Hyperinflation: Inflation Tax and the Economic Policy Regime

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This paper addresses the issue of money essentiality. It shows that Obstfeld-Rogoff infeasibility condition is an empirical question, rather than a theoretical issue, and should be tested by using inflation tax data from hyperinflation experiments. Furthermore, the paper shows that this infeasibility condition is very much feasible, because this is a necessary condition for the occurrence of hyperinflation in a fiscal regime, where money is used to finance a fiscal deficit.

1. Introduction

Hyperinflation is a monetary disease through which the value of money is destroyed by society. Economic theory tries to explain this phenomenon in three different ways. In the first, the model yields a hyperinflation steady state equilibrium in which the real quantity of money is equal to zero and the price level is infinite. In the second way, hyperinflation is a bubble, the actual real cash balance goes to zero but it is different from the real quantity of money given by the fundamentals of the model.[1] The third way economic theory has explained hyperinflation hypothesizes that the origin of the problem is in the production of the money asset, since the stock of money increases at an increasing pace because the financing of the public deficit requires printing money. Monetary decontrol, caused by the fiscal situation of the government, ends up destroying the currency, which, in the end of the process, loses its value completely. The model has no equilibrium because the public deficit can not be financed permanently by issuing money.

This paper addresses these issues by using Brock's (1975) model; based on it I will present, in the next section, an argument which casts doubt on the conclusion reached by Obstfeld and Rogoff (1983), henceforth called O-R, based on the same theoretical framework. According to O-R, speculative hyperinflation in the first sense above -under a pure fiat money regime-can be ruled out only when severe restrictions are placed on individual preferences, e.g., agents must have infinitely negative utility when their real balances are zero. We argue that this restriction can be tested with data from hyperinflation experiments by looking at the behavior of the inflation tax, as real balances approaches zero. Furthermore, the severe restrictions stressed by O-R are necessary for the occurrence of hyperinflation in a fiscal regime, where money is issued to finance a fiscal deficit, the monetary-policy rule that has been observed in countries which have gone through the experience of hyperinflation. Our conclusion based on the theoretical framework presented

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[1] Cagan's(1956) seminal work has shown that hyperinflation could occur as a bubble. If the product of two parameters of his model, the price semi-elasticity (a) and the adaptive coefficient (b), is less than one there is a possi bilty of a bubble hyperinflation in the fiscal regime. According to Cagan[opus cit, p.79] "... the condition that $a\ b < 1$found to hold for all hyperinflations."

is that money is essential, otherwise hyperinflation would not occur; however, this hypotheses remains to be tested.

This paper is organized as follows: Section 2 reexamines Brock's model under both a monetary and a fiscal regime and it shows that O-R infeasibility conditions are indeed very much feasible; Section 3 presents the same model of the previous section with continuous variables, and it shows that in this set up the conclusions reached by Buiter (1987) with respect to some surprising monetarist arithmetic will not hold; Section 4 concludes the paper.

2. Hyperinflation and the price-elasticity of the demand for money

The representative agent maximizes the present discounted value of his utility stream, according to,

$$U = \sum_{t=0}^{\infty} b^t [u(c_t) + v(m_t)] \quad , \quad b < 1$$

where c is the consumption level, m is the real cash balance, and b is the discount rate. The functions $u(\ )$ and $v(\ )$ are increasing in their arguments, strictly concave, and they obey the Inada conditions.

The representative agent budget constraint is given by:

$$y_t + h_t = c_t + \frac{M_t - M_{t-1}}{P_t}$$

where y denotes consumer's real income, h units of the consumption good are received by each agent at the beginning of each period as government transfers, M is the nominal stock of money, and P is the price level, and the initial stock of money M_{-1} is given.

The first order conditions to solve this problem yields the following Euler equation:

$$\frac{1}{P_t} u'(c_t) = \frac{1}{P_t} v'(m_t) + \frac{b}{P_{t+1}} u'(c_{t+1})$$

and the optimum solution has to obey the transversality condition:

$$\lim_{t \to \infty} b^t u'(c_t) m_t = 0$$

The stock of money expands proportionately with factor m:

$$M_s - M_{s-1} = m M_{s-1} , \quad s = 1, 2, 3, \ldots$$

The government transfers to the consumers the resources obtained by the printing of money. Thus:

$$h_s = \frac{M_s - M_{s-1}}{P_s} \quad , \; s = 1, 2, 3, \ldots$$

There is no production sector in this economy and the amount of perishable output by unit of time is equal to y. Equilibrium in the market for goods and services requires that demand for goods and services are equal to supply:

$$c_t = y \quad , \; t = 1, 2, 3, \ldots$$

The model solution is obtained by multiplying both sides of the Euler equation by the stock of money at period t, and by taking into account the behavior of the government and the equilibrium in the market for goods and services. The result is the following non-linear first order difference equation:

$$m_t \left[u'(c) - v'(m_t) \right] = \frac{b \; u'(c)}{1+m} \, m_{t+1}$$

This equation can be analyzed by using the auxiliary equations $A(m)$ and $B(m)$, introduced by Brock(1975). That is:

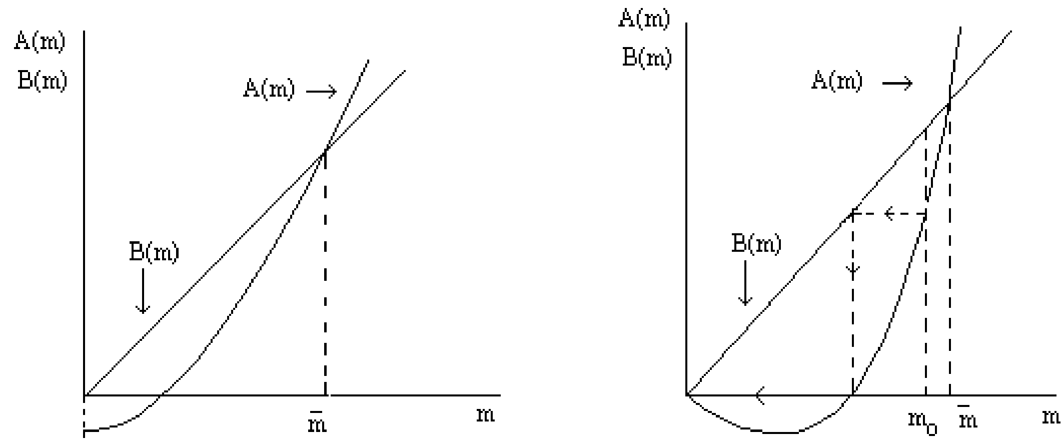
$$A(m) = m \left[u'(c) - v'(m) \right]$$

$$B(m) = \frac{b \; u'(c)}{1+m} \, m$$

Thus, the finite difference equation can be written as:

$$A(m_t) = B(m_{t+1})$$

Figure 1 shows the steady state equilibrium of the model in two situations. Firstly, (Figure 1a), when $\lim_{m \to 0} m v'(m) > 0$, the model has only one steady state equilibrium and there is no chance of occurring a process of hyperinflation. Secondly, (Figure 1b), when $\lim_{m \to 0} m v'(m) = 0$, the model has two steady state equilibrium points, one of which corresponds to a real cash balance equal to zero. Therefore, in this case there is a hyperinflation path, such as the one drawn in the Figure 1b, which can be better understood with some algebra.



a) $\lim_{m\to 0} m\, v'(m) > 0$ b) $\lim_{m\to 0} m\, v'(m) = 0$

Figure 1

By multiplying both sides of the Euler equation by M_t we get:

$$\frac{M_t}{P_t} u'(y) = \frac{M_t}{P_t} v'(m_t) + b \quad \frac{M_{t+1}}{P_{t+1}} \frac{M_t}{M_{t+1}} u'(y)$$

or:

$$m_t \; u'(y) = m_t \; v'(m_t) + \frac{b}{1+m} \; m_{t+1} \; u'(y)$$

where it was taking into account the fact that in equilibrium $u'(c_t) = u'(c_{t+1}) = u'(y)$. This equation can be written forward one period to obtain:

$$m_{t+1} \; u'(y) = m_{t+1} \; v'(m_{t+1}) + \frac{b}{1+m} \; m_{t+2} \; u'(y)$$

Thus, the usual iterated substitution yields,

$$m_t = \sum_{s=t}^{t+T} \left(\frac{b}{1+m} \right)^{s-t} \frac{m_s \; v'(m_s)}{u'(y)} + \left(\frac{b}{1+m} \right)^{t+T+1} m_{t+T+1} \; u'(y)$$

When the transversality condition is met, the real stock of money at period *t* is equal to the present value of the flow of money services:

$$m_t = \sum_{s=t}^{\infty} \left(\frac{b}{1+m} \right)^{s-t} \frac{m_s v'(m_s)}{u'(y)}$$

It is easy to conclude from this expression that there is a steady state equilibrium at the point m=0, when $\lim_{m\to 0} m\, v'(m) = 0$. Let us assume that during period *T* the price level becomes infinite, real cash balance is equal to zero, and according to the fundamentals of the model the real stock of money should be equal to zero*(m_T=0)* because the discounted value of the cash flow of money services is equal to zero. When $\lim_{m\to 0} m\, v'(m) > 0$, real cash balance is equal to zero at the period *T* and the price level *(P)* is infinite, but the real stock of money according to the fundamentals of the model is positive:

$$m_T = \sum_{S=T}^{\infty} \left(\frac{b}{1+m}\right)^{s-t} \frac{m_s\ v'(m_s)}{u'(y)} > 0$$

Therefore, this is a bubble and not a steady state equilibrium. The phase diagram of Figure 1a suggests that under this circumstance the real stock of money in the following period T+1 would be negative. Indeed, that is not true.[2] For simplicity, let us take the particular case where the nominal stock of money is constant *(M_t=M)* and the utility function is logarithmic $(v\,(m) = \log m)$. The Euler equation is given by:

$$\frac{1}{P_t} = \frac{1}{M} + b\ \frac{1}{P_{t+1}}$$

where we use the normalization $u'(y) = 1$. Suppose that at period T-1, $P_{T-1} = M$. Thus, the real stock of money at periods *T-1* and *T* are equal to: $m_{T-1} = 1$ e $m_T = 0$. The real quantity of money at period *T+1* is not negative because a bubble has destroyed money value and the Euler equation is no long valid.

The wrong conclusion that the price level would be negative at period *T+1* comes from the following transformation:

$$q = \frac{1}{P} \quad , q \neq 0$$

where *q* is the price of money in terms of goods. In this transformation, negative values of q and P, in the third quadrant of Figure 2, is not part of the model, because there is free disposal.

The nonlinear difference equation of *P* becomes the linear difference equation of the money price *q*:

$$q_t = \frac{1}{M} + b\ q_{t+1}$$

Of course in this equation *q* can be negative, but a negative value is not a solution of the model, because the solutions of this equation are not necessarily solutions of the price level equation, as shown in Figure 2. When $\lim_{m\to 0} m\, v'(m) > 0$, this model has no hyperinflation steady state equilibrium, but hyperinflation can occur as a bubble

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[2] Several textbooks have presented wrong arguments for this case. For example,Walsh[(1998), p. 59] states that: "When $\lim_{m\to 0} A(m) < 0$, paths originating to the left of m converge to $m < 0$; but this result is clearly not possible, since real balances cannot be negative". A similar statement can be found in Gray[(1984), p. 100].

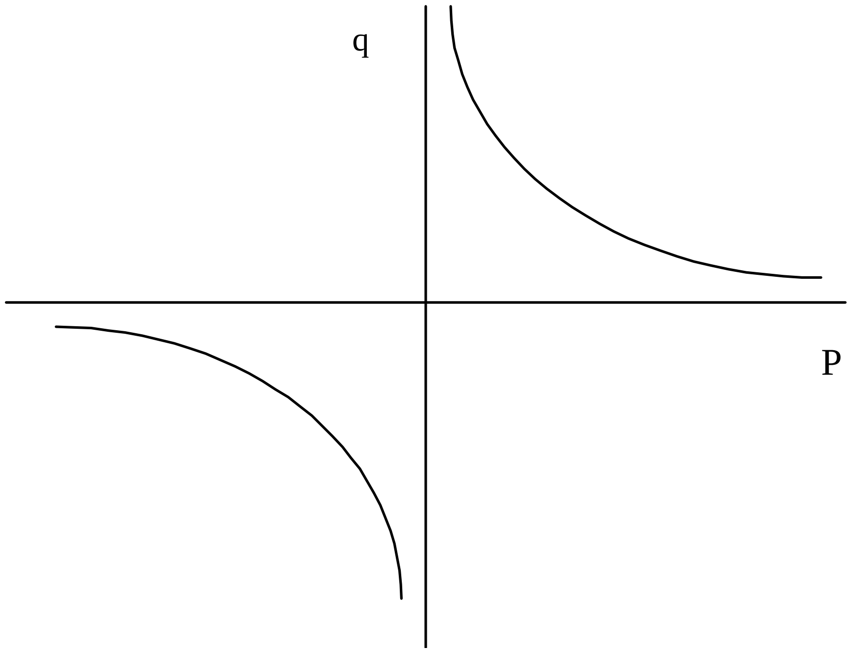

Figure 2

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Inflation Tax and Hyperinflation

We may conclude that a necessary condition to have hyperinflation is that the limit of the product of real cash balance by its marginal utility be equal to zero, when the real quantity of money approaches zero. It follows, then, that we should devise a means to find out the value of this limit, based on observed economic behavior of the society. The best way to do this is to analyze what happens with the inflation tax during the hyperinflationary process.

The seigniorage obtained by printing money can be decomposed into two components, the change in the real stock of money and the inflation tax. That is:

$$s_{t+1} = \frac{M_{t+1} - M_t}{P_{t+1}} = m_{t+1} - m_t + m_t \left(1 - \frac{1}{P_{t+1} / P_t} \right)$$

The last part of this expression corresponds to the inflation tax t_{t+1}, which can be written, after taking into account the price index ratio given by the Euler equation, in the following way;

$$t_{t+1} = m_t \left(1 - \frac{1}{b} \right) + \frac{m_t \, v'(m_t)}{b \, u'(c)}$$

It is easy to verify that:

$$\lim_{m \to 0} t_{t+1} = 0 \, , \; if \quad \lim_{m \to 0} m \, v'(m) = 0$$

and:

$$\lim_{m \to 0} t_{t+1} > 0, \quad if \quad \lim_{m \to 0} m v'(m) > 0$$

Thus, if the inflation tax collected by the government does not tend to zero, when the rate of inflation increases, money is essential. In such a case there will be no hyperinflation when the central bank controls the stock of money.

Fiscal Regime

The model just presented considered a monetary policy regime in which the central bank controls the stock of money. That is not what happens in hyperinflation. The economic policy regime in countries that have experienced hyperinflation is a regime that is called a fiscal regime, where the central bank finances the public deficit according to:

$$g_s = g = \frac{M_s - M_{s-1}}{P_s}, \quad s = 1, 2, 3, \ldots$$

By combining the Euler equation, the market for goods and services equilibrium condition ($y = c + g$), and the economic policy regime equation (we impose in this case the condition $h_s = 0$), we obtain the following difference equation:[3]

$$m_t [u'(c) - v'(m_t)] = b\, u'(c) \left(m_{t+1} - g \right)$$

This equation can be analyzed by using the function $A(m)$ defined before, and the function,

$$C(m) = b\ u'(c) \left(m - g \right)$$

Figure 3 shows the solution of this model for the two cases just presented. There is no possibility of occurring hyperinflation in the fiscal regime when $\lim_{m \to 0} m v'(m) = 0$. Otherwise, hyperinflation can occur when this limit is positive and it is equal to a fraction of the public deficit to be financed by printing money.[4]

For the purpose of understanding the conditions that supports a hyperinflation steady state equilibrium under a fiscal regime we write down real cash balances at period *t*

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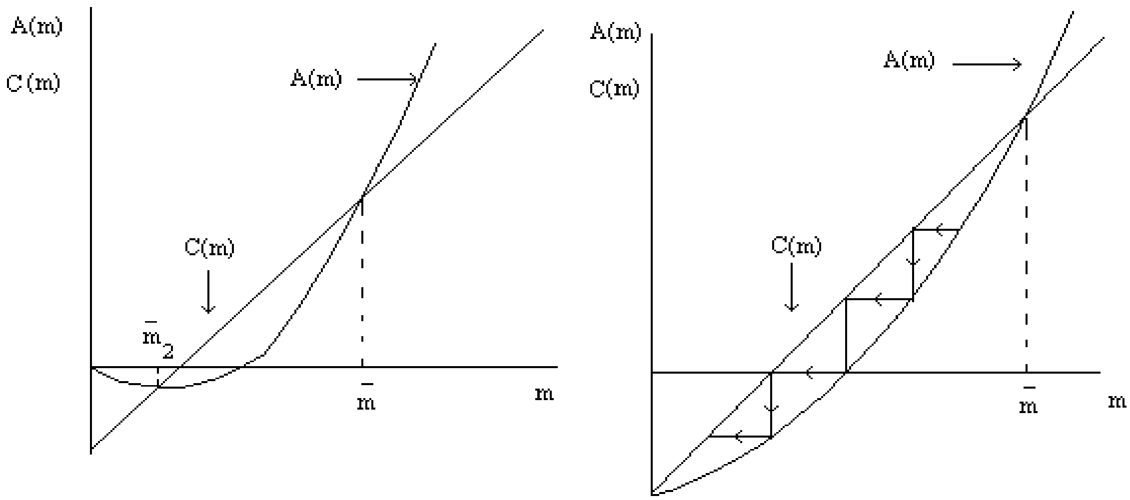
[3] This equation can also be written as: $m_{t+1} - m_t = g - t_{t+1}$. If g is always greater (lesser) than t there will be no steady state equilibrium.

[4] The difference equations corresponding to the two economic policy regimes can be written as functions of the inflation tax. For the monetary regime the equation is given by: $m_{t+1} = (1+m)(m_t - i(m_t))$. For the fiscal regime the difference equation is: $m_{t+1} = m_t + g - i(m_t)$, where in both cases *i(m$_t$)* is a function that assigns a unique tax to each real cash balance. For the monetary regime the steady state *m=0* exists when *i(o)=o*. For the fiscal regime the steady state m=0 exists when g= i(o). Let us define the function *D(m)=m-i(m)*. The two difference equations can be written as: $m_{t+1} = (1+m)\, D(m_t)$ and $m_{t+1} = D(m_t) + g$, and it is straightforward to draw the corresponding phase diagrams.

as a function of the discounted cash flow of money services. By combining Euler equation with the fiscal regime monetary policy we get:

$$m_t = \frac{m_t \; v'(m_t)}{u'(y)} - b \;\; g + b \;\; m_{t+1}$$



a) $\lim_{m \to 0} m\, v'(m) = 0$ b) $\lim_{m \to 0} m\, v'(m) > 0$

Figure 3

It follows that m_{t+1} depends upon m_{t+2} according to:

$$m_{t+1} = \frac{m_{t+1} \; v'(m_{t+1})}{u'(y)} - b \;\; g + b \;\; m_{t+2}$$

By iterating forward, the real stock of money at t is :

$$m_t = \sum_{s=t}^{t+T} b^{\,s-t} \left(\frac{m_s v'(m_s)}{u'(y)} - b \;\; g \right) + b^{\,t+T+1} m_{t+T+1}$$

When the transversality condition is met, the real stock of money is given by:

$$m_t = \sum_{s=t}^{\infty} \; b^{\,s-t} \left[\frac{m_s v'(m_s)}{u'(y)} - b \;\; g \right]$$

This expression shows that if $\lim_{m \to 0} m\, v'(m) = 0$, the fiscal regime model has no hyperinflation steady state equilibrium. However, if

$$\lim_{m \to 0} \; \frac{m\, v'(m)}{u'(y)} = b \;\; g$$

there is a hyperinflation steady state equilibrium.

Analyzing an economy where the economic policy regime is a monetary one, O- R state that "speculative hyperinflation can be excluded only through severe restrictions on individual preferences"[Obstfeld and Rogoff(1983), p.675]. The severe restrictions on the representative agent utility function which they refer to is that the value of the money services, measured in utilities, should be greater than zero, when real cash balances approach zero. There is no a priori reason to agree with O-R that this restriction is not feasible, and that the hypotheses that the value of the money services approaches zero when inflation increases without bounds is more tenable. The choice between the two hypotheses is an empirical question, rather than a theoretical issue. Data from hyperinflation experiments can be used to test whether or not money is essential. The first hypothesis implies, as O-R have shown, that money is essential and that its utility is minus infinite when the real stock of money goes to zero.[5]

The hyperinflation experiments that have occurred in several countries can be used in empirical investigations to draw conclusions with regard to what did happen with money services value in extreme situations. If, in hyperinflation, the inflation tax does not tend to zero when the real stock of money approaches zero, then money is essential and the restriction considered by O-R as non intuitive is indeed the relevant restriction. We would like to call attention to the fact that this hypothesis requires that the demand for money elasticity with respect to the rate of inflation be less than one in absolute value. This hypothesis is more likely than the hypothesis implicitly adopted in the functional form used by Cagan's demand for money [see Barbosa(1993)].[6]

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[5] Burmeister, Flood and Garber[(1994), p.159] agree with OR and state: "Is hard to imagine any circumstance when na investigator would feel confortable assuming a money to be essential to the degree implied by $[v(0) = -\infty]$." Farther on they add: " Since the impllication of the assumption which underlies Brock's work is unpalatable $[\lim_{m \to 0} m\, v'(m) > 0]$ the exclusion of bubbles must rest on posterior beliefs formed entirely from experience with data."

[6] Bailey[(1956), p. 109] already has observed that Cagan's specification of the money demand equation could not be appropriate: " All but one (Hungary II) of Cagan's regression showed a high degree of serial correlation in the residuals(with respect to time); this suggests the possibility of inappropriate specification of the estimating system."

If the inflation tax does not converge to zero when the rate of inflation increases unboundedly, there is no possibility of occurring hyperinflation when the economic policy regime is a monetary regime in which the central bank controls the (rate of growth of the) stock of money. Therefore, a change of regime, from a fiscal to a monetary regime, is fundamental to end hyperinflation.[7] There is no reported experience until today of hyperinflation occurring under a monetary regime. This empirical evidence is consistent with the hypotheses that the price-elasticity of the demand for money is less than one in absolute value, a fact well-known and indeed common knowledge in the demand for money empirical studies literature under normal situations.[8] The analysis presented here casts doubt on empirical studies that have a priori adopted specifications for the demand for money equation that does not allow a test of the price-elasticity hypothesis. Table I presents a summary of the model conclusions.

Table I

Hyperinflation: Price Elasticity of the Demand for Money x Regime of Economic Policy

Economic Policy Regime	Price-Elasticity of the Demand for Money (ε)	
	$\lvert\varepsilon\rvert < 1$	$\lvert\varepsilon\rvert > 1$
Monetary	NO	YES
Fiscal	YES	NO

3. Hyperinflation model with continuous variables

The economy has a representative agent with an infinite life that maximizes the functional

$$\int_o^\infty e^{-r\,t}\left[u\,(c) + v(m)\right]\,d\,t$$

where ρ is the rate of time preference, the utility function depends on consumption (c) and the services provided by money (m = M/P, is the real stock of money). The utility function has the traditional properties and it is also assumed to be of the separable type.

[7] This hyperinflation model offers a theoretical explanation for the empirical evidence found out by Sargent (1982), with regard to the ends of several European hyperinflations, in the first half of the twentieth century. The ends of these hyperinflations occurred through a credible change of the economic policy regime, from a fiscal to a monetary regime.

[8] The empirical evidence presented by Cagan[opus cit. p.79] is not consistent with the hypothesis that the inflation tax goes to zero when inflation goes to infinite: " This fact[lag in expectations] helps to explain why a similar time pattern of [tax] revenue emerged in all the seven hypeerinflations. The [tax] revenue was high at the start, when the expected rate of price increase was still low; tended to decline in the middle, as the expected rate started to rise considerably; and rose near the end, when the rate of new issues skyrocketed." Farther on in the footnote 36(p.79) he added: "Part of this rise in revenue resulted from the failure of real cash balances to make further declines in the final months, apparently because the end of hyperinflation appeared imminent."

The income (y) of the consumer is given, he receives from the government a transfer of h units of consumption goods and he spends all his resources purchasing consumption goods, and increasing his stock of money $(\dot{M}/P)$. That is:

$$y + h = c + \frac{\dot{M}}{P}$$

The representative agent solves the following problem:

$$\text{maximize } \int_0^{\infty} e^{-rt}\left[\, u(c) + v(m)\right] dt$$

subject to the following restrictions:

$$\dot{m} = y + = h - c - mp$$
$$m(0) = m_0, m_0 \; given.$$

The current value Hamiltonian equals:

$$H = u(c) + v(m) + l\left[y + h - c - mp\,\right]$$

The first order conditions are:

$$\frac{\P H}{\P c} = \frac{\P u}{\P c} - \; l = 0$$
$$\dot{l} = r\; l - \frac{\P H}{\P m} = (r + p)\; l - \frac{\P\; v}{\P\; m}$$
$$\frac{\P H}{\P\; l} = y - c - mp = \dot{m}$$

The transversality condition requires that the limit of the present value of the real stock of money, evaluated at the marginal utility of consumption, be equal to zero:

$$\lim_{t \to \infty} l \quad m\, e^{-rt} = 0$$

This model can be analysed under two alternatve economic policy regimes. Under the regime that will be called monetary regime, the central bank controls the rate of growth of the money supply, and the government transfers the resources of issuing money to the public, according to:

$$m = \frac{\dot{M}}{M} = cons\tan t$$
$$h = \frac{\dot{M}}{P}$$

The economy model under the monetary regime is given by the following system of equations:

$$\begin{cases} l = u'(c) \\ \dot{l} = (r + p)\, l - v'(m) \\ y = c \\ \dot{m} = m\,(m - p) \\ \lim_{t \to \infty} l \quad m\, e^{-rt} = 0 \end{cases}$$

The third equation assumes equilibrium in the market for goods and services, and we have adopted the following notation: $u'(c) = \partial u / \partial c \quad and \quad v'(m) = \partial v / \partial m.$

Under the fiscal regime the central bank has to finance the public deficit, which is assumed constant and equal to g:

$$g = \frac{\dot{M}}{P} = m\ m \quad , \quad m = \frac{\dot{M}}{M}$$

where (μ) is the endogenous rate of growth of money supply, which is determined by the public deficit parameter (g) and the real quantity of money demanded by the representative agent. In this case the government transfer is equal to zero (h=0). The economy model, under the fiscal regime, is given by the following system of equations:

$$\begin{cases} l = u'(c) \\ \dot{l} = (r + p)\, l - v'(m) \\ \dot{m} = g - m\, p \\ y = c + g \\ \lim_{t \to \infty} l \quad m\, e^{-rt} = 0 \end{cases}$$

The equation $y=c+g$ assumes that the market for goods and services is in equilibrium. Under both regimes marginal utility of consumption is constant, because consumption is constant. We now analyze the model under the fiscal regime.

Steady state equilibrium

The equilibrium steady state values of *p*, *m* and *c* in the model are obtained by making $\dot{l} = \dot{m} = 0$, and solving the following system of equations:

$$r + p = \frac{v_m}{u_c}$$

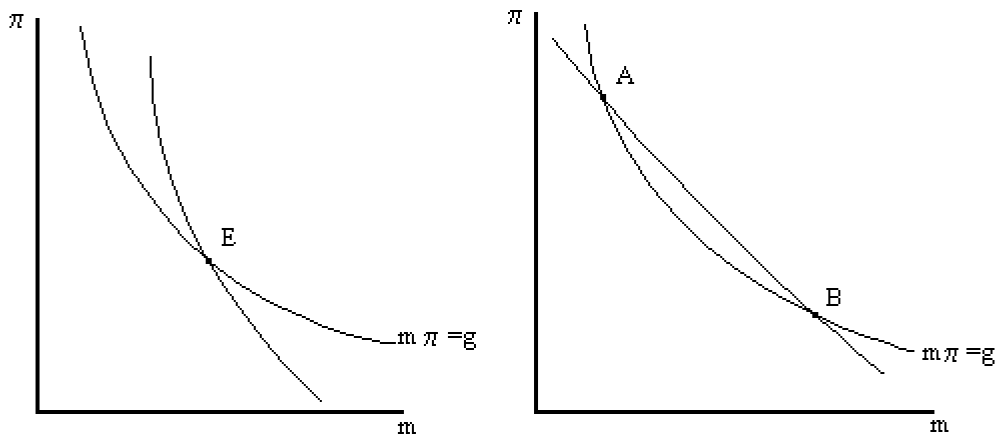
$$m\, p = g$$

$$c = y - g$$

The real stock of money (m) and the equilibrium rate of inflation (π) are negatively correlated, since:

$$\frac{\P m}{\P p} = \frac{u_c}{v_{mm}} < 0$$

This model may have a single or multiple equilibrium, as indicated in Figure 4. In both cases, the transversality condition is met by the values of the equilibrium points. In the case of multiple equilibrium, the lower inflation equilibrium corresponds to a higher welfare level because the real stock of money is larger.



a) Unique equilibrium b) Multiple equilibrium

Figure 4. Steady state equilibrium

Hyperinflation

Hyperinflation may occur under three different circumstances. In the first one, the model has a steady state hyperinflation equilibrium point(m=0); in the second one, there is an steady state equilibrium with m>0, but the economy enters an explosive path that results in hyperinflation; in the third one there is no steady state equilibrium because the public deficit can not be financed by the inflation tax . We will analyze each of these possibilities.

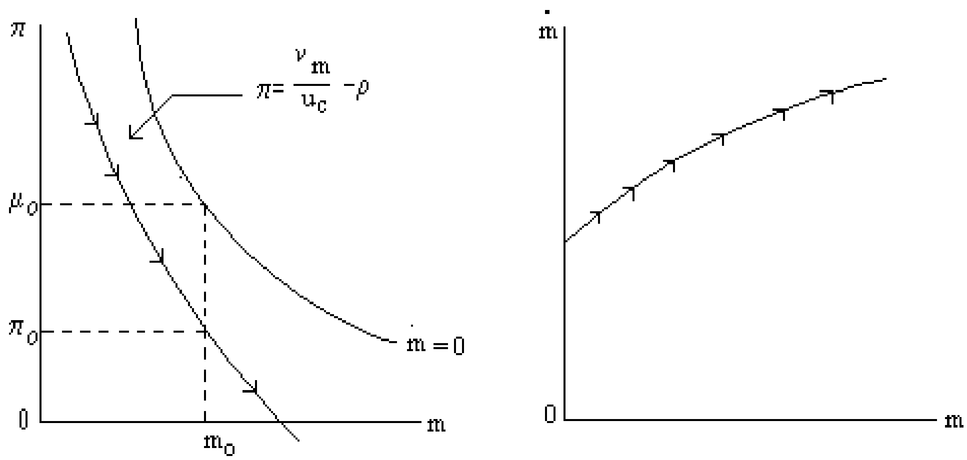
The equilibrium may not exist because the public deficit to be financed by printing money may always be greater than the inflation tax that can be collected, that is: g> m π. Figures 5a and 5b show two phase diagrams of the model in this situation; Figure 5a represents the model in the (m, π) phase plane and Figure 5b in the $(m, \dot{m})$ phase plane.[9] Both figures show that the real liquidity of the economy (m) would tend to increase indefinitely (m→∞), yielding hyperdeflation- and not hyperinflation, as Buiter (1987) has already observed. Nevertheless, the path of hyperdeflation is not feasible because it does not satisfy the transversality condition of the model, since

[9] It is easy to verify that the partial derivative of $\dot{m}$ with respect to m is given by:

$$\frac{\P \dot{m}}{\P m} = -p\left(1 + \frac{1}{e}\right) = -p\left(1 - \frac{1}{|e|}\right)$$

$$\lim_{m\to\infty}\frac{\dot m}{m} = \lim_{m\to\infty}\left(\frac{g}{m} - \frac{v_m}{u_c} + r\right) = r$$

taking into account that $\lim_{m\to\infty}\frac{v_m}{u_c} = 0$. Therefore, as m grows at a rate of ρ, the transversality condition is not satisfied.



a) Phase plane diagram (m, p) b) Phase plane diagram (m, $\dot m$)

Figure 5. Steady state equilibrium does not exist: g> m π

The solution of the model under the assumption that g> m π cannot yield hyperdeflation, as stated by Buiter(1987), but an instantaneous hyperinflation. The economic agents know beforehand that the value of the services provided by money when the inflation rate reaches high levels (π→∞, m →o) would not be enough to finance the public deficit, since

$$\lim_{m\to 0} m\frac{v_m}{u_c} < g$$

Individuals, therefore, will try to get rid immediately of the stock of money they hold. This attempt will cause an unbounded rise in prices, yielding instantaneous hyperinflation in the economy.

The second possibility of hyperinflation occurs when the public deficit to be financed by money is equal to the limit of the inflation tax, when the rate of inflation goes

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where ε is the elasticity of the quantity demanded of money with respect to the rate of inflation: $e = \frac{\P m}{\P p}\cdot\frac{p}{m}$. Thus, if $|e| < 1$, $\frac{\P \dot m}{\P m} > 0$ *and if* $|e| > 1$, $\frac{\P \dot m}{\P m} < 0$. When $e = -1$, $\frac{\P \dot m}{\P m} = 0$.

to infinity,as described in Figure 6. To analyze this case let us make explicit the fundamentals that explain the real stock of money. By taking the value of π given by this equation,

$$r + p = \frac{v'(m)}{u'(c)}$$

in the $\dot{m}$ equation, we obtain:

$$\dot{m} = r\ m + g - \frac{mv'(m)}{u'(c)}$$

The solution of this differential equation is given by:

$$m(t) = m\ (T)\ e^{-r\,(T-t)} + \int_t^T\ e^{-r\,(t-t)} \left[\frac{m\ v'(m)}{u'(c)} - g \right] dt$$

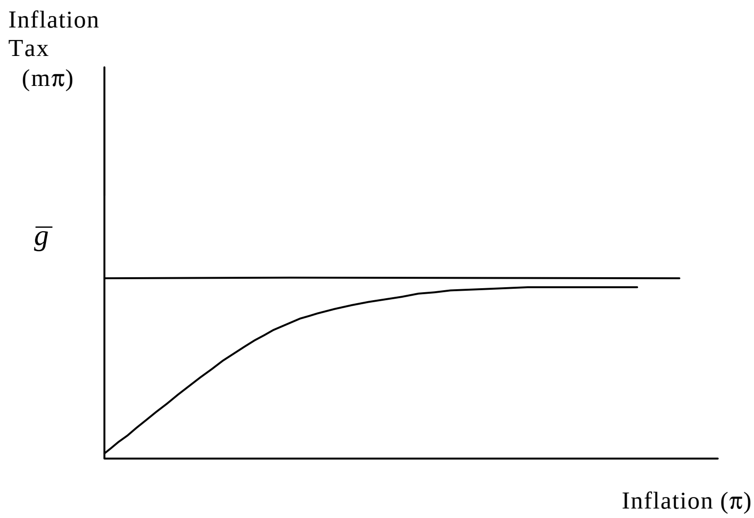


Figure 6

When the transversality condition is met,

$$\lim_{T \to \infty}\ m\ (T)\, l\ \ e^{-r\,(T-t)} = 0$$

the real stock of money at t is given by:

$$m(t) = \int_t^{\infty}\ e^{-r\,(t-t)} \left[\frac{mv'(m)}{u'(c)} - g \right] dt$$

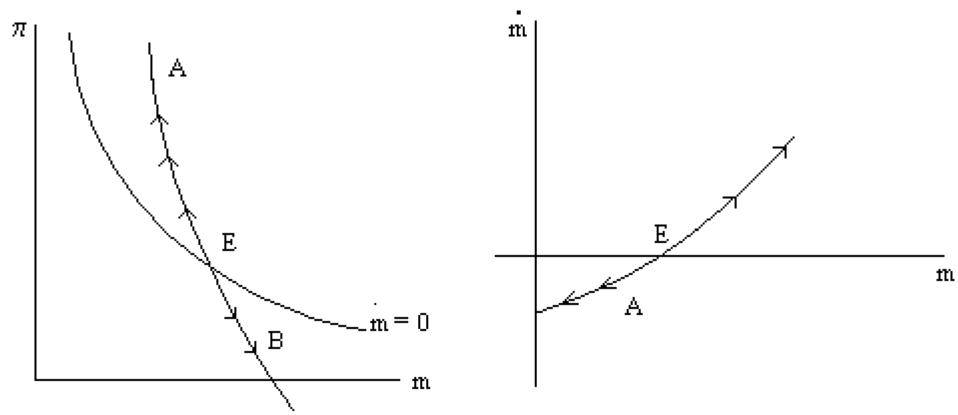
Therefore, it is easy to conclude that if

$$\lim_{m \to 0} \frac{mv'(m)}{u'(c)} = g$$

the real stock of money is equal to zero. Thus, the model has a hyperinflation steady state equilibrium.

We will analyze now the possibilities of hyperinflation when the model has a steady state equilibrium with the real stock of money greater than zero. Figures 7a and 7b show the phase diagrams in this case.[10] The path EB is a path of hyperdeflation, but it is not feasible, since it does not satisfy the transversality condition.

The path EA is a path of hyperinflation and does violate the transversality condition of the model. In this model it is not feasible for the real cash balance to become



a) Phase plane diagram (m, π)   b) Phase plane diagram $(m, \dot{m})$

Figure 7. Steady state equilibrium and hyperinflation

negative as one would conclude by looking at Figure 7b. In the limit when *m* tends to zero, the limit of $\dot{m}$ is a negative number. It is easily verifiable, through the phase diagram of Figure 7a, that the rate of inflation will always be greater than the rate of growth of the stock of money but the real stock of money will always be positive. However, it is important to clarify this point because it has some practical implications regarding the span of time of the hyperinflationary process. In order to be as simple as possible we will assume that the model utility function is logarithmic and the units are chosen in such way that the marginal utility of consumption is equal to one. Then, the model of the economy would consists of the following equations:

$$\dot{m} = g - m\ p$$

$$(\ r + p\ )\ m = a$$

where α is a parameter of the utility function. By taking the value of $p\ m$ given by the second equation in the first, we get the following first order differential equation:

$$\dot{m} = -(\ a - g\ ) + \ r\ m$$

Therefore, its solution is given by:

[10] Figure 9 supposes that:

$$\lim_{m \to o} \dot{m} = g - \lim_{m \to o} m \frac{u_m}{u_c} < 0$$

$$m = \overline{m} + (m_0 - \overline{m}) e^{r t}$$

The fact that m becomes negative is a mathematical property because the model has also a representation on the third quadrant, where both the rate of inflation and the real quantity of money are negative but their product is positive, which has no economic meaning since there is free disposal. By taking into account the value of m given by the demand for money equation, the rate of inflation is:

$$p = \frac{a}{\overline{m} + (m_0 - \overline{m}) e^{r t}} - r$$

When $m_0 < \overline{m}$, there is a time T for which $m (T) = 0$. Hence, we have:

$$T = -\frac{1}{r} \log \left(1 - \frac{m_0}{\overline{m}} \right)$$

We conclude, then, that the limit of p when $t \rightarrow T$, is infinite. Thus hyperinflation occurs in a finite time that depends upon the size of the shock in relation to the steady state equilibrium and the real rate of interest; the greater the shock the lesser the length of hyperinflation; by the same token, the real rate of interest and the length of hyperinflation are negatively correlated.

This hyperinflation path is a bubble because at instant T the real stock of money based on the fundamentals of the model is given by

$$m (T) = \int_t^{\infty} e^{-r(t-t)} \left[\frac{m\, v'(m)}{u'(c)} - g \right] dt > 0$$

which is different from the real stock observed in the economy.

4. Conclusion

This paper has analyzed the occurrence of hyperinflation – a phenomenon characterized by an explosive rate of inflation and by a real stock of money that approaches zero – in an economy inhabited by a representative agent who allocates his resources intertemporally in order to maximize his welfare. In this economy, all markets are in equilibrium and two economic policy regimes are considered; in the first, the central bank finances the government deficit; in the second, it effectively controls the (growth of the) money supply.

Hyperinflation may occur instantaneously when the government deficit financed by issuing money is greater than the inflation tax that can be permanently levied upon individuals in this society. On the other hand, even if the government deficit is financed on a permanent basis by issuing money – i.e. when the model has a positive steady-state equilibrium point – hyperinflation is still possible: the rate of inflation may gradually increase and the real stock of money and its price will gradually approach zero as a conventional bubble. When the path is explosive, hyperinflation occurs in finite time, which is a function of the shock, relative to the steady-state equilibrium, and the real interest rate. The larger the shock, the lesser the duration of hyperinflation. Likewise, the real interest rate and the duration of hyperinflation are negatively correlated.

Within the fiscal regime in which monetary policy is passive and the central bank is forced to finance the public deficit, steady state equilibrium hyperinflation may occur if the inflation tax does not disappear when inflation grows unboundedly $(\lim_{m \to 0} m\, p > 0)$. This fact implies that money is essential for the agents in this economy and that it is impossible for hyperinflation to occur as a result of self-fulfilling prophecies in the monetary regime where the central bank effectively controls the growth rate of the monetary base. Therefore, when money is essential hyperinflation is a phenomenon typical of the fiscal regime in this class of models. Hyperinflation is caused, according to the fundamentals, by the increasing of the fiscal deficit to be financed by issuing money.

The use of money is grounded on social convention and it depends upon the institutional framework, which establishes conditions under which contracts are liquidated. Money essentiality does not depend only on the transactions financial technology but also on the institutions of each country. In Argentina, for instance, after the Conversion Stabilization Plan of 1991, the peso and the American dollar are legal tender. Therefore, we should not expect that the peso is essential, and any lack of confidence in the actual institutional set up can lead to a flight from peso through an instantaneous hyperinflation.

Money essentiality in a hyperinflation environment depends upon the degree of enforcement of the legal system. However, in any case, the essentiality of money should be tested, and not taken as a given, even if the argument looks sound and is appealing from a theoretical perspective. The theoretical framework presented in this paper and the fragmented empirical evidence suggests that money is essential in normal situations and in hyperinflation environments. This hypotheses has to be submitted to the verdict of the data.

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