ESTIMATOR CHOICE AND FISHER'S PARADOX: A REEVALUATION OF THE EVIDENCE

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April 12, 2000

Abstract

This paper argues that Fisher's paradox can be explained away in terms of estimator choice. We analyse by means of Monte Carlo experiments the small sample properties of a large set of estimators (including virtually all available single-equation estimators), and compute the critical values based on the empirical distributions of the t-statistics, for a variety of Data Generation Processes (DGPs), allowing for structural breaks, ARCH effects etc. We show that it is precisely the estimator most commonly used in the literature, namely OLS, which (together with the FMLS estimator) has the worst performance in small samples, and produces rejections of the Fisher hypothesis. If one employs the estimators with the most desirable properties (i.e., the smallest downward bias and the minimum shift in the distribution of the associated t-statistics), or if one uses the empirical critical values, the evidence based on US data is strongly supportive of the Fisher relation, consistently with many theoretical models.

Keywords: Fisher's Paradox, Cointegration, Single-Equation Estimators, Monte Carlo Analysis, Small-Sample Properties

JEL Classification: C15, C22, E43

*We are grateful to Mark Wohar for kindly supplying the monthly dataset used in Crowder and Wohar (1997), and to Marco Barassi and Nicola Spagnolo for research assistance. Corresponding author: Professor Guglielmo Maria Caporale, Department of Economics, University of East London, Longbridge Road, Dagenham, Essex RM8 2AS, UK. Tel. +44 (0)20 8233 2965. Fax:\ +44 (0)20 8233 2849. E-mail: g.m.caporale@uel.ac.uk

1 Introduction

Ex ante real interest rates are a key variable for understanding investment and savings decisions and asset prices determination. Their long-run behaviour is often analysed in the context of the Fisher relationship, linking nominal rates to expected inflation and requiring full adjustment of the former to the latter. In the absence of one-to-one adjustment, any permanent shocks to either inflation or nominal rates would be translated into permanent disturbances to real rates themselves, a result which is inconsistent with standard models of intertemporal asset pricing. The reason is that in such models permanent shocks to real rates imply permanent shocks to consumption growth rates as well, a hypothesis which is not supported by the empirical evidence (see Rose,1988). Furthermore, without a one-to-one response, and assuming that expected inflation is essentially driven by monetary growth, superneutrality does not hold, as changes in the rate of growth of the money supply permanently affect inflationary expectations and, in turn, real rates. [1] However, the empirical evidence is far from being unambiguously supportive of the Fisher relationship. Numerous studies have found that the slope coefficient in a regression of inflation against nominal rates is significantly different from one, at least over certain periods (see, e.g., Mishkin, 1992, and Evans and Lewis, 1995), the implication being that real rates contain the same unit root component as inflation.

Alternative explanations have been provided for this "paradox" (see Carmichael and Stebbing, 1983), i.e. the apparent inconsistency between the wide acceptance of the Fisher equation as an interest rate theory, and the failure to find corroborating evidence. This has been attributed, in turn, to the inadequate treatment of inflationary expectations (see, e.g. Woodward, 1992); the fact that the effects of taxation have not been taken into account (see Darby, 1975); the use of short rather than long rates, the latter being more relevant for investment and savings decisions (see Gilbert and Yeoward, 1994); the need to distinguish between short- and long-run Fisher effects (see Mishkin, 1992); shifts in expected inflation (see Evans and Lewis, 1995). More recently, Crowder and Hoffman (1996) have suggested that the estimator choice might account for the contradictory evidence gathered so far. In particular, they argue that differences in the small sample properties of the OLS, DOLS (see Stock and Watson, 1993) and Johansen's (1988, 1991) maximum likelihood (ML) estimators are responsible for the vastly different conclusions reached in the literature about the relationship between inflation and interest rates. They report that the existence of a tax-adjusted Fisher effect is supported by the data when the ML estimator (which is shown to outperform the others) is used.

[1] If superneutrality is assumed, the Tobin (1969) effect, which occurs when agents shift out of nominal assets into real assets in response to increases in (expected) inflation, is only a short-run phenomenon (see Lucas, 1981).

This paper also argues that the disagreement in the literature about the empirical validity of the Fisher effect can be put down to differences in the estimation procedures adopted. In order to substantiate this claim, we analyse by means of Monte Carlo experiments the small sample properties of a much larger set of estimators (including virtually all available single-equation estimators) for a variety of Data Generation Processes (DGPs) which are likely to have produced the inflation and interest data employed. We show that a number of them, even if asymptotically efficient, are affected in small samples (of the size typically used in empirical studies) by biases and shifts in the distribution of the associated t-statistics which yield misleading inference. Moreover, we are able to demonstrate that it is precisely the estimator most commonly used in the literature, namely OLS, which (together with the Fully Modified Least Squares (FMLS) estimator - see Phillips and Hansen, 1990) has the worst performance in small samples, and produces rejections of the Fisher hypothesis. When the estimators with the highest ranking in terms of small sample properties are chosen, and the correct critical values we compute are applied, the evidence based on US data is strongly supportive of the Fisher relation, consistently with many theoretical models. Finally, the qualitative results are the same even when one allows for structural breaks or uses monthly data.

The layout of the paper is as follows. Section 2 provides a more extensive discussion of the Fisher equation and of the existing empirical literature. Section 3 outlines the various estimation methods which are later used for the empirical analysis. Section 4 presents estimates of the Fisher equation obtained using the alternative estimators, and then examines their small sample properties by means of Monte Carlo techniques. Section 5 investigates whether the results are robust to the presence of structural breaks and to alternative data frequencies, and also considers tax effects. Section 6 summarises the main findings and offers some concluding remarks.

2 The Fisher Effect: A Brief Literature Review

Fisher (1930) stressed the distinction between nominal and real interest rates, and pointed out that nominal rates adjust, even though not instantaneously, to reflect changes in expected inflation. Formally, the "Fisher effect" can be expressed as:

$$i_t(m) = \pi_t^e(m) + r_t^e(m) \tag{1}$$

where $i_t(m)$ is the m-period nominal interest rate at time t, $\pi_t^e(m)$ denotes the expected rate of inflation from time t to t+m, and $r_t^e(m)$ is the ex-ante real interest rate. Under the usual assumption of rational expectations (see, e.g., Mishkin, 1992),

$$\pi_t(m) = \pi_t^e(m) + e_t \tag{2}$$

where e_t is a white noise process, orthogonal to $\pi_t^e(m)$. Equations (**??**) and (**??**) along with the additional assumption that the real interest rate process follows a white noise process with a mean equal to r, lead to testing for the Fisher effect in the context of the following regression:

$$i_t(m) = r + \theta\pi_t(m) + u_t \tag{3}$$

It is normally found that $i_t(m)$ and $\pi_t(m)$ are I(1) processes, regardless of the maturity m. Therefore, regression (**??**) involves non-stationary regressors, which in turn implies that a necessary condition for the Fisher effect to hold is cointegration. Specifically, the null hypothesis to be tested can take the form:

Fisher effect holds $\leftrightarrow$ (i) u_t is I(0) and (ii) $\theta = 1$.

The early empirical literature stressed that a one-to-one adjustment of nominal rates to expected inflation implies constancy of the real rate. Fama (1975) focused on nominal rates as predictors of inflation, and concluded that the US bond market was efficient, in the sense that nominal rates summarised all the information about future inflation contained in past inflation rates. Efficiency, combined with the observed constancy of real returns, implied a full adjustment of nominal rates to changes in expected inflation. Subsequently, Nelson and Schwert (1977) showed that Fama's tests were not powerful enough to reject the hypothesis that real rates are constant, and Garbade and Wachtel (1978) using variable parameter regressions did reject it, although they found an inflationary premium in nominal rates.

In regressions of inflation against nominal rates the coefficient on the latter was generally estimated to be significantly different from one, and hence inconsistent with the standard formulation of the so-called "Fisher hypothesis". Gilbert and Yeoward (1994) argue that the reason is that most studies focus on short-term interest rates, whereas long-term rates are the relevant variable to consider in relation to savings and investment. If the former are not strongly correlated to the latter, evidence based on them cannot lead to any conclusions with regard to the Fisher effect. For instance, Summers (1983) analysed rates on short-term bills, and concluded that the variance of nominal rates is not accounted for by inflation. Barsky (1987) considered three-month rates, and found a Fisher relation (though with a coefficient far from one) in more recent decades, when there was more variation in expected inflation. [2]

[2] It is standard to assume rational expectations. Woodward (1992), though, stresses that the lack of direct measures of inflation expectations is a problem in testing the Fisher effect. He derives expected inflation from the price of UK indexed and non-indexed gilt-edged stocks, and concludes that there is evidence for the Fisher effect in its tax-adjusted form.

Subsequent studies have taken a cointegration approach. Mishkin (1992) used the Engle and Granger (1987) method to analyse both one-month and three-month rates, and argued that the Fisher effect is a long-term (rather than short-term) phenomenon, changes in expected inflation being associated with changes in nominal rates when these two variables exhibit trends (whereas no evidence can be found when they do not display trends). However, the estimated coefficient is far below one. Similarly, Evans and Lewis (1995) reported that the long-run response (estimated with the DOLS method of Stock and Watson, 1993) is less than unity, which they attributed to the "changing dynamics of inflation" over the sample. An exception is the study by Wallace and Warner (1993), who applied Johansen (1988, 1991) and found a one-to-one relationship between one-period inflation and both short- and long-term interest rates.

Tests of the "inverted Fisher hypothesis", where real rates are regressed against inflation, tend to be more supportive of the Fisher effect (see Carmichael and Stebbing, 1983), though not always (see Groenewold, 1989). Another way to formulate the Fisher hypothesis is in terms of the spread between the long-term (multi-period) interest rate and the one-period inflation rate, changes of which signal a change in future one-period inflation. Using a VAR methodology, Engsted (1995) found considerable cross-country differences in this relationship.

A possible explanation for the failure to find evidence of a Fisher effect was put forward by Darby (1975), who emphasised that rational agents will require nominal rates to adjust in response to movements in "tax-adjusted" expected inflation, and that, as a result, a coefficient in the range of 1.3. to 1.5 is consistent with the Fisher effect. This point was further investigated by Crowder and Wohar (1997), who used both taxable US Treasury and tax exempt municipal bond interest rates and various estimation techniques, and found a "Darby effect" in the post-war data.

In fact, as pointed out by Crowder and Hoffman (1996), the choice of estimator could be an important reason for the lack of empirical support for the Fisher relation. In their study, they apply Johansen's (1988, 1991) maximum likelihood (ML) estimation procedure to quarterly data, and find that nominal interest rates increase by 1.34 percent in response to a 1 percent increase in inflation, i.e. the ex-post real interest rate is stationary around a constant mean. Furthermore, the coefficient is insignificantly different from 1 when taking into account the effect of a time-varying average marginal tax rate on the Fisher equilibrium. By contrast, the ML estimates obtained using the same monthly data as in Mishkin (1992) and Evans and Lewis (1995) are negative and highly implausible, whilst the OLS and DOLS estimates are more meaningful from an economic viewpoint. Crowder and Hoffman (1996) then infer that "... the estimator choice (not data frequency or sample) may be responsible for the observed differences". They illustrate this point by means of Monte Carlo experiments, which show that the OLS and DOLS estimators suffer from considerable (downward) bias in small samples, whereas the ML estimator does not.

We extend their analysis to cover a variety of estimators and to allow for structural breaks and alternative data frequencies, and examine whether (i) differences in the estimates of θ from one and among themselves can be solely attributed to small sample bias, and (ii) rejections of the null reflect the use of asymptotic critical values, rather than appropriately computed empirical ones. In other words, the Fisher hypothesis may hold, but might not receive empirical support because of the inability of the estimators to estimate the true θ (which is equal to one) unbiasedly, and owing to the fact that the correct critical values for hypothesis testing are not employed. It is our contention that this in fact explains "Fisher's paradox", namely the finding that nominal interest rates and inflation do not move one-to-one in the long run, contrary to what interest rate theory would normally predict.

3 Estimation Methods

In this section we consider a number of cointegration estimators (most of which are asymptotically efficient) which can be used to estimate θ. It is useful to discuss these estimators and their properties with reference to Phillips's triangular representation of cointegration. Let z_t and u_t be two bivariate processes, with $\mathsf{z}_t = [y_t, x_t]^\top$ and $\mathsf{u}_t = [u_{1t}, u_{2t}]^\top$. We further assume that u_t is an I(0) process and the generating mechanism for y_t is given by the system

$$y_t = \theta x_t + u_{1t} \tag{4}$$

$$\Delta x_t = u_{2t} \tag{5}$$

for $t = 1, 2, ...T$. The long-run covariance matrix $\Omega = [\omega_{ij}], i, j = 1, 2$ of the process u_t is

$$\Omega = \lim_{T \Rightarrow \infty} T^{-1} \sum_{i=1}^{T} \sum_{j=1}^{T} E(\mathsf{u}_i \mathsf{u}_j^\top) \tag{6}$$

which, under stationarity, reduces to

$$\Omega = G + \Lambda + \Lambda^\top \tag{7}$$

where

$$G = E(\mathsf{u}_0 \mathsf{u}_0^\top) \tag{8}$$

and

$$\Lambda = \sum_{k=1}^{\infty} E(\mathsf{u}_0 \mathsf{u}_k^\top) \tag{9}$$

Also, we define the one-sided covariance matrix $\Delta = [\delta_{ij}]\,, i, j$ to be

$$\Delta = G + \Lambda \tag{10}$$

An early result by Stock (1987) shows that the OLS estimator of θ obtained from (**??**) is super-consistent, (in the sense that the rate of convergence is T, instead of $\mathrm{T}^{1/2}$) regardless of the presence of temporal and/or contemporaneous correlation between the regression error u_{1t} and the error that drives the regressor, u_{2t}. On the other hand, in general, the asymptotic distribution of the OLS estimator of θ falls outside the Local Asymptotic Mixture of Normals (LAMN) family and contains nuisance parameters. Non-standard asymptotics reflect the presence of contemporaneous and temporal correlation between the elements of u_t. Two types of second-order asymptotic effects are present in the limiting distribution of the OLS estimator (see Phillips and Loretan, 1991): The first is the nuisance parameter, ω_{12}/ω_{22} that describes the "long-run correlation" effect, due to non-diagonality of the long-run covariance matrix $\Omega = [\omega_{ij}]\,, i, j = 1, 2$. The second is the nuisance parameter $\delta_{21} = \sum_{k=0}^{\infty} E(u_{20}u_{1k})$ that describes the "endogeneity" effect.

Two alternative single-equation estimation methods dealing with these second-order effects have been suggested in the literature. The first method attempts to estimate the long-run parameters in the context of a dynamic model, in which the regression error forms a martingale difference sequence with respect to a selected information set. Models of this type fall into the category of the Hendry-style Autoregressive Distributed Lag (ADL) models, which encompass the Error Correction models (ECM) as a special case (see Hendry et al, 1984, and Banerjee et al, 1993). The second method, suggested by Phillips and Hansen (1990), permits direct estimation of the long-run relationship by correcting the simple OLS estimator for serial dependence and endogeneity in a semiparametric way. The resulting estimator is usually referred to as the Fully-Modified LS (FMLS) estimator.

The two methods have been proven to be asymptotically equivalent, if and only if there are no feedbacks from the cointegrating error to the error that drives the regressor (see, for example, Phillips, 1988, and Phillips and Loretan, 1991). This in turn implies that asymptotic inference procedures, which are based on single-equation error correction models, are likely to achieve optimality. If, however, the cointegrating error Granger-causes the regressor's error, then the generating mechanism for the latter is not fully estimated. This loss of information has an accumulative effect which results in the presence of second-order asymptotic bias effects.

In such a case, further augmentation of the ADL models by the leads of the regressor restores strong exogeneity and removes the second-order asymptotic bias (see Phillips and Loretan, 1991, Saikkonen, 1992, and Stock and Watson, 1993).

Based on the above discussion, the estimators that we consider are the following:

1) The OLS estimator

This is the simple OLS estimator, applied to the static regression (**??**). This estimator will be used as the benchmark estimator.

2) The Autoregressive Distributed Lag, ADL(p,q), estimator

This estimator is based on the following Autoregressive Distributed Lag, ADL(p,q), model:

$$y_t = \sum_{i=1}^{p} \pi_i y_{t-i} + \sum_{j=0}^{q} \gamma_j x_{t-j} + u_t \tag{11}$$

The parameter of interest θ is equal to the long-run multiplier of y_t with respect to x_t. A direct estimate of θ along with its standard error may be obtained by transforming the ADL model into the Bewley form (see Bewley, 1979, Wickens and Breusch, 1988, and Banerjee et. al. 1993):

$$y_t = \sum_{i=0}^{p-1} \rho_i \Delta y_{t-i} + \theta x_t + \sum_{j=0}^{q-1} \delta_j \Delta x_{t-j} + \nu_t \tag{12}$$

with the cointegrating parameter θ being equal to

$$\theta = \left(1 - \sum_{i=1}^{p} \pi_i\right)^{-1} \left(\sum_{j=0}^{q} \gamma_j\right) \tag{13}$$

Estimates of the coefficients of the transformed model can be obtained by using the Instrumental Variables estimator, with the original matrix of regressors as the instrumental variables.

3) The Augmented-by-Leads Autoregressive Distributed Lag, AADL(p,q,r) estimator

This estimator corrects for feedbacks from the cointegrating error to the regressor, by employing the ADL model, augmented by the leads of the regressor, which can be written, in the Bewley form, as follows:

$$y_t = \sum_{i=0}^{p-1} \rho_i \Delta y_{t-i} + \theta x_t + \sum_{j=0}^{q-1} \delta_j \Delta x_{t-j} + \sum_{k=1}^{r-1} \phi_k \Delta x_{t+k} + e_t \tag{14}$$

4) The Fully-Modified Least Squares, FMLS, estimator

The FMLS estimation procedure involves consistent estimators of Ω and Δ. While any consistent estimates of Ω and Δ will produce the same asymptotics, Phillips and Hansen (1990) consider the following class of kernel estimates:

$$\widehat{\Omega} = \widehat{G} + \widehat{\Lambda} + \widehat{\Lambda}^\top \tag{15}$$

$$\widehat{\Delta} = \widehat{G} + \widehat{\Lambda} \tag{16}$$

where

$$\widehat{G} = \left(\frac{1}{T-1}\right)\sum_{t=2}^{T} \widehat{\mathsf{u}}_t \widehat{\mathsf{u}}_t^\top \tag{17}$$

and

$$\widehat{\Lambda} = \sum_{j=1}^{T-2} k(j/S_T)\widehat{\Gamma}(j), \tag{18}$$

$$\widehat{\Gamma}(j) = \left(\frac{1}{T-1}\right)\sum_{t=2}^{T-j} \widehat{\mathsf{u}}_t \widehat{\mathsf{u}}_{t+j}^\top \; , j > 0, \tag{19}$$

$$\widehat{\Gamma}(j) = \widehat{\Gamma}(-j)^\top, \; j < 0 \tag{20}$$

The kernel weights $k(\cdot)$ are selected from the set of kernels, K, that necessarily generate positive semi-definite (psd) estimators of Ω in finite samples (see Andrews, 1991). This set includes the kernels employed in the present study, namely the Bartlett, Parzen and Quadratic Spectral (QS) ones.

Andrews (1991) shows that for unconditionally fourth or eighth order stationary random variables and for any given bandwidth sequence $\{S_T\}$, such that $S_T \to \infty$ and $S_T/T^{1/2} \to 0$, the estimators $\widehat{\Omega}$ and $\widehat{\Delta}$, as defined above, are consistent estimators of Ω and Δ respectively. Consistency and rate of convergence results have been established for more general cases, including unconditional heteroskedasticity and trending moments (see Andrews, 1991, Hansen, 1992, and De Jong and Davidson, 1996). Moreover, the QS kernel is best with respect to ATMSE in the class K. As far as the choice of S_T is concerned, Andrews (1991) provides sequences of fixed bandwidth parameters that are optimal in the sense of minimizing the ATMSE. Specifically, for the kernels discussed above, the optimal bandwidth parameters $\{S_T^+\}$are:

Bartlett: $S_T^+ = 1.1447[a(1)T]^{1/3}$

Parzen: $S_T^+ = 2.6614[a(2)T]^{1/5}$

Q.S. $\quad S_T^+ = 1.3221[a(2)T]^{1/5}$

where $a(q), q = 1, 2$ is a function of the unknown spectral density matrix of u_t at frequency zero, its q-th generalized derivative and a 4×4 weighting matrix of known constants. This means that $a(1)$, $a(2)$ and hence S_T^+ are also unknown in practice. Estimates of $a(1)$, and $a(2)$ may be obtained either by estimating simple parametric models, as suggested by Andrews (1991), or non-parametrically as suggested by Newey and West (1994). Once these estimates are obtained, they may be plugged into the formulas given above, to yield an estimator $\widehat{S}_T^+$ of S_T^+. The latter is usually referred to as the "automatic bandwidth estimator". If each element of u_t is approximated by an AR(1) model, then the corresponding estimates $\widehat{a(1)}$ and $\widehat{a(2)}$ are respectively given by the following relationships:

$$\widehat{a(1)} = \sum_{i=1}^{2} w_i \frac{4\widehat{\rho}_i^2\widehat{\sigma}_i^4}{(1-\widehat{\rho}_i)^6(1+\widehat{\rho}_i)^2} \text{Á} \sum_{i=1}^{2} w_i \frac{\widehat{\sigma}_i^4}{(1-\widehat{\rho}_i)^4} \qquad (21)$$

$$\widehat{a(2)} = \sum_{i=1}^{2} w_i \frac{4\widehat{\rho}_i^2\widehat{\sigma}_i^4}{(1-\widehat{\rho}_i)^8} \text{Á} \sum_{i=1}^{2} w_i \frac{\widehat{\sigma}_i^4}{(1-\widehat{\rho}_i)^4} \qquad (22)$$

where ρ_i and σ_i^2 $i = 1, 2$ denote the autoregressive coefficient and the conditional variance in the univariate model for u_{it} respectively and w_i, $i = 1, 2$ denote the weights assigned to the two diagonal elements of Ω.

Alternatively, the non-parametric method of Newey and West (1994) does not make any specific assumptions on the structure of u_t, but instead utilizes truncated sums of the sample autocovariances to obtain estimates of $a(1)$, and $a(2)$, as follows:

$$\widehat{a}(q) = \left(\frac{w^\top \widehat{\Omega}^{(q)} w}{w^\top \widehat{\Omega}^{(0)} w}\right)^2, q = 1, 2 \qquad (23)$$

where

$$\widehat{\Omega}^{(q)} = \sum_{j=-l}^{l} |\, j \,|^q\, \widehat{\Gamma}(j),\ q = 0, 1, 2 \qquad (24)$$

and l is equal to $c_1(T/100)^{2/9}$, $c_2(T/100)^{4/25}$ and $c_3(T/100)^{2/25}$ for the Bartlett, Parzen and QS kernel respectively. The choice of l depends on the choice of $c_i, i = 1, 2, 3$ which in turn implies that an element of subjective choice is built in this procedure as well. Newey and West (1994) consider the values 4 and 12 for c_1 and c_2 and the values 3 and 4 for c_3. They also consider weight vectors w which are more general than those of Andrews (1991) in the sense that they assign positive (instead of zero) weights to the off-diagonal elements of $\widehat{\Omega}^{(0)}$ and $\widehat{\Omega}^{(q)}$.

In the present study we utilize the QS kernel, and both the Andrews and Newey and West procedures for the estimation of the optimal bandwidth.

5) The Prewhitened Fully Modified Least Squares, PFMLS, estimator

In an extensive Monte Carlo study, Andrews (1991) reports cases where the kernel estimators of Ω yield confidence intervals whose coverage probabilities are too low. This problem is not associated with a poor choice of a specific kernel or bandwidth parameter and is particularly severe when there is considerable temporal dependence in the data. In the context of cointegration, the problem is likely to be more evident, since in many macroeconomic applications the equilibrium error seems to exhibit a rather long memory. In such a case, data filtering before estimating Ω may yield more accurately sized test statistics than standard kernel estimators. Andrews and Monahan (1990) suggest prewhitening of the OLS residuals $\widehat{\mathsf{u}}_t$ with a low-order vector autoregression (VAR), such as a VAR(1), in order to obtain an uncorrelated sequence of residual vector $\widehat{\mathsf{u}}_t^*$,

$$\widehat{\mathsf{u}}_t = \widehat{B}\widehat{\mathsf{u}}_{t-1} + \widehat{\mathsf{u}}_t^* \tag{25}$$

for $t = 2, ...T$. By replacing $\widehat{\mathsf{u}}_t$ with $\widehat{\mathsf{u}}_t^*$ in equations (**??**) to (**??**), we obtain kernel estimates of the long-run covariance matrix $\widehat{\Omega}^*$ of the prewhitened residuals $\widehat{\mathsf{u}}_t^*$. These estimates, in conjuction with $\widehat{B}$, give rise to the "VAR -prewhitened kernel estimators" $\widehat{\Omega}_{pw}$ and $\widehat{\Delta}_{pw}$:

$$\widehat{\Omega}_{pw} = \widehat{D}\widehat{\Omega}^*\widehat{D}^\top \tag{26}$$

$$\widehat{\Delta}_{pw} = \widehat{G}_{pw} + \widehat{\Lambda}_{pw} = \widehat{G}_{pw}\widehat{D}$$

where

$$vec\widehat{G}_{pw} = \left(I_2 - \widehat{B} \otimes \widehat{B}\right)^{-1} vec\widehat{\Omega}^*$$

$$\widehat{D} = \left(I_2 - \widehat{B}\right)^{-1} \tag{27}$$

According to the asymptotic results of Andrews and Monahan (1990), the prewhitened kernel estimators, $\widehat{\Omega}_{pw}$ and $\widehat{\Delta}_{pw}$ converge to Ω and Δ respectively, at the same rate as the corresponding standard kernel estimators $\widehat{\Omega}$ and $\widehat{\Delta}$.

6) The Fully Modified ADL, FMADL(p,q), and the Fully Modified Augmented ADL, FMAADL(p,q,r) estimators

Inder (1993) suggests that the semi-parametric corrections of the fully modified procedure could be applied to the estimate of θ, the coefficient of the I(1) variable, in the context of the dynamic specification (**??**). The resulting estimator uses both parametric and non-parametric corrections in order to remove the second-order bias effects. In fact, asymptotic theory predicts that such corrections are necessary only for the estimators which are based on dynamic specifications without leads, and only if there are feedbacks from the cointegrating error to the first difference of the regressor. On the other hand, the estimators based on dynamic specifications with leads hardly need these corrections asymptotically, even if the cointegrating error Granger-causes the regressor. However, in finite samples, the combination of the parametric and non-parametric corrections may further reduce the second-order bias.

Following Inder (1993), we obtain this estimator in two stages:

i). Obtain estimates of θ and the coefficients of the short-term dynamics, ρ_i, δ_j, ϕ_k, namely $\widehat{\theta}$, $\widehat{\rho}_i, \widehat{\delta}_j, \widehat{\phi}_k$ by employing the Bewley dynamic specifications (**??**) or (**??**).

ii). Remove the short-run effects, by defining $y_t^* = y_t - \sum_{i=0}^{p-1} \widehat{\rho}_i \Delta y_{t-i} - \sum_{j=0}^{q-1} \widehat{\delta}_j \Delta x_{t-j} - \sum_{k=1}^{r-1} \widehat{\phi}_k \Delta x_{t+k}$.Then the Fully Modified Instrumental Variable Estimator is obtained by applying the Phillips-Hansen non-parametric corrections to the OLS estimator of y_t^* on x_t. Needless to say, the 'prewhitened' version of these corrections is also applicable.

Summary of the Estimators that will be used:

Here, we sum up the estimators that we shall use for the estimation of the Fisher parameter θ, and specify the exact orders, p, q, r, in the dynamic specifications. The various combinations that result from alternative lag-lead specifications, and options regarding the bandwidth estimation and/or prewhitening of the fully modified procedures result in a total of 25 estimators:

1) OLS

2) ADL(1,2)

3) AADL(1,2,1)

4) ADL(4,4)

5) AADL(4,4,4)

Using Standard FMLS

6) Andrews-FMLS

7) NW-FMLS

8) Andrews-FM-ADL(1,2)

9) NW-FM-ADL(1,2)

10) Andrews-FM-AADL(1,2,1)

11) NW-FM-AADL(1,2,1)

12) Andrews-FM-ADL(4,4)

13) NW-FM-ADL(4,4)

14) Andrews-FM-AADL(4,4,4)

15) NW-FM-AADL(4,4,4)

Using Prewhitened FMLS

16) Andrews-FMLS

17) NW-FMLS

18) Andrews-FM-ADL(1,2)

19) NW-FM-ADL(1,2)

20) Andrews-FM-AADL(1,2,1)

21) NW-FM-AADL(1,2,1)

22) Andrews-FM-ADL(4,4)

23) NW-FM-ADL(4,4)

24) Andrews-FM-AADL(4,4,4)

25) NW-FM-AADL(4,4,4)

4 Empirical Analysis

4.1 Estimation Results

As in Crowder and Hoffman (1996), we use quarterly data on the US three-month T-bill rate and the implicit price deflator for total consumption expenditure covering the period 1960:1 to 1999:4. The data are taken from Datastream. Annualised log changes in the price serve as a proxy for expected inflation. Monthly data are also used, as in Evans and Lewis (1995) and Crowder and Wohar (1997), but the analysis of these data is presented in a separate section since they exhibit different statistical properties from the quarterly ones. In particular, although monthly data allow the asymptotic effects to work better owing to a larger sample, they do so at the expense of a lower signal-to-noise ratio. A natural signal-to-noise ratio in the present context may be defined as the ratio of the long-run variance of the innovation of the regressor to the long-run variance of the cointegrating error, that is ω_{22}/ω_{11}. We estimate this ratio non-parametrically using a quadratic spectral kernel and a prewhitened estimator, and we find that in the case of quarterly data it is equal to 1.25, whereas for monthly data (as discussed in Section 5.2) it is equal to 0.92. Consequently, we mainly focus on quarterly data to draw inference on the validity of the Fisher hypothesis, albeit we also present estimates based on monthly data for comparison with other studies.

Some preliminary results (not reported) confirm the widely held view that interest and inflation rates are I(1) processes and cointegrated, meaning that the necessary condition for the Fisher effect to hold, that is cointegration between inflation and interest rates, is satisfied. Next, we focus on the second and most debated part of the Fisher hypothesis, namely whether the estimate of the slope coefficient is significantly different from one. The estimates of θ are reported in table 1 along with t-tests for the null hypothesis of interest $\theta = 1$. The results are mixed. When the most commonly used estimator, namely OLS (see, e.g., Barsky, 1987, Mishkin, 1992) and the Standard FMLS are employed the estimates of θ are significantly less than one. On the other hand, when any of the remaining estimators are used, the Fisher hypothesis easily survives the empirical evidence. The point estimates of θ are very close to one, which in turn leads to non rejections of the null hypothesis $\theta = 1$. In other words, it appears that the decision whether to accept or reject the Fisher hypothesis depends entirely on the properties of the estimation methods used. Therefore, the next logical step is to examine the small-sample performance of the alternative estimators in order to establish whether it is those with the most desirable properties which produce results supporting the Fisher relation. For this purpose, we shall estimate the magnitude of the negative bias that is associated with each estimator, the associated shift in the distribution of the t-statistic, and the correct empirical critical values to be used for testing the Fisher hypothesis.

5 Monte Carlo Analysis

In this subsection, we use Monte Carlo techniques to evaluate the performance of each of the estimators considered above in the context of a DGP which aims at mimicking the true DGP which gave rise to the observed data.

We assume that the errors $\mathsf{u}_t = [u_{1t}, u_{2t}]^\top$ follow a bivariate VAR(1) process:

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} u_{1t-1} \\ u_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \tag{28}$$

and

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \tilde{} IID \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right] \tag{29}$$

Both eigenvalues of the matrix $A = [a_{ij}], i, j = 1, 2$ are assumed to be less than one in modulus, in order for y_t and x_t to be I(1) variables, and the cointegrating error to be an I(0) process. In such a case the long-run covariance matrix Ω is equal to $(I - A)^{-1}\Sigma(I - A^\top)^{-1}$, where Σ denotes the innovations covariance matrix of the VAR. A sufficient condition for diagonality of Ω and for $\delta_{21} = 0$ amounts to x_t being strictly exogenous, that is $a_{12} = a_{21} = \sigma_{12} = 0$. As mentioned above, in the absence of a strictly exogenous inflation rate, second-order asymptotic bias effects are encountered when θ is estimated by simple OLS. In view of these effects, the performance of the twenty-five estimators introduced above is assessed by means of the following statistics:

1) **Bias**, computed as:

$$\widehat{\theta} - \theta_0$$

where:

$$\widehat{\theta} = \sum_{i=1}^{r} \widehat{\theta}_i / r$$

$i = 1, ..., r$ and r is the number of replications and $\theta_0 = 1$.

2) **Absolute Bias**, computed as

$$AB(\widehat{\theta}) = \sum_{i=1}^{r} \left| \widehat{\theta}_i - \theta \right| / r$$

3) Average **mean square error,** rmse$(\widehat{\theta})$, computed according to the "bias" formula in which $\overline{\theta}$ has been replaced by θ_0

4) average t-statistics, **mean-t**, for testing $\theta = \theta_0$

5) **standard deviations** of the t-statistics, for testing $\theta = \theta_0$

6) **skewness and kurtosis** coefficients of the t-statistics, denoted by $a_3(t_{\widehat{\theta}})$ and $a_4(t_{\widehat{\theta}})$ respectively.

7) The **2.5% and 97.5% points** of the empirical distribution of the t-statistic.

The parameters of this model are calibrated using the interest and inflation rate data in hand. Table 2 reports the results from these simulations. The first part of Table3 ranks all the estimators, for the DGP under consideration, according to three alternative criteria, namely bias, non-centrality of the associated t-statistics and distributional divergence of the t-statistics from the normal. OLS appears to be the worst estimator of all, since it exhibits the largest bias, mislocation of its t-statistic and deviation from the normal. The mean value of the OLS t-statistic is -3.339 instead of 0, which in turn produces a $t_{0.025}$ value of -8.07 instead of -1.96. It is obvious that a rejection of the null hypothesis of interest $\theta = 1$, based on a t-test in which the reference distribution is the asymptotic one, is totally misleading. It is worth noting that if, instead of the asymptotic $t_{0.025}$ critical value, we use the corresponding value from the empirical distribution, the null hypothesis of $\theta = 1$ is not rejected. The fully modified procedures remove some of this bias, especially when the Andrews (1991) procedure for the selection of the bandwidth parameter is used. However, both the A-FM and the NW-FM estimators produce highly non-central t-statistics whose mean values are -0.691 and -1.233 respectively. As in the OLS case, this non-centrality produces $t_{0.025}$ and $t_{0.975}$ points of the empirical distribution, which are totally different from the ones corresponding to the asymptotic distribution. When using the critical values from the empirical distribution, the null hypothesis of the Fisher effect still survives the empirical evidence. The dynamic specifications fare much better than the OLS and Standard FMLS. The prewhitened FMLS performs significantly better than the standard FMLS and is only slightly outperformed by the dynamic specifications. The best estimator, however, is the one that combines the dynamic models with the FMLS procedure. According to Table 3, this appears to be the A-FM-ADL(1,2) one, by any of the three criteria used. The empirical critical values are almost identical to the corresponding asymptotic ones, thus producing reliable inferences. On the other hand, at the bottom of the ladder we find the OLS and the standard FMLS estimators. It should be stressed at this point that, as already mentioned, the former is precisely the one which is most commonly used in the literature and which leads to rejections of the Fisher effect (see, e.g., Barsky, 1987, Mishkin, 1992).

6 Further Empirical Results

6.1 The Effects of Structural Breaks

Evans and Lewis (1995, EL henceforth) argue that small sample bias in the estimates of θ may arise because of infrequent shifts in the inflation process. In the case of the series under consideration, a shift might have occurred as a result of changes in monetary policy. For instance, during the Federal Reserve (Fed) experiment of 1979 to 1982, the Fed adopted an alternative target instrument, namely non-borrowed reserves (NBR's), rather than interest rates, with effects on the volatility of interest rates and inflation.[3] In order to investigate the effects of such shifts (or more general structural breaks) on the performance of cointegration estimators, we carry out two additional simulation experiments. In the first, we assume a Data Generation Process in which the variance of the cointegrating vector is not constant over the whole sample. In particular, we assume that during the period 1960.I to 1981.IV the variance of the cointegrating error is σ_{11}. Over the remanining period, that is 1982.I to 1999.IV, the variance of the cointegrating error is increased to twice as much as σ_{11}. In the second Data Generation Process (DGP3) we assume that the shift in the variance of the cointegrating error was not permanent, but lasted only from 1982.I to 1984.IV, and then (from 1985.I to 1999.IV) it returned to its previous level of σ_{11}. The results are reported in Tables 3 and 4 for the DGP2 and DGP3 respectively. The first thing to notice is that the overall performance of all estimators is negatively related to the number of structural breaks. In other words, the estimators perform better in the case of a permanent shift in the variance of the cointegrating error than in the case of a short-lived shift,

[3]Other changes which might have had an impact on the volatility of inflation are wars involving the US, the OPEC oil crisis, and the October 1987 stock market crash.

with the variance then returning to its previous level. In particular, the presence of structural breaks increases the bias of all estimators, shifts the distribution of the t-statistics further to the left, and increases the skewness of these distributions. The overall effect is reflected in the empirical points of the distribution of the t-statistics which are moved further away from their corresponding asymptotic ones. However, importantly, the null hypothesis that the Fisher effect holds is not rejected by any of these estimators, when the empirical critical values are used instead of the asymptotic ones.

The second interesting observation is that, even in the presence of breaks, the relative performance of the twenty-five estimators remains largely the same (see the second and third parts of Table 5). As in the no-breaks case, OLS is the worst estimator by any of the three criteria used before, followed by the NW-FM and A-FM estimators. Concerning the best estimators in the presence of structural breaks, the results seem to favour once again the A-FM-ADL(1,2) estimator at least in terms of minimum negative shifts in the distribution of the t-statistic. In terms of distributional divergence from the standard normal, this estimator also fares well, although the ADL(1,2) and the AADL(4,4,4,) seem to emerge as the best performers (by this criterion) for the DGP2 and DGP3 respectively. The analysis in this subsection confirms therefore that the crucial issues are the choice of estimator and the use of appropriate critical values, rather than the presence of structural breaks in the inflation process as suggested by EL (1995).

6.2 Monthly Data - ARCH effects

One respect in which the analysis conducted so far differs from the study by EL and a more recent one by Crowder and Wohar (1997, CW henceforth) is the data frequency. These authors employ monthly instead of quarterly data for different time periods, that is January 1947 to February 1987 and January 1950 to December 1995 in the EL and CW studies respectively. EL employ the Stock and Watson's (1993) dynamic OLS estimator which is equivalent to our AADL family of estimators, and report an estimate of θ equal to 0.739 which leads to rejection of the null hypothesis $\theta = 1$. On the other hand, CW utilise the dynamic OLS and FMLS estimators and employ Treasury bill (TB) rates as well as tax-free Municipal bond rates and provide estimates of θ which are greater than one in the TB case and smaller than one in the Municipal bond case. They interpret these results as evidence that "taxes have a profound influence on the size of the estimated Fisher effect".

In order to make our results more directly comparable to those of EL and CW, in this subsection we also use monthly data, specifically the same data set as CW (see CW, 1997, for further details), and reinterpret their findings in the light of the performance of the employed estimators in the context of the DGP which is most likely to have given rise to the observed data. In the case of monthly data, massive dynamic heteroskedasticity in both inflation and interest rates is detected by some preliminary testing. Therefore, when comparing alternative estimators, the considerable GARCH effects must be taken into account. For brevity, we report estimates of θ using the standard FMLS where the bandwidth parameter is selected via the Andrews (1991) procedure. The prewhitened FMLS and the NW procedure provide similar results. The estimates of θ for the TB rates case are reported in the first column of table 6. It can be seen that the Fisher hypothesis is rejected only by the OLS and FMLS estimators by applying asymptotic criteria, something that occurred in the case of quarterly data as well. Before reinterpreting these estimates using the correct empirical distributions, it is important to compare our estimates with those of EL and CW. The slight difference between the DOLS point estimates of EL and our corresponding AADL estimates can plausibly be attributed to the different sample periods. Concerning the CW study, our estimates are considerably lower than theirs, which might look strange at first sight since we employ exactly the same data set. For example, our OLS and FMLS estimates of θ are 0.37 and 0.57 respectively, whereas the corresponding CW estimates are 0.89 and 1.45. The reason for these discrepancies has to do with the way in which the deterministic components are treated in each case. CW do not include a constant in the cointegrating regression. This is a sensible assumption if the nonstationary inflation and interest rate series contain no drifts, or if they contain drifts which cancel out each other. For the data in hand, our results show that drifts are very significant for both series, and so is the constant in the cointegrating regression. Moreover, some additional Monte Carlo results (not reported) indicate that the omission of a non-superfluous constant from the cointegrating regression may have detrimental effects on the estimates of the cointegrating vector. In view of this, we chose to include a constant in the cointegrating regression, which explains the discrepancies between our estimates and those of CW.

Next, we investigate the performance of the estimation methods in the presence of GARCH effects and derive the empirical distributions of the relevant t-statistics. These issues are analysed in the context of a DGP in which the errors continue to follow a VAR(1) process, such as (**??**), but now the innovations are not NIID but instead they follow a bivariate GARCH process. In specifying the exact form of this process, we adopt the Bollerslev's (1990) Constant Conditional Correlation (CCC) specification. In particular, we assume that $\mathsf{e}_t \mid I_{t-1} \sim (\mathsf{0}, H_t)$, where

$$H_t = \begin{bmatrix} h_{1t} & h_{12t} \\ h_{12t} & h_{2t} \end{bmatrix}$$

and

$$h_{1t} = k_1 + \gamma_1 h_{1t-1} + a_1 u_{1t-1}^2 \tag{30}$$

$$h_{2t} = k_2 + \gamma_2 h_{2t-1} + a_2 u_{2t-1}^2 \tag{31}$$

$$h_{12t} = \rho_{12}(h_{1t} h_{2t})^{1/2} \tag{32}$$

As can be seen from these equations, the conditional covariance, h_{12t}, is proportional to the square root of the product of the conditional variances, h_{1t} and h_{2t}, thus leaving the conditional correlation, ρ_{12}, constant over time. We calibrate this model with the monthly data and derive the following estimates: $a_{11} = 0.92$, $a_{12} = 0.18$, $a_{21} = 0.14$, $a_{22} = -0.47$, $k_1 = 0.08$, $\gamma_1 = 0.82$, $a_1 = 0.12$, $k_2 = 0.68$, $\gamma_2 = 0.8$, $a_2 = 0.13$, $\rho = -0.9$. The 0.025 and 0.975 critical values, calculated by means of a Monte Carlo experiment based on 5000 replications, are reported in the last two columns of table 6. As in the quarterly data case, the Fisher hypothesis is not rejected even by OLS or FMLS t-statistics when the critical values from the empirical distributions are used instead of those from the asymptotic distribution.

Some additional estimates were obtained and the corresponding Monte Carlo simulations were conducted for the case where the Municipal bond rates are used instead of the TB rates. For brevity, we do not report these results but we briefly describe them: The Fisher hypothesis, when tested by means of the empirical distributions of the relevant t-statistics, is still given empirical support, although, as reported by CW, the point estimates of θ are less than the ones corresponding to the TB rates. This may be thought of as weak evidence that tax effects have a role to play in the Fisher equation.

However, the main result emerging from all the Monte Carlo simulations conducted in this paper is that there is a negative small sample bias in all the estimators used, which in turn shifts the distributions of the associated t-statistics to the left. This effect is extremely large for the OLS and the standard FMLS estimation procedures. As a result, the appropriate critical values to test the null hypothesis are vastly different from those predicted by the relevant asymptotic theory.

7 Conclusions

This paper has shed some new light on "Fisher's paradox" by examining the small-sample properties of various estimators. The thrust of our argument is the following. The reason why numerous empirical studies have not succeeded in finding empirical support for the Fisher effect lies in their adopting estimation methods which, despite their asymptotic efficiency, perform poorly in samples of the size typically available to applied researchers. In particular, our Monte

Carlo experiments show that the estimators with the least desirable small-sample properties include the one most frequently used in empirical studies which reject the Fisher hypothesis, namely OLS, as well as the FMLS estimator. When one chooses (out of the large set we consider) the estimators with the best properties, namely the smallest downward bias and the minimum shift in the distribution of the associated t-statistics, the evidence is strongly supportive of the Fisher effect in the US. Moreover, we compute the correct critical values, based on the empirical distributions of the t-statistics, and show that the Fisher hypothesis survives even when less satisfactory estimators are employed, provided the appropriate critical values are used. Although an earlier study by Crowder and Hoffman (1996) had already put forward the suggestion that the choice of estimator might be crucial when testing the validity of the Fisher equation, their analysis was much more limited, as it only compared three estimators in terms of small-sample bias. By contrast, our study covers virtually all available single-equation estimators, and assesses their relative performance, as well as calculating the empirical critical values for the distribution of the t-statistic, for a variety of DGPs (including structural breaks and ARCH processes) which are likely to have given rise to the observed data.

Other explanations offered in the literature for the fact that the Fisher hypothesis is often not borne out by the data (for instance, the effects of taxation, the use of short rates, difficulties in measuring inflationary expectations, etc.) might be relevant in some cases. In particular,we do find some slight evidence that taxation might play a role, as suggested, e.g., by Crowder and Wohar (1997). However, as our analysis demonstrates, even more crucial is the estimation method. Provided appropriate estimators are used, the empirical evidence does not contradict economic theory, which generally predicts that the real rate of interest follows a stationary process. For instance, neoclassical models of dynamic growth have the property that consumption growth and the real rate of interest are both constant in the steady state. Stationarity of the real rate is also a prediction of standard asset pricing models, and it is consistent with superneutrality (to the extent that inflation is a monetary phenomenon). It appears that, in fact, these theories are not at odds with the data.

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TABLE 1 ESTIMATION RESULTS: quarterly data

	ESTIMATORS	$\hat{\theta}$	$H_0 : \theta = 1$
1	OLS	0.671 (0.063)	-5.22*
2	ADL (1,2)	0.944 (0.231)	-0.242
3	AADL (1,2,1)	1.069 (0.268)	0.257
4	ADL (4,4)	0.838 (0.258)	-0.627
5	AADL (4,4,4)	0.961 (0.323)	-0.120
	STANDARD-FM		
6	A-FM	0.678 (0.132)	-2.439*
7	NW-FM	0.739 (0.132)	-2.139*
8	A-FM-ADL (1,2)	0.853 (0.258)	-0.569
9	NW-FM-ADL (1,2)	0.972 (0.224)	−0.125
10	A-FM-AADL (1,2,1)	0.935 (0.159)	-0.408
11	NW-FM-AADL (1,2,1)	1.024 (0.241)	0.099
12	A-FM-ADL (4,4)	0.808 (0.142)	-1.352
13	NW-FM-ADL (4,4)	0.897 (0.230)	-0.447
14	A-FM-AADL (4,4,4)	0.871 (0.136)	-0.948
15	NW-FM-AADL (4,4,4)	0.911 (0.249)	-0.357

Notes: 1) standard errors in parentheses

2) an asterisk indicates rejection of the null at 5% level

	TABLE 1 cont. ESTIMATION RESULTS: quarterly data		
	ESTIMATORS	$\hat{\theta}$	$H_0 : \theta = 1$
	PREWHITENED-FM		
16	A-FM	0.946 (0.232)	-0.232
17	NW-FM	0.879 (0.230)	-0.526
18	A-FM-ADL (1,2)	0.982 (0.229)	-0.078
19	NW-FM-ADL (1,2)	0.972 (0.224)	-0.125
20	A-FM-AADL (1,2,1)	1.066 (0.247)	0.267
21	NW-FM-AADL (1,2,1)	1.024 (0.241)	0.099
22	A-FM-ADL (4,4)	0.864 (0.232)	-0.586
23	NW-FM-ADL (4,4)	0.897 (0.230)	-0.442
24	A-FM-AADL (4,4,4)	0.904 (0.250)	-0.384
25	NW-FM-AADL (4,4,4)	0.911 (0.249)	-0.357

Notes: 1) standard errors in parentheses

2) an asterisk indicates rejection of the null at 5% level

TABLE 2. Monte Carlo Results

DGP: $\theta = 1$, $\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} = \begin{bmatrix} 0.88 & 0.13 \\ 0 & -0.25 \end{bmatrix}$, $\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 0.96 & -0.7 \\ -0.7 & 1.3 \end{bmatrix}$

Number of breaks: 0

5000 Replications

Estimators	Bias	Abs. Bias	MSE	Mean-t	s.d.(t)	$\alpha_3(t)$	$\alpha_4(t)$	t $_{0.05}$	t $_{0.95}$
OLS	-0.123	0.131	0.028	-3.339	2.771	-0.380	3.276	-8.07	1.11
ADL (1,2)	-0.011	0.087	0.015	-0.016	0.948	-0.483	5.162	-1.66	1.5
AADL (1,2,1)	-0.011	0.089	0.016	-0.121	0.958	-0.429	4.853	-1.64	1.55
ADL (4,4)	-0.010	0.090	0.016	-0.122	0.941	-0.323	4.402	-1.73	1.46
AADL (4,4,4)	-0.010	0.097	0.019	-0.138	0.946	-0.315	4.177	-1.73	1.42
Standard-FM									
A-FM	-0.064	0.094	0.018	-0.691	1.242	-0.689	4.524	-2.91	1.17
NW-FM	-0.085	0.106	0.021	-1.233	1.546	-0.377	3.442	-3.93	1.38
A-FM-ADL (1,2)	-0.002	0.084	0.014	-0.042	1.169	-0.271	5.065	-1.94	2.02
NW-FM-ADL (1,2)	-0.009	0.089	0.015	-0.106	1.045	-0.485	5.497	-1.76	1.66
A-FM-AADL (1,2,1)	-0.003	0.085	0.015	-0.049	1.185	-0.222	4.834	-1.94	2.11
NW-FM-AADL (1,2,1)	-0.009	0.091	0.016	-0.111	1.057	-0.449	5.133	-1.85	1.68
A-FM-ADL (4,4)	-0.002	0.087	0.016	-0.045	1.228	-0.267	4.790	-2.04	2.25
NW-FM-ADL (4,4)	-0.010	0.092	0.017	-0.127	1.077	-0.221	4.431	-1.78	1.70
A-FM-AADL (4,4,4)	-0.004	0.092	0.018	-0.076	1.276	-0.376	4.60	-2.30	2.14
NW-FM-AADL (4,4,4)	-0.012	0.095	0.018	-0.149	1.113	-0.326	4.149	-1.93	1.70
Pre–whitened-FM									
A-FM	-0.028	0.094	0.018	-0.266	1.078	-0.598	4.917	-2.198	1.493
NW-FM	-0.022	0.092	0.017	-0.226	1.080	-0.690	4.598	-2.12	1.46
A-FM-ADL (1,2)	-0.012	0.087	0.015	-0.119	1.017	-0.397	5.085	-1.75	1.65
NW-FM-ADL (1,2)	-0.009	0.089	0.015	-0.106	1.045	-0.485	5.497	-1.76	1.67
A-FM-AADL (1,2,1)	-0.011	0.090	0.016	-0.118	1.038	-0.372	4.811	-1.77	1.65
NW-FM-AADL (1,2,1)	-0.009	0.091	0.016	-0.111	1.057	-0.449	5.133	-1.84	1.68
A-FM-ADL (4,4)	-0.011	0.090	0.017	-0.128	1.062	-0.167	4.418	-1.84	1.69
NW-FM-ADL (4,4)	-0.01	0.092	0.017	-0.127	1.077	-0.221	4.431	-1.77	1.70
A-FM-AADL (4,4,4)	-0.012	0.095	0.018	-0.151	1.1125	-0.303	4.038	-1.94	1.71
NW-FM-AADL (4,4,4)	-0.012	0.095	0.018	-0.149	1.113	-0.326	4.149	-1.93	1.70

TABLE 3. Monte Carlo Results

DGP: $\theta = 1, \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} = \begin{bmatrix} 0.88 & 0.13 \\ 0 & -0.25 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & -0.7 \\ -0.7 & 1.3 \end{bmatrix}$

Number of breaks: 1 at 1981:01

Regime 1: $\sigma_{11} = 0.96$; Regime 2: $\sigma_{11} = 1.92$;

Estimators	Bias	Abs. Bias	MSE	Mean-t	s.d.(t)	$\alpha_3(t)$	$\alpha_4(t)$	t $_{0.05}$	t $_{0.95}$
OLS	-0.195	0.218	0.077	-3.565	3.297	-0.367	3.875	-9.21	1.74
ADL (1,2)	-0.045	0.193	0.071	-0.286	1.291	-0.726	5.329	-2.25	1.86
AADL (1,2,1)	-0.045	0.200	0.075	-0.296	1.312	-0.652	5.279	-2.23	1.83
ADL (4,4)	-0.040	0.200	0.079	-0.290	1.300	-0.682	5.406	-2.28	1.84
AADL (4,4,4)	-0.033	0.222	0.101	-0.296	1.345	-0.383	4.481	-2.34	1.91
Standard-FM									
A-FM	-0.121	0.171	0.056	-1.060	1.618	-0.619	4.611	-3.98	1.44
NW-FM	-0.142	0.198	0.070	-1.417	2.016	-0.467	4.301	-4.81	1.87
A-FM-ADL (1,2)	-0.026	0.166	0.056	-0.244	1.600	-0.716	5.472	-2.72	2.38
NW-FM-ADL (1,2)	-0.037	0.200	0.073	-0.279	1.459	-0.736	5.892	-2.66	2.27
A-FM-AADL (1,2,1)	-0.027	0.172	0.060	-0.261	1.628	-0.629	5.090	-2.88	2.42
NW-FM-AADL (1,2,1)	-0.037	0.203	0.077	-0.287	1.513	-0.675	5.672	-2.65	2.25
A-FM-ADL (4,4)	-0.022	0.172	0.062	-0.257	1.701	-0.623	5.732	-2.96	2.07
NW-FM-ADL (4,4)	-0.041	0.205	0.080	-0.324	1.604	-0.519	5.364	-2.65	2.48
A-FM-AADL (4,4,4)	-0.023	0.196	0.083	-0.314	1.921	-0.283	5.062	-3.22	2.74
NW-FM-AADL (4,4,4)	-0.037	0.221	0.100	-0.347	1.739	-0.300	4.845	-3.03	2.78
Pre–whitened-FM									
A-FM	-0.072	0.202	0.082	-0.509	1.533	-0.635	4.903	-3.38	1.94
NW-FM	-0.069	0.188	0.070	-0.468	1.511	-0.918	6.050	-3.10	1.95
A-FM-ADL (1,2)	-0.046	0.191	0.070	-0.298	1.415	-0.626	5.223	-2.59	2.07
NW-FM-ADL (1,2)	-0.037	0.200	0.073	-0.279	1.489	-0.736	5.892	-2.66	2.27
A-FM-AADL (1,2,1)	-0.044	0.199	0.075	-0.304	1.461	-0.561	5.097	-2.59	2.14
NW-FM-AADL (1,2,1)	-0.037	0.203	0.077	-0.287	1.513	-0.675	5.672	-2.65	2.25
A-FM-ADL (4,4)	-0.043	0.199	0.079	-0.327	1.558	-0.519	5.259	-2.64	2.40
NW-FM-ADL (4,4)	-0.041	0.205	0.080	-0.324	1.604	-0.519	5.364	-2.65	2.49
A-FM-AADL (4,4,4)	-0.038	0.220	0.101	-0.344	1.724	-0.270	4.680	-3.04	2.79
NW-FM-AADL (4,4,4)	-0.037	0.221	0.100	-0.347	1.739	-0.300	4.841	-3.03	2.78

TABLE 4. Monte Carlo Results

DGP: $\theta = 1, \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} = \begin{bmatrix} 0.88 & 0.13 \\ 0 & -0.25 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & -0.7 \\ -0.7 & 1.3 \end{bmatrix}$

Number of breaks: 2 at 1981:1 and 1984:1

Regime 1: $\sigma_{11} = 0.96$; Regime 2: $\sigma_{11} = 1.92$; Regime 3: $\sigma_{11} = 0.96$

Estimators	Bias	Abs. Bias	MSE	Mean-t	s.d.(t)	$\alpha_3(t)$	$\alpha_4(t)$	t $_{0.05}$	t $_{0.95}$
OLS	-0.202	0.211	0.066	-4.470	3.197	-0.587	4.573	-10.08	0.611
ADL (1,2)	-0.052	0.153	0.046	-0.431	1.316	-1.023	7.581	-2.54	1.61
AADL (1,2,1)	-0.051	0.161	0.050	-0.443	1.349	-1.043	7.715	-2.57	1.64
ADL (4,4)	-0.045	0.158	0.049	-0.404	1.330	-0.801	6.378	-2.62	1.78
AADL (4,4,4)	-0.036	0.173	0.066	-0.402	1.388	-0.724	5.801	-2.52	1.82
Standard-FM									
A-FM	-0.126	0.155	0.044	-1.350	1.690	-1.226	6.745	-4.39	1.14
NW-FM	-0.143	0.175	0.053	-1.778	1.977	-0.586	4.630	-5.20	1.53
A-FM-ADL (1,2)	-0.031	0.133	0.037	-0.367	1.693	-0.871	6.351	-3.11	2.30
NW-FM-ADL (1,2)	-0.044	0.156	0.047	-0.423	1.597	-0.914	6.875	-2.82	2.21
A-FM-AADL (1,2,1)	-0.031	0.138	0.041	-0.385	1.773	-0.838	6.038	-3.25	2.48
NW-FM-AADL (1,2,1)	-0.043	0.159	0.049	-0.426	1.563	-0.862	6.722	-2.93	2.23
A-FM-ADL (4,4)	-0.028	0.139	0.042	-0.351	1.826	-0.720	5.707	-3.313	2.69
NW-FM-ADL (4,4)	-0.044	0.160	0.049	-0.457	1.652	-0.611	5.327	-3.17	2.37
A-FM-AADL (4,4,4)	-0.022	0.152	0.055	-0.390	2.068	-0.763	6.094	-3.58	3.08
NW-FM-AADL (4,4,4)	-0.038	0.173	0.064	-0.472	1.804	-0.517	4.818	-3.29	2.73
Pre–whitened-FM									
A-FM	-0.073	0.161	0.057	-0.678	1.589	-0.997	6.518	-3.50	1.76
NW-FM	-0.063	0.150	0.047	-0.580	1.582	-1.155	6.761	-3.26	1.83
A-FM-ADL (1,2)	-0.054	0.152	0.045	-0.467	1.460	-0.950	7.127	-2.59	1.87
NW-FM-ADL (1,2)	-0.044	0.156	0.047	-0.423	1.537	-0.914	6.875	-2.82	2.12
A-FM-AADL (1,2,1)	-0.051	0.159	0.048	-0.466	1.515	-0.899	6.897	-2.92	1.96
NW-FM-AADL (1,2,1)	-0.043	0.159	0.049	-0.426	1.563	-0.862	6.722	-2.93	2.23
A-FM-ADL (4,4)	-0.047	0.155	0.047	-0.465	1.600	-0.649	5.694	-3.01	2.21
NW-FM-ADL (4,4)	-0.044	0.160	0.049	-0.457	1.652	-0.611	5.327	-3.17	2.37
A-FM-AADL (4,4,4)	-0.039	0.174	0.064	-0.469	1.790	-0.473	4.789	-3.15	2.73
NW-FM-AADL (4,4,4)	-0.038	0.173	0.064	-0.472	1.804	-0.517	4.818	-3.29	2.73

Table 5.1

	Estimator	DGP 1 Bias	Rank	Mean - t	Rank	Distr	Rank
1	OLS	-0.123	25	-3.339	25	6.96	25
2	ADL(1.2)	-0.001	13	-0.116	9	0.76	20
3	ADL(1.2.1)	-0.011	13	-0.121	12	0.73	18
4	ADL(4.4)	-0.010	9	-0.122	13	0.73	18
5	AADL(4.4.4)	-0.010	9	-0.138	17	0.77	21
	With Standard - FM						
6	A-FM	-0.064	23	-0.691	23	1.74	23
7	NW-FM	-0.085	24	-1.233	24	2.55	24
8	A-FM-ADL(1.2)	-0.002	1	-0.042	1	0.08	1
9	NW-FM-ADL(1.2)	-0.009	5	-0.106	5	0.5	13
10	A-FM-AADL(1.2.1)	-0.003	3	-0.049	3	0.17	2
11	NW-FM-AADL(1.2.1)	-0.009	5	-0.111	7	0.39	7
12	A-FM-ADL(4.4)	-0.005	1	-0.045	2	0.37	6
13	NW-FM-ADL(4.4)	-0.010	9	-0.127	14	0.44	10
14	A-FM-AADL(4.4.4)	-0.004	4	-0.076	4	0.52	15
15	NW-FM-AADL(4.4.4)	-0.012	17	-0.149	18	0.29	4
	With Prewhitened FM						
16	A-FM	-0.028	22	-0.266	22	0.775	22
17	NW-FM	-0.022	21	-0.226	21	0.66	17
18	A-FM-ADL(1.2)	-0.012	17	-0.119	11	0.52	15
19	NW-FM-ADL(1.2)	-0.009	5	-0.106	5	0.49	12
20	A-FM-AADL(1.2.1)	-0.011	13	-0.118	10	0.5	13
21	NW-FM-AADL(1.2.1)	-0.009	5	-0.111	7	0.4	9
22	A-FM-ADL(4.4)	-0.011	13	-0.128	16	0.39	7
23	NW-FM-ADL(4.4)	-0.010	9	-0.127	14	0.45	11
24	A-FM-AADL(4.4.4)	-0.012	17	-0.151	20	0.27	3
25	NW-FM-AADL(4.4.4)	-0.012	17	-0.149	18	0.29	4

Table 5.2							
		DGP 2					
	Estimator	Bias	Rank	Mean - t	Rank	Distr	Rank
1	OLS	-0.195	25	-0.565	25	7.47	25
2	ADL(1.2)	-0.045	18	-0.286	6	0.39	1
3	ADL(1.2.1)	-0.045	18	-0.296	10	0.4	2
4	ADL(4.4)	-0.04	13	-0.29	9	0.44	4
5	AADL(4.4.4)	-0.033	5	-0.296	10	0.43	3
	With Standard - FM						
6	A-FM	-0.121	23	-1.06	23	2.54	2
7	NW-FM	-0.142	24	-1.417	24	2.94	24
8	A-FM-ADL(1.2)	-0.026	3	-0.244	1	1.18	13
9	NW-FM-ADL(1.2)	-0.037	6	-0.279	4	1.01	9
10	A-FM-AADL(1.2.1)	-0.027	4	-0.261	3	1.38	16
11	NW-FM-AADL(1.2.1)	-0.037	6	-0.287	7	0.98	7
12	A-FM-ADL(4.4)	-0.022	1	-0.257	2	1.71	18
13	NW-FM-ADL(4.4)	-0.041	14	-0.324	15	1.21	14
14	A-FM-AADL(4.4.4)	-0.023	2	-0.314	14	2.04	22
15	NW-FM-AADL(4.4.4)	-0.037	6	-0.347	19	1.89	19
	With Prewhitened FM						
16	A-FM	-0.072	22	0.509	22	1.44	17
17	NW-FM	-0.069	21	-0.468	21	1.15	12
18	A-FM-ADL(1.2)	-0.046	20	-0.298	12	0.74	5
19	NW-FM-ADL(1.2)	-0.037	6	-0.279	4	1.01	9
20	A-FM-AADL(1.2.1)	-0.044	17	-0.304	13	0.81	6
21	NW-FM-AADL(1.2.1)	-0.037	6	-0.287	7	0.98	7
22	A-FM-ADL(4.4)	-0.043	16	-0.327	17	1.12	11
23	NW-FM-ADL(4.4)	-0.041	14	-0.324	15	1.22	15
24	A-FM-AADL(4.4.4)	-0.038	12	-0.344	18	1.91	21
25	NW-FM-AADL(4.4.4)	-0.037	6	-0.347	19	1.89	19

	Table 5.3						
		DGP 3					
	Estimator	Bias	Rank	Mean - t	Rank	Distr	Rank
1	OLS	-0.202	25	-4.47	25	9.469	25
2	ADL(1.2)	-0.052	19	-0.431	11	0.93	3
3	ADL(1.2.1)	-0.051	17	-0.443	12	0.93	3
4	ADL(4.4)	-0.045	15	-0.404	6	0.84	2
5	AADL(4.4.4)	-0.036	5	-0.402	5	0.7	1
	With Standard - FM						
6	A-FM	-0.126	23	-1.35	23	3.25	23
7	NW-FM	-0.143	24	-1.778	24	3.67	24
8	A-FM-ADL(1.2)	-0.031	3	-0.67	2	1.49	13
9	NW-FM-ADL(1.2)	-0.044	11	-0.423	7	1.02	7
10	A-FM-AADL(1.2.1)	-0.031	3	-0.385	3	2.155	21
11	NW-FM-AADL(1.2.1)	-0.043	9	-0.426	9	1.24	9
12	A-FM-ADL(4.4)	-0.028	2	-0.351	1	1.9	17
13	NW-FM-ADL(4.4)	-0.044	11	-0.457	13	1.62	14
14	A-FM-AADL(4.4.4)	-0.022	1	-0.39	4	2.74	22
15	NW-FM-AADL(4.4.4)	-0.038	6	-0.472	19	2.1	19
	With Prewhitened FM						
16	A-FM	-0.073	22	-0.678	22	1.74	16
17	NW-FM	-0.063	21	-0.58	21	1.43	12
18	A-FM-ADL(1.2)	-0.054	20	-0.467	17	0.94	5
19	NW-FM-ADL(1.2)	-0.044	11	-0.423	7	1.02	7
20	A-FM-AADL(1.2.1)	-0.051	17	-0.466	16	0.96	6
21	NW-FM-AADL(1.2.1)	-0.043	9	-0.426	9	1.24	9
22	A-FM-ADL(4.4)	-0.047	16	-0.465	15	1.3	11
23	NW-FM-ADL(4.4)	-0.044	11	-0.457	13	1.62	14
24	A-FM-AADL(4.4.4)	-0.039	8	-0.469	18	1.96	18
25	NW-FM-AADL(4.4.4)	-0.038	6	-0.472	19	2.1	19

TABLE 6. Estimation and Monte Carlo results: Monthly data - ARCH effects				
Estimators	$\hat{\theta}$	$H_0 : \theta = 1$	$t_{0.025}$	$t_{0.975}$
OLS	0.370 (0.629)	-21.7	-22.12	-3.35
ADL (1,2)	0.762 (0.239)	-0.99	-3.11	0.51
AADL (1,2,1)	0.883 (0.230)	-0.50	-3.21	0.46
ADL (4,4)	0.712 (0.274)	-1.05	-3.08	0.54
AADL (4,4,4)	1.053 (0.262)	0.20	-3.04	0.59
A-FM	0.532 (0.143)	-3.27	-5.09	0.28
A-FM-ADL (1,2)	0.878 (0.148)	-0.82	-3.46	1.12
A-FM-AADL (1,2,1)	0.946 (0.135)	-0.40	-3.44	1.05
A-FM-ADL (4,4)	0.856 (0.146)	-0.87	-3.42	1.23
A-FM-AADL (4,4,4)	1.042 (0.135)	0.31	-3.13	1.36