The threat of insurance
A note on the robustness of principal-agent models

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Abstract

The traditional principal-agent model assumes that the principal o¤ers an exclusive contract to the agent. This paper studies the robustness of the traditional results to the potential entry of risk-neutral players, not interested in the agent's e¤ort, that o¤er contracts to the agent. We …nd that the standard equilibrium is not robust to the presence of these players. Although these new players do not get any utility, the total welfare of the economy decreases with free entry (and so does the surplus of the principal, while the agent remains in his reservation level of utility).

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1 Introduction.

Most principal-agent models tend to assume that an agent signs an exclusive contract with a principal. In this paper we investigate to what extent the standard results are robust to the introduction of other contracting possibilities for the agent. In particular, we assume the presence of some other players who are not interested in the agent´s effort, but are less risk averse than him – more specifically, we model those "insurers" as risk neutral.

We study the sequential game when there is a finite number of potential insurers and when there is free entry . Under the former specification, the results do not diverge substantially from the standard case. The principal gives more risk to the agent but that extra risk is taken by an insurer; in particular the "last" insurer is the one who takes the agent to the second-best contract.

When there is free entry the results are quite different. The second-best contract cannot be implemented. The lack of exclusive contracts reduces social welfare, either because low effort will be provided or because the agent will face too much risk.

Our results cast some doubts on the appropiateness of the use of the standard results in many applied problems. They might provide a rationale for the seemingly scarce utilization of performance contracts.

The results share the spirit of some other papers which have explored moral hazard with non-exclusive contracts. Kahn and Mookherjee (1998) study the effects of nonexclusive credit or insurance contracts from multiple risk neutral firms with sequential free entry. They find that competition between firms can induce a reduction in customer´s effort, and that the lack of coordination among insurers may affect the cost of implementation even without affecting effort levels.

Bisin and Guaitoli (1999) analyze a case where "intermediaries" design and offer contracts simultaneously. They show that the optimal action is not implemented in equilibrium for an open set of economies. They also show that whenever the equilibrium contracts implement the optimal action, intemediaries make positive profits and equilibrium allocations are inefficient.

Our paper is also related to a recent literature ("common agency") that studies problems where there are more than one principal interested in the agent´s action. (Bernheim and Whinston 1986 is the pioneer). In this paper we develop one variation of common agency, in which we allow the agent to choose among the set of contracts he is offered,[2] and in which all but the first "principal" are uninterested in the agent´s action.

The rest of the paper is organized as follows. Section 2 presents the "traditional" principal agent problem and shows that it is not an equilibrium for the principal to offer the second-best contract when entry of insurers is allowed. Section 3 introduces the model with nonexclusive contracts. Section 4 studies the case of sequential offers by a finite number of "insurers" (4.1) and in the case of free entry (4.2). Section 5 concludes and proposes some extensions.

2 Traditional P-A

It is well known that the principal agent problem arises when there is asymmetric information (the effort is not contractable) and the agent is risk averse. Whenever the principal wants to implement high effort it is needed to make the agent face some risk. There is a problem between risk sharing and incentives.

Following Mas Collel et all (1995), the problem of the principal can be decomposed in two steps

1) find the optimal incentive scheme for each level of effort

2) choice of the optimal level of effort.

The optimal incentive scheme for implementing e must solve

$$Min_{w(\pi)} \int w(\pi)\, f(\pi|e)\, d\pi$$

s.t.

$$(IR) \quad \int v(w(\pi))\, f(\pi|e)\, d\pi - g(e) \geq \bar{u}$$

[2] The literature refers to this case as "delegated" common agency. See Tommasi and Weinschelbaum (2000) for a discussion of that literature and for a critique of the terminology.

$$(IC)\ e \text{ solves } Max_{\mathbf{e}} \int v(w(\pi))f(\pi|\mathbf{e})d\pi - g(\mathbf{e})$$

When the principal wants to implement low effort (e_l) the cost is $\int w^p(\pi)f(\pi/e_l)d\pi = \phi[g(e_l) + \overline{u}].$

While when he wants to implement high effort (e_h), the cost is $\int w^p(\pi)f(\pi/e_l)d\pi = \int w^{2ndBest}(\pi)f(\pi/e_l)d\pi.$

Comparing the costs we can show the following lemma.

Lemma 1 The cost of implementing e_h is strictly greater than the cost of implementing e_l, that is,

$$\int w^{2ndBest}(\pi)f(\pi/e_h)d\pi > \phi[g(e_l) + \overline{u}].$$

Proof. The agent will be at his reservation utility in both cases. So since the cost of doing high effort is higher than the one of doing low effort $(g(e_h) > g(e_l))$ the utility that the agent receives form the payment in the high effort case should be greater than the one implementing low effort. The fact that a flat wage will implement the low effort, while for the high effort the agent will face some risk will increase the difference. ■

The equilibrium will depend on whether the principal can make higher profits, inducing the agent to make High or low effort.

Proposition 1 Subgame perfect equilibria of the traditional principal-agent game:

i) If $\int \pi f(\pi/e_l)d\pi - \phi[g(e_l) + \overline{u}] > \int \pi f(\pi/e_h)d\pi - \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi$, then the principal offers the flat wage $w^p(\pi) = \phi[g(e_l) + \overline{u}]$, the agent accepts this contract, chooses e_l and makes its reservation level of utility $\overline{u}$.

ii) If $\int \pi f(\pi/e_l)d\pi - \phi[g(e_l) + \overline{u}] < \int \pi f(\pi/e_h)d\pi - \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi$, then the principal offers the contract $w^p(\pi) = w^{2ndBest}(\pi)$, the agent accepts this contract, chooses e_h and makes its reservation level of utility $\overline{u}$.

iii) If $\int \pi f(\pi/e_l)d\pi - \phi[g(e_l) + \overline{u}] = \int \pi f(\pi/e_h)d\pi - \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi$ then this game has two SPE. They are the ones in i) and ii).

Proof. It follows directly from the fact that there are two possible levels so the principal has to compare the optimal scheme of one case with the other ■

The interesting case is when the principal wants to induce high effort level (e_h) in this case the incentive compatibility constraint becomes

$$\text{(IC)} \quad \int v(w(\pi)) f(\pi|e_h)\, d\pi - g(e_h) \geq \int v(w(\pi)) f(\pi|e_l)\, d\pi - g(e_l)$$

It can be shown (see Mas Collel Lemma 14.B.1) that both restrictions will be binding. This implies that the agent is indifferent between low and high effort

$$\int v\left(w^{2ndBest}(\pi)\right) f(\pi|e_h)\, d\pi - g(e_h) = \int v\left(w^{2ndBest}(\pi)\right) f(\pi|e_l)\, d\pi - g(e_l)$$

This result is implicitly assuming either that the agent has an “exclusivity” contract or that there is no other player who is risk neutral and the same informational structure that the principal has. Otherwise this “third” player will offer an insurance and the agent will make low effort.

Proposition 2 If there is another player who is risk neutral and have the same informational structure that the principal has, he can get positive profits offering the agent an insurance.

Proof. Using Jensen’s inequality we know that

$$v\left(\int w^{2ndBest}(\pi) f(\pi|e_l)\, d\pi\right) - g(e_l) > \int v\left(w^{2ndBest}(\pi)\right) f(\pi|e_l)\, d\pi - g(e_l)$$

Hence there is a value $p > 0$ such that

$$v\left(\int w^{2ndBest}(\pi) f(\pi|e_l)\, d\pi - p\right) - g(e_l) = \int v\left(w^{2ndBest}(\pi)\right) f(\pi|e_l)\, d\pi - g(e_l)$$

Therefore if the third player pays $\int w^{2ndBest}(\pi) f(\pi|e_l) - p - w^{2ndBest}(\pi')$ when the profits are π' the agent will accept the contract and the insurer will get an expected profit of p (the risk premium). ■

This is because $\int w^{2ndBest}(\pi) f(\pi/e_l) d\pi > \phi[g(e_l) + \bar{u}]$, the expected payment that the principal would do under the 2ndBest scheme when the agent makes low effort is higher than the flat wage, the difference is the risk premium:

So the question is how the results change? when we "relax" the assumption of exclusivity in the contracts, to where we turn now

3 The Model.

We assume that there are risk neutral players the "insurers" not interested in the agent's action. Since we allowed the agent to accept or reject each contract, this game belongs to the delegated common agency problems.

The traditional principal-agent problem is solved using backward induction,we could use the same technique to solve for the subgame perfect equilibria of this game, but this is too cumbersome, especially when the quantity of insurers (N) is large: the agent can take any of $(2^{N+2} - 1)$ possible actions.

Instead, we investigate the properties of subgame perfect equilibria of the game.

The original principal's strategy space contains every possible contract. However, this space is split into three parts; so that the principal's final strategy consists of either implementing e_l, implementing e_h; or making no offer at all. We say the principal implements e if he offers a contract such that the agent accepts it and chooses e.

A sort of indirect utility function is then constructed, representing the benefits of choosing the optimal contract from one of these three categories. These functions when there are exclusive contracts are[3]

$$\begin{aligned} \Pi^p(e_l) &= \int \pi f(\pi/e_l) d\pi - \phi[g(e_l) + \bar{u}]; \\ \Pi^p(e_h) &= \int \pi f(\pi/e_h) d\pi - \int w^{2ndBest}(\pi) f(\pi/e_h) d\pi; \\ \text{and } \Pi^p(\text{ no offer }) &= 0: \end{aligned}$$

[3] We will show that these values are upper limits to the values when the contracts are not exclusive.

These functions are very useful because when we test whether a certain strategy profile $(w^p(\pi); e)$ is a Nash equilibrium, all we have to do is consider a deviation in the principal's reduced strategy space: $\{$implement e_l; implement e_h; make no offer$\}$. This is, precisely, the best the principal can do if he decides to deviate.

This is a useful technique for the traditional principal-agent problem; but as we will see, it is much more useful when there are many players. We will study this game under two different sequences. a) Sequential: the principal makes the first offer the agent accepts or rejects, then the "insurers" make the offers in order knowing which offers the agent has accepted and b) Simultaneous: the principal and all the "insurers" make simultaneously the offers..

We say a player $j \in P$ is active if he chooses to participate and his contract is accepted by the agent. Denote by $A \subset P$ the set of active players.

To apply the technique described above, we must be careful with the definition of implementation.

Definition 1 We say a player $i \in P$ implements e if he offers a contract such that the agent accepts it and chooses e; and if that contract is a best reply to the contracts offered by every other player $j \in P_{-i}$.

Notice that in the sequential version of this game, such a contract must be a best reply both to all other previous players contracts and to all other following players best responses.

Again, we can simplify the strategies available to players $i \in P$: each can choose not to participate (i.e. offer $w(\pi) = 0$), or can participate and choose to implement e_l or e_h.

As in the traditional problem, denote by $\Pi^i(e)$ the maximum benefits of a player $i \in P$ who follows the strategy of implementing e.

Proposition 3 In a Nash equilibrium, all active players choose to implement the same level of effort.

Proof. The agent can only choose one level of effort. If some active players are implementing different levels of effort, then some of them are playing best responses to all other player's contracts that give the agent incentives to choose a level of effort that is not finally chosen in equilibrium. Therefore, they are better off by switching to a contract that implements the level of effort chosen by the agent in equilibrium. (If these best replies coincide, then the player is implementing either one or the other and the proposition still holds.) ■

Let $W_e^{aggregate}(\pi)$ be the aggregate contract accepted by an agent who chooses e. We have a lower limit for the cost of this aggregate contract.

Proposition 4 i) $\int W_{e_l}^{aggregate}(\pi) f(\pi/e_l) d\pi \geq \phi[g(e_l) + \bar{u}]$:
ii) $\int W_{e_h}^{aggregate}(\pi) f(\pi/e_h) d\pi \geq \int w_{e_h}^{2ndBest}(\pi) f(\pi/e_h) d\pi$:

Proof. From Section 2, we know that $\phi[g(e_l) + u]$ and $\int w_{e_h}^{2ndBest}(\pi) f(\pi/e_h)$ are the minimum costs for a principal who implements e_l and e_h; respectively. This proposition says that it is not possible for our aggregate player (the principal and the insurers) to do better. Suppose the contrary, so that the inequalities were reversed. Then the principal can mimic such an aggregate contract in the traditional principal-agent problem and pay less to implement e than the minimum found in Section 2. So we have a contradiction that tell us that this is impossible. ■

Proposition (4) has a natural interpretation. Insurers have an information structure identical to the principal's: they cannot observe the agent's effort. Thus, the introduction of these new players to the game can add nothing to our previous economy.

Another peculiarity of insurers allows us to establish the following result. Ex-ante, insurers are not interested in the agents's effort. This implies that, by themselves, they can never offer a profitable contract that gives an agent incentives to choose e. The principal must be active in every such case.

Proposition 5 In any case where the agent chooses e_l or e_h; the principal is always an active player. ($p \in A$).

Proof. From Proposition 4, the aggregate cost of implementing e is greater than zero, so that $\int W_e^{aggregate}(\pi) f(\pi/e) = \int \sum_{j \in A} w^j(\pi) f(\pi/e) = \int w^p(\pi) f(\pi/e) - \sum_{i \in A \cap I} \Pi^i(e) > 0$: If the principal is not active, then $\int w^p(\pi) f(\pi/e) = 0$, so that $\sum_{i \in A \cap I} \Pi^i(e) < 0$. But then at least one insurer is making negative profits (and this is not possible in equilibrium). ■

These three propositions are true both in the simultaneous and sequential versions of the game. Next, we exploit the sequential property of the game to characterize subgame perfect equilibria.

4 Sequential.

4.1 N insurers.

4.1.1 Implementing e_l in equilibrium

We now consider SPE where the agent chooses low effort, e_l. From Proposition 4 the aggregate contract that implements e_l satisfies

$$\int w^p(\pi) f(\pi/e_l) d\pi - \sum_{i \in A \cap I} \Pi^i(e_l) \geq \phi[g(e_l) + \bar{u}]; \quad (1)$$

and from Proposition 5 the principal makes an offer that is accepted by the agent.

Consider the case where the principal offers $\int w^p(\pi) f(\pi/e_l) d\pi < \phi[g(e_l) + \bar{u}]$: Then $\Pi(e_l) < 0$ for at least one potentially active insurer, the aggregate contract is the principal's contract, and the agent prefers not to accept it.

Therefore, if there is an equilibrium, we must have $\int w^p(\pi) f(\pi/e_l) d\pi \geq \phi[g(e_l) + \bar{u}]$:

Because the principal is moving first, if he can guarantee an equilibrium with $\int w^p(\pi) f(\pi/e_l) d\pi = \phi[g(e_l) + \bar{u}]$ then we know that there can be no equilibrium where the agent chooses e_l and the principal pays a higher cost, i.e. $\int w^p(\pi) f(\pi/e_l) > \phi[g(e_l) + \bar{u}]$: It turns out that this is the case.

Lemma 2 There is no equilibrium in which the agent provides low effort and the insurer gets positive utility.

Proof. We know that in equilibrium

$$\Pi^p(e_l) + \sum_{i \in A \cap I} \Pi^i(e_l) \quad \int \pi f(\pi | e_l) d\pi - \phi(g(e_L) + \bar{u})$$

Which is the total profit minus the minimum payment that the agent requires to accept to participate. Note that to achieve this value we require a) The agent receive no surplus, b) The agent receive no risk, so the total amount to split between the principal and the insurer is maximum.

But if the principal offers a flat contract $w^P(\pi) = \phi(g(e_L) + \overline{u})$ the insurer has no incentive to offer any contract. So the principal can guarantee himself $\int \pi f(\pi|e_l)\,d\pi - \phi(g(e_L) + \overline{u})$ hence $\Pi^i(e_l) = 0 \; \forall i \in A \cap I$. ■

Can any insurer be better off by taking the agent to e_h instead of e_l?

Let stage h be the stage of the game where, given the aggregate contract up to that moment, insurer number h must make his offer. We have N of these stages in this sequential version of the game. Define $W^h(\pi)$ as stage h aggregate contract; that is, the aggregate contract (including the principal's contract) at the end of stage h. Notice, for example, that $w^p(\pi) = W^0(\pi)$ and $W^{aggregate}(\pi) = W^N(\pi)$.

Lemma 3 For $h = 1, \ldots, N$, if $\int W^{h-1}(\pi) f(\pi/e_l)d\pi = \phi[(g(e_l) + \overline{u}]$; then it is a best response for insurer number h either to stay out or to implement e_l. In either case, he makes zero profits.

Proof. We have already shown that given such a wage scheme the benefits of an insurer who either implements e_l or stays out are equal to zero. It remains to be seen that any insurer that implements e_h makes negative profits.

Consider such a deviating insurer. If he decides to implement e_h but leaves the principal out, at least one insurer has to make negative profits, as shown in the proof of Proposition 5. Therefore, any deviating insurer h must offer a contract such that the agent accepts his contract, the principal's contract, and (maybe) other insurer's contracts and chooses the level of effort e_h. But by Proposition 4 the cost of doing so, given the aggregate contract at the beginning of stage h, is

$$\phi[g(e_l) + u] - \Pi^h(e_h) - \sum_{i \in F \cap A} \Pi^i(e_h) \geq \int w^{2ndBest}(\pi) f(\pi/e_h);$$

where F is the set of insurers moving after insurer h.

Using Lemma 1, this results in $\Pi^j(e_h) + \sum_{i \in N} \Pi^i(e_h) < 0$. Therefore, a player h can deviate and make nonnegative profits only at the expense of an insurer who moves next making negative profits. But this is impossible given the sequential nature of the game. The player moving next will never choose to implement e_h and make negative profits; he'd rather stay out. ■

Notice that given the wage scheme, any insurer offers a contract that, in essence, does not change the cost of the aggregate contract up to that moment: he either offers no contract or offers one such that he makes zero profits; that is, $\int w^i(\pi) f(\pi/e_l) d\pi = 0$. Therefore, by initially choosing the flat wage the principal can guarantee that this will be the final aggregate contract and that no insurer will have incentives to interfere with the contract and take the agent to e_h. We summarize these results in the following Proposition.

Proposition 6 In any SPE where the agent chooses e_l;

i) The principal offers $w^p(\pi)$ such that $\int w^p(\pi) f(\pi/e_l) d\pi = \phi[g(e_l) + \bar{u}]$ and the agent accepts this contract.

ii) $k \in [0, N]$ insurers are active and offer contracts such that each of their profits are zero.

iii) The aggregate contract satisfies $W_{e_l}^{aggregate}(\pi) = \phi[g(e_l) + \bar{u}]$.

iv) The agent accepts this contract, chooses e_l, and makes its reservation level of utility $\bar{u}$.

In short, the introduction of these new risk-neutral players does not modify our results: the principal can implement e_l at the same cost as before, the agent gets its reservation utility $\bar{u}$, and every insurer makes zero profits. The insurers participate if the principal gives any risk to the agent; that is, in any case where the insurer does not offer the flat wage $\phi[g(e_l) + \bar{u}]$ directly.

4.1.2 Implementing e_h in equilibrium

Next we proceed similarly and characterize SPE where the agent chooses high effort, e_h. From Proposition 4 the aggregate contract that implements e_h satisfies

$$\int w^p(\pi)f(\pi/e_h)d\pi - \sum_{i\in A\cap I} \Pi^i(e_h) \geq \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi, \tag{2}$$

and from Proposition 5 the principal makes an offer that is accepted by the agent.

Consider the case where the principal offers $\int w^p(\pi)f(\pi/e_h)d\pi < \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi.$ Then $\Pi^i(e_h) < 0$ for at least one potentially active insurer. Since an insurer will never make negative profits in equilibrium, $\Pi^i(e_h) \geq 0\ \forall i$, if there is an equilibrium, we must have $\int w^p(\pi)f(\pi/e_h)d\pi \geq \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi.$

Because the principal is moving first, if he can guarantee an equilibrium with $\int w^p(\pi)f(\pi/e_h)d\pi = \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi$ then we know that there can be no equilibrium where the agent chooses e_h and the principal pays a higher cost, i.e. $\int w^p(\pi)f(\pi/e_h)d\pi > \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi.$ It turns out that this is the case; that is, we are looking for an equilibrium with

$$\int w^p(\pi)f(\pi/e_h)d\pi = \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi.$$

Notice that in such a case all insurers that implement e_h make zero profits, and the aggregate contract will always satisfy $\int W^{h-1}(\pi)f(\pi/e_h)d\pi = \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi$ for all $h = 1, ..., N-1.$

Lemma 4 Given $\int W^{h-1}(\pi)f(\pi/e_h)d\pi = \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi$ then it is a best response for insurer h to implement e_h if and only if $\int W^{h-1}(\pi)f(\pi/e_l)d\pi \leq \phi[g(e_l)+\bar{u}]$, for $h = 1, ..., N$.

Proof. From Proposition 4 we know that,

$$\int w^{2ndBest}(\pi)f(\pi/e_h) - \Pi^h(e_h) - \sum_{i\in F\cap A} \Pi^i(e_h) \geq \int w^{2ndBest}(\pi)f(\pi/e_h).$$

This means that $\Pi^h(e_h) + \sum_{i\in F} \Pi^i(e_h) \leq 0$. Insurer h can offer a contract such that $\Pi^h(e_h) = 0$ and the following players implement e_h and make zero profits too. These players do not have incentives to implement e_l unless insurer h himself has incentives to do so. This is what we check next.

Insurer h can deviate and offer a contract such that

$$\int W^{h-1}(\pi) f(\pi/e_l) - \Pi^h(e_l) - \sum_{i \in F \cap A} \Pi^i(e_l) \geq \phi[g(e_l) + u].$$

This can be rewritten as

$$\Pi^h(e_l) + \sum_{i \in F \cap A} \Pi^i(e_l) \quad \int W^{h-1}(\pi) f(\pi/e_l) - \phi[g(e_l) + u].$$

Insurer h can leave insurers $i \in F \cap A$ out, set $W^h = \phi[g(e_l) + u]$ and make benefits

$$\Pi^h(e_l) = \int W^{h-1}(\pi) f(\pi/e_l) - \phi[g(e_l) + u].$$

Insurers $i \in F \cap A$ will either stay out or make zero profits. If they deviate to e_h, they make strictly negative profits (the proof is in Lemma 3). The agent will accept such a contract and choose e_l.

In order to avoid this; that is, in order to be a best response for all insurers to implement e_h, we need $\Pi^h(e_l) = \int W^{h-1}(\pi) f(\pi/e_l) - \phi[g(e_l) + u] \quad 0$ for $h = 1, ..., N$. This establishes the result. ■

Together with $\int w^p(\pi) f(\pi/e_h) d\pi = \int w^{2ndBest}(\pi) f(\pi/e_h) d\pi$, Lemma 4 implies, for $h = 1$, that the principal's contract must satisfy

$$\int w^p(\pi) f(\pi/e_l) d\pi \quad \phi[g(e_l) + \bar{u}]. \tag{3}$$

Notice that is feasible, because

$$\begin{aligned} \int w^p(\pi) f(\pi/e_h) d\pi &= \int w^{2ndBest}(\pi) f(\pi/e_h) d\pi \\ &> \phi[g(e_l) + \bar{u}] \geq \int w^p(\pi) f(\pi/e_l) d\pi. \end{aligned}$$

The interpretation is that the principal offers a riskier contract to the agent, but still pays the cost of the second best contract. We know (see Proposition 2) that the second best does not satisfy equation 3. Indeed, the second best contract does not satisfy any of the conditions $\int W^{h-1}(\pi) f(\pi/e_l) d\pi \quad \phi[g(e_l) + \bar{u}]$ for $h = 1, ..., N$ when $\int W^{h-1}(\pi) f(\pi/e_l) d\pi =$

$\int w^{2ndBest}(\pi) f(\pi/e_l)d\pi$: This implies that the aggregate contracts $W^{h-1}(\pi) \neq w^{2ndbest}(\pi)$ for $h = 1, \ldots N$.

However, in equilibria of the kind we are considering, the aggregate contract must be equal to the second best contract. To see this, notice that the contract $W^{N-1}(\pi)$ gives the agent greater risk than the second best contract. Therefore, the agent will not accept such a contract. The last insurer must always be active in equilibrium, and he will make zero profits.

This implies that the cost of the aggregate contract is $\int W^{aggregate}(\pi) f(\pi/e_h)d\pi = \int w^{2ndBest}(\pi) f(\pi/e_h)d\pi$. The unique contract that both satisfies this and gives incentives to the agent to choose e_h is the second best contract itself. That is, we must have $W^{aggregate}(\pi) = w^{2ndBest}(\pi)$. Therefore, the last insurer takes the agent to the second best, and he is the only one who can do so because there is no other insurer moving next with incentives to, given the second best contract, take the agent to e_l:

We summarize our results in the following proposition.

Proposition 7 In a SPE where the agent chooses e_h,

i) The principal offers a contract w^p such that $\int w^p(\pi) f(\pi/e_h)d\pi = \int w^{2ndBest}(\pi) f(\pi/e_h)d\pi$ and $\int w^p(\pi) f(\pi/e_l)d\pi \geq [g(e_l) + \bar{u}]$; and the agent accepts this contract.

ii) $k \in [0, N-1]$ insurers are active, make zero profits and offer contracts such that $\int W^h(\pi) f(\pi/e_l)d\pi \geq [g(e_l) + \bar{u}]$ for all $h = 1, \ldots, N-1$:

iii) The last insurer (insurer N) is always active, makes zero profits, and takes the agent to the second best contract, $W^{aggregate}(\pi) = w^{2ndBest}(\pi)$:

iv) The agent accepts the aggregate contract, chooses e_h and makes its reservation level of utility $\bar{u}$.

The presence of risk-neutral players uninterested (ex-ante) in the agent's effort does not alter most of the results from the traditional principal-agent model. The principal pays the same cost for implementing e_h; the agent gets her reservation utility, and every insurer makes zero profits. The only difference is that the principal offers a contract which gives more risk to the agent than the second best contract, as shown by equation (3). This contract by itself leaves the agent at a utility lower than $\bar{u}$. The insurers reduce this risk and o¤er an aggregate contract that gives the agent $\bar{u}$ once again.

The intuition is simple. Knowing that some insurers have incentives to reduce any risk the agent might take and eventually give her incentives to choose e_l, the principal o¤ers a riskier contract[4] such that the insurers are just willing to participate and insure the agent up to the second best contract.

We have characterized equilibria where e_l and e_h are implemented. To …nd out the equilibria of this game, we must compare the bene…ts of the principal in cases where e_l and e_h are implemented. These are identical to the ones obtained under the traditional principal-agent problem. Therefore, the introduction of N risk neutral insurers has not altered our results signi…cantly, in particular, the spirit of Proposition 1 remains true.

This might seem surprising at …rst but in fact it is not. The only di¤erence between this problem and the traditional principal-agent problem is that the principal is now an aggregate player o¤ering a contract to the same agent. Proposition 3 says that in equilibrium the principal and the insurers have the incentive to implement the same level of e¤ort. Besides, the sequential nature of the game guarantees us that the cost of the aggregate contract must be identical to the cost in the traditional problem. This allows us to treat the principal and the insurers as an aggregate player and obtain results identical to the ones obtained in the traditional problem. What we have simply worked out in this section is how these $N + 1$ players reach such an aggregate contract. In particular, we were interested on whether the presence of insurers increases the principal's costs of implementing e_h. The answer was no in this environment.

In the case of implementing e_h a key player in the result is the "last" insurer, as we saw he will always be active and the responsible to "convert" the aggregate contract in the traditional 2ndBest: The distinct characteristic of this player is that he knows that the agent

[4] The payment is higher in the "good" states and lower in the "bad" ones in such a way that is the same on average when the agent makes high e¤ort, but lower when he makes low e¤ort.

will not receive another offer after his. This player is such important that if he were not there the results would substantially change. In the following subsection we study the problem when there is no "last" insurer.

4.2 Free entry

In this section we consider the case where there is free entry of insurers who make offers sequentially; that is, the set of insurers is the set of natural numbers, $N = \{1; 2; 3 \dots\}$ When the aggregate contract implements low effort, nothing changes, when it implements high effort we need to make an extension of the analysis from the previous section. Results differ substantially when the agent chooses high effort, and this time the potential entry of insurers makes the principal worse off.

If there is a time where an insurer will decide not to enter, we still have a game where an aggregate player (consisting of the principal and a subset of insurers $I \subseteq N$) makes an offer to an agent. The slight and crucial diﬀerence with the previous section is that this aggregate player is now made up of an unknown number of insurers; that is, the set I is endogenous to the game. Nevertheless, we can still speak of the aggregate contract $W^{aggregate}(\pi) = w^p(\pi) + \sum_{i \in I} w^i(\pi)$.

Every single result of the previous section holds; except for the fact that the second best is no longer the contract that minimizes the aggregate cost of implementing e_h. Therefore, the set of equilibria where the agent implements e_h changes; and this is what we show in the present section.

The fact that the second best cannot be an equilibrium aggregate contract is easily seen from Lemma 4. With free entry, a new insurer would have incentives to enter, offer a flat wage and take the agent to e_l.

As before, let us begin by looking at the principal and the insurers as one player. Now the contract that minimizes this player's cost of implementing e_h must deal with a further constraint: there must be no profit opportunities for an outside insurer from implementing

e_l: Therefore, the cost minimizing contract W solves:

$$\max -\int W(\pi)f(\pi/e_h)d\pi$$

subject to

$$\int v(W(\pi))f(\pi/e_h)d\pi - g(e_h) \geq \bar{u} \quad (4)$$

$$\int v(W(\pi))f(\pi/e_h)d\pi - g(e_h) \geq \int v(W(\pi))f(\pi/e_l)d\pi - g(e_l) \quad (5)$$

$$\int W(\pi)f(\pi/e_l)d\pi \quad \phi[g(e_l)+\bar{u}]: \quad (6)$$

Constraint 6 comes from Lemma 4. It says that whenever previous insurers are implementing e_h as a whole, there are no incentives for a new insurer to take the agent to e_l:

Call this cost minimizing contract the third best, and denote it by $w^{3rdBest}$:

Proposition 8 The third best $w^{3rdBest}$ is the solution of the first order condition

$$\frac{1}{v'(w^{3rdBest}(\pi))} \equiv \frac{\lambda}{1+\delta\frac{f(\pi/e_l)}{f(\pi/e_h)}};$$

and constraints (4) and (6) with equality.

Proof. See Appendix 1.

Proposition 9 The minimum aggregate cost of implementing e_h with sequential free entry of insurers is strictly greater than the cost of implementing e_h without insurers or with a finite number of sequential insurers; that is

$$\int w^{3rdBest}(\pi)f(\pi/e_h)d\pi > \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi:$$

Proof. A revel preference argument show us that $\int w^{3rdBest}(\pi)f(\pi/e_h)d\pi > \int w^{2ndBest}(\pi)f(\pi/e_h)d\pi$, to find the third best there is a new restriction to hold. The strict inequality comes from the fact that the solution to the second best is unique and this solution does not satisfy the "new" restriction. ■

Once we know the cost of implementing e_h for a single aggregate player facing the potential entry of an insurer, we can characterize SPE under e_h in exactly the same way we did in Subsection 4.1.2

A version of Proposition 4 implies that the aggregate contract that implements e_h satisfies

$$\int w^p(\pi) f(\pi/e_h)d\pi - \sum_{i \in A \cap I} \Pi^i(e_h) \geq \int w^{3rdBest}(\pi) f(\pi/e_h)d\pi.$$

Analogously, we look for equilibria where $\int w^p(\pi) f(\pi/e_h)d\pi = \int w^{3rdBest}(\pi) f(\pi/e_h)d\pi.$

Notice that in such a case all insurers that implement e_h make zero profits, and the aggregate contract will always satisfy $\int W^h(\pi) f(\pi/e_h)d\pi = \int w^{3rdBest}(\pi) f(\pi/e_h)d\pi$ for all $h \in I.$

In this case we need that $\int W^{h-1}(\pi) f(\pi/e_l)d\pi \geq \phi[g(e_l) + \bar{u}]$; for $h = 1, ..., N$. If it did not, then by the same argument given in the proof of lemma 4 an insurer would have incentives to enter and take the agent to e_l.

By following exactly the same steps as in the previous section we arrive at the following proposition, which is simply a modified version of Proposition 7.

Proposition 10 In a SPE where the agent chooses e_h,

i) The principal offers a contract w^p such that $\int w^p(\pi) f(\pi/e_h)d\pi = \int w^{3rdBest}(\pi) f(\pi/e_h)d\pi$ and $\int w^p(\pi) f(\pi/e_l)d\pi \geq \phi[g(e_l) + \bar{u}]$; and the agent accepts this contract.

ii) $k \in \{0, 1, ...\}$ insurers are active, make zero profits and offer contracts such that $\int W^h(\pi) f(\pi/e_l)d\pi \geq \phi[g(e_l) + \bar{u}]$ for all $h = 1, ..., L$; where is the last insurer to offer a contract.

iii) The aggregate contract satisfies $W^{aggregate}(\pi) = w^{3rdBest}.$

iv) The agent accepts the aggregate contract, chooses e_h and makes its reservation level of utility $\bar{u}$.

Notice that the set of equilibria include as a particular case the one where the principal offers the first best, $w^p(\pi) = w^{3rdBest}(\pi)$; no insurer enters; and the agent accepts the third best and gets its reservation utility $\bar{u}.$ There are also a continuum of equilibria where the principal o¤ers a payment scheme such that has the same expected payment of the third best (when the agent makes high e¤ort) but riskier, n 2 N insurers enter and each o¤ers a contract such that gives them zero pro…ts and the aggregate is the third best, and the agent accepts this contracts and gets its reservation level of utility $\bar{u}$.

Proposition 11 Taking all the parameters randomly the expected welfare of the society is lower when there is free entry of insurers.

Proof. We can split the values of the parameters in two groups, a) Cases where with no insurers (or …nite number of insurers) the contract implements low e¤ort and b) Cases where with no insurers (or …nite number of insurers) the contract implements high e¤ort.

For the values of the parameters corresponding to a) nothing change. But for the ones corresponding to b) the welfare of the society decreases, the agent and insurers always get the same but the utility of the principal decreases, there are two cases. 1) Even with free entry the contract implements high e¤ort, in this cases the agent will face more risk (third best instead of second best). 2) The contract with free entry of insurers implements low e¤ort. The free entry of insurers make less pro…table to implement high e¤ort in such a way that implementing low e¤ort the principal will be better than implementing high e¤ort. ■

5 Conclusions and Extensions

In this paper we study the robustness of the traditional principal-agent results, to the existence of risk-neutral players not interested in agent´s e¤ort but able to o¤er contracts to the agent. We found that when there are a …nite number of insurers most of the results are not altered. The e¤ort implemented, social welfare, principal´s and agent´s utility do not change. The only di¤erence is that when the contract implements high e¤ort the principal o¤ers a riskier contract to the agent, but the insurers take this new risk. A key player is the last insurer, who is the one that takes the agent to the second best contract.

When there is no "last" insurer, i.e. in the free-entry case, the results change substantially. In this case, the presence of these insurers hurts the economy. The second best contract is no longer feasible. Whenever the principal would implement high e¤ort with exclusive contracts, social welfare is smaller in the absence of exclusive contracts. This could be either because high e¤ort is not implemented or because the cost of implementation is higher.

A natural next step is to analyze the case in which the principal and all the insurers simoultaneously o¤er contracts to the agent. Preliminary computations suggest that in that case, the result could be the third-best contract even when the number of insurers is …nite.

Our results could be used to revisite a large applied principal-agent literature, to ponder whether the assumption of no insurability or of exclusive contracts (necessary for the standard results to obtain) is a reasonable one in each of those contexts.

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Appendix 1

Proof of Proposition 8 : The Problem is

$$\max \; -\int W(\pi) f(\pi/e_h) d\pi$$

subject to

$$\int v(W(\pi)) f(\pi/e_h) d\pi - g(e_h) \geq \overline{u} \tag{7}$$

$$\int v(W(\pi)) f(\pi/e_h) d\pi - g(e_h) \geq \int v(W(\pi)) f(\pi/e_l) d\pi - g(e_l) \tag{8}$$

$$\int W(\pi) f(\pi/e_l) d\pi \geq \phi [g(e_l) + \overline{u}] : \tag{9}$$

The Lagrange mutlpliers associated with the restrictions, the three must be greater than or equal to zero, are $\lambda; \gamma; \delta$.

The first order condition is

$$\frac{1}{v'(W(\pi))} = \frac{\lambda + \gamma \left[1 - \frac{f(\pi/e_l)}{f(\pi/e_h)} \right]}{1 + \delta \frac{f(\pi/e_l)}{f(\pi/e_h)}}$$

As in the traditional principal agent problem the first restriction must hold with equality, following the same arguments of Mas-Collel et al. (1995) see footnote 8, p.485.

We know that $\delta > 0$, otherwise we would have the 2nd best solution but from Proposition 2 we know that equation 9 does not hold for 2nd best.

So equation 9 is:

$$\int W(\pi) f(\pi/e_l) d\pi = \phi [g(e_l) + \overline{u}]$$

which implies that

:

$$v \left(\int W(\pi) f(\pi/e_l) d\pi \right) = v(\phi [g(e_l) + \overline{u}]) = g(e_l) + \overline{u}$$

by Jensen´s inequality we know that

$$\int v(W(\pi)) f(\pi/e_l) d\pi - g(e_l) < \overline{u}$$

using equation 7 (that holds with equality) we know that

$$\int v(W(\pi))\, f(\pi/e_l)\,d\pi - g(e_l) < \int v(W(\pi))\, f(\pi/e_h)\,d\pi - g(e_h)$$

Hence, equation 8 holds with equality and $\gamma = 0$. Therefore the first order condition becomes

$$\frac{1}{v'(W(\pi))} = \frac{\lambda}{1 + \delta \frac{f(\pi/e_l)}{f(\pi/e_h)}}$$

■