

# High Inflation and Processes of Evolutionary Self-

## Organization \*

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**Abstract** — We study some features of the processes that have generated high inflation in Latin-American countries. The statistical evidence shows that these inflationary experiences are fractional brownian noises. Several authors showed that self-organized criticality (SOC) processes can constitute an the best explanation of the origin of such noises, but this hypothesis implies that the underlying structure must remain invariant. We conjecture instead that the economic structures evolve in time being at each stage of their evolution self-organized structures. We find that these ESO (evolutionary self-organized) processes still

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generate fractional brownian noises. Thus, this seems to be a better explanation for the economic phenomenon of high inflation.

**Keywords:** Inflation - fractional brownian noise - self-organized structure - evolution - structural change.

## 1. Introduction

The Latin-American inflation episodes of the last decades showed a remarkable persistence accompanied by periods of extremely high inflation. This makes them interesting objects of study. Despite the fact that those levels of inflation seem byproducts of a past era, they still deserve a careful analysis. In a previous work London, Dabús and Tohmé (1995) presented a hypothesis about the behavior of the notorious Argentinean inflation: that it was the result of a critically self-organized (SOC) process. The main phenomenological property of such a system is that it generates time series behaving like  $1/f$  noises. The hypothesis is that the underlying economic structure that determines the price levels in the economy is in a permanent critical state, such that any shock can generate a non-correlated and rather unbounded inflationary response. We showed data that made this contention plausible. Nevertheless, there are two main problems with this analysis. On one hand, it lacks a solid econometric support. On the other, to accept the SOC hypothesis involves to assume the validity of some properties which have a very low likelihood when interpreted in the context of real economies: SOC processes can generate fluctuations of all sizes in response to uniformly distributed exogenous shocks. If this were true, it would imply that an economy like the Argentinean is prone *at any moment* to generate wild and unexpected bursts of inflation, contrary to the fact that prices remained stable for almost the entire decade of 1990. This is because of another feature of a SOC process, namely that a *single process* generates the entire series of its outcomes. Again, this seems counterintuitive in the case of a real world economy for a period of more than

thirty years since it would mean, if true, that the institutional, political, social and economic rules that determine the behavior of inflation remains invariant during such a long period of time. Therefore, to accept the SOC hypothesis for inflation means to assume that the economy has a strong tendency to generate inflation. Moreover, if this is so, *any* level of inflation, no matter how high, should be expected.

In order to widen the domain of evidence we analyze four Latin-American high inflation experiences: Argentina, Chile, Colombia and Ecuador. The first issue to address is whether the inflation rates behave like  $1/f$  noises, the key signature of SOC processes. We find that this kind of behavior is indeed a common trait of *all* the inflationary economies mentioned before. This may mean that all the inflation of these economies behave like SOC processes, at least with respect to inflation. The common characteristic of being  $1/f$  noises makes the different inflationary experiences very similar in a crucial aspect: the statistical features of their respective time series of observations. However, this seems odd, especially when we consider the remarkable historical, political and economic differences among them.

Hence, even if our database has been expanded, we have to face a hard-to-believe set of conclusions. Moreover, we still lack a strong econometric tool to make sure that this is not just a statistical artifact. In other words, we face the necessity of justifying our methodology of analysis. In this sense, we follow the method of making conjectures and looking for refutations similar to Lakatos (1976). That is, we just make a claim without being “ontologically” committed to it and then we confront that conjecture with data and theoretical arguments.

The alternative hypothesis we will confront to the SOC hypothesis is based on a simple insight: it may be possible that the underlying economic structure that generates inflation consists of a *sequence of SOC processes*, instead of a single one. We call such a process an Evolutionary Self-Organized (ESO) system. In fact,  $1/f$  noises are generated also in

this case, so that the ESO hypothesis is able to explain the same statistical information that can be concluded from the SOC hypothesis. Nonetheless, instead of obeying a single set of behavioral rules, an ESO process goes through an evolutionary process, characterized by sudden (but sporadic) changes of structure. This seems to be a better explanation of the behavior of the economies under study, since these have exhibited a history of erratic and unstable political and economic conditions. In other words, the ESO hypothesis supersedes the SOC conjecture since it explains equally well the same set of data being, at the same time, far less vulnerable, because of its generality, to attacks based on political or institutional criteria.

In this we analyze the four inflationary series under the alternative SOC and ESO hypotheses. We show that both are similar in their main predictions and that those are verified by the Latin-American inflations. The next step in our argument consists in applying a version of Occam's Razor, in order to keep only the less demanding of both hypotheses and drop the other. This procedure lead us to a general explanation of inflationary processes, with less precision and predictive capabilities than any of the known theories of inflation in macroeconomics. This seems to go beyond the limits of economic analysis, and directly into the field of System Theory.

In the next section we make precise the methodological foundations that allow us to support the idea that the high inflation processes in Latin-America have been ESO processes. Section 3 presents a quick refresher of the key notion of  $1/f$  noise and its relation to SOC processes. Section 4 defines and characterizes what we call ESO processes, the core of our alternative hypothesis. In Section 5 we analyze and discuss the price data, the regimes of inflation of each economy and the overall behavior of the inflation series. We look for evidence that may favor either the SOC or ESO hypothesis. Finally, section 6 presents the conclusions. We find the data compatible with both SOC and ESO hypotheses. Since most of the inflationary processes under study have clearly

suffered structural changes, the ESO hypothesis appears to be a better explanation for the statistical properties of the series.

## 2. Methodological Foundations

Economics has long ago adopted a clear and precise method for accepting or rejecting hypothesis. It begins with the formulation of the hypothesis, that should be made quantitatively precise (i.e. it must be a sentence about the magnitude of certain variable). Then, a sample from the population over which the hypothesis is predicated must be provided. The magnitude of the variable under analysis is measured for every element in the sample, so that a sample distribution of its values obtains. Then, the hypothetical value is placed in this distribution. If it lies more than a given number of standard deviations from the sample's average the hypothesis is rejected (in a probabilistic sense). On the other hand, the two traditional ways of making deductive inferences are *modus ponens* (from  $A \rightarrow B$  and  $A$  it follows the validity of  $B$ ) and *modus tollens* (from  $A \rightarrow B$  and  $\neg B$  it follows that  $\neg A$ ). If we make  $A$  the statement "the real value of the variable  $X$  is  $x$ " and  $B$ , "the sample value of  $X$  is  $x$ ", from the rejection of  $B$  follows the rejection of  $A$ , while from the non-rejection of  $B$  does not follow anything about the validity of  $A$ . This explains why the main hypothesis to be tested is the one that the analyst is interested in rejecting.

This discussion shows implicitly that in order to test an hypothesis, we need first a clear statement of the hypothesis (which allows to define also clearly all its complementary statements). Second, a sample sufficiently big is needed to ensure that the sample distribution found approximates (asymptotically) the real distribution of the population. These requirements can be fulfilled in case the hypothesis, as said, is about the magnitude of a variable. But when the hypothesis is of a higher order, say about the real model underlying to a set of observations, this is no longer so clear. In fact, a certain number of crucial assumptions about the distribution of values in the real population, which are

assumed typically in empirical analyses, cannot be either assumed nor verified. Known examples of how very different hypotheses about the distribution of a single population can be all accepted with the same degree of precision, show that this is not a minor problem when working with data [e.g. Tufte (1983)].

In that sense, if different hypotheses about the structure are all verified with the same degree of precision by the same database, then the method based on *modus tollens*, i.e. trying to reject the complement of the hypothesis of interest, is no longer tenable. This is because, unlike the magnitudes of a variable, a set of possible structures does not always can be matched by observable partitional information. To make this clear, consider just an example. The statement “variable  $X$  has value  $\mathbf{x}$ ” induces a partition on the domain of values of the measurable variable  $X$  in two sets:  $\{\mathbf{x}\}$  and  $\{x \in \text{Dom}(X): x \neq \mathbf{x}\}$ . Instead, a more complex statement, like “ $f(x)$  is concave”, on a population where only the first derivative at each point can be evaluated generates a partition on its domain (a functional space  $F$ ):  $\{f \in F : f \text{ is concave}\}$  and  $\{f \in F : f \text{ is not concave}\}$ . The measurability of the first derivative leads to the following partition of the domain of observable variables:  $\{f \in F : f' < 0\}$  and  $\{f \in F : f' > 0\}$ , which is clearly not a refinement nor a coarsening of the partition induced by the hypothesis. Then, just rejecting  $f' < 0$  does not allow to say much about the concavity of  $f$ .

Therefore, the statement of hypotheses involving theoretical entities without a straightforward real world counterpart does not allow the application of any test, except for just concluding that the hypothesis is consistent with the data. This does not mean that the hypothesis is accepted, but in Popper’s (1992) approach it would mean that its *degree of confirmation* has increased. Of course, even this assertion is not quite certain, since a precondition for the definition of degrees of confirmation is the existence of a  $\sigma$ -algebra on the set of possible hypotheses. In the case of the concavity hypothesis of a function  $f$  it is

ensured, since it is trivial that such an algebra can be defined over the functional space  $F$ , but for more elaborated hypothesis this is completely unclear.

In fact, except for a few exceptions, most discussions in economics involve hypotheses that are not *directly* translatable to observational terms. In several areas of economic analysis it is common to base explanations on the behavior of non-observable entities like preferences, beliefs, expectations, goals, etc. Great efforts have been made to measure them. But, since they are all prone to strategic manipulation the possibility of testing hypotheses requires relying on the validity of approximations to measurable variables. Even so, empirical studies are performed and their conclusions are used for evaluating and designing policies.

A possible justification for doing so, is based on the following points:

- Any first-order sentence  $\Phi$ , involving theoretical concepts, can be translated into a proposition  $\Phi'$  involving only observational terms [see Craig (1953)].
- Since  $\Phi'$  is also a first-order statement, it can be decomposed as the conjunction of binary and unary terms (i.e. as the conjunction of predicates involving either one or two observable variables) [see Quine (1954)].
- Since the binary and unary terms are observable, they must be testable, then  $\Phi'$  and indirectly  $\Phi$  are also testable (the translation from the theoretical sentence to the observable is not in general one-to-one).
- If the tests indicate that  $\Phi'$  cannot be rejected,  $\Phi$  is accepted until a better hypothesis is proposed.

That is, all the non-rejected hypotheses must be deemed as temporarily accepted. On the other hand, if  $\Phi \rightarrow \Lambda$  and  $\Lambda$  is such that the observational  $\Lambda'$  is identical to  $\Phi'$ , the accepted hypothesis becomes  $\Lambda$ . This is because from the two hypotheses, the more

general one is preferable, since it is the least constrained. That is, it requires less extra assumptions while providing the same set of observable conclusions.

But, even if the previous description already characterizes what we will do in the next sections, there is an additional point that requires clarification. Assume that an hypothesis  $\Phi$  is temporarily accepted. Despite this, a way of rejecting it is to argue that even as a theoretical construction  $\Phi$  is not feasible because  $\Phi \wedge \Gamma \rightarrow \perp$ , where  $\Gamma$  is another accepted hypothesis. That is, both hypotheses cannot be true at the same time because their conjunction leads to inconsistencies. Then, one of them must be dropped. If there exists a plausibility ordering of them, i.e.  $\Phi \angle \Gamma$ , meaning that  $\Gamma$  is more plausible than  $\Phi$ , it is clear that  $\Phi$  must be dropped. This ordering  $\angle$  may be purely subjective, based on the appraisal of the specialist or it can be objective, say because  $\Gamma$  is a more precise claim that has already been tested. For example, suppose that  $\Phi$  is “f is concave”, and  $\Phi'$  is “f' > 0”. Then, if  $\Phi'$  is accepted, we assume temporarily  $\Phi$ . But then, if  $\Gamma$  is “f' > 0”, and has been accepted, we have to keep  $\Gamma$  and drop  $\Phi$ . Of course, what happened here is that the translation from  $\Phi$  to  $\Phi'$  was poor and so the acceptance of the later should not imply that of the former.

As Rescher (1976) reported, no matter how a plausibility ordering arises, it should be linear since it must allow the analyst to decide over every pair of incompatible hypotheses. So, once decided which one to drop, a replacement for it must be provided. Although there does not exist a mechanical procedure that provides the alternative, some guidelines are rather obvious. The new hypothesis  $\Theta$  should be such that  $\Gamma \rightarrow \Theta$ , i.e. it must be implied by the validity of the defeating hypothesis, while at the same time,  $\Phi' \rightarrow \Theta'$ . That is, its observational consequences should be implied by those that have been already tested. In our example,  $\Theta$  could be “f is increasing for  $x \in \text{Dom}(f)$ ,  $x \geq x'$ ”, since it implies that  $f' > 0$  for an upper segment of its domain (in particular for the non-negative segment). In fact, this

shows that an extra-logical requirement is the *minimality* of  $\Theta$ . This means that from all the elements that verify the desired conditions we should choose the one that “differs the less” from  $\Phi$ . This is certainly a very tricky point, but it means that  $\Phi \rightarrow \Theta$ . That is, that  $\Phi$  is a particular instance of  $\Theta$ .

In the next sections we present a discussion about two possible hypotheses that could explain the same statistical evidence for the inflation processes in some Latin-American countries. Our argument for accepting one of them and dropping the other is based on the methodological principles presented here. In these terms we confront two candidate hypotheses,  $\Phi$  and  $\Theta$ .  $\Phi$  represents that the inflation processes are SOC, while  $\Theta$  represents that they are ESO processes. As said, there exist reasons to think that the SOC hypothesis is inconsistent with the fact that all these inflation processes underwent structural changes, which will be represent by  $\Gamma$ . Moreover, these reasons indicate that  $\Gamma$  is more plausible than  $\Phi$ , because the SOC hypothesis implies that a single process explains the entire series.

### **3. SOC Processes**

Given a time series  $\{a_t\}_{t=0\dots T}$ , two functions, derived from the series provide information about the structure of the process that generates its values :

- $S_x$  : spectral density of the series. To each frequency it associates the mean of the corresponding Fourier coefficients.
- $P_x$  : sample density of the series.

$S_x$  indicates the degree of dependence among values of the series.  $P_x$  approximates the theoretical density function of the generating mechanism. A taxonomy of possible cases is the following:

- $S_x \approx \lambda^0$  (where  $\lambda \in [0, 1/T]$  is the frequency) the series is a `white noise`
- $S_x \approx \lambda^{-2}$  it is a `brownian noise`
- $S_x \approx \lambda^{-\alpha}$  with  $0 < \alpha < 2$ , it is a `1/f noise`

1/f noise is the most interesting, because it is an intermediate case which covers the widest variety of situations. A white noise is a series in which observations are non-correlated. A brownian noise, instead, is a series in which observations are highly correlated. A 1/f noise shows a certain correlation among data, but it is diminished by the effect of random movements.

Fractal behavior in time series can be evidenced by the properties of  $P_x$ , if it has a variance which changes with the size of the sample (`infinite variance property`). Among the distributions that verify this property, the most representative is the Pareto-Levy distribution, where

$$P_x \approx |x|^{-\beta} \text{ with } 0 < \beta < 2$$

On the other hand, according to Casti (1995), additional evidence for 1/f noise and fractal behavior in a time series can be detected by means of the Hurst coefficient. Given the average of the series  $\mathbf{a} = \sum_{i=0}^T (a_i/T)$  the minimal and maximal “accumulated flux” of the series are, respectively:

- $\mathbf{min} = \min_{t=0..T} (\sum_{i=0}^t (a_i - \mathbf{a})/t)$
- $\mathbf{max} = \max_{t=0..T} (\sum_{i=0}^t (a_i - \mathbf{a})/t)$

A new variable  $R_T$  is defined as  $\max - \min$ . Finally, given the standard deviation

$$S_T = (\sum_{i=0}^T (a_i - a)^2 / T)^{1/2}$$

Hurst's coefficient  $H$  is

$$H = \log (R_T / S_T) / \log T$$

This number provides another form to determine the degree of correlation among data in the time series. In particular,  $H > 0.5$  indicates that an increase of magnitude in an earlier stage implies an increase at later stages.

The fact is that there exists a great number of processes in nature and society that show  $1/f$ , fractal and  $H \neq 0.5$  behavior, which we characterize as *fractional brownian noises*, by an abuse of language [see Mandelbrot and Van Ness (1968) and Voss (1988)]. Attempts to explain these phenomena have been highly ad-hoc. The first systematic foundation proposed as an explanation is the existence of Self-Organized Criticality (SOC). The idea is that fractional brownian noises are generated by underlying complex processes, such that the magnitude of the connections among components of the support system can be affected by exogenous shocks that modify its structure. The result is that any such system goes from one meta-stable configuration to another. In the case that the fluctuations are non-correlated with the magnitude of the shocks, it seems natural to assume that they have been endogenously generated by the system [see Bak et al. (1993) and Scheinkman and Woodford (1994)]. A clear representation that captures this property is given by a *sandpile*, in which a constant amount of sand is added, and there exists a critical value for the slope of the pile. Once the sandpile attains that critical state, avalanches of all sizes can be expected, even some of a size far greater than the amount of sand added. This is because, at the critical slope, any overflow due to local configurations can generate a cascade [see Bak and Chen (1991)].

A property of these processes is that the generated series are  $1/f$  noises, the density functions approximate the Pareto-Levy distribution and  $H \neq 0.5$ . SOC is the most systematic and reliable model explaining those behaviors. Thus, if evidence is given in form of series behaving like fractional brownian noises, the existence of an underlying self-organized critical system generating the series cannot be discarded. According to Krugman (1996), this does not mean that no other explanation may exist or that a general hypothesis (instead of a particular representation for each phenomenon) is a theoretical necessity. It just means that the SOC hypothesis is not implausible. No statistical test exists to detect its presence since there is no well-defined class of reference for processes generating fractional brownian noise.

#### 4. ESO Processes

The shortcomings of SOC processes, in order to explain phenomena that develop over long periods, are all related to their static features: the dimension of the underlying structures and the systems of rules must remain unchanged. Moreover, the size of the shocks does not affect those parameters. But it is relatively easy to relax those stringent conditions and create a model in which shocks are able to transform the very nature of the system. London and Tohmé (1998) present an alternative:

**Definition:** an ESO system,  $S$ , is  $E=(T, R, M)$ , where  $T=(T_0, T_1, \dots)$  is a sequence of topologies,  $R=(R_0, R_1, \dots)$  is a sequence of local uniform rules and  $M$  is a set of meta-rules. A topology  $T_i$  is  $T_i=(S_i, C_i)$ , where  $S_i$  is a finite set of sites and  $C_i$  a regular structure of connections among them.

The notion of topology here implies the existence of neighborhoods for sites. For example, an  $N$ -sites complete graph is a topology where the neighborhood of a site

consists of the entire set of nodes. Instead, if we consider a 2-dimensional orthogonal arrangement of sites (a "chessboard"), the neighborhood of a site consists of the set of eight adjacent sites.

Each  $s$  in  $S_i$  can be in a state  $|s|$ , a numerical value in a discrete interval  $\{0,1,\dots,n_i\}$ . Given a site  $s$  and the values in the neighborhood in time  $t$ ,  $|N_s|_t$ , the value of the site at  $t+1$  is the result of the application of the rule  $R_i$ :  $|s|_{t+1}=R_i(|N_s|_t, |s|_t)$ . That is, the state of the site depends on the previous state of the neighborhood and the state of site. We add the requirement that each  $R_i$  must be a non-linear function, to avoid the trivial case in which the asymptotic result consists of all sites having the same (maximum) value.

A shock is a random variation of the state of one or more sites. Assume that site  $s$  is in a state  $|s|$  and the shock is  $D_s$ . The result of the shock depends on the rule  $R_i$ . In the case of a sandpile, any excess over  $n_i$  is transferred to the neighborhood. In the Game of Life, for example, the shock generates a new state with values in the legal interval. Instead, if potential variations are not limited they can change the structure of the system. Variations in excess over the admissible value of a site can provoke that the meta-rules transform both the topology and the set of rules.

A shock is applied in time  $t$ , and it propagates until it settles down to a definite pattern. This pattern may be periodic or non-periodic. In any case, the result is recorded, and in time  $t+1$  a new shock is applied to the system.

If shocks in time  $t$  are within the "stable" range, i.e.  $|s| + D_s < n_i$  then  $T_{t+1} = T_t$ , and  $R_{t+1} = R_t$ . If not, any meta-rule  $M_k$  is selected at random, and  $(T_{t+1}, R_{t+1}) = M_k(T_t, R_t, D_s - n_i)$ . That is, the meta-rule generates a new topology and a new set of rules, depending on the previous structure and the magnitude of the shock over the limit value. We add a rule of parsimonious behavior:  $S_t$  and  $S_{t+1}$  cannot be disjoint (which provides continuity to the history of the system).

According to Bak et al. (1989), if shocks are always within their normal range, the behavior of the system is analogous to the Game of Life, a cellular automaton that constitutes a paradigmatic example of a SOC system. The non-linearity of R provides for a version of the “birth-reproduction-death” rule of the Game of Life, and therefore, the propagation of shocks generates  $1/f$  noise, fractal structures and universal computability [see Berlekamp et al. (1982)]:

**Proposition 1** : if for every  $t$ ,  $|s| + D_s < n_i$ , then  $E$  is a SOC system.

**Proof:** if for every  $t$   $|s| + D_s < n_i$ , then  $T_{t+1} = T_t$  and  $R_{t+1} = R_t$ . That means that  $E$  can be represented as  $E(T_0, R_0)$  since the initial topology and rules are permanent. The set of meta-rules  $M$  is irrelevant, since these rules are never activated.

As  $R_0$  is non-linear,  $E$  is an extension of the Game of Life. It is equivalent to a sandpile model: each site's state depends non-linearly on the state of its neighbors and the range of values is discrete, with more than two possible values. Therefore, as a sandpile is a SOC system it follows that  $E$  is also a SOC.

When shocks are not in range, the whole system "mutates", adding new sites, new connections or new rules. The overall behavior of the system is no longer easy to describe, but we have this immediate result that characterizes an ESO system as a sequence of SOCs:

**Theorem 1:** the sequence of average values of sites is a  $1/f$  noise.

**Proof :** First, let us show that for any site  $s$ , the sequence of its values in time,  $\mathbf{s}=(|s|_0, |s|_1, \dots)$  must be a  $1/f$  noise. The Fourier transform  $\mathcal{F}(\mathbf{s})$  of the time series of this sequence, which gives the **spectral density** of the time series, must be, for the SOC stage  $T$ ,

$F(\mathbf{s}) \approx \lambda^{-\alpha^T}$  with  $0 < \alpha^T < 2$ , where  $\lambda \in [0, \infty]$  is the frequency, while it must be  $F(\mathbf{s}) \approx \lambda^0$  or  $F(\mathbf{s}) \approx \lambda^{-2}$  for the transients between SOC stages. Since  $F(\cdot)$  is a linear operator the transform for the entire sequence must be the average of all the stages. Then, we have that  $0 < -\text{Log}(F(\mathbf{s}^*)) / \text{Log } \lambda < 2$  (because an average always falls between the extreme values). That is,  $\mathbf{s}$  is a 1/f noise. Then, for the sequence of average values over sites,  $\mathbf{s}^* = (s^*_0, s^*_1, \dots)$ ,  $F(\mathbf{s}^*)$  is the average of the transforms of the individual sites. Call  $\alpha_h$  the highest value of the  $\alpha$  parameter (for site  $i_h$ ) while  $\alpha_l$  is the lowest value among all the sites. Then  $\alpha_l < -\text{Log}(F(\mathbf{s}^*)) / \text{Log } \lambda < \alpha_h$ . This means that  $F(\mathbf{s}^*)$  cannot behave as a power law with a parameter outside the interval  $[\alpha_l, \alpha_h] \subset [0, 2]$ . I.e. it is, in average, a 1/f noise.

We can consider a "degree of homogeneity" represented for each  $t$  and the corresponding  $s^* = \text{average}(|s|)$  by the standard deviation of values  $\sigma(s_t)$ , or  $\sigma_t$  in short. This measure allows us to understand the scope of 1/f noisiness in the behavior of an ESO system:

**Proposition 2** : the sequence  $(\sigma_0, \sigma_1, \dots)$  is, in average, a 1/f noise.

**Proof** : the standard deviation  $\sigma$  is such that  $-2s^* < \sigma < 2s^*$  and according to Theorem 1 the average  $\mathbf{s}^*$  follows a power law. Any power law is preserved under a multiplication by a constant, therefore the sequence of standard deviations,  $(\sigma_0, \sigma_1, \dots)$  must be also, in average, a 1/f noise.

Similar arguments can be presented to show that an ESO system must exhibit, in average, a spatial distribution that approximates a Pareto-Levy distribution. The identification of regions of meta-stability in the history of an ESO (a sequence of SOC processes, according to Theorem 1) provides regions of robustness for the relevant

parameters. In particular for the signature parameter  $\phi$ , such that if  $\omega$  represents the spectral weight and  $\lambda$  the frequency  $\omega \approx 1/\lambda^\phi$ . If so, the identification of meta-stable stages in the history of a ESO system is equivalent to the detection of stages of robustness (i.e. periods with little change) for the  $\phi$  parameter. Besides, an interesting consequence of assuming that an economic system can be represented as an ESO system is the following:

**Theorem 2:** an ESO system may never reach a stationary state.

**Proof:** *trivial. Since the sequence of standard deviations  $(\sigma_0, \sigma_1, \dots)$  is a  $1/f$  noise, there may not exist (except for a degenerate case) a  $t^*$  such that for each  $t > t^*$ ,  $h_{t+1} = h_t$ , even if the number of sites remains fixed.*

In short, an ESO system never settles down in a specific pattern of diversity. That is, there may exist a permanent variation of the set of rules.

## 5. Evidence from Inflationary Series

The inflationary phenomenon is generally a consequence of economic instability. Its behavior changes with the level of inflation. Situations of more instability show a more erratic evolution of the inflation rate, its effects being in turn stronger on the inner workings of the economy. The performance of the economy becomes affected by that kind of processes.

To describe the actual behavior of inflation we use a version of the Leijonhufvud's (1990) taxonomy of *regimes*. Each regime represents an economic context characterized by an inflation rate fluctuating in a certain range of values, and to which a system of

expectations and a degree of uncertainty are associated. According to Leijonhufvud (1981) and Heymann and Leijonhufvud (1995), these features are related to the degree of coherence of expectations. The concept of inflationary regime represents, therefore, how economic agents respond to both the available information and to unexpected shocks. When the inflation is higher, the individual expectations are not uniform, leading to a diversity of decisions, and then to a dispersion of values. The four regimes we consider here are the following:

- moderate inflation (less than 2% of monthly inflation)
- high inflation (between 2% and 10% )
- very high inflation (between 10% and 50 %)
- hyperinflation (over 50 %).

The definition of inflationary regime, therefore, can be stated in the framework of SOC and ESO processes, given that each regime may correspond to a pattern of connections in the economy. This pattern is a result of the previous performance of the system and also of exogenous shocks. Each regime corresponds to a state of the system, and each one generates inflation episodes of different magnitude, according to the past experience and the shocks of economic policy.

In order to see if the series behave like fractional brownian noises, we analyze the inflation of four Latin-American countries: Argentina, Chile, Colombia and Ecuador. For the four countries we use inflation series derived from the Consumer Price Index (CPI).

The periods are as follows:

- Argentina : January 1957-December 1993
- Chile : January 1934-September 1995
- Colombia : January 1954-December 1992
- Ecuador : January 1960-June 1989.

These four countries exhibit different inflationary processes (see Figures 1, 2, 3 and 4 in the Appendix). Argentina experienced moderate inflation during the 1960's, high and very high inflation in the 70's and 80's, with hyperinflationary episodes in 1989 and 1990. Since 1991 the country returned to the regime of moderate inflation. In turn, Chile experienced moderate and high inflation during almost all the period, with very high inflation episodes in the 70's. Colombia showed a pattern of chronic moderate inflation, while Ecuador had a bounded inflation, with several episodes of deflation.

Despite the differences among these inflationary histories, our results are quite homogeneous. Table 1 shows the values of  $S_x$ ,  $P_x$  and Hurst's coefficient for the four countries. In the case of the spectral density, it can be approximated by the function  $\lambda^\alpha$  with the following values of  $\alpha$  for each country (see figures 5, 7, 9 and 11 in the Appendix). This shows that even if all four series fall in the category of  $1/f$  noises, there are some differences. Chile, for example, seems closer to a white noise, while Argentina reveals a heavier participation of low frequencies in the explanation of the series. This means that long range correlation is more important in this case than they are for the other countries. Nonetheless, this conclusion is a bit deceptive, since it is an artifact of the U shape of the spectral density. This means that both low frequencies (i.e. trends) and very high frequencies (i.e. shocks) are relevant as determinants of the behavior of the series.

In the case of the sample density, it can be approximated by the function  $x^\beta$  (see Figures 6, 8, 10 and 12 in the Appendix), being typically close to a power law, with small differences among the countries. Finally, for the Hurst's coefficients the result is, again, that all the countries show long range correlation, all positive. In other words, an increase (decrease) of the inflation level in a moment leads to an increase (decrease) in its level in the future. In brief, all the evidence suggests that inflation in the four countries is a fractional brownian noise.

**Table 1: Coefficients of Spectral Density ( $\lambda^\alpha$ ), Sample Density ( $\chi^\beta$ ) and Hurst (H)**

Countries/Coefficients	$\lambda^\alpha$	$\chi^\beta$	H
Argentina	-0.19	-1.5	0.74
Chile	-0.09	-1.1	0.73
Colombia	-0.15	-0.7	0.76
Ecuador	-0.14	-1.2	0.76

The causes of fractional brownian noises can be understood in terms of two kinds of processes presented in Sections 3 and 4: SOC and ESO. The latter are the more general and encompass the former as particular cases. If inflation is generated by a SOC system, the existence of meta-stable states (inflationary regimes) may be a warning. Hence, the possibility of very high and hyperinflation would be *always* present, although with a very low probability.

However, the SOC hypothesis seems to be hard to accept: the bottom line of this assumption is that inflation is a result of a process that remains unaltered during the entire period of analysis. In other words, the structure of the economy must be assumed invariant and the evolution of inflation must be explained by its interaction with non-correlated shocks. Despite the fact that certain underlying mechanisms may remain “constant”, the variations of policy affecting inflation could not be considered as uniformly distributed external shocks. To incorporate the changes of economic policies (for example the application of different stabilization programs), one should consider the possibility that they may generate drastic structural changes. In this sense, while ESO and SOC processes exhibit the same statistical properties, the former allows to consider the changes of

structure that are discarded in the SOC processes while preserving the self-organized nature of the phenomena. The evidence is consistent with such hypotheses, but economic intuition indicates that the first one should be chosen: it is more natural to assume that the social, political and economic rules operating in an economy evolve in time. In particular, the Latin-American economies under study have suffered political, social and economical shocks that changed suddenly their structures, or at least their rules of operation. For example, Dabús (2000) reports evidence of significant structural change in the relation between inflation and relative price variability for Argentina.

In terms of the characterizations presented in Section 2, we have the following statements:

- $\Phi$  = “ The inflationary process is a SOC”
- $\Phi'$  = “ The inflation series is a fractional brownian noise”
- $\Theta$  = “ The inflationary process is an ESO”
- $\Theta'$  = “ The inflation series is a fractional brownian noise”
- $\Gamma$  = “ The inflationary process underwent structural changes”

We argued that  $\Phi \not\subset \Gamma$ , i.e. that the SOC hypothesis seems incompatible with the idea that the inflationary process suffered structural changes. This is supported both by evidence and by economic intuition. On the other hand, by definition we have that  $\Phi \rightarrow \Theta$ ,  $\Gamma \rightarrow \Theta$ ,. Finally, the evidence provided by the inflationary series indicates that  $\Phi' = \Theta'$  (a particular instance of  $\Phi' \rightarrow \Theta'$ ). In other words, the ESO hypothesis must be favored instead of the SOC one.

A finer analysis would detect the stages of meta-stability of the series (which indicate the presence of different SOC components in a single ESO process). The long range features of the series are the result of the overlapped action of different constituent economic structures.

## 6. Conclusions

This paper shows two results, one strong and the other weaker. The strong result is that several, rather different, Latin-American economies show inflation processes that behave like fractional brownian noises. In other words, all of them exhibit power laws in the domain of values (revealed by the sample density), in the frequencies domain (seen in the spectral density) and in the domain of “fluxes” (shown by H, Hurst’ s coefficient).

The weak result is that ESO processes could provide an explanation for the emergence of those fractional brownian noises. Moreover, they allow a natural explanation in terms of periods of relevant analysis. If economies in Latin America are structures in evolution, with sudden changes and jumps, they could be explained to an explanation in terms of ESO processes.

A natural step in this analysis is to detect the periods of meta-stability in the time series, each one associated to a SOC system. If our hypothesis is close to be true, those periods, should have endpoints characterized by sudden social, political or economic changes. Otherwise we have to look for other explanations.

The inflation rates of other inflationary economies could also constitute  $1/f$  noises. An interesting case must be the European episodes of hyperinflation, which were followed by decades of price stability. It would be possible to distinguish periods of meta-stability in terms of both the inflation rate and the economic structure. Then, if a strong correlation could be found between them (say using dummy variables for the structure) support for the hypothesis of ESO behavior would receive a strong empirical support.

## Appendix

### Evolution of the Monthly Inflation Rate (CPI)

Figure I: Inflation rate in Argentina, January 1957-December 1993  
High Inflation and Processes of Evolutionary Self-Organization  
Fernando Tohmé, Carlos Dabús and Silvia London

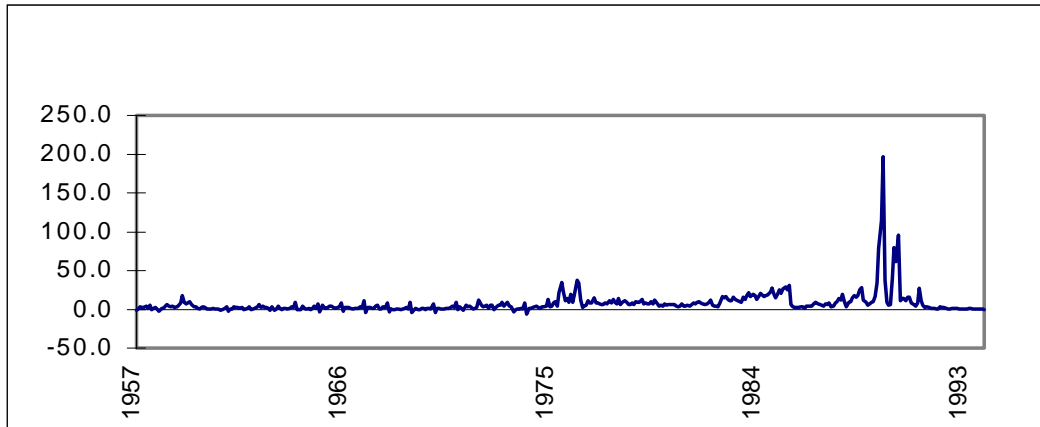


Figure II: Inflation rate in Chile, January 1934-September 1995

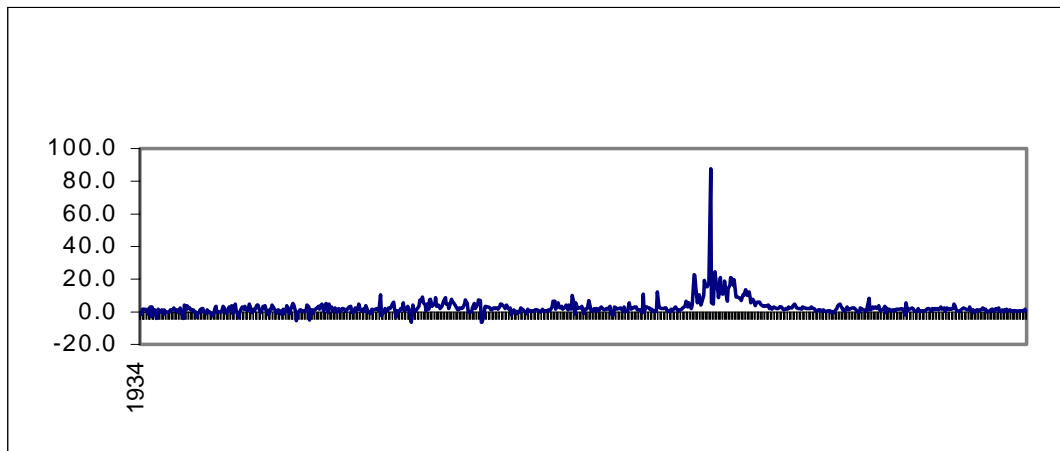


Figure III: Inflation rate in Colombia, January 1954-December 1992

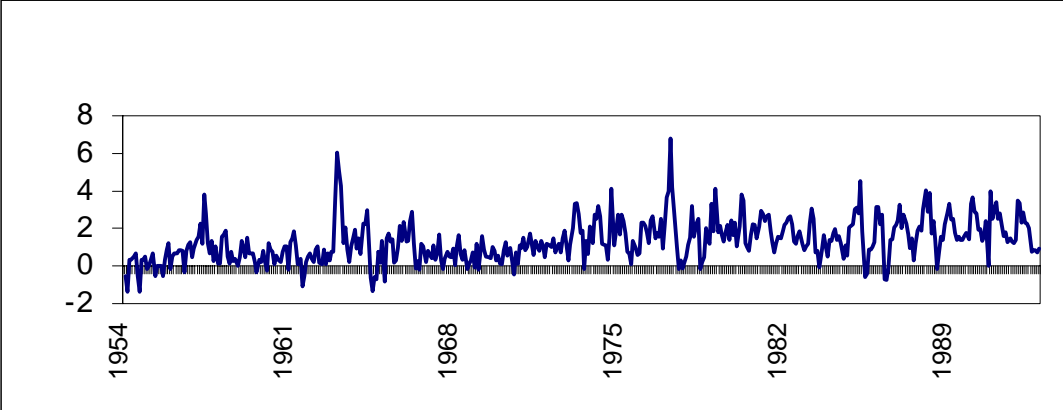
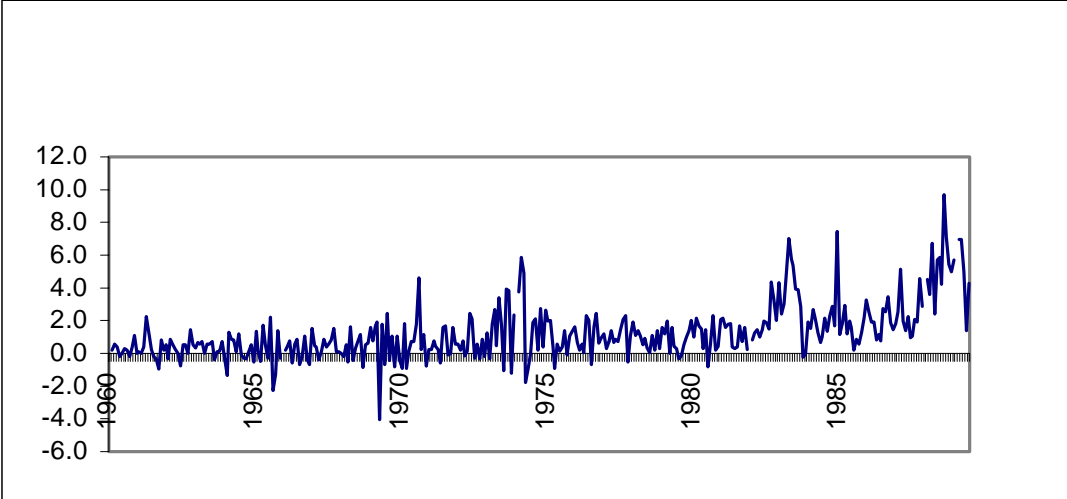


Figure IV: Inflation rate in Ecuador, January 1960-June 1989



## Spectral and Sample Density

Figure 5: Spectral Density of Argentinean Inflation, January 1957-November 1993

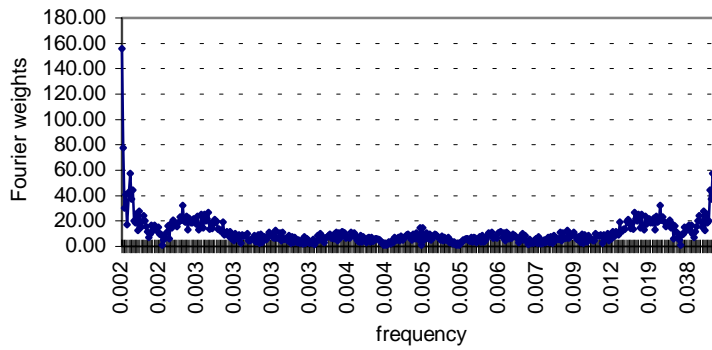


Figure 6: Sample Density of Argentinean Inflation, January 1957-November 1993

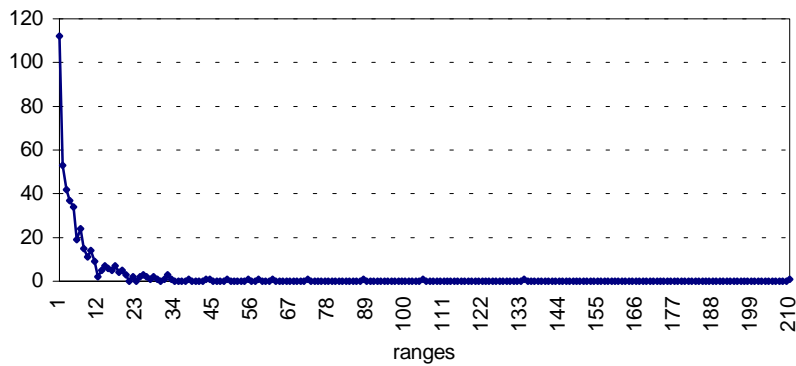


Figure 7: Spectral Density of Colombian Inflation, January 1954-December 1992

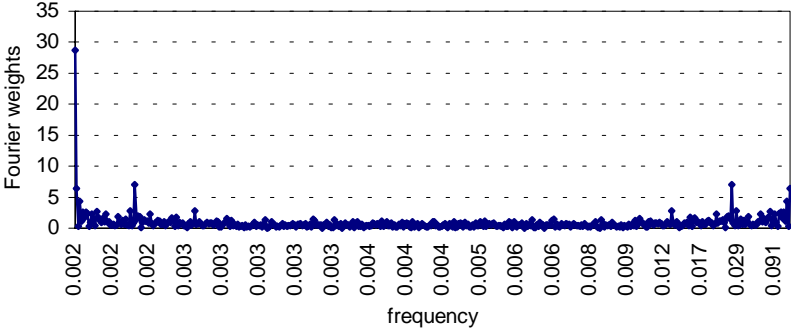


Figure 8: Sample Density of Colombian Inflation, January 1954-December 1992

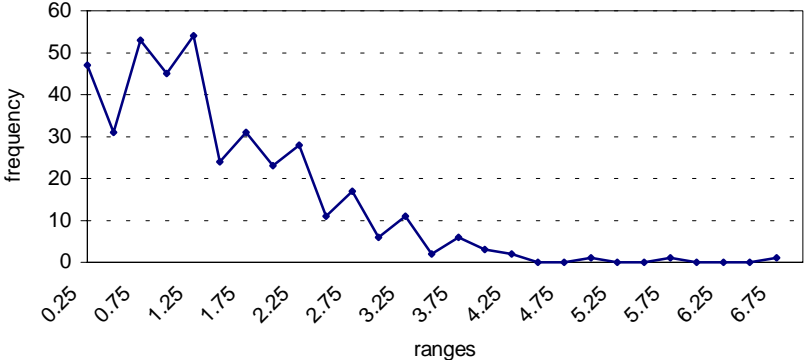


Figure 9: Spectral Density of Chilean Inflation, January 1934-September 1995

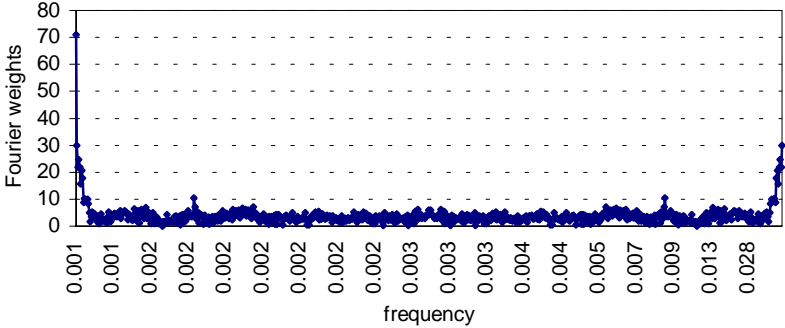


Figure 10: Sample Density of Chilean Inflation, January 1934-September 1995

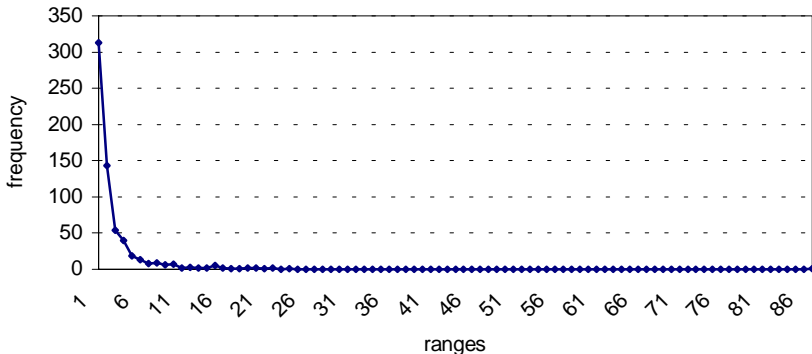


Figure 11: Spectral Density of Ecuadorian Inflation, January 1960-June 1989

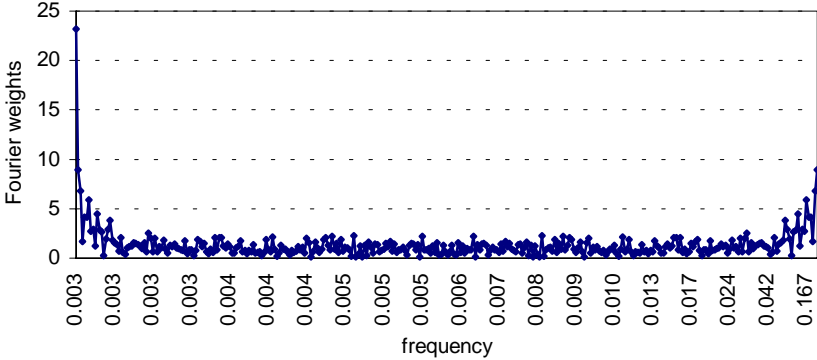
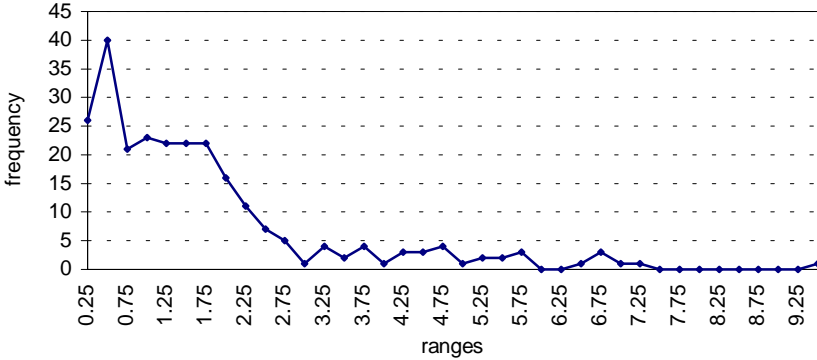


Figure 12: Sample Density of Ecuadorian Inflation, January 1960-June 1989



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