Interest Rate Sustainability Risk[¤]

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Abstract

We analyze the joint determination of interest rate risk and debt sustainability for governments with …scal imbalances. Because higher interest rates imply increased debt services they worsen a debtor's …nancial situation, increasing the probability of default. We derive an "interest rate La¤er curve", and we …nd that higher interest rates imply increased growth volatility. Using the "…scal theory of price" we show why countries with public …scal imbalances are more vulnerable to currency attacks.

1 Introduction

Government bond rates in Less Developed Countries (LDCs) are typically higher than in Developed Countries because LDC bonds rates typically re‡ect a premium due to the risk of default. Open Macroeconomics textbooks generally write the (arbitrage) condition between LDC interest rates and the (riskless) international interest rate as,

$$i = i^{¤} + ¢E^{e}=E + °$$

where i is the country's interest rate, $i^{¤}$ is the (riskless) international interest rate, and $¢E^{e}=E + °$ is the default risk premium, which is the sum of the [expected] devaluation of the country's currency and the explicit (sovereign) default. This premium is itself a function of the interest rate, especially in countries with …scal imbalances. Higher interest rates imply higher debt services, which worsen …scal imbalances and decrease the ability of the government to rollover its debt. We examine this endogeneity and develop a theory of sovereign risk for …nancially troubled governments. We …nd that this endogeneity undermines the use of interest rates as a monetary policy instrument for …nancially distressed governments. As a consequence, countries with public …scal imbalances are more vulnerable to currency attacks.

The same endogeneity problem arises in the analysis of public debt solvency. To determine how the Debt/GDP ratio will evolve through time one must know the future path of the interest rates

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on the accumulated debt. This is so because interest rates a¤ect the amount of services to be paid, and therefore the amount of new debt issuing. However, interest rates are themselves functions of the evolution of Debt/GDP: if Debt/GDP is growing then interest rates should be relatively higher (and growing), re‡ecting higher probabilities of default; if, in contrast, Debt/GDP is stable, then interest rates should be relatively lower. By explicitly modeling default risk we show how solvency and interest rates are jointly determined.

Some previous literature has argued that, in the context of international capital markets [see Obstfeld and Rogo¤ (1996)], the notion of solvency – which is satis…ed when the government is able to generate su¢cient surpluses in the future to repay existing debt – is not the appropriate yardstick for evaluating sustainability. Because lenders have only limited powers to punish sovereign borrowers (e.g., governments) directly, the binding constraint on debt repayment may be willingness to pay rather than ability to pay. The literature [see Eaton and Gersovitz (1981), Grossman and Huyck (1988) and Chari and Kehoe (1993)] has typically modeled willingness to pay based on reputational considerations. The government is willing to repay its debt in order to ensure access to capital markets in the future. Thus the level of debt is sustainable when the bene…ts of defaulting do not exceed the costs of being unable to borrow in the future.

This paper has a somewhat di¤erent approach to the notion of sustainability. In contrast with the literature our main focus is on households' willingness to lend. In other words, we examine how the availability of funds imposes constraints on sustainability in addition to the constraints imposed by pure inter-temporal solvency. In our model, the proximate cause of a default is the result of the borrower's decision not to make a loan payment that is due. But that decision, in turn, is the result of a lender's decision not to extend further credit.

To reproduce a …scal imbalanced government decision not make a loan payment we have a stripped down model of willingness to pay. Although the present value of feasible surpluses may theoretically be su¢cient to repay debt, it may not be politically feasible (in particular periods) to cut government expenditures in order to service the debt. This is so because the political-economic situation a¤ects the ability of the government to implement drastic policies without causing social and political upheaval. Because there is uncertainty about the government willingness to meet its obligation, investors may be more or less willing to lend to the government. We then study how the "investment under risk" results are shaped by a inter-temporal constraint.

In recent Balance of Payment (BOP) crises, some governments tried, unsuccessfully, to defend their currencies through sharp raises in the interest rates. One reason for this lack of success seems to have been that the new interest rates were so high that the market did not expect that the government would be able to make payments at those rates. For example, a Deutsche Bank Research Report (1/29/99) reported "In Brazil, the existing link between high interest rates and …scal de…cit through the domestic debt pose a limit to the policy of tight monetary policy. High interest rates are reaching a point where they are ine¢cient to prevent economic agents from running out of the currency". Similarly, The Economist magazine (1/23/99) wrote "The Government clearly hopes that maintaining a tough monetary stance will check the real's fall, while acting as a stick to encourage recalcitrants in Congress to approve tough …scal measures... Nearly all debt pays ‡oating interest rates – adding further to the Government's …scal troubles, and creating what Alan Greenspan has called "a vicious cycle"... If things go well falling interest rates will soon ease the debt burden; if they go badly, it could become unmanageable". We provide a formalization for what was called "the interest rate La¤er curve" in that episode. This result suggests how interest rates may be of limited use especially when they are already high.

Using the Leeper-Sims-Woodford [see Woodford (1995)] "…scal theory of price" we determine the equilibrium price that guarantees debt sustainability. We can then interpret increases in the initial price as currency attacks. Experiments with parameter changes can explain why countries with fiscal imbalances are more vulnerable to currency attacks, even when government have interest rates as an instrument.

Our paper also relates to other strains in the literature. Stiglitz and Weiss (1981) show that credit rationing may arise in the context of banks willingness to lend. In their model the interest rate a bank charges may affect the riskiness of loans by affecting the actions and types of borrowers (i.e., there are both moral hazard and adverse selection problems). In our paper the interest rate on government bonds affect the riskiness of these bonds by changing, through inter-temporal solvency, the probability of default. This in turn may generate a lower amount of sustainable debt, that is the analogous to a credit constraint. Sargent and Wallace (1981) show that there is a trade-off between current and future inflation. In our model the same type of trade-off occurs, but between current price (devaluation) and permanent interest rates.

2 Model

We consider an economy that is an extension of a simple endogenous growth model with linear technology [Rebelo (1991)]. It is populated by a continuum of identical infinitely-lived households with mass one and a government. In each period, households choose how to allocate their resources among consumption (c_t), capital (k_{t+1}), and (risky) nominal government bonds (B_{t+1}/p_{t+1}). Then nature plays the "reservation surplus" (α_{t+1}^{res}) of the government determining whether government defaults or not. Defaulting is reflected in the interest rate that government bonds effectively pay (r_{t+1}) and in the implied government consumption (g_{t+1}). Actions take place in the order depicted in Figure 1:

- Taxes are levied, leaving $(1-\tau)w_t$ units of resources for each household.
- Households allocate their portfolio among c_t, k_{t+1}, B_{t+1}/p_{t+1}.
- Nature plays α_{t+1}^{res}, which determines r_{t+1} and g_{t+1}.
- Production takes place as a function of capital, which determines the amount of resources available to households in the next period (w_{t+1}).

Household preferences are given by,

$$U_h = E\sum_{t=0}^{\infty}\beta^t[u(c_t) + \gamma u(g_t)] \tag{1}$$

where,

$$\begin{aligned} u(c) &= \frac{c^{1-\sigma}}{(1-\sigma)} \text{ for } \sigma > 0 \text{ and } \sigma \neq 1 \\ u(c) &= \ln(c) \text{ for } \sigma = 1 \\ \beta &\in (0,1) \end{aligned}$$

The household budget constraint is,

$$c_t + B_{t+1}/p_{t+1} + k_{t+1} = (1 - \tau)w_t \tag{2}$$

where w_t denotes the real value of household wealth at the beginning of period t, c_t denotes consumption, B_{t+1} denotes the nominal value of government bonds, p_{t+1} denotes the price of the consumption (and investment) good in terms of the price of bonds, and k_{t+1} denotes the level of capital allocated to production.

Real wealth in period $t + 1$ follows from the household's portfolio decision according to,

$$w_{t+1} = (1 + r_{t+1})B_{t+1}/p_{t+1} + Ak_{t+1} \tag{3}$$

where A denotes a technology parameter and r_{t+1} denotes a (stochastic) real interest rate.

We can rewrite the budget constraint in a more usual form by defining product (y_t) and taxes (T_t) as,

$$y_t = [A + (1 - \delta)]k_t \tag{4}$$

$$T_t = \tau[(1 + r_t)B_t/p_t + Ak_t] \tag{5}$$

where δ represents the depreciation rate. We can write the capital formation law as,

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{6}$$

where i_t denotes investment. These definitions imply,

$$c_t + B_{t+1}/p_{t+1} + i_t = (1 + r_t)B_t/p_t + y_t - T_t \tag{7}$$

The government's budget constraint is

$$B_{t+1}/p_{t+1} = (1 + r_t)B_t/p_t + g_t - T_t \tag{8}$$

Summing equations 7 and 8 we obtain the standard economy-wide resource constraint,

$$c_t + i_t + g_t = y_t \tag{9}$$

In order to focus our analysis on interest rate risk we assume that monetary policy is determined exogenously. Because in our model there is no money, monetary policy is specified by the "latent" interest rate, ρ. This is as if we had assumed that there is a Monetary Authority that independently sets the "face value" of the (real) rate of interest. We could incorporate money into the model via a "cash-in-advance" specification. If so, the Monetary Authority would set both nominal interest rates and inflation (money growth). However, the "real interest rate rule" that we have chosen is a convenient simplification and does not affect our analysis.

To model the occurrence of default, we assume that government primary surplus is constrained to be lower than a certain fraction of the product,

$$s_t \equiv (T_t - g_t)/y_t < \sigma_t^{res} \tag{10}$$

where s_t is primary surplus as a fraction of GDP, and σ^{res}, the "reservation surplus", is a random variable, serially uncorrelated and identically distributed over time according to the cumulative distribution function $F(.)$. Where government decisions are affected by interest groups and the executive power is constrained by the legislature, σ^{res} can be interpreted as a restriction on the amount the government must spend in the period, given its budgetary resources. More generally, we interpret σ^{res} as a measure of the political-economic environment, which reflects the ability of the government to implement policies without causing social and political upheaval.

The occurrence of default is assumed to be completely determined by the "reservation surplus" constraint. Government defaults if and only if honoring the latent interest rate would imply having a surplus higher than σ_t^{res}. That is, government evaluates how much the surplus would have to be in order to pay a interest rate equal to ρ on its debt. If this surplus $[s_t(\rho)]$ is smaller than the reservation surplus than the government actually pays the interest rate: $1 + r_{t+1} = 1 + \rho$. Otherwise the government defaults, generating a lower surplus. Notice that $s_t(r_{t+1})$ must be a decreasing function for this government modeling make sense.

To obtain a closed form solution, we further assume that the "effective" interest rate, r_{t+1}, can only take two values. Either there is no default (and then $1 + r_t = 1 + \rho$) or there is default, in which case the effectively paid interest rate is given by $1 + r_t = (1 + \rho)(1 - \theta)$. We can think of $\theta \in [0; 1]$ as a default rate.When $\theta = 1$ the government "fully" defaults on its debt.

Government "decision" to default can be written as

$$\begin{aligned} 1 + r_{t+1} &= 1 + \rho \text{ if } s_t(\rho) < \sigma_t^{res} \\ 1 + r_{t+1} &= (1 + \rho)(1 - \theta) \text{ otherwise} \end{aligned}$$

We assume that government policy $(\rho; \theta; F; \tau)$ is exogenous and set at the beginning of time and study the resulting "Stationary Competitive Equilibrium". A competitive equilibrium is the set of allocations that result when households maximize utilility given their resource constraint and government policy and market clears. The competitive equilibrium is stationary if the household's portfolio allocations don't change with time. That is, the fraction of wealth allocated to consumption, production and government bonds remain the same. We can represent an stationary equilibrium by the set $(\lambda_1; \lambda_2; \pi)$, where λ_1 and λ_2 are the portfolio allocation,

$$\lambda_1 \equiv \frac{B_{t+1}/p_{t+1}}{(1 - \tau)w_t} \tag{11}$$

$$\lambda_2 \equiv \frac{k_{t+1}}{(1 - \tau)w_t} \tag{12}$$

and π is the probability of default:

$$\begin{aligned} 1 + r_{t+1} &= 1 + \rho \text{ with probability } (1 - \pi) \\ 1 + r_{t+1} &= (1 + \rho)(1 - \theta) \text{ with probability } \pi \end{aligned} \tag{13}$$

3 Characterization of Equilibrium

3.1 Dynamics

Households' portfolio allocate ther resources among consumption, a riskless asset with rate of return A, and a risky asset with stochastic rate of return given by 13, where ¼ is taken as exogenous (price taking). This portfolio allocation joint with government budget constraint 8 imply s_t(½), what in its turn determines ¼. More formally,

Proposition 1 Equilibrium allocations are given by,

$$
\begin{aligned}
B_{t+1}=p_{t+1} &= ¸_1(1 ¡ ¿)w_t \\
k_{t+1} &= ¸_2(1 ¡ ¿)w_t \\
c_t &= (1 ¡ ¸_1 ¡ ¸_2)(1 ¡ ¿)w_t \\
w_{t+1} &= [A¸_2 + (1 + ½)¸_1](1 ¡ ¿)w_t; \quad \text{if } ¾_1 > ¾^{res}_{t+1} \\
w_{t+1} &= [A¸_2 + (1 + ½)(1 ¡ µ)](1 ¡ ¿)w_t; \quad \text{if } ¾_1 < ¾^{res}_{t+1}
\end{aligned}
$$

where

$$
\begin{aligned}
¸_1 &= (¯A)^{1="}[(1 + ½)µ]^{(1 ¡ ")="} f[(1 ¡ ¼)=(A ¡ (1 + ½)(1 ¡ µ))]^{1="} ¡ [¼=(1 + ½ ¡ A)]^{1="} g \\
¸_2 &= ¯^{1="} A^{(1 ¡ ")="}[(1 + ½)µ]^{(1 ¡ ")="} f(1 + ½)[¼=(1 + ½ ¡ A)]^{1="} ¡ \\
&\quad (1 + ½)(1 ¡ µ)[(1 ¡ ¼)=(A ¡ (1 + ½)(1 ¡ µ))]^{1="} g \\
¼ &= F(¾_1) \\
¾_1 &= f(1 + ½) ¡ [(1 + ½)¸_1 + A¸_2](1 ¡ ¿)gf¸_1=[¸_2(A ¡ 1 + ±)]g
\end{aligned}
$$

Proof. See appendix. ■

The dynamics of this equilibrium are displayed in Figure 2, where, by hypothesis, there is default in period t. Notice that before default (up to t ¡ 1), all variables grow together, maintaining the same proportions (the vertical axis scale is logarithmic). In period t there is a drop in private wealth, w_t, because $1 + r_t = (1 + ½)(1 ¡ µ)$. As a consequence, households consume less and allocate less to capital and bonds. Government consumption increases in period t, since government does not have to pay interest on its debt, and private investment decreases to adjust capital to its new path. Output is delayed one period with respect to wealth, and therefore only su¤ers in period t + 1. From period t + 1 on, the economy is adjusted again, and the variables maintain the same proportions they had before the default. In particular, the debt/GDP ratio is always constant.

3.2 Sustainability

We now examine how the equilibrium changes when interest rates are increased. This requires that we …rst examine how government debt can be sustained under di¤erent interest rate rules.

In order for debt to be sustainable, the growth rate of debt must be no larger than the growth rate of private wealth. If debt grows more quickly than private wealth, the magnitude of the debt would eventually surpass the money available to service it (which is bounded above by private wealth), which is clearly not sustainable.

Debt consists of two components: debt services and government primary de…cit. Thus the debt growth rate is a function of interest rates and the de…cit/debt ratio,

$$(B_{t+1}=p_{t+1})=(B_t=p_t) = (1 + r_t) + (g_t ¡ T_t)=(B_t=p_t)$$

Private wealth also consists of two components: physical output and debt services. Thus the private wealth growth rate is a weighted average of productivity and interest rates, where the weights are determined by the portfolio allocation between production and debt,

$$w_{t+1}/w_t = (A\lambda_2 + (1 + r_t)\lambda_1)(1 - \tau)$$

In order to maintain equality between these two growth rates when interest rates increase (which increases debt services), at least one of two things must happen: (i) government surplus relative to debt must increase (offsetting the increase in debt services and thereby preventing an increase in the growth rate of debt), or (ii) the household portfolio allocation must change (implying an increase in the growth rate of private wealth). The second possibility suggests that, in principle, it is possible to raise interest rates and wealth growth without reducing government surplus. But, to really understand the problem, and investigate this possibilities one must study households' portfolio allocation under risk.

In order to simplify the analysis we initially restrict our attention to an economy with $\varepsilon = 1$ (logarithmic preferences) and $\theta = 1$. Although there is no consensus among economists regarding the [aggregate] elasticity of savings with respect to the rate of return, a large segment of the literature argues that this elasticity is very low - i.e., savings do not vary significantly with the rate of return [see Hall (1988)]. In our model, $\varepsilon = 1$ implies that this elasticity is zero. In this case, the fraction of resources allocated to consumption by households is $(1 - \lambda_1 - \lambda_2) = 1 - \beta$. The remaining fraction, β, is allocated either to government bonds or production, depending on the values for $(1 + \rho)$ and π.

With these assumptions, government surplus when there is no default (σ_1)and when there is default (σ_2) are given by,

$$\sigma_1 = [(1 + \rho)(1 - \pi) - A][1 - \beta(1 - \pi)(1 - \tau)]/[\pi(A - 1 + \delta)] \tag{14}$$

$$\sigma_2 = -A\beta(1 - \tau)[(1 + \rho)(1 - \pi) - A]/[(1 + \rho - A)(A - 1 + \delta)] \tag{15}$$

The assumption of full default ($\theta = 1$) is algebrically convenient and does not seem do reduce the generality of government policies, if we don't restrict $F(.)$ to a special form. Notice, in particular, that expected surplus, $E[s]$, remains very general:

$$E[s] = [1 - F(\sigma_1)]\sigma_1 + F(\sigma_1)\sigma_2 = \sigma_1 + (\sigma_2 - \sigma_1)F(\sigma_1)$$

From 14 and 15 one can see that $\sigma_1 > 0$, $\sigma_2 < 0$, $\partial\sigma_1/\partial\rho > 0$ and $\partial\sigma_2/\partial\rho < 0$[1]: $F(.)$ must be non decreasing, but the magnitude of $\partial F(\sigma_1)/\partial\rho$ is to be determined by the form of $F(.)$. These information alltogether indicate that $\partial E[s]/\partial\rho$ has indeterminate sign and dimension.

To have some intuition for the political-economical environment behind this government modeling we can define,

Definition 2 (Whorishness) Government I is more whorish than government II if σ_{II}^{res} first order stochastically dominates σ_I^{res}. Equivalently, $F_I(\sigma^{res}) \geq F_{II}(\sigma^{res})$ for any σ^{res}.

Figures 3 depicts the equilibrium determination when interest rates are increased. The two curves depicted are equation 14, and equation,

$$\pi = F(\sigma_1) \tag{16}$$

[1] Note that to have a positive amount of government bonds it is necessary to have $(1 - \pi)(1 + \rho) > A$.

Equation 14 guarantees the inter-temporal solvency of the government. It says that government expenditures must be such that it is possible to rollover the debt. In other words, the growth of real debt must be equal to the growth of the economy, so that the debt can continuously be rolled over. Notice that $\partial\sigma_1/\partial\pi < 0$.

Equation 16 indicates how the solvency expenditures level affects the chances of default. It says that when the government must spend less (i.e., when s_1 is larger) then there is a higher probability of default. Equilibrium is determined by the intersection of the two curves. $\lim_{\pi\to 0}(\sigma_1) = \infty$ and $\lim_{\pi\to 1}(\sigma_1) < 0$ imply that $\lim_{\pi\to 0}[\sigma_1 - F(\sigma_1)] > 0$ and $\lim_{\pi\to 1}[\sigma_1 - F(\sigma_1)] < 0$. Therefore there is at least one equilibrium. Because $\partial\sigma_1/\partial\pi < 0$ and $\partial F(\sigma_1)/\partial\sigma_1 > 0$, the equilibrium is unique. ****Fix assumptions on F here****

We can thus state the comparative statics that follow:

Proposition 3 Higher ("latent") interest rates (ρ), lower return of production (A), higher tax rates (τ), lower intertemporal discount parameter (β), lower depreciation rate (δ) and more whorish government [$F(\cdot)$] all imply higher probability of default (π).

Proof. Notice that $\partial\sigma_1/\partial\rho > 0$, $\partial\sigma_1/\partial A < 0$, $\partial\sigma_1/\partial\tau > 0$, $\partial\sigma_1/\partial\beta < 0$, $\partial\sigma_1/\partial\delta < 0$. Therefore the indicated changes imply that curve 14 shifts to the right. A more whorish government is represented by a shift of 16 to the right. ■

3.3 The Interest Rate Laffer Curve

Maintaining the assumptions $\varepsilon = 1$ and $\theta = 1$, we have that the fraction of resources allocated to government bonds is given by,

$$\lambda_1 = \beta\{1 - \pi(1+\rho)/[1+\rho-A]\} \tag{17}$$

The derivative of this fraction with respect to the interest rate is,

$$\frac{\partial\lambda_1}{\partial\rho} = \frac{\beta[A\pi - (1+\rho)(1+\rho-A)\partial\pi/\partial\rho]}{[(1+\rho-A]^2} \tag{18}$$

Because $\partial\pi/\partial\rho > 0$, the sign of the derivative is indeterminate, but we can prove that

Proposition 4 If $\pi \in (0,1)$ and $\partial\pi/\partial\rho \in [L_1, L_2]$, where L_1 and L_2 are two positive constants, then:

(i) $\lim_{1+\rho\to A}[\partial\lambda_1/\partial\rho] > 0$
(ii) $\lim_{1+\rho\to\infty}[\partial\lambda_1/\partial\rho] < 0$

This proposition indicates that the demand for bonds eventually decreases with ρ, what sugests the presence of an "Interest Rate Laffer Curve", as the one depicted in Figure 4.

To further examine this phenomenon consider an example in which, for the relevant region of equilibrium determination, $\pi = a\rho + b$, where a and b are positive constants. We know from proposition 2 that π is increasing on ρ, so this is the simplest possible case. The demand for government bonds becomes $\lambda_1 = \beta\{1 - (a\rho + b)(1+\rho)/[1+\rho-A]\}$. Note that $\partial\lambda_1/\partial A < 0$, $\partial\lambda_1/\partial a < 0$ and $\partial\lambda_1/\partial b < 0$. An increase in A means that the opportunity cost of holding bonds increases. An increase in a or b means a more whorish government.

From $\frac{\partial\lambda_1}{\partial\rho} = 0$ and equation 18 we get the parabola

$$-a\rho^2 + 2a(A - 1)\rho + Ab - a + Aa = 0$$

One of the roots correspond to a interest rate lower than A, and therefore is not relevant. The other root, which we call ρ^{Laffer}, exactly as in figure 4, has the properties (see appendix): $\partial\rho^{Laffer}/\partial A > 0$, $\partial\rho^{Laffer}/\partial b > 0$ and $\partial\rho^{Laffer}/\partial a < 0$.

[**** Intuition ??? ***]

3.4 Currency Attacks

In this section we "shock" the previous economy in order to simulate a currency attack. In order to find the equilibrium for the new economy we use the Leeper-Sims-Woodford "fiscal theory of price." For example, suppose that at time $t = 0$, just after households consume and make their portfolio allocation decisions, (i) the political-economic environment of the country changes, yielding a higher value for the parameters a or b; or (ii) international interest rates increase, increasing A. In both cases the demand for bonds as a fraction of wealth and the amount of sustainable bonds are reduced. In order to have the economy in equilibrium we need,

$$(B_1/p_1)/[(B_1/p_1) + k_1] = \lambda_1/(\lambda_1 + \lambda_2)$$

Because $\lambda_1/(\lambda_1 + \lambda_2)$ decreases with the change, and because B_1 and k_1 are fixed, the economy must re-equilibrate via an increase in p_1. We can interpret this change in prices as a devaluation of the currency.

An alternative interpretation for the portfolio allocation problem is that households decide whether to invest in government (domestic) bonds or in international riskless bonds. The international bonds take the place of the physical capital and have the same riskless return, equal to A. Under this interpretation, a currency attack corresponds to a portfolio reallocation and a sudden search for international assets (e.g., dollars) in exchange for domestic assets (i.e., government bonds or the local currency), with a consequent change in the exchange rate (or prices).

Suppose that monetary authority wants to prevent the devaluation by raising ρ. Our Laffer curve analysis indicate that it may be impossible to prevent an attack, because after a certain level the demand for bonds decreases with ρ. More importantly, countries with fiscal imbalances are more vulnerable to currency attacks even when governments can use interest rates as a policy instrument.

3.5 Interest Rate Rules

Next we examine the dynamics of the economy in response to changes in interest rates. Suppose the economy is similar to that depicted in Figure 3a and that the parameters are such that the economy is on the "right side" of the laffer curve, i.e., $\rho < \rho^{Laffer}$.

In this case an increase in ρ implies an increase in π and in σ_1. Because we assumed that the economy is on the right side of the Laffer curve there is an increase in the demand for government bonds which implies, by the fiscal theory of price, a decrease in the price level (or a reduction in inflation).

Since the growth rate of the economy can be expressed as $w_{t+1}/w_t = A\lambda_2 + (1 + \rho)\lambda_1$, and for $\varepsilon = 1$, $\lambda_2 + \lambda_1 = \beta$, the new growth rate of the economy after an increase in ρ is larger than before when there is no default, and smaller than before when there is a default. In addition, defaults happen more frequently. In other words, growth is more volatile.

3.6 General Case

In Figure 5 we show how the government budget constraint is changed for general Constant Absolute Risk Aversion (CARA) utility functions ($\gamma \neq 1$) and when default is not full ($\theta < 1$).(*****to be done****)

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5 Appendix: Proof of Proposition 1

Because the public good is separable and households are atomistic, a household's problem can be written as,

$$V(w) = \text{Max}_{\lambda_1, \lambda_2} \{u(c) + \beta E V(w')\}$$

such that

$$\begin{aligned} c &= (1 - \lambda_1 - \lambda_2) w (1 - \tau) \\ w' &= [A\lambda_2 + (1+\rho)\lambda_1] w (1 - \tau) \text{ with } (1 - \pi) \\ w' &= [A\lambda_2 + (1+\rho)(1 - \theta)\lambda_1] w (1 - \tau) \text{ with } \pi \end{aligned}$$

In order to simply the algebra, we transform the variables λ_1 and λ_2 into α_1 and α_2 according to,

$$\begin{aligned} [A\lambda_2 + (1+\rho)\lambda_1] &= \alpha_1 \\ [A\lambda_2 + (1+\rho)(1 - \theta)\lambda_1] &= \alpha_2 \end{aligned}$$

This implies,

$$
\begin{aligned}
\lambda_1 &= (\alpha_1 - \alpha_2)/[(1+\rho)\theta] \\
\lambda_2 &= [(1+\rho)\alpha_2 - (1+\rho)(1-\theta)\alpha_1]/[A(1+\rho)\theta] \\
1 - \lambda_1 - \lambda_2 &= 1 - \alpha_1[A - (1+\rho)(1-\theta)]/[A(1+\rho)\theta] \\
&\quad - \alpha_2[(1+\rho) - A]/[A(1+\rho)\theta]
\end{aligned}
$$

We define C_1 and C_2 as,

$$
\begin{aligned}
C_1 &= [A - (1+\rho)(1-\theta)]/[A(1+\rho)\theta] \\
C_2 &= [(1+\rho) - A]/[A(1+\rho)\theta]
\end{aligned}
$$

We guess the functional form for V as $V(w) = Fw^{1-\varepsilon}/(1-\varepsilon)$ and using the transformed variables obtain,

$$
\begin{aligned}
Fw^{1-\varepsilon} &= \mathrm{Max}_{\alpha_1;\alpha_2}\{(1 - C_1\alpha_1 - C_2\alpha_2)(w(1-\tau))^{1-\epsilon} + \\
&\quad \beta(1-\pi)F\alpha_1(w(1-\tau))^{1-\varepsilon} + \beta\pi F\alpha_2(w(1-\tau))^{1-\varepsilon}\}
\end{aligned}
$$

This maximization implies two first order conditions and a parameter identification equation given by,

$$
\begin{aligned}
C_1(1 - C_1\alpha_1 - C_2\alpha_2)^{-\varepsilon} &= \beta(1-\pi)F\alpha_1^{-\varepsilon} && (19) \\
C_2(1 - C_1\alpha_1 - C_2\alpha_2)^{-\varepsilon} &= \beta\pi F\alpha_2^{-\varepsilon} && (20) \\
(1 - C_1\alpha_1 - C_2\alpha_2) + \beta(1-\pi)F\alpha_1 + \beta\pi F\alpha_2 &= F && (21)
\end{aligned}
$$

These three equations can be solved for the variables α_1, α_2 and F. Dividing the first equation 19 by equation 20,

$$\alpha_1 = [(C_2(1-\pi))/(C_1\pi)]^{1/\varepsilon}\alpha_2 \qquad (22)$$

Plugging 22 back in 20, and 20 and 22 in 21, we get respectively,

$$
\begin{aligned}
\{C_1[(C_2(1-\pi))/(C_1\pi)]^{1/\varepsilon} + C_2 + [C_2/(\beta\pi F)]^{1/\varepsilon}\}\alpha_2 &= 1 \\
\beta\pi/C_2\{C_1[(C_2(1-\pi))/(C_1\pi)]^{1/\varepsilon} + C_2 + [C_2/(\beta\pi F)]^{1/\varepsilon}\}\alpha_2^{1-\varepsilon} &= 1
\end{aligned}
$$

Eliminating α_2, we solve for F,

$$F^{1/\varepsilon}[1 - \beta^{1/\varepsilon}(1-\pi)^{1/\varepsilon}C_1^{-(1-\varepsilon)/\varepsilon} - \beta^{1/\varepsilon}\pi^{1/\varepsilon}C_2^{-(1-\varepsilon)/\varepsilon}] = 1$$

And then for α_1 and α_2,

$$
\begin{aligned}
\alpha_1 &= [\beta(1-\pi)/C_1]^{1/\varepsilon} \\
\alpha_2 &= [\beta\pi/C_2]^{1/\varepsilon}
\end{aligned}
$$

Returning for the solutions in λ_1 and λ_2,

$$
\begin{aligned}
\lambda_1 &= (\beta A)^{1/\varepsilon}[(1+\rho)\theta]^{(1-\varepsilon)/\varepsilon}\{[(1-\pi)/(A - (1+\rho)(1-\theta))]^{1/\varepsilon} - [\pi/(1+\rho-A)]^{1/\varepsilon}\} \\
\lambda_2 &= \beta^{1/\varepsilon}A^{(1-\varepsilon)/\varepsilon}[(1+\rho)\theta]^{(1-\varepsilon)/\varepsilon}\{(1+\rho)[\pi/(1+\rho-A)]^{1/\varepsilon} - \\
&\quad (1+\rho)(1-\theta)[(1-\pi)/(A - (1+\rho)(1-\theta))]^{1/\varepsilon}\}
\end{aligned}
$$

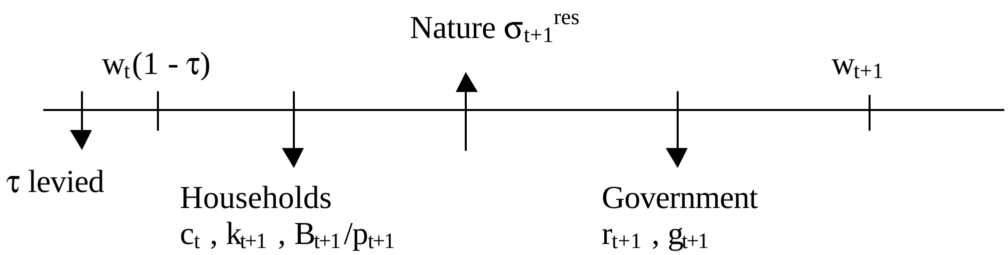

Figure 1: Timeline

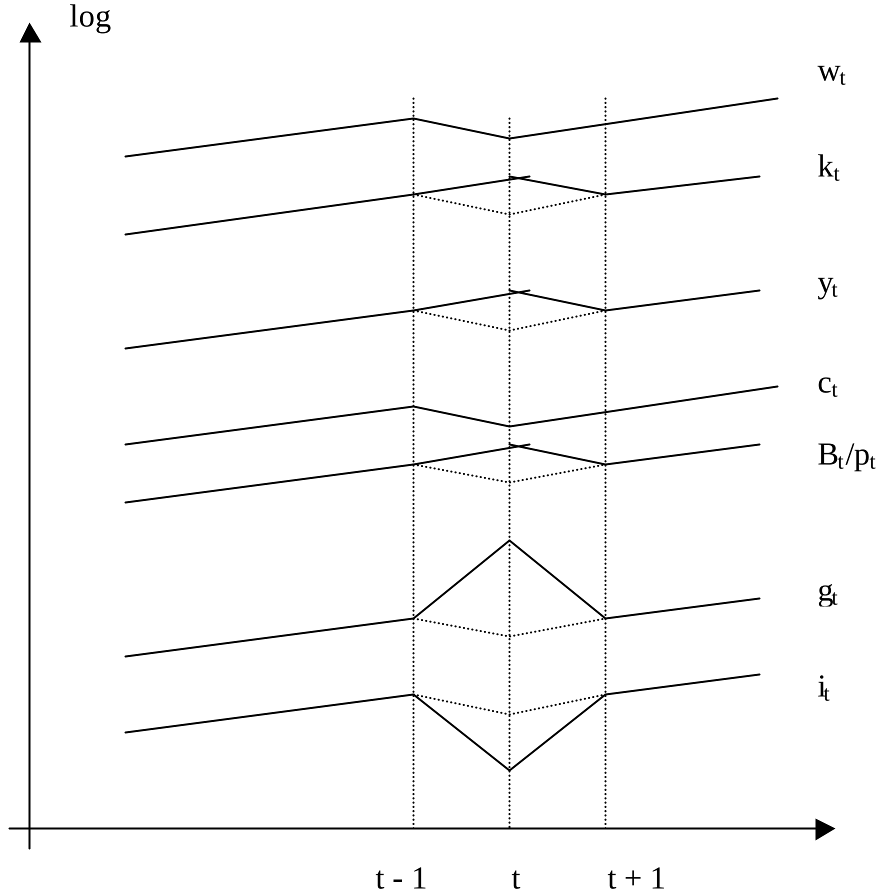

Figure 2: Dynamics

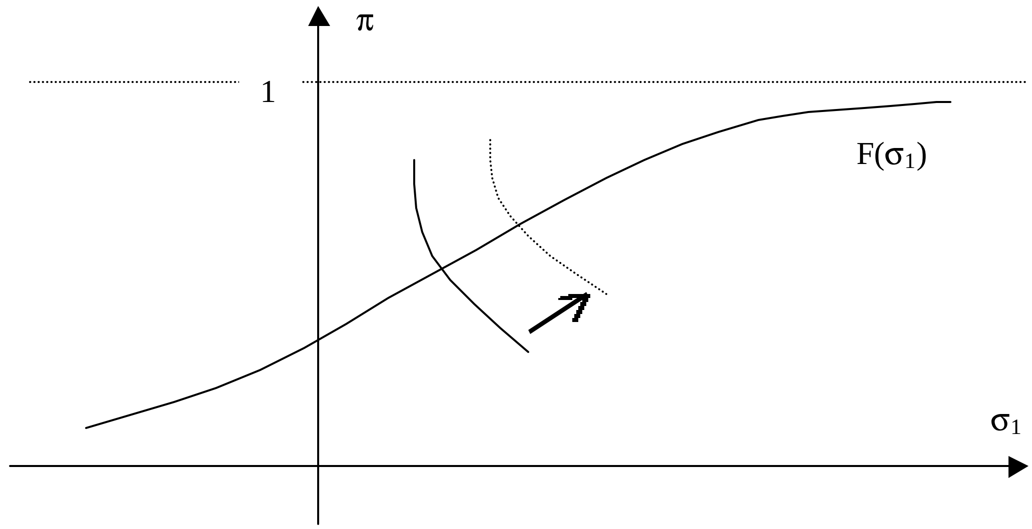

Figure 3: Sustainability

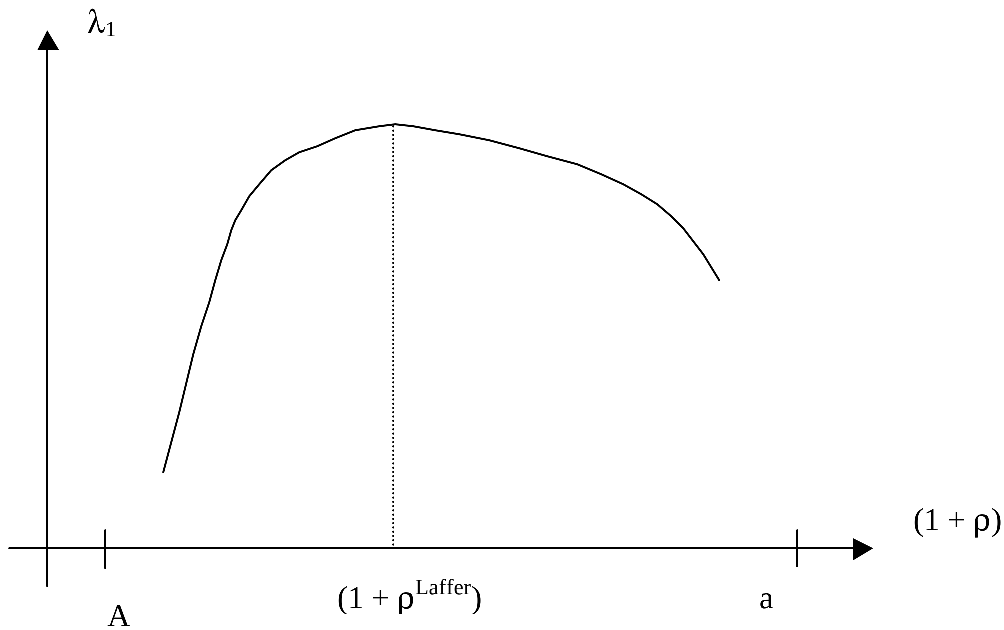

Figure 4: Interest rate Laffer curve