-looking and rule-of-thumb consumers exist such that differences in the consumption choices of the two groups do not lead to the accumulation of claims by one group on the other. Importantly, this does not affect the pricing of contingent contracts because all agents are identical from an *ex ante* perspective.

[3] More specifically, all log-linearizations are taken around a steady state with zero inflation. Hence, π_t is by definition the percent deviation from its steady-state value, while r_t denotes the percent deviation of the interest rate from its steady state value associated with zero inflation.

$\delta \equiv \frac{1}{2-\psi}$, $\tilde{\sigma}^{-1} \equiv \frac{\psi}{2-\psi}\sigma^{-1}$, and $\tilde{g}_t \equiv \frac{\psi}{2-\psi}g_t$. The parameter σ^{-1} measures the intertemporal elasticity of substitution in consumption. The term g_t represents variation in spending that is not caused by changes in the real interest rate, such as disturbances to the marginal utility of consumption caused by fluctuations in ξ_t.

In the case where all households are able to choose consumption optimally, i.e. $\psi = 1$, (5) reduces to the standard intertemporal IS equation

$$y_t - g_t = E_t[y_{t+1} - g_{t+1}] - \sigma^{-1}[r_t - E_t\pi_{t+1}] \tag{6}$$

that characterizes the demand side in the models of Woodford (1996, 1999b), McCallum and Nelson (1999), Clarida et al. (1999), and many other studies. The key difference between (5) and (6) is the effect of lagged on current output in the presence of rule-of-thumb consumers. Furthermore, smaller values of ψ reduce the effects of both the expected real interest rate (through smaller values of $\tilde{\sigma}^{-1}$) and of the disturbance g_t on current output.

A variable that plays an important role in the analysis below is the output gap $x_t \equiv y_t - y_t^n$, where y_t^n denotes the "natural rate of output", the level of output that would obtain if prices were completely flexible. The IS equation (5) can be expressed in terms of the output gap as

$$x_t = (1-\delta)x_{t-1} + \delta E_t x_{t+1} - \tilde{\sigma}^{-1}E_t[r_t - \tilde{r}_t^n - \pi_{t+1}] \tag{7}$$

where

$$\tilde{r}_t^n \equiv \tilde{\sigma}E_t[\delta y_{t+1}^n + (1-\delta)y_{t-1}^n - y_t^n - (\tilde{g}_{t+1} - \tilde{g}_t)] \tag{8}$$

is the Wicksellian "natural rate of interest", the real interest rate that would obtain if all prices were flexible, and that would correspond to the equilibrium nominal interest rate in the case of price stability. The natural rate of interest in the special case that $\psi = 1$, which we denote by r_t^n, is defined by

$$r_t^n \equiv \sigma E_t[y_{t+1}^n - y_t^n - (g_{t+1} - g_t)]. \tag{9}$$

2.2 Rule-of-Thumb Price Setters and Aggregate Supply

Real effects of monetary policy arise in this model from the assumption that within a given period not all suppliers are able to adjust their prices in response to fluctuations in demand. Specifically, we follow Calvo (1983) in assuming that each period a fraction $1-\alpha$ of suppliers is offered the opportunity to choose a new price, while the remaining suppliers have to maintain whichever price they charged before. Moreover, suppliers are drawn randomly and independent of their own history, in particular independent of the time they were last offered the opportunity to adjust their price.

Since every supplier faces the same demand function (2), all forward-looking suppliers chosen in period t to adjust their price will choose the same price, which we denote by p_t^f, and which maximizes

$$E_t \left\{ \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\frac{u_c(Y_{t+j};\xi_{t+j})}{P_{t+j}} \left(\frac{p_t^f}{P_{t+j}} \right)^{-\theta} Y_{t+j} p_t^f - v\left(\left(\frac{p_t^f}{P_{t+j}} \right)^{-\theta} Y_{t+j}; \xi_{t+j} \right) \right] \right\} \tag{10}$$

The first term in brackets represents the household's utility from consumption in period $t+j$ if it chooses price p_t^f in the current period. It is the product of its revenue in period $t+j$ conditional on its price being p_t^f, and the marginal utility of income in period $t+j$, $u_c(Y_{t+j};\xi_{t+j})/P_{t+j}$. The second term represents the disutility incurred from supplying the amount of its product demanded in period $t+j$ if its price is still p_t^f. Since the price chosen in period t will still be in effect in period $t+j$ with probability α^j, the household discounts the stream of future utilities conditional on its choice of price today by the factor $\alpha\beta$.

To introduce rule-of-thumb behavior, we assume that only a fraction λ of suppliers is able to set its price in the forward-looking manner described in (10), while the remaining suppliers follow a simple rule of thumb, in which they learn from the prices chosen in the previous period by both forward-looking and rule-of-thumb price setters. We follow Gali and Gertler (1999) by assuming that the rule-of-thumb price setters chosen in period t to adjust their price are setting this price, which we denote by p_t^b, according to

$$p_t^b = P_{t-1}^* \cdot \frac{P_{t-1}}{P_{t-2}} \tag{11}$$

where P_t^* is the aggregate of the prices newly chosen in period t by both forward-looking and rule-of-thumb price setters. While we do not propose any particular explanation for this behavior, one might consider a situation in which it is costlier for some suppliers than for others to gather the information necessary to maximize (10). Like the rule (3), however, the rule (11) has the property that deviations of p_t^b from p_t^f are temporary since (11) relies on prices chosen by forward-looking price setters in the previous period.

The assumption that suppliers are offered the opportunity to reset their price independent of their history leads to a simple law of motion for the aggregate price index. Combining this law of motion with equations (10) and (11) then leads to a modified version of the new Keynesian Phillips curve (derived in appendix A)

$$\pi_t = \tilde{\kappa} x_t + \gamma^b \pi_{t-1} + \gamma^f E_t \pi_{t+1} \tag{12}$$

The coefficients $\tilde{\kappa}, \gamma^b$, and γ^f are functions of the structural parameters $\beta, \sigma, \theta, \alpha, \lambda$, and the parameter ω measuring the elasticity of the disutility function v.[4] In the case that all price setters are forward-looking, i.e. $\lambda = 1$, (12) reduces to the "new-Keynesian Phillips curve"

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{13}$$

where the coefficients $\tilde{\kappa}$ and κ are related by $\tilde{\kappa} = \frac{\lambda\alpha}{\alpha+(1-\lambda)(1-\alpha(1-\beta))}\kappa$. Comparing (12) and (13), the most important effect of allowing for rule-of-thumb behavior is that current inflation is now in part determined by lagged inflation. For smaller values of λ, the importance of the lagged inflation term relative to expected inflation increases. Moreover, smaller values of λ reduce the sensitivity of current inflation to fluctuations in the current output gap by reducing $\tilde{\kappa}$.

2.3 Welfare Implications of Rule-of-Thumb Behavior

One effect of allowing for rule-of-thumb behavior is that lagged endogenous variables appear in equations (5) and (12). A second implication of allowing for rule-of-thumb behavior, not previously noted in the literature, is that it affects the objective that monetary policy seeks to achieve. The early literature on desirable stabilization policy, such as Taylor (1979), postulates that the objective for monetary policy should be to minimize some combination of the variances of inflation and either output or the output gap. Within the context of an optimization-based model similar to the one above, but absent rule-of-thumb behavior, Rotemberg and Woodford (1997) and Woodford (1999a) show that maximization of the representative household's welfare indeed implies minimization of the variances of inflation and the output gap, where the relative weights on inflation and output gap stabilization are determined by the model's structural parameters.

[4] Many authors have utilized a Phillips curve equation similar to (12), except that they put weights on expected and lagged inflation that sum to one, with each typically equal to one half. However, in our model it is not possible for the condition $\gamma^f = \gamma^b = 1/2$ to hold, since this would require either $\beta = 1$ or $\lambda = 0$ (see Appendix A). The coefficients γ^f and γ^b have the properties that they are positive, sum to less than one, and are equal to each other only when $\lambda = 1 - \alpha\beta = 0.3466$, the latter equality arising under our choices for α and β, as discussed in section 3.1.

To describe how rule-of-thumb behavior affects the objective for monetary policy, suppose that monetary policy chooses at some point $t = 0$ a plan that maximizes the representative household's welfare, defined by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\psi u(C_t^f; \xi_t) + (1-\psi) u(C_t^b; \xi_t) - \int_0^1 v(y_t(i); \xi_t) di \right] \right\} \tag{14}$$

The integral over $v(y_t(i); \xi_t)$ is taken in order to abstract from the effects on an individual household's supply of the particular date at which it last had the opportunity to adjust its price. We evaluate alternative policies by taking the unconditional expectation of (14) with respect to the distribution of exogenous shocks, and under the assumption that all endogenous variables in the initial period are at their unconditional expectation of zero, to ensure that the desirability of the chosen plan does not depend upon initial conditions at time 0.[5]

In appendix A, we show that a second-order Taylor series expansion of (14) around the same steady state used to derive (5) and (12) can be expressed in the form of a loss function

$$W = \sum_{t=0}^{\infty} \beta^t E_0 L_t \tag{15}$$

where the period-loss L_t depends on the values of the parameters ψ and λ. We first discuss the loss function in the case $\psi = \lambda = 1$, and then consider the cases in which either ψ or λ are different from 1.[6]

In the special case without rule-of-thumb behavior, i.e. $\lambda = \psi = 1$, the period-loss function L_t is given by

$$L_t \equiv \pi_t^2 + \frac{\kappa}{\theta} x_t^2$$

Consequently, after taking the unconditional expectation of (15) to abstract from initial conditions (as discussed above), the loss function becomes

$$E(W) = V[\pi_t] + \frac{\kappa}{\theta} V[x_t] \tag{16}$$

where the measure of variability for any variable z that is used here is defined by

$$V[z] \equiv E\left[(1-\beta) \sum_{t=0}^{\infty} \beta^t E_0 z_t^2 \right]$$

[5]This allows us to view our time-invariant rules as approximations to an optimal plan that does not exploit initial conditions. See Woodford (1999a) for further discussion of this point.

[6]For clarity of exposition, we do not consider the situation in which both ψ and λ are different from 1, but the extension of the current analysis to this case is straightforward.

Except for discounting, this measure corresponds to the unconditional variance of z_t. The objective (16) is analyzed, e.g., by Clarida et al. (1999).

The model presented above has the property that fluctuations in the output gap are the only source of inflation variability, as can be seen from (12). It is therefore possible to completely stabilize both inflation and the output gap in response to fluctuations in r_t^n, and hence no tradeoff exists between the goals of output gap stabilization and inflation stabilization. As Rotemberg and Woodford (1997) point out, however, there may exist a tradeoff between the goals of output gap and *price level* stabilization due to the zero lower bound on nominal interest rates. For a sufficiently variable process $\{r_t^n\}$, complete stabilization of the output gap requires interest rate variability sufficiently high such that a positive steady-state rate of inflation is necessary to avoid the zero lower bound. In fact, the second-order approximation of welfare given by (16) assumes that both π_t and x_t have steady-state values of 0. To account for the possibility that steady-state inflation $\bar{\pi}$ will be non-zero under the optimal policy requires (16) to be augmented by the term $\bar{\pi}^2$.[7] Furthermore, we follow Rotemberg and Woodford (1997) and penalize interest rate variability by assuming that, for any level of interest rate variability, all realizations of the interest rate will be distributed within the interval $[E[r] - kV[r]^{1/2}, E[r] + kV[r]^{1/2}]$. Hence, only interest rate policies such that $kV[r]^{1/2}$ exceeds the steady-state net real interest rate $\beta^{-1} - 1$ cause positive steady-state inflation rates. Thus, the criterion for evaluating alternative policies is based on a modification of (16) given by

$$\hat{W} = V[\pi_t] + \frac{\kappa}{\theta} V[x_t] + \Big(\max\{kV[r]^{1/2} - (\beta^{-1} - 1), 0\}\Big)^2 \tag{17}$$

In the presence of rule-of-thumb price setters (but absent rule-of-thumb consumers), the period-loss function becomes

$$L_t^p \equiv \pi_t^2 + \frac{\kappa}{\theta} x_t^2 + \frac{1-\lambda}{\alpha\lambda} (\pi_t - \pi_{t-1})^2 \tag{18}$$

while the criterion for evaluating alternative policies is given by

$$\hat{W}^p = V[\pi_t] + \frac{\kappa}{\theta} V[x_t] + \frac{1-\lambda}{\alpha\lambda} V[\pi_t - \pi_{t-1}] + \Big(\max\{kV[r]^{1/2} - (\beta^{-1} - 1), 0\}\Big)^2 \tag{19}$$

The fact that now a fraction $1 - \lambda$ of price setters is learning about the optimal price by observing the average of prices set in the previous period creates a rationale for reducing variability in the

[7] In addition, the proper loss function contains a similar term related to the steady-state level of the output gap $\bar{x}$ equal to $\frac{\kappa}{\theta}\bar{x}^2$. By equation (13), $\bar{x} = \frac{1-\beta}{\kappa}\bar{\pi}$. As will be seen, the weight on this term is close to zero, so it is omitted in the calculations that follow.

change of inflation. The only effect of this additional term is to increase the weight on inflation stabilization relative to output gap stabilization, since any reduction in inflation variability reduces the variability of the change in inflation as well. The additional term in the loss function in particular does not generate any new tradeoffs between stabilization of the various variables, since price stability still achieves the minimum of all three terms entering (18). Hence, even in the presence of rule-of-thumb price setters, the only potential tradeoff is due to the lower bound on nominal interest rates.

In the presence of rule-of-thumb consumers (but absent rule-of-thumb price setters), the period-loss function becomes

$$L_t^c \equiv \pi_t^2 + \frac{\kappa}{\theta}\left[x_t^2 + \frac{\sigma}{\sigma+\omega}\frac{1-\psi}{\psi}(y_t - y_{t-1})^2\right] \tag{20}$$

while the objective that monetary policy seeks to minimize is given by

$$\hat{W}^c = V[\pi_t] + \frac{\kappa}{\theta}\left[V[x_t] + \frac{\sigma}{\sigma+\omega}\frac{1-\psi}{\psi}V[y_t - y_{t-1}]\right] + \left(\max\{kV[r]^{1/2} - (\beta^{-1}-1), 0\}\right)^2 \tag{21}$$

When a fraction $1-\psi$ of households is choosing consumption following the rule of thumb (3), fluctuations in output (not only in the output gap) create welfare losses, because changes in output cause larger departures of rule-of-thumb from optimal consumption. In contrast to the case of rule-of-thumb price setting, the presence of rule-of-thumb consumers *does* give rise to an additional tension among the various stabilization goals in (20). In particular, even if complete output gap (and hence inflation) stabilization is feasible without inducing positive steady-state inflation, the output variability that is necessary to keep output at its natural rate implies welfare losses according to the criterion (21).

3 Optimal Monetary Policy

This section characterizes how optimal monetary policy differs as the fraction of forward-looking agents varies. We conduct the analysis separately under the two types of rule-of-thumb behavior modelled to make clear the implications of each for optimal policy.[8] We perform our analysis in the context of simple interest rate rules that belong to the class of generalized Taylor rules.[9] We

[8] It is plausible that the fractions of forward-looking consumers and price setters are not equal in actual economies. Price setters are only a small subset of the total population, for which the incentives and costs of using rules of thumb likely differ from typical consumers.

[9] By using a rule we restrict ourselves to the case of commitment.

have two reasons for considering simple rules directly, as opposed to the optimal plan. First, one of the main purposes of this paper is to address whether super-inertial policy remains optimal in the presence of backward-looking agents. This type of policy in the context of a simple rule is represented by a coefficient greater than one on the lagged interest rate. The calculation of the optimal response to the lagged instrument is necessary because a measure of interest rate inertia from an unconditional autocorrelation function is misleading for this purpose. Such a measure captures persistence in interest rates due to persistence in other variables that enter the policy rule, as well as from responses to lagged interest rates.[10] Furthermore, the calculation of optimal simple rules within a class of generalized Taylor rules allows us to make direct comparisons to similar rules recommended by other authors (e.g. the various contributors to Taylor (1999)). Second, as Woodford (1999b) shows in the purely forward-looking case, simple rules closely approximate the properties of the optimal plan.[11] Since our model contains lagged endogenous variables, we may expect Woodford's result to carry through here with due account taken to include the appropriate lagged state variables in our rules.

3.1 Calibration

Before proceeding to analyze the properties of optimal simple interest rate rules, we briefly discuss how we calibrated the model. The two equations (7) and (12) contain seven structural parameters ($\alpha, \beta, \sigma, \omega, \theta, \lambda$ and ψ), for which values must be specified. Furthermore, imposition of the zero lower bound on nominal interest rates requires us, through our formulation of this constraint in (19), to pick k. When we focus upon the implications of introducing rule-of-thumb price setters (subsection 3.2), we fix $\psi = 1$ and vary λ across its feasible range of (0,1]; in subsection 3.3, we instead focus on rule-of-thumb consumers by fixing $\lambda = 1$ and varying ψ in the interval (0,1]. The remaining six parameters were chosen to equal those used by Woodford (1999a, 1999b), which he obtained based on the estimation results of Rotemberg and Woodford (1997). These values are

[10] Persistence in these other variables, e.g. inflation, may arise due to endogenous persistence in the other structural relations of the model or from exogenous shocks.

[11] In general, many different rules implement the optimal plan. In the purely forward-looking case, there is no loss in generality in restricting oneself to rules involving lag polynomials in inflation and interest rates. Furthermore, Woodford shows that one can closely approximate the optimal rule in this class by considering responses to only current inflation and the interest rate lagged once.

given in Table 1.[12] The parameterization and calibration of the shock processes assumes that g_t and y_t^n, and hence r_t^n, are AR(1) processes. The standard deviations (τ) and correlation coefficients (ρ) of g_t and y_t^n are given in Table 1, as is the correlation between the two processes, ρ_{gy}. These parameter choices imply a standard deviation τ_r of r_t^n of 3.07, and a correlation ρ_r of 0.35. Details of the calibration are described in Appendix B.

Table 1: Parameter Values

Structural Parameters	
α	0.66/quarter
β	0.99/quarter
σ	0.157
ω	0.473
θ	7.88
Shock Processes	
ρ_g	0.35
τ_g	29.46 percent/year
ρ_y	0.35
τ_y	4.29 percent/year
ρ_{gy}	-0.082
τ_u	1.535 percent/year
Interest-Rate Bound	
k	2.26

We have three main reasons for calibrating the model in this way. First, Rotemberg and Woodford estimate a version of a fully specified general equilibrium model which comes closest to matching our model equations and in which all of our structural parameters (except λ and ψ) are present.[13] Gali and Gertler (1999) estimate an equation for inflation in their model of rule-of-thumb price setters, thus providing estimates of the parameters α, β and λ. Our choice of $\alpha = 0.66$ is slightly lower than their estimates, which are in the interval from 0.8 to 0.87, although our value is

[12]The parameters were chosen so that one period equals a quarter.

[13]Unlike in the present case, Rotemberg and Woodford's model involves decison lags in consumption planning and time delays for a fraction of price setters in establishing newly chosen prices. However, by taking two-period expectations of their structural equations, one obtains (6) and (13).

well within the range of existing survey evidence (e.g. Blinder, 1994). Their estimates of λ (0.48 to 0.92) show that a wide range of values for this parameter are consistent with their data, but it is also clear that focusing on the range $\lambda > 0.5$ is most realistic from an empirical perspective.[14] Second, Rotemberg and Woodford's sequential estimation approach suggests that the values of the structural parameters they obtained remain empirically valid even if the processes for the exogenous shocks are misspecified. Third, one of our primary motivations is to assess whether Woodford's (1999b) finding about the desirability of inertial policy remains valid in the presence of backward-looking behavior, so using the same values for common parameters affords a direct comparison to his results.

3.2 Results with Rule-of-Thumb Price Setters

In this subsection, we consider the case of rule-of-thumb price setters, i.e. when the model's structural equations are given by (6) and (12). In much of the recent literature that examines simple rules for conducting monetary policy, the variables focused upon typically only include a short-term interest rate as the policy instrument and inflation, output, and lagged interest rates as the feedback variables.[15] We analyze similar rules, with the exception that we omit responses to the current observation on output, as discussed above. However, as we showed in section 2, the presence of rule-of-thumb price setters alters the policymaker's welfare-based objective by supplementing the standard objective with a term that is proportional to the discounted variance of the change in inflation, as shown in (19). Consequently, here we allow for the possibility that optimal policy is better characterized by a rule that includes a term for the change in inflation:

$$r_t = ar_{t-1} + b\pi_t + c(\pi_t - \pi_{t-1}) \tag{22}$$

The coefficients a, b and c in (22) are chosen to minimize the objective $\hat{W}^p$ subject to the constraints imposed by the model and the policy rule.[16] In addition to examining optimal policy under the

[14] One reason for not using Gali and Gertler's results is they estimate only a single equation model, and thus they do not provide the full set of parameters needed. An additional reason is they estimate their model using data on real unit labor costs instead of the output gap, x_t.

[15] In some cases, output has been replaced with the output *gap*.

[16] In the absence of rule-of-thumb behavior, our model reduces to the one analyzed by Woodford (1999b), who shows that a rule of this form with $a = 14.3, b = 50.5$ and c equal to zero approximates the optimal plan.

theoretically preferred measure of welfare $\hat{W}^p$, we also characterize policy under an objective in which the discounted variance of the change in inflation is omitted, given by $\hat{W}$ in (17). This latter objective is closer to the standard form of loss function assumed in many other studies. By considering this more restricted objective, we can separate the implications for optimal policy from introducing endogenous inflation persistence apart from changing the policymaker's loss function as well. Finally, for both objectives, optimal response coefficients are found, in turn, by varying the proportion of forward-looking price setters.[17]

Table 2: Welfare and Variances with Rule-of-Thumb Price Setting *

	$V[\pi_t]$		$V[\pi_t - \pi_{t-1}]$		$V[x_t]$		$V[r_t]$		$\hat{W}$		$\hat{W}^p$	
	Objective:		Objective:		Objective:		Objective:		Objective:		Objective:	
λ	$\hat{W}^p$	$\hat{W}$	$\hat{W}^p$	$\hat{W}$	$\hat{W}^p$	$\hat{W}$	$\hat{W}^p$	$\hat{W}$	$\hat{W}^p$	$\hat{W}$	$\hat{W}^p$	$\hat{W}$
1.0	0.062	0.062	0.050	0.050	5.474	5.474	1.880	1.880	0.337	0.337	0.337	0.337
0.9	0.060	0.059	0.040	0.040	5.522	5.539	1.882	1.880	0.337	0.337	0.343	0.343
0.8	0.056	0.057	0.031	0.031	5.617	5.627	1.885	1.881	0.338	0.338	0.350	0.350
0.7	0.052	0.053	0.024	0.024	5.767	5.756	1.888	1.882	0.342	0.341	0.357	0.357
0.6	0.046	0.049	0.018	0.018	5.982	5.944	1.891	1.884	0.346	0.346	0.365	0.365
0.5	0.038	0.042	0.012	0.013	6.268	6.209	1.895	1.887	0.353	0.353	0.372	0.373
0.4	0.029	0.033	0.008	0.008	6.622	6.560	1.899	1.891	0.362	0.362	0.380	0.381
0.3	0.020	0.023	0.004	0.004	7.021	6.976	1.903	1.897	0.373	0.373	0.388	0.388
0.2	0.011	0.012	0.002	0.002	7.411	7.394	1.907	1.902	0.383	0.383	0.394	0.394
0.1	0.004	0.004	0.000	0.000	7.722	7.723	1.910	1.908	0.392	0.392	0.397	0.397

* Notes: The table shows the levels of welfare (last 4 columns) and its component discounted variances (columns 2 through 9) achieved under the optimal rule (22) when $\hat{W}^p$ is minimized (left column under each statistic) or $\hat{W}$ is minimized (right column) for different values of λ. Inflation and the interest rate are expressed in annualized percentages, the output gap in percentages.

Table 2 shows the values of the objective functions and their components, discounted variances of the endogenous variables, obtained under the various cases described above. Under each statistic, the first column corresponds to when $\hat{W}^p$ is used as the objective, the second column when $\hat{W}$ is the objective. The fact that perfect stabilization is not possible once the zero lower bound on interest

[17] The optimization was performed for λ from 0.1 to 1, at a step of 0.1.

rates is taken account of is evident by non-zero values for all of the loss components. The result that the two variances involving inflation decrease as λ decreases, whereas the variances of the output gap and the interest rate increase, is only in part due to this tradeoff.[18] Inspection of (12) reveals that there are two effects on the AS equation from varying λ. On the one hand, the degree of endogenous inflation persistence increases as λ decreases, which, *ceteris paribus*, would imply a higher $V[\pi_t]$. On the other hand, the coefficient $\tilde{\kappa}$ declines with smaller values of λ, implying that fluctuations in the output gap (due to discrepancies between the natural rate and the real rate of interest) have a weaker impact on inflation. However, because any disturbances to inflation are long-lived for small values of λ, the incentive to minimize them increases as λ declines, which is evident in declining values for $V[\pi_t]$. The fact that smaller values of $\tilde{\kappa}$ make it more difficult for policy to counteract the prolonged effects of shocks on inflation — through changes in current and expected future output gaps — thus requires $V[x_t]$ and $V[r_t]$ to increase. One consequence is that it is optimal to incur higher interest rate variability — and, therefore, higher steady-state inflation — for small λ than would be needed to keep $V[\pi_t]$ constant across different values of λ. However, the cost in terms of increased interest rate variability necessary to offset these persistent fluctuations apparently exceeds the benefits from stabilizing the output gap, which, as stated above, has an increasingly smaller effect on $V[\pi_t]$. This is shown in Table 2 by relatively larger increases in $V[x_t]$ compared to $V[r_t]$.

By contrast, the higher weight on inflation stabilization in the objective $\hat{W}^p$ as λ decreases is only of minor importance. This is seen in the results under the objective $\hat{W}$. For this objective, the total weight on inflation terms does not increase as λ decreases, so the differences in outcomes solely reflect the degree of endogenous inflation persistence, the responsiveness of inflation to the output gap, and the resultant appropriate adjustments of optimal policy in response. Nevertheless, under the restricted objective $\hat{W}$, the variance of inflation is higher and the variance of the interest rate is lower for most values of λ, which implies that the higher implicit weight on the variance of

[18]The standard variability frontiers presented in the literature are mapped out by changing the relative weights in the objective on the terms involved in the tradeoff. For an example of this type of frontier in the purely forward-looking version of our model, see Figure 6.1 in Woodford (1999a). But λ is a structural parameter of the model here, not a weight in the welfare function. While it is true that the weight on $V[\pi_t - \pi_{t-1}]$ depends upon λ, it is also true that the zero lower bound constraint implicitly depends upon this parameter, with the resulting relative effect in the welfare function being unclear *a priori*. See the discussion below.

inflation in the proper loss function $\hat{W}^p$ generally leads to lower inflation variation in equilibrium.[19] It is worth noting that the level of welfare achieved, regardless of which objective is used by the policymaker, is almost independent of λ. Thus it appears for rule-of-thumb price setters that the choice of objective simply changes the *mix* between the loss components, but not the overall level of welfare.

Corresponding to the statistics presented in Table 2 are optimal values for the three feedback coefficients. Strikingly, each of the optimal feedback coefficients changes monotonically with respect to λ and there is no discrete change in policy in moving from $\lambda = 1$ to $\lambda < 1$. The optimal coefficient on inflation ranges from 8.24 for $\lambda = 0.1$ to 70.36 for $\lambda = 1$, while the coefficient on the change in inflation varies from 96.49 to -2.70. More importantly, the optimal values of a range from 1.98 to 13.32.[20,21] That is, for even a very small fraction of forward-looking price setters, super-inertial policy is optimal. Thus, the suggestion that the desirability of this policy behavior is highly dependent upon the sole existence of purely forward-looking agents seems to be unfounded. Additional evidence in support of this conclusion is provided by considering the impact on welfare of changing the degree of inertial behavior in policy (i.e. the size of a) for various values of λ. To do so, we fixed a at different values in a grid from 0 to 2 and found the optimal values for the other response coefficients.[22] Figure 1 shows a plot of the level of welfare, $\hat{W}^p$, obtained across different values of a for $\lambda = (0.2, 0.4, 0.6, 0.8, 1.0)$. The plot confirms our conclusion that highly inertial policy is desirable. In fact, there is little difference in welfare for any value of a greater than one in the range considered. Also, while super-inertial policy is optimal for all λ, difference rules with $a = 1$, as in the work of Fuhrer and Moore (1995b) and others, may entail little welfare loss if enough price setters are rule-of-thumbers.[23] Finally, the figure also shows that the greater

[19]To see that the implicit weight on the discounted variance of inflation in $\hat{W}^p$ is bounded below by one, note that

$$V[\pi_t] + \frac{1-\lambda}{\alpha\lambda} V[\pi_t - \pi_{t-1}] = (1 + \frac{2(1-\lambda)(1-\rho_\pi)}{\alpha\lambda}) V[\pi_t]$$

where $\frac{1-\lambda}{\alpha\lambda} \geq 0$ and ρ_π is the first-order autocorrelation of inflation in equilibrium. Since $|\rho_\pi| < 1$, $1 + \frac{2(1-\lambda)(1-\rho_\pi)}{\alpha\lambda} \geq 1$.

[20]One exception to the monotonicity is the optimal $a = 13.39$ for $\lambda = 0.9$.

[21]The (minor) differences between our results in the $\lambda = 1$ case and the results of Woodford (1999b) are due to our slightly different choice of the variance of r_t^n and the fact that the first difference of inflation enters our feedback rule.

[22]The step size in the grid for a was 0.1.

[23]Amato and Laubach (1999) make a similar point in the context of a purely forward-looking model with sticky prices and wages.

the fraction of forward-looking agents, the more beneficial is an inertial policy versus a policy with little or no smoothing. Therefore, while it is true that the welfare gains from super-inertial policy become smaller as the proportion of forward-looking price setters decreases, it remains an optimal policy even when most price setters are backward looking.

Further insight into the differences between optimal policy for different degrees of forward-looking behavior can be obtained by investigating the impulse responses to a shock in the natural rate of interest. Figure 2 shows impulse response functions for π_t, x_t and r_t over a horizon of 12 quarters. Overall, the dynamic responses of the endogenous variables are broadly similar across the three values of λ chosen, in particular, in the magnitude of responses and the trajectories of the paths. Importantly, the mechanism for exploiting expectations for stabilization purposes as described by Woodford (1999b) is in effect even when a small percentage of price setters are forward looking. Essentially, the highly inertial character of policy (represented by $a > 1$) ensures that r_t remains high for many quarters after it initially increases in response to the shock. Persistently tight policy — both in the current level and expected future values of r_t — forces x_t to quickly become negative and slowly return to its steady-state. In turn, this also causes inflation to become negative and converge back to its steady-state slowly after the initial inflationary impact of the shock. The most appreciable differences in the impulse responses are for inflation: for lower values of λ, the responses are smaller in the first few quarters,[24] but more persistent. The more muted, but also more persistent response of inflation for lower values of λ reflects the more inertial behavior of inflation in this case, as (12) makes clear.

3.3 Results with Rule-of-Thumb Consumers

We next deal with the case when all price-setters are forward-looking, but some fraction of agents may be rule-of-thumb consumers, i.e. the model equations are (7) and (13). We undertake the analysis in the same way as in the previous subsection. Note that the standard objective in this case is supplemented by a term proportional to the variance of the change in output. Thus, by analogy to the previous subsection, we consider rules here that allow a response to the change in

[24]The exception is that both inflation and the output gap move less at *t=0* when $\lambda = 1$, although the interest rate response is nonetheless stronger since the rule coefficients in this case are considerably larger.

output, in addition to inflation and the lagged interest rate:

$$r_t = ar_{t-1} + b\pi_t + c(y_t - y_{t-1}) \tag{23}$$

Similarly, a, b and c in (23) are chosen to minimize, in turn, the full objective $\hat{W}^c$ given by equation (21) and the standard form of objective $\hat{W}$ given by equation (17).

For a fixed value of ψ, there is a tradeoff in this model between $V[\pi_t]$ and $V[y_t - y_{t-1}]$, in addition to the tradeoff created by the zero lower bound on interest rates described above. As ψ declines, the weight on $V[y_t - y_{t-1}]$ increases, so one might expect to see $V[\pi_t]$ and $V[r_t]$ increasing as ψ declines, and the opposite for $V[y_t - y_{t-1}]$. However, changes in ψ also affect elements of the model, as can be seen clearly from equation (7). On the one hand, smaller values of ψ increase the degree of endogenous persistence in the output gap — as captured by δ converging to 0.5 from above as ψ goes to 0 — thereby engendering greater variability in x_t. This mechanism is similar to the one described in the previous subsection for inflation holding constant the variance of the interest rate. The consequence is a higher level of the welfare function and the discounted variances.

On the other hand, smaller values of ψ dampen the impact of shocks on x_t, reducing its variance. To see this, note that as ψ declines, $\tilde{\sigma}$ increases, so the size of the elasticity of output with respect to the one-period short term expected real interest rate becomes small. At the same time, this change in elasticity requires that the variance of $\tilde{r}_t^n$ be increased to ensure that aggregate demand would equal the natural rate of output in a flexible price equilibrium. However, there is a second effect on $\tilde{r}_t^n$ which acts to reduce its variance. As equation (8) shows, the transmission of the shocks y_t^n and g_t to $\tilde{r}_t^n$ is dampened as ψ declines. The quantitative effect of this dampening can be seen from the dashed line in Figure 3, which shows $var(\tilde{r}_t^n)/\tilde{\sigma}^2$ as a function of ψ.[25] The solid line is $var(\tilde{r}_t^n)$ and the dash-dotted line is the value of $var(\tilde{r}_t^n)$ at which the zero lower bound becomes a constraint on policy. If $var(\tilde{r}_t^n)$ were below this threshold, then complete stabilization would be possible. Overall, $var(\tilde{r}_t^n)$ increases as ψ declines, but $\tilde{r}_t^n$ induces smaller fluctuations in x_t, as shown by $var(\tilde{r}_t^n)/\tilde{\sigma}^2$. Put another way, it is not beneficial from a welfare perspective to match the increases in $var(\tilde{r}_t^n)$ by increases in $var(r_t)$ because $\tilde{\sigma}^{-1}$ falls sufficiently fast with ψ.

The net effect between the change in the transmission of shocks and stronger lagged dynamics is a reduction in the variance of the output gap as ψ declines until $\psi = 0.1$, when the persistence effect dominates. This is shown in Table 3, which gives the variances of the endogenous variables,

[25] The operator $var(\bullet)$ is an (undiscounted) unconditional variance.

along with the levels of the restricted and unrestricted objective functions, under optimal rules of the form (23). A consequence of $V[x_t]$ being an increasing function of ψ (for $\psi > 0.1$) is that the variance of inflation also increases, which follows directly from equation (13).

Now, consider the effects of changing the welfare function. Use of the restricted objective $\hat{W}$ hardly makes a difference for the true level of welfare achieved, $\hat{W}^c$; the difference is decreasing in ψ, and is very small except for $\psi = 0.1$. Thus, for both rule-of-thumb price setters and consumers, taking account of the additional terms in the welfare function is ultimately unimportant for the actual level of $\hat{W}^c$ achieved. Yet, as before, the mix, in terms of altering the relative variability of the endogenous variables, is affected. By placing a higher weight on $V[y_t - y_{t-1}]$, this quantity is lower in equilibrium and $V[\pi_t]$ is generally higher. But achieving lower variability in the change in output also requires inducing more variability in short-term interest rates, which can be seen in (5).

Table 3: Welfare and Variances with Rule-of-Thumb Consumption *

	$V[\pi_t]$		$V[y_t - y_{t-1}]$		$V[x_t]$		$V[r_t]$		$\hat{W}$		$\hat{W}^c$	
	Objective:		Objective:		Objective:		Objective:		Objective:		Objective:	
ψ	$\hat{W}^c$	$\hat{W}$	$\hat{W}^c$	$\hat{W}$	$\hat{W}^c$	$\hat{W}$	$\hat{W}^c$	$\hat{W}$	$\hat{W}^c$	$\hat{W}$	$\hat{W}^c$	$\hat{W}$
1.0	0.063	0.063	11.489	11.489	5.485	5.485	1.880	1.880	0.338	0.338	0.338	0.338
0.9	0.054	0.053	9.181	9.340	4.730	4.770	1.869	1.865	0.291	0.291	0.303	0.303
0.8	0.046	0.044	7.214	7.460	4.055	4.113	1.857	1.851	0.249	0.248	0.270	0.271
0.7	0.039	0.036	5.551	5.834	3.455	3.514	1.846	1.839	0.211	0.210	0.240	0.240
0.6	0.033	0.030	4.160	4.449	2.926	2.975	1.835	1.827	0.178	0.177	0.211	0.213
0.5	0.027	0.025	3.020	3.293	2.471	2.504	1.824	1.815	0.149	0.148	0.186	0.187
0.4	0.022	0.021	2.112	2.357	2.101	2.111	1.814	1.805	0.126	0.124	0.164	0.166
0.3	0.019	0.018	1.425	1.633	1.837	1.822	1.804	1.796	0.109	0.107	0.149	0.152
0.2	0.018	0.017	0.957	1.124	1.740	1.695	1.796	1.789	0.103	0.099	0.149	0.153
0.1	0.025	0.029	0.749	0.941	2.065	1.786	1.795	1.786	0.125	0.115	0.206	0.217

* Notes: The table shows the levels of welfare (last 4 columns) and its component discounted variances (columns 2 through 9) achieved under the optimal rule (23) when $\hat{W}^c$ is minimized (left column under each statistic) or $\hat{W}$ is minimized (right column) for different values of ψ. Inflation and the interest rate are expressed in annualized percentages, the output gap in percentages.

When $\hat{W}^c$ is minimized, the patterns of optimal coefficients are broadly similar to those in the model with rule-of-thumb price setters. In particular, there is continuity in the pattern of optimal coefficients, even when ψ changes from 1.0 to 0.9. Especially important is the fact that a is always above one. In fact, the coefficients on the lagged interest rate and inflation are monotonically increasing in ψ (a ranges from 1.49 to 18.16 and b from 8.03 to 96.67).[26] The coefficient on the change in output is hump-shaped with respect to ψ, although it is always close to zero.

As before, it is informative to consider the level of welfare achieved when a is fixed at various values in the range [0,2] and b and c are chosen optimally. As an analogue to Figure 1, Figure 4 plots $\hat{W}^c$ for five different values of ψ. Our conclusions under rule-of-thumb price setters remain valid here; namely, the higher is ψ, the more beneficial is a super-inertial policy, and the greater the gain from increasing the super-inertial character of policy. Unlike in the price setting case, however, the level of welfare obtainable is strictly *decreasing* in ψ uniformly over this range of a, reflecting the results presented in Table 3.

Lastly, it is once again useful to examine the impulse response functions of π_t, x_t and r_t with respect to the exogenous shocks. Figure 5 presents the case of a perturbation to the marginal utility of consumption of size one percent of steady-state output.[27] This shock acts much like the one to r_t^n that we considered in the previous subsection, and the responses are very similar. Notably, there is little qualitative difference in the trajectories of the variables across different values of ψ. However, the magnitudes of the responses vary with ψ, which is another explanation of the differences in equilibrium variability and welfare captured in Table 3.

4 "Cost-Push" Shocks

An important assumption underlying the model is that fluctuations in the natural rate y_t^n are identical with fluctuations in the efficient level of output, that is, the level of output that would obtain absent any frictions, not just those associated with price setting. As a consequence, there exists no tradeoff between inflation stabilization and stabilization of output around its efficient level.

[26]One exception is a is slightly larger when ψ equals 0.1 versus 0.2, where the optimal value in the latter case is 1.36.

[27]Specifically, we consider a shock to g_t and we assume that the source of this shock has a direct impact only on the marginal utility of consumption. Using the definitions of g_t and z_t in Appendix A, the shock is an innovation in the component of g_t that is orthogonal to z_t.

However, much of the literature on stabilization policy is concerned with optimal policy responses in the presence of such tradeoffs. In this section, we examine the robustness of our results when we allow for the presence of "cost-push shocks" that cause a tradeoff between output gap and inflation stabilization. Since these shocks affect only the model's AS equation, we consider their policy implications in the presence of rule-of-thumb price setters.

We maintain the assumption that the *steady-state* level of output is efficient, but we now consider disturbances, such as variation in the degree of market power by firms, which may cause *fluctuations* in the natural rate to be inefficient. To this end, we define the output gap as the difference between y_t and the efficient level of output y_t^e (both expressed as percent deviations from steady state). The AS equation (12) is then given by

$$\pi_t = \tilde{\kappa} x_t + \gamma^b \pi_{t-1} + \gamma^f E_t \pi_{t+1} + u_t \tag{24}$$

where $x_t \equiv y_t - y_t^e$ and $u_t \equiv \tilde{\kappa}(y_t^e - y_t^n)$. Clarida et al. (1999), for example, analyze the implications of the presence of the "cost-push shock" u_t for monetary policy.[28] Since it is this definition of x_t that enters the objectives discussed in section 2, the case of an inefficient natural rate of output implies the existence of a tradeoff between inflation and output gap stabilization. However, the relative weights in the objective do not change (see Woodford, 1999b). With this change in the definition of x_t, the IS equation expressed in terms of the output gap becomes

$$x_t = E_t x_{t+1} + \sigma^{-1} E_t[r_t - r_t^n - \pi_{t+1}] \tag{25}$$

where it is now y_t^e, not y_t^n, that enters r_t^n.

To characterize optimal monetary policy in an economy with cost-push shocks, we must choose both the standard deviation of u_t (denoted by τ_u) and the correlation between u_t and r_t^n. Unfortunately, there does not exist any empirical evidence on these parameters from an estimated model consistent with our structural equations. Thus, as an example, we set τ_u equal to half of the standard deviation of the natural rate of interest, and the correlation between the shocks to zero. This latter choice is consistent with the interpretation of u_t as representing shocks to firms' desired

[28] In recent independent work, Steinsson (2000) also considers the effects of cost-push shocks in a related model. However, there are important differences in our approaches. For example, Steinsson does not allow for shocks to the natural rate of interest, nor does he account for the zero lower bound on nominal interest rates, which plays a crucial role in our results.

markups that are uncorrelated with the natural rate of output or shocks to the marginal utility of consumption.[29] We furthermore assume that u_t is serially uncorrelated.

To approximate optimal policy, we again find optimal values for the parameters in a simple interest rate rule. However, since there exists a tradeoff between inflation and output gap stabilization in the presence of cost-push shocks, we add the output gap to a rule of the form (22):

$$r_t = ar_{t-1} + b\pi_t + c(\pi_t - \pi_{t-1}) + dx_t \tag{26}$$

The introduction of cost-push shocks leaves our key result unchanged, that highly inertial policy is desirable irrespective of λ. To be specific, we perform the simulation again of fixing the response to the lagged interest rate at various points in the range (0,2] and finding optimal values for the other coefficients. Figure 6 shows a plot of $\hat{W}^p$ for the same values of λ used before. With a cost-push shock, the optimal values of a are increasing with λ (as before), but a highly inertial policy is now more beneficial for *smaller* λ. The presence of the cost-push shock makes simultaneous stabilization of inflation and the output gap impossible. As discussed before, however, a large fraction of rule-of-thumb price setters imparts substantial inertia to the inflation rate, which means that the inability to perfectly insulate inflation and the output gap from cost-push shocks produces large welfare losses. The welfare gains from an inertial response of the interest rate to a cost-push shock are larger for smaller values of λ, since in that case the tradeoff between stabilizing inflation (which is highly inertial) and the output gap becomes particularly acute.

5 Conclusions

In this paper, we investigated the implications for monetary policy of a fraction of agents pursuing rule-of-thumb behavior. We found that the presence of these types of agents changes the welfare function that policymakers seek to optimize. In addition to the usual objectives of minimizing inflation and output gap variability, we found that the existence of rule-of-thumb price setters motivates a desire to minimize variation in the change in inflation; the existence of rule-of-thumb consumers leads to an objective of stabilizing variation in the change of output itself. In terms of the characterization of optimal monetary policy, one of our main results was that Woodford's

[29] The results reported in this section remain qualitatively unchanged when we allow u_t to be positively or negatively correlated with r_t^n.

finding about the desirability of highly inertial policy in a purely forward-looking model continues to hold even when most agents adopt rules of thumb.

Overall, we find the character of optimal monetary policy to be qualitatively similar across the range of rule-of-thumb behavior considered. Of course, in practice, the exact specification of optimal policy will depend upon the estimated values obtained from macroeconomic time series for the various structural parameters and parameters describing the shock processes. The results based on our calibrated parameters and shocks are suggestive, but further work is needed. Among the more important empirical issues relevant to the subject matter here is determining what proportion of the persistence evident in inflation and output should be attributed to endogenous dynamics versus exogenous shocks.

A Log-linear Approximations

A.1 Derivation of Equations (5) and (12)

For forward-looking households, the optimal intertemporal allocation of consumption is characterized by the usual Euler equation

$$\frac{u_c(C_t^i;\xi_t)}{P_t} = \beta R_t E_t \left[\frac{u_c(C_{t+1}^i;\xi_{t+1})}{P_{t+1}}\right] \tag{27}$$

where R_t denotes the gross nominal interest rate on a one-period riskless bond. As discussed in section 2, households are assumed to insure themselves against all idiosyncratic risk such that, within type, their consumption paths are identical, and hence C_t^i in (27) can be replaced by C_t^f, the forward-looking households' consumption.

In deriving (5) and (12) we take first-order Taylor-series expansions of the exact non-linear equilibrium conditions. If $\|\xi\|$ is a measure of the magnitude of fluctuations of the process $\{\xi_t\}$, the expansions in this subsection are accurate up to a remainder of size $\mathcal{O}(\|\xi\|^2)$, which we omit from the equations. For ease of exposition, we assume that the steady-state level of output is at its efficient level Y^*, characterized in our model by

$$\frac{v_y(Y^*;0)}{u_c(Y^*;0)} = 1 \tag{28}$$

and all Taylor series expansions are taken around Y^*. This assumption does not affect the results presented in this article. For a generalization of the analysis to the case in which the steady-state output level is inefficient, see Woodford (1999a). Moreover, all expansions are developed around a zero inflation steady state, and we assume that the steady state value of ξ_t is zero as well. A log-linear approximation of (27) around this steady state can then be written as

$$-\sigma(c_t^f - g_t) = r_t - E_t\pi_{t+1} - \sigma E_t(c_{t+1}^f - g_{t+1}) \tag{29}$$

where

$$c_t^f \equiv \log\left(\frac{C_t^f}{Y^*}\right), \ \sigma \equiv -\frac{u_{cc}(Y^*;0)Y^*}{u_c(Y^*;0)}, \ g_t \equiv -\frac{u_{c\xi}(Y^*;0)}{u_{cc}(Y^*;0)Y^*}\xi_t$$

and $r_t \equiv \log R_t - \log\beta^{-1}$, where β^{-1} is the steady-state gross real interest rate. Market clearing requires that $C_t \equiv \psi C_t^f + (1-\psi)C_t^b$ equals Y_t, and hence the rule of thumb (3) implies that $C_t^b = Y_{t-1}$. Solving for C_t^f and taking logs, we obtain $c_t^f = \psi^{-1}(y_t - (1-\psi)y_{t-1})$, where $y_t \equiv \log(Y_t/Y^*)$. Substituting for c_t^f in (29) and rearranging yields (5).

In deriving (12) we assume that all households are able to solve the problem (27), so that $C_t^f = C_t = Y_t$. The first step in deriving (12) is to take the first-order condition of (10) with respect to p_t^f and approximate it around the same steady state as before:

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left\{ -\sigma(y_{t+j} - g_{t+j}) + \hat{p}_t^f - \sum_{k=1}^{j} \pi_{t+k} - \omega[y_{t+j} - z_{t+j} - \theta(\hat{p}_t^f - \sum_{k=1}^{j} \pi_{t+k})] \right\} = 0 \tag{30}$$

where

$$\hat{p}_t^f \equiv \log\left(\frac{p_t^f}{P_t}\right), \ \omega \equiv \frac{v_{yy}(Y^*;0)Y^*}{v_y(Y^*;0)}, \text{ and } z_t \equiv -\frac{v_{y\xi}(Y^*;0)}{v_{yy}(Y^*;0)Y^*}\xi_t$$

Solving (30) for $\hat{p}_t^f$ yields

$$\hat{p}_t^f = (1-\alpha\beta) E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\frac{\sigma+\omega}{1+\omega\theta}(y_t - y_t^n) + \sum_{k=1}^{j} \pi_{t+k} \right] \tag{31}$$

where $y_t^n \equiv (\sigma+\omega)^{-1}(\sigma g_t + \omega z_t)$ is the "natural rate of output", i.e. the level of output that would obtain if all prices were flexible.

Log-linearizing equation (11) and the price indices

$$P_t^* = [\lambda(p_t^f)^{1-\theta} + (1-\lambda)(p_t^b)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{32}$$

and

$$P_t = [\alpha P_{t-1}^{1-\theta} + (1-\alpha)(P_t^*)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{33}$$

yields respectively

$$\hat{p}_t^b = \hat{p}_{t-1}^* + \pi_{t-1} - \pi_t \tag{34}$$

$$\hat{p}_t^* = \lambda \hat{p}_t^f + (1-\lambda)\hat{p}_t^b \tag{35}$$

$$\hat{p}_t^* = \frac{\alpha}{1-\alpha}\pi_t \tag{36}$$

where $\hat{p}_t^b \equiv \log(p_t^b/P_t)$, and $\hat{p}_t^* \equiv \log(P_t^*/P_t)$. Substituting (34) and (36) into (35), we obtain

$$\hat{p}_t^f = \frac{1-\lambda(1-\alpha)}{\lambda(1-\alpha)}\pi_t - \frac{1-\lambda}{\lambda(1-\alpha)}\pi_{t-1} \tag{37}$$

Furthermore, the double sum in (31) can be simplified as

$$\sum_{j=0}^{\infty} (\alpha\beta)^j \sum_{k=1}^{j} \pi_{t+k} = (1-\alpha\beta)^{-1} \left[\sum_{j=0}^{\infty} (\alpha\beta)^j \pi_{t+j} - \pi_t \right] \tag{38}$$

Substituting (37) and (38) into (31), rearranging, and quasi-differencing the resulting expression, we obtain (12), where

$$\begin{aligned}\tilde{\kappa} &\equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha+(1-\lambda)(1-\alpha(1-\beta))}\cdot\frac{\lambda(\sigma+\omega)}{1+\omega\theta} \\ \gamma^b &\equiv \frac{1-\lambda}{\alpha+(1-\lambda)(1-\alpha(1-\beta))} \\ \gamma^f &\equiv \frac{\alpha\beta}{\alpha+(1-\lambda)(1-\alpha(1-\beta))}\end{aligned}$$

For $\lambda = 1$, $\gamma^b = 0, \gamma^f = \beta$, and $\tilde{\kappa}$ reduces to

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\cdot\frac{\sigma+\omega}{1+\omega\theta}$$

A.2 The Representative Household's Welfare

In this section we derive the second-order approximation (15) to the representative household's welfare (14), using methods discussed in more detail in Woodford (1999a). Specifically, we form a second-order Taylor series expansion of (14) around the steady state characterized by the efficient output level Y^* defined in (28) and zero inflation. Hence, we form the approximation around the same steady state around which the model's exact equilibrium conditions have been log-linearized.

Let $U_t \equiv \psi u(C_t^f;\xi_t) + (1-\psi)u(C_t^b;\xi_t) - \int_0^1 v(y_t(i);\xi_t)di$. Using again that $C_t^b = Y_{t-1}$ and the expression (4) for C_t^f, the first two terms in U_t can be approximated as

$$\begin{aligned}&\psi u(C_t^f;\xi_t) + (1-\psi)u(C_t^b;\xi_t) \\ &= u_c Y^*\left[y_t + \frac{1}{2}(1-\sigma)y_t^2 - \frac{1-\psi}{2\psi}\sigma(y_t - y_{t-1})^2 + \sigma y_t g_t\right] + t.i.p. + \mathcal{O}(\|\xi\|^3) \qquad (39)\end{aligned}$$

where the partial derivative u_c is evaluated at $(Y^*;0)$, and y_t, g_t, and σ are defined as in (29). The notation $t.i.p.$ stands for terms that are independent of monetary policy, and the approximation is exact up to a remainder of order $\|\xi\|^3$.

Following the same steps as, e.g., in Rotemberg and Woodford (1997), we obtain

$$\begin{aligned}&\int_0^1 v(y_t(i);\xi_t)di \\ &= v_y Y^*\left[y_t + \frac{1}{2}(1+\omega)y_t^2 + \frac{1}{2}(\theta^{-1}+\omega)var_i(\hat{y}_t(i)) - \omega y_t z_t\right] + t.i.p. + \mathcal{O}(\|\xi\|^3) \qquad (40)\end{aligned}$$

where z_t and ω are defined as in (30), $\hat{y}_t(i) \equiv \log(y_t(i)/Y^*)$, and var_i measures the variance of $\hat{y}_t(i)$ across i. Subtracting (40) from (39), and using from (28) that $u_c = v_y$, and from (2) that

$var_i(\hat{y}_t(i)) = \theta^2 var_i(\log p_t(i))$, we obtain

$$U_t = -\frac{Y^* u_c}{2}\left[(\sigma+\omega)x_t^2 + \sigma\frac{1-\psi}{\psi}(y_t - y_{t-1})^2 + \theta(1+\omega\theta)var_i(\log p_t(i))\right] + t.i.p. + \mathcal{O}(\|\xi\|^3) \quad (41)$$

Let $\bar{P}_t \equiv E_i[\log p_t(i)]$ and $\Delta_t \equiv var_i(\log p_t(i))$. Then

$$\begin{aligned}\Delta_t &= var_i(\log p_t^i - \bar{P}_{t-1}) \\ &= E_i\left[(\log p_t(i) - \bar{P}_{t-1})^2\right] - \left(E_i[\log p_t(i) - \bar{P}_{t-1}]\right)^2 \\ &= \alpha\Delta_{t-1} + (1-\alpha)\lambda(\log p_t^f - \bar{P}_{t-1})^2 + (1-\alpha)(1-\lambda)(\log p_t^b - \bar{P}_{t-1})^2 - (\bar{P}_t - \bar{P}_{t-1})^2 \quad (42)\end{aligned}$$

We now wish to substitute for each of the squared terms in (42) in terms of inflation. First note that $\bar{P}_t = \log P_t + \mathcal{O}(\|\xi\|^2)$, so that $\bar{P}_t - \bar{P}_{t-1} = \pi_t + \mathcal{O}(\|\xi\|^2)$. Furthermore, taking logs on both sides of (33) and rearranging yields

$$\log P_t^* - \log P_t = \frac{\alpha}{1-\alpha}\pi_t \quad (43)$$

Taking logs on both sides of (11) yields

$$\log p_t^b = \log P_t^* + \log P_{t-1} - \log P_{t-2} \quad (44)$$

and, using (43),

$$\log p_t^b - \log P_{t-1} = \frac{1}{1-\alpha}\pi_{t-1} \quad (45)$$

Finally, taking logs on both sides of (32), rearranging, and substituting from (43) and (44), we obtain

$$\log p_t^f - \log P_{t-1} = \frac{1}{\lambda(1-\alpha)}\pi_t - \frac{1-\lambda}{\lambda(1-\alpha)}\pi_{t-1} \quad (46)$$

Substituting (45) and (46) into (42) then yields

$$\begin{aligned}\Delta_t &= \alpha\Delta_{t-1} + \frac{\alpha}{1-\alpha}\pi_t^2 + \frac{1-\lambda}{\lambda(1-\alpha)}(\pi_t - \pi_{t-1})^2 + \mathcal{O}(\|\xi\|^3) \\ &= \alpha^{t+1}\Delta_{-1} + \sum_{s=0}^{t}\alpha^{t-s}\left[\frac{\alpha}{1-\alpha}\pi_s^2 + \frac{1-\lambda}{\lambda(1-\alpha)}(\pi_s - \pi_{s-1})^2\right] + \mathcal{O}(\|\xi\|^3) \quad (47)\end{aligned}$$

Since we consider evaluating policies adopted at time 0, the term Δ_{-1} is treated as independent of the policy under consideration. Hence,

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{1}{1-\alpha\beta}\sum_{t=0}^{\infty}\beta^t\left[\frac{\alpha}{1-\alpha}\pi_t^2 + \frac{1-\lambda}{\lambda(1-\alpha)}(\pi_t - \pi_{t-1})^2\right] + t.i.p. + \mathcal{O}(\|\xi\|^3) \quad (48)$$

Taking the discounted sum $\sum_{t=0}^{\infty}\beta^t U_t$ using (41) and (48), we obtain the loss functions presented in the text.

B Calibration of Shock Processes

In this appendix we present the calibration of the shock process r_t^n as defined in (9) and its components y_t^n and g_t. Recall the definitions

$$y_t^n \equiv \frac{\sigma}{\sigma+\omega} g_t + \frac{\omega}{\sigma+\omega} z_t,\ g_t \equiv -\frac{u_{c\xi}(Y^*;0)}{u_{cc}(Y^*;0)Y^*}\xi_t,\ \text{and } z_t \equiv -\frac{v_{y\xi}(Y^*;0)}{v_{yy}(Y^*;0)Y^*}\xi_t \tag{49}$$

Rotemberg and Woodford (1997) present empirical estimates of the shock processes g_t and y_t^n (and hence implicitly of z_t) based on their study of U.S. data for the period 1980-95. The standard deviations of their processes $E_{t-2}\hat{G}_t$ and $E_{t-2}\bar{Y}_t$, corresponding to g_t and z_t and expressed as percentages of steady-state output, are 29.46 and 11.72 respectively, while the first-order serial correlation of their processes is 0.92 and 0.68. Furthermore, the two processes are highly negatively correlated with each other, with a correlation coefficient of -0.87.

We would like to model g_t and z_t as AR(1) processes

$$g_t = \rho_g g_{t-1} + \epsilon_t \tag{50}$$

$$z_t = \rho_z z_{t-1} + \eta_t \tag{51}$$

where $[\epsilon_t, \eta_t]$ are i.i.d. mean zero disturbances with covariance matrix

$$V \equiv \begin{bmatrix} \tau_\epsilon^2 & \tau_{\epsilon\eta} \\ \tau_{\epsilon\eta} & \tau_\eta^2 \end{bmatrix}$$

By choosing $\rho_g = .92, \rho_z = .68, \tau_\epsilon^2 = 139.37, \tau_\eta^2 = 73.60$, and $\tau_{\epsilon\eta} = -113.42$, we would be able to replicate the statistical properties of Rotemberg and Woodford's estimated processes. Moreover, the standard deviation of the process r_t^n obtained by using this parametrization of g_t and z_t is 3.76, and matches closely that of Rotemberg and Woodford's corresponding process, which is 3.72.

However, this parametrization of the processes (50) and (51) would imply that the process r_t^n is nearly serially uncorrelated (the first-order correlation coefficient ρ_r would be 0.01). By contrast, as discussed in Woodford (1999b), a value for ρ_r of 0.35 would imply in our model (which is identical to his model when $\psi = \lambda = 1$) a degree of concern for reduction of interest rate variability similar to that obtained in the estimated model of Rotemberg and Woodford. We therefore choose values for ρ_g and ρ_z that are consistent with $\rho_r = 0.35$ while implying a level of variability of r_t^n as close to the one found in Rotemberg and Woodford's estimate as possible. The parameters that achieve these two objectives most closely are $\rho_g = \rho_z = 0.35$, in which case $\rho_r = 0.35$ as well. The implied standard deviation of r_t^n is now 3.07, and hence our natural rate process is slightly less variable than that of Rotemberg and Woodford. Finally, in order to preserve the covariance properties of g_t and z_t found in the estimates despite our choices for ρ_g and ρ_z, it is now necessary to choose the elements of V as $\tau_\epsilon^2 = 761.59, \tau_\eta^2 = 120.59$, and $\tau_{\epsilon\eta} = -264.90$.

References

[1] Amato, Jeffery D., and Thomas Laubach. "Monetary Policy in an Estimated Optimization-Based Model with Sticky Prices and Wages." Federal Reserve Bank of Kansas City Working Paper no. 99-09, October 1999.

[2] Blinder, Alan S. "On Sticky Prices: Academic Theories Meet the Real World." In N. Gregory Mankiw (ed), *Monetary Policy*, Chicago: University of Chicago Press 1994, 117-50.

[3] Calvo, Guillermo A. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (1983), 383-398.

[4] Campbell, John Y., and N. Gregory Mankiw. "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence." In Olivier J. Blanchard and Stanley Fischer (eds), *NBER Macroeconomics Annual* 1989, 185-216.

[5] Clarida, Richard, Jordi Gali, and Mark Gertler. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (1999), 1661-1707.

[6] Coenen, Günter, and Volker Wieland. "A Small Estimated Euro Area Model with Rational Expectations and Nominal Rigidities." Manuscript, European Central Bank, February 2000.

[7] Deaton, Angus. *Understanding Consumption.* Oxford: Clarendon Press, 1992.

[8] Fuhrer, Jeffrey C. "Comment." In Ben S. Bernanke and Julio J. Rotemberg (eds), *NBER Macroeconomics Annual* 1997a, 346-355.

[9] Fuhrer, Jeffrey C. "Towards a Compact, Empirically-Verified Rational Expectations Model for Monetary Policy Analysis." *Carnegie-Rochester Conference Series on Public Policy* 47 (1997b), 197-230.

[10] Fuhrer, Jeffrey C., and George Moore. "Inflation Persistence." *Quarterly Journal of Economics* 110 (1995a), 127-159.

[11] Fuhrer, Jeffrey C., and George Moore. "Monetary Policy Trade-offs and the Correlation Between Nominal Interest Rates and Real Output." *American Economic Review* 85 (1995b), 219-39.

[12] Gali, Jordi, and Mark Gertler. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* 44 (1999), 195-222.

[13] Ireland, Peter N. "A Small, Structural, Quarterly Model for Monetary Policy Evaluation." *Carnegie-Rochester Conference Series on Public Policy* 47 (1997), 83-108.

[14] Mankiw, N. Gregory, Julio J. Rotemberg and Lawrence H. Summers. "Intertemporal Substitution in Macroeconomics." *Quarterly Journal of Economics* 100 (1985), 225-251.

[15] McCallum, Bennett T., and Edward Nelson. "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis." *Journal of Money, Credit, and Banking* 31 (1999), 296-316.

[16] Roberts, John M. "New Keynesian Economics and the Phillips Curve." *Journal of Money, Credit, and Banking* 27 (1995), 975-984.

[17] Roberts, John M. "Is Inflation Sticky?" *Journal of Monetary Economics* 39 (1997), 173-196.

[18] Rotemberg, Julio J. "Sticky Prices in the United States." *Journal of Political Economy* 90 (1982), 1187-1211.

[19] Rotemberg, Julio J., and Michael Woodford. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." In Ben S. Bernanke and Julio J. Rotemberg (eds), *NBER Macroeconomics Annual* 1997, 297-346. Expanded version circulated as NBER Technical Working Paper no. 233, May 1998.

[20] Rotemberg, Julio J., and Michael Woodford. "Interest-Rate Rules in an Estimated Sticky Price Model." In John B. Taylor (ed), *Monetary Policy Rules*, NBER and Chicago University Press 1999, 57-119.

[21] Sbordone, Argia M. "Prices and Unit Labor Costs: A New Test of Price Stickiness." Manuscript, Rutgers University, October 1998.

[22] Steinsson, Jon. "Optimal Monetary Policy in an Economy with Inflation Persistence." Manuscript, Princeton University, May 2000.

[23] Taylor, John B. "Estimation and Control of a Macroeconomic Model with Rational Expectations." *Econometrica* 47 (1979), 1267-1286.

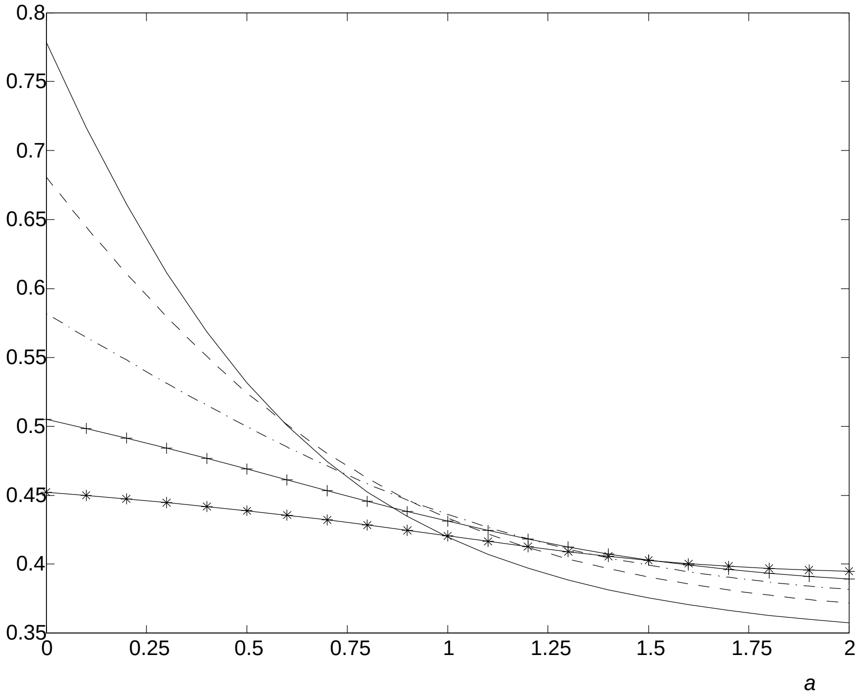
[24] Taylor, John B. "Introduction." In John B. Taylor (ed), *Monetary Policy Rules*, NBER and Chicago University Press 1999.

[25] Woodford, Michael. "Control of the Public Debt: A Requirement for Price Stability?" NBER Working Paper no. 5684, July 1996.

[26] Woodford, Michael. "Inflation Stabilization and Welfare." Ch. 6 in *Interest and Prices*, manuscript, May 1999a.

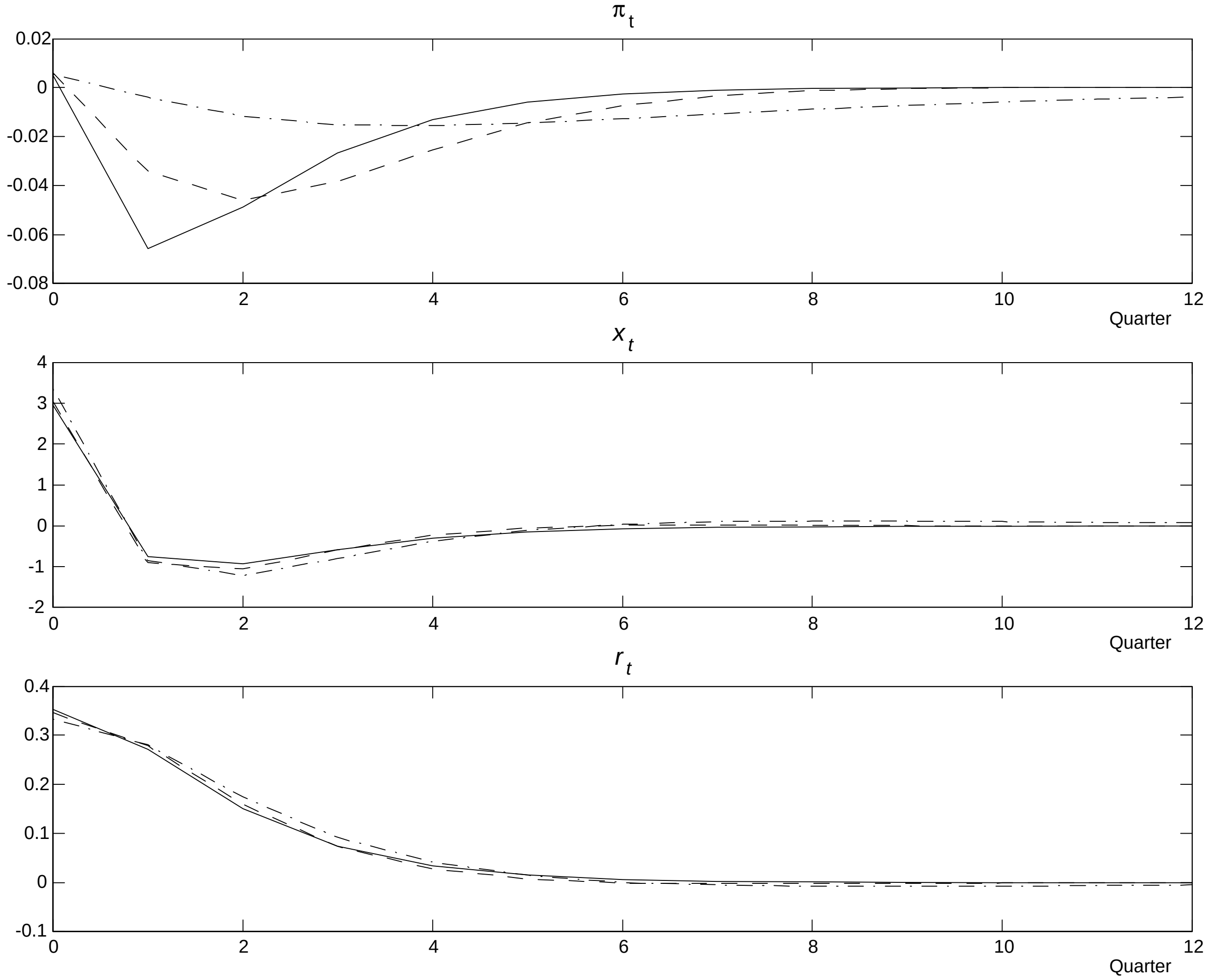
[27] Woodford, Michael. "Optimal Monetary Policy Inertia." NBER Working Paper no. 7261, July 1999b.

Figure 1: Welfare Under Different Degrees of Inertial Policy with Rule-of-Thumb Price Setting



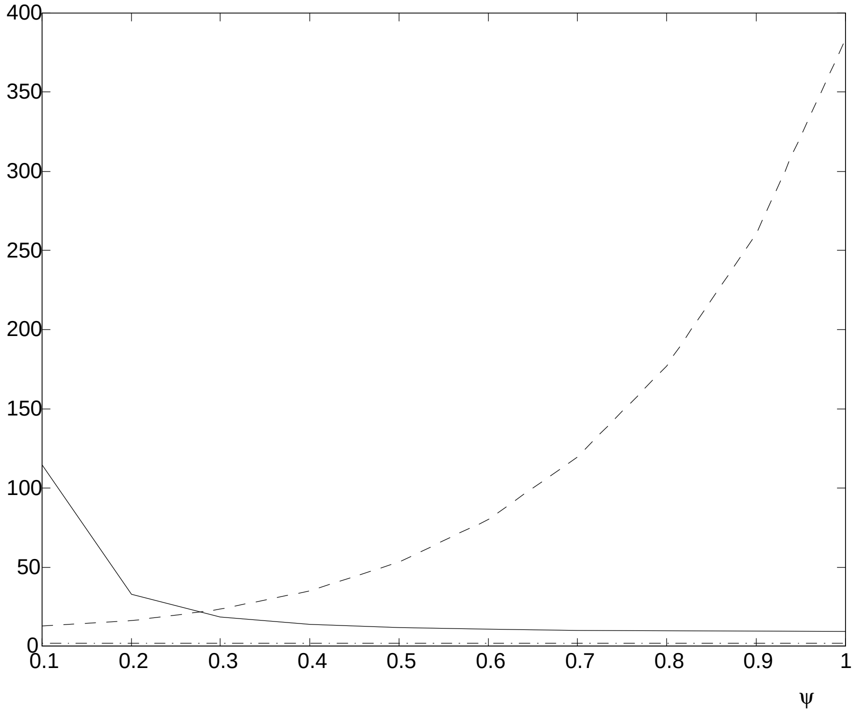
Notes: The figure shows the level of welfare $\hat{W}^p$ obtained for different degrees of inertial policy, as represented by the response coefficient on the lagged interest rate, a, in a rule of the form (22). For each fixed value of a in the range 0 to 2 (calculated at a step of 0.1), optimal values of the coefficients b and c were found to minimize $\hat{W}^p$. The different lines correspond to different values of λ: 1.0 (solid), 0.8 (dash), 0.6 (dash-dot), 0.4 (solid-plus) and 0.2 (solid-star).

Figure 2: Impulse Responses to a Natural Rate of Interest Shock with Rule-of-Thumb Price Setting



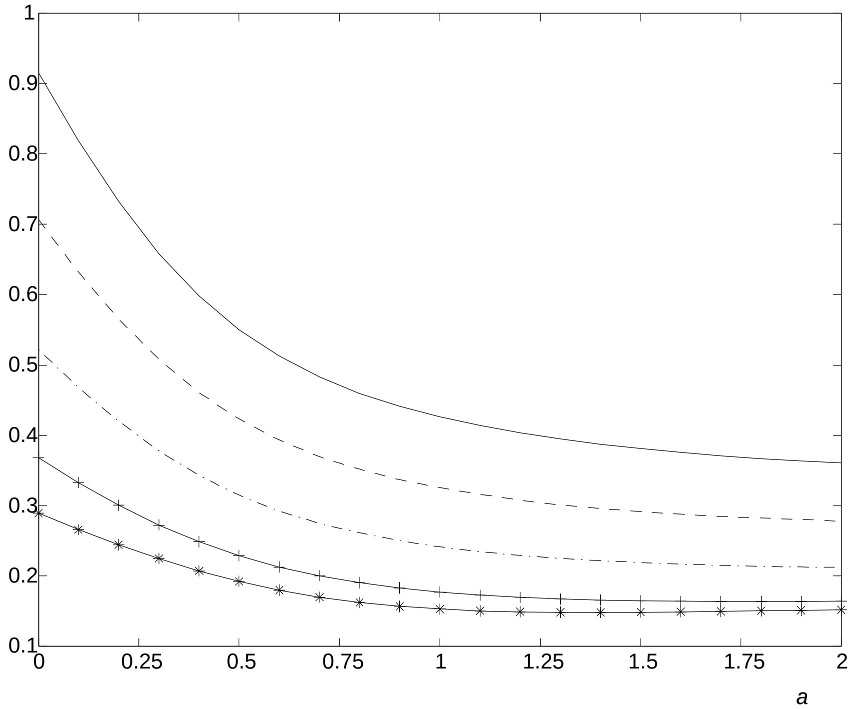
Notes: The figure maps out the impulse responses of the endogenous variables to a one-standard deviation shock to the innovation in the natural rate of interest at time 0. The different lines correspond to different values of λ: 1.0 (solid), 0.6 (dash) and 0.2 (dash-dot). One period is equal to a quarter. Inflation and the interest rate are expressed in annualized percentages, the output gap in percentages.

Figure 3: Variance of the Natural Rate of Interest with Rule-of-Thumb Consumption



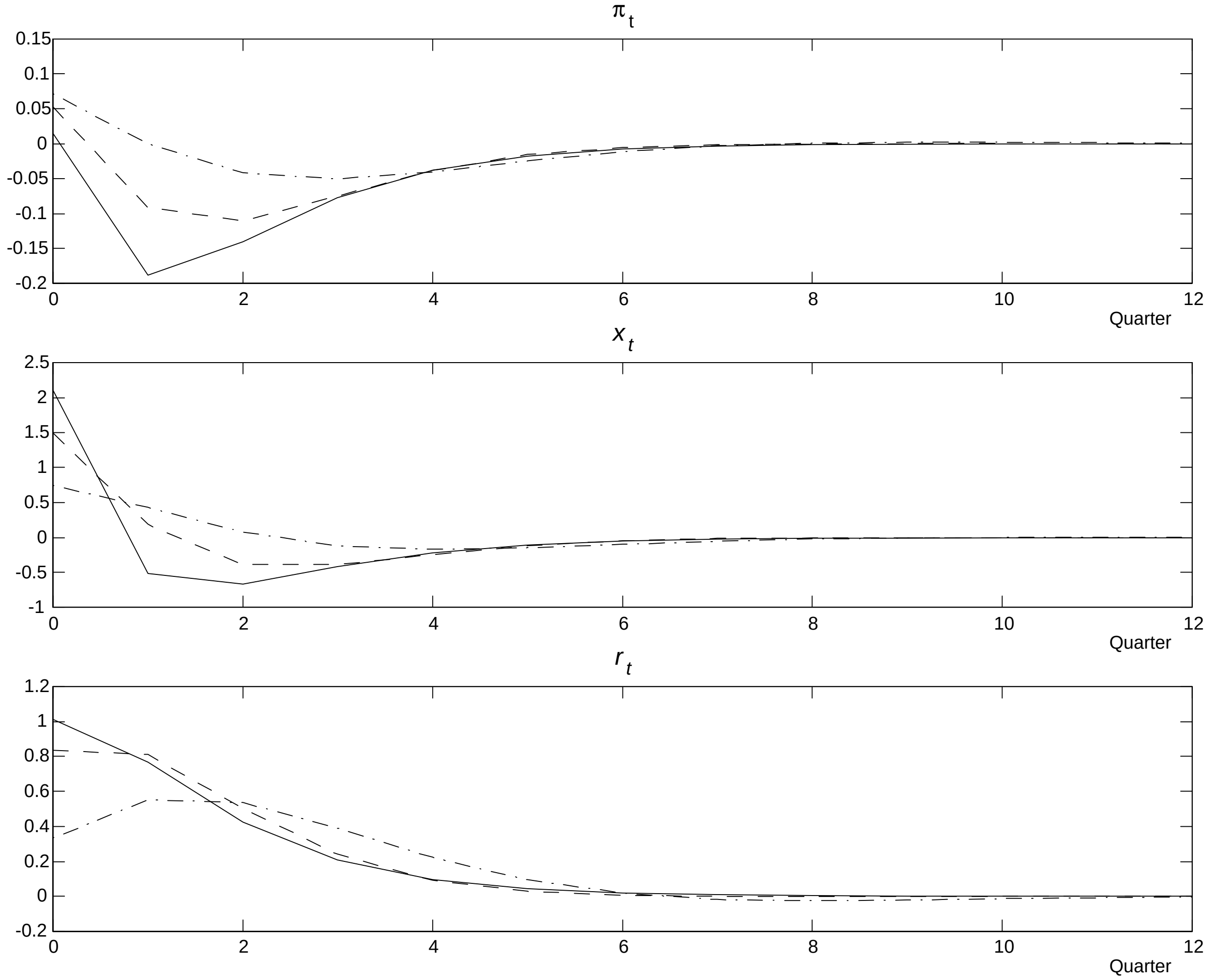
Notes: The figure shows the unconditional variance of the natural rate of interest $\tilde{r}_t^n$ (solid), given in equation (8), across different values of ψ. Also shown is the component of this variance related to the conditional expectation of the shock processes, i.e. $var(\tilde{r}_t^n)/\tilde{\sigma}^2$ (dash), and the threshold at which the zero lower bound on nominal interest rates becomes binding, i.e. $var(\tilde{r}_t^n) = 1.76$ (dash-dot). The natural rate of interest is expressed in annualized percentages.

Figure 4: Welfare Under Different Degrees of Inertial Policy with Rule-of-Thumb Consumption



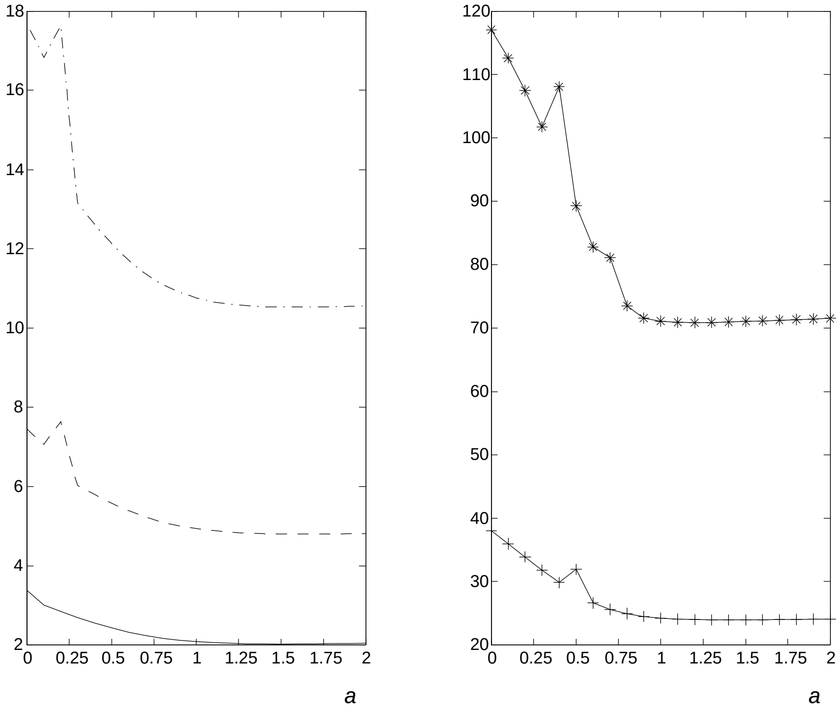
Notes: The figure shows the level of welfare $\hat{W}^c$ obtained for different degrees of inertial policy, as represented by the response coefficient on the lagged interest rate, a, in a rule of the form (23). For each fixed value of a in the range 0 to 2 (calculated at a step of 0.1), optimal values of the coefficients b and c were found to minimize $\hat{W}^c$. The different lines correspond to different values of ψ: 1.0 (solid), 0.8 (dash), 0.6 (dash-dot), 0.4 (solid-plus) and 0.2 (solid-star).

Figure 5: Impulse Responses to a Marginal Utility of Consumption Shock with Rule-of-Thumb Consumption



Notes: The figure maps out the impulse responses of the endogenous variables to a one-unit shock to the innovation in g_t, i.e. ϵ_t in (50), at time 0. It is assumed that ϵ_t reflects a shock to the marginal utility of consumption that has no immediate impact on the marginal utility of production. The different lines correspond to different values of ψ: 1.0 (solid), 0.6 (dash) and 0.2 (dash-dot). One period is equal to a quarter. Inflation and the interest rate are expressed in annualized percentages, the output gap in percentages.

Figure 6: Welfare Under Different Degrees of Inertial Policy with Rule-of-Thumb Price Setting and Cost-Push Shocks



Notes: The figure shows the level of welfare $\hat{W}^p$ obtained for different degrees of inertial policy, as represented by the response coefficient on the lagged interest rate, a, in a rule of the form (26). For each fixed value of a in the range 0 to 2 (calculated at a step of 0.1), optimal values of the coefficients b, c and d were found to minimize $\hat{W}^p$. The different lines correspond to different values of λ: 1.0 (solid), 0.8 (dash), 0.6 (dash-dot), 0.4 (solid-plus) and 0.2 (solid-star).