International and Local Lenders of Last Resort in a small open economy with aggregate risk [¤]

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Abstract

In a two - currency, …xed exchange rates version of the Bryant - Diamond and Dybvig, small open economy model with aggregate liquidity shocks I study the role of an international and a local lenders of last resort in both implementing the …rst best allocation and preventing self ful…lling bank runs. The …rst best allocation implies that perfect insurance against aggregate liquidity risk, contrary to the partial suspension of convertibility result in Wallace (1988). It is shown that an international liqiudity-providing institution may implement the …rst best without runs. in contrast with the well-known result by Chang and Velasco (1998, 2000), when a local lender of last resort is needed. It is also shown that the International Lender of Last Resort in general prevent runs, independent of whether it implements the …rst best or not. An interim-date borrowing constrained social optimum is also characterized. A second best problem arises when borrowing constraints are introduced. Here Wallace' s (1990) partial suspension of convertibility is obtained. The …rst best results on implementation as well as the prevention of runs remains true also in the second best case.

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1. Introduction

Since the Asian Financial Crisis in 1997 [1] many discussions about banking regulation have taken place at di¤erent levels. One of the most recent ones has been related to the role of international institutions (such as the IMF) in preventing …nancial crises. In October 1998 a new credit line was approved by the G7 to help "vulnerable but essentially healthy nations" (see New York Times [16]). Indeed, the G7 group declared on October 30, 1998 that

> The statement...re‡ects the shared determination of the U.K. and G7 to modernize the …nancial system and to put in place new rules and procedures that will promote stability and growth. It a¢rms that the G7 commit themselves to: (...)
>
> - develop improved procedures for managing crises and preventing them from spreading, including an enhanced IMF …nancing mechanism supported by private and bilateral …nance as appropriate..

The chancellor 's statement following the G7 declaration a¢rms explicitly the creation of a "supplementary reserve facility which would provide a contingent short term line of credit for countries pursuing strong IMF approved policies. This facility could be drawn upon in times of need and would entail appropriate interest rates along with shorter maturities. "[2]

Although this statement also includes issues such as global regulatory regimes; which are not clari…ed yet, it is explicit in terms of creating a reserve for short term credit in order to prevent / solve …nancial distress for those economies following an IMF approved policy. This implicitly states that the distress usually should be mostly associated with liquidity rather than solvency issues, at least from an o¢cial point of view. If the IMF tends to approve policies that do not induce risky investments by the …nancial sector, this then leads to the conclusion that this line would hardly be applied to countries with …nancial crises due to solvency problems.

Although the last remark is debatable, it certainly states that if a …nancial sector has only a liquidity problem, then it is likely to be bene…ciary of such a credit line. The objective of the present work is to show that this credit line would be useful in preventing bank runs in small open economies with short run liquidity shocks. It is also a question analyzed in this paper the implementation

[1] For surveys on the Asian Crisis see [6] and [8], among others.

[2] See Chancellor's statement, including in the G7 statement.

of a social optimal solution by a system with such an institution. The idea is that, in general, unless coupled with a local lender of last resort, the international institution may fail to implement the …rst best. However this feature may not always be necessarily true.

This paper uses a version of the Bryant - Diamond - Dybvig model ([3] and [11]) in a small open economy [3], in the spirit of Chang and Velasco [5], with the addition of aggregate liquidity shocks. I incorporate them using the same device as in Wallace [18]. I …rst study the socially e¢ cient allocation with unbounded foreign credit in the interim period. I demonstrate that the social optimum is characterized by an absence of partial suspension of deposits (in the sense of Wallace [18]). The main result is the fact that the per-capita consumption for impatient agents is non-random, regardless of the type and liquidity shock realization. This is obvious since the …rst best implies perfect risk sharing among all consumers.

Regarding implementation issues, I construct a banking system that shares many features with Chang and Velasco [4]. The only di¤erence is the fact that both types of consumers (impatient and patient) derive utility from local currency holdings. The …rst result in this regard states that, when the number of impatient consumers is high, an international lender may implement the …rst best allocation. However this is not always true. The main condition is that, when the optimal consumption of local currency is su¢ ciently low, then the presence of the international lender ensures implementation of the …rst best. Otherwise a local lender of last resort is essential for a banking system to implement the social optimum.

The illiquidity conditions for bank runs to arise are also analyzed. They are similar to those presented in Wallace [18]. I introduce an international lender of last resort (what the G7 group ultimately had created). I show that this institution always prevents runs, regardless of implementation. If there are patient consumers who behave as impatient in the interim period the international lender takes care of them. This implies that the banking system does not have to make early liquidations of the long asset. An international lender of last resort providing liquidity in foreign currency coupled with a local lender of last resort (providing liquidity in the local currency) always prevents runs as well as implements the …rst best solution.

Since credit lines in reality are far from being as perfect as in the …rst model, I introduce exogenous borrowing constraints in the social planner's problem. The main di¤erence is the re-appearance of the partial suspension result. That is,

[3] For a survey on the bank runs literature see [13], chapter 7:

with a borrowing constraint at date 1 the optimal allocation implies more date-1 consumption for the …rst group of impatient agents that for those in the second order in line. This states then that even though in the interim period the country has credit available to cover a higher withdrawal pattern, borrowing constraints are enough to reintroduce a di¤erence in period 1 consumption. In terms of implementation similar results hold relative to the …rst best allocation.

Section 2 discusses the literature on bank runs in closed and small open economies. Section 3 studies the economy as well as the …rst best solution. Section 4 discusses illiquidity conditions as well as the role of lenders of last resort to prevent runs. Section 5 adds borrowing constraints to the planner's problem of the economy in section 3. Section 6 discusses some policy implications. Finally section 7 presents concluding remarks and points of future research.

2. Related Literature

As mentioned before, I follow the Bryant - Diamond and Dybvig tradition ([3], [11]). The main feature in the current paper is the presence of two currencies instead of the typical unique type of money in the literature. However the two papers share with the standard literature the potential existence of two equilibria, one involving runs.

The two main antecessors are the papers by Chang and Velasco (see [4] and [5]). They construct a Bryant - Diamond - Dybvig model in a small open economy. In [5] the long term investment is …nanced partially by international borrowing (to be paid in the last period). The consumption for impatient is …nanced by short term international funds to be paid at the end of the economy. In my model the utility functions of both types depend upon real local currency holdings.

The device to treat the aggregate uncertainty case is taken from Wallace ([17] and specially [18]). The amount of short run withdrawals is itself stochastic. In the …rst paper [17] he shows that the two run-preventing regulatory regimes studied by Diamond and Dybvig cannot be implemented, due to the non-observability of proportion of impatient consumers. In the second paper [18] Wallace presents a special case in which the banking system 's manager can learn the proportion through the order in which consumers withdraw in the interim period. He shows that declaration of partial suspension of convertibility of deposit contracts is part of the planner's allocation. I use this special case to redo the exercise with two currencies.

3. The Economy.

The economy lasts for three periods: $t = 0, 1, 2.$ There are two currencies, called the home currency (called pesos) and the foreign currency (called dollars). I also use the term money interchangeably with the term currency. There is one consumption good, which is the numeraire. To simplify I identify this good directly with the foreign currency. There is a storage technology for the good. This technology returns one unit in period $t + 1$ for each unit of the currency invested in period t, where $t = 0, 1.$ On the other hand there is a long-term technology. For each unit of the good invested in the long term technology at date 0 it returns $R > 1$ units of the same type of money in period 2, but only $r \in (0, 1]$ in period $1.$ The economy takes the price of the foreign currency as given since it is small. I assume that this price to be one.

There is a measure one of ex-ante identical consumers. At the beginning of period 1 each person receives an idiosyncratic preference shock. This determines whether the consumer survives until period 2 or dies at period $1.$ The ex-ante probability of dying in period 1 is π. The person who survives until date 1 is called impatient, otherwise she is patient. However, unlike the traditional model, this probability is stochastic and unknown ex-ante. In period 0 the proportion of impatient is a random variable. For simplicity I adopt the device presented by Wallace ([17] and specially [18]). Assume that π can be either $p\alpha + (1 - p)$, with probability q_1 and $p\alpha$, with probability $q_2.$ The economy knows that at least a proportion αp of the people are impatient (and $(1 - \alpha)\, p$ are patient consumers). There could be still other $(1 - p)$ impatient consumers (with probability q_2). Otherwise they are all impatient (with probability q_1). Any person is within the first group with probability p and within the second group with probability $(1 - p)$.

If a person is impatient her utility function is

$$u(c_1) + v(m_1)$$

while if she is patient it is

$$u(c_2) + v(m_2)$$

Here c_t denotes consumption of dollars at date t, while m_t is the consumption of pesos. The function u is C^2, strictly increasing and strictly concave. The function v is C^2 and strictly concave. However it possesses a satiation level $\bar{m} > 0.$ This is consistent with Chang and Velasco [4]. The difference is that not only patient agents, but also the impatient ones derive utility from local currency real holdings. In some sense utility functions are symmetric unlike preferences in the cited paper.

Hence the ex-ante utility is now:

$$
\begin{aligned}
& p\left[\alpha\left(u\left(c_1^1\right)+v\left(m_1^1\right)\right)+(1-\alpha)\left(\sum_{s=1}^{2} q_s\left(u\left(c_2^1(s)\right)+v\left(m_2^1(s)\right)\right)\right)\right] \\
& +(1-p)\left(q_1\left(u\left(c_1^2(1)\right)+v\left(m_1^2(1)\right)\right)+q_2\left(u\left(c_2^2(2)\right)+v\left(m_2^2(2)\right)\right)\right)
\end{aligned}
\tag{3.1}
$$

where $c_t^j(s)$ denotes consumption of dollars in period t, state s and position-in-line j; and similarly for $m_t^j(s)$. Here $s = 1$ denotes the state in which all the people in the second group are impatient, and $s = 2$ corresponds to the state in which all of them are patient. Similarly, $j = 1$ denotes the (individual) state in which the consumer is in the first group, while $j = 2$ denotes the state in which the consumer is in the second group. Note that consumption of impatient consumers within the first group does not depend on s; i.e., c_1^1 and m_1^1 are both independent of s. This is because I assume that the planner does not observe the aggregate state s. She must infer it from the number of impatient trying to withdraw at date 1. If only αp impatient consumers show up, the planner infers (correctly) that the proportion of impatient people is αp and then the state is $s = 1$. If the proportion exceeds αp, this must be clearly equal to $(\alpha p + (1 - p))$ (provided that nobody lies). Hence whenever the planner observes that there are more impatient consumers than αp, then she infers that the state is $s = 2$. However, in any case, people who were lucky showing up first (among the first αp) should receive the same consumption in period 1 since the planner cannot know the state at that stage, due to the sequential service constraint.

3.1. The planner's problem.

The planner borrows an amount of d^1 units of the consumption good at date 0. Immediately after this she decides how much to invest in the short run investment and how much in the long run asset. At the beginning of period 1 the preference shock is realized at the individual as well as at the aggregate level. The planner does not know at that time whether the proportion of impatient agents is high or low. She must learn it through the actual amount of agents withdrawing at date 1. The planner is able to differentiate patient consumers from impatient ones, though. Impatient consumers are ordered through a queue. If the proportion of impatient agents is αp (state 2), then all receive the same amount of consumption c_1^1; as well as the satiation level of local currency, $\bar{m}$: The cost of issuing pesos is assumed to be zero. If the actual proportion is $p\alpha + 1 - p$ (state 1), then the first $p\alpha$ consumers also get c_1^1; but the rest gets a (potentially different) amount $c_1^2(1)$: This is because the planner must learn through the queue whether the true state 1 or 2. In each case the planner potentially borrows from abroad an additional amount of $\lambda(s)$ dollars to be repaid at date 2. After this, at date 2, the planner pays off an amount of $c_2^j(s)$ dollars to those patient consumers in state s; as well as the amount $\bar{m}$ of pesos. This is done after repaying the total debt, at a gross rate 1:

The problem for the planner is then to maximize 3.1 subject to the constraints

$$x + y \leq \bar{d} \tag{3.2}$$

$$p\alpha c_1^1 + (1 - p)\, c_1^2(1) \leq y + \lambda(1) \tag{3.3}$$

$$p\alpha c_1^1 \leq y + \lambda(2) \tag{3.4}$$

$$p(1 - \alpha)\, c_2^1(1) + \left(\bar{d} + \lambda(1)\right) \leq Rx \tag{3.5}$$

$$p(1 - \alpha)\, c_2^1(2) + (1 - p)\, c_2^2(2) + \left(\bar{d} + \lambda(2)\right) \leq Rx \tag{3.6}$$

for every t; s and j:

The (sufficient) first order conditions give the following result.

Proposition 3.1. The planner's solution implies that:

$$c_1^1 = c_1^2(1) = c_2^j(2) = c_2(1)$$

That is, there is perfect risk sharing at the solution of the planner's problem: Moreover the first level consumption of dollars is equal to

$$\bar{e} = (R - 1)\,\bar{d}$$

Proof. See Appendix. ■

This proposition contrasts with the optimality-of-partial-suspension result by Wallace [18]. In his article he showed that partial suspension is optimal in a one currency economy. In this economy perfect capital markets in period 1 are assumed. This is enough to ensure perfect risk sharing for all agents. This cannot be surprising since the presence of perfect borrowing markets at date 1 implies market completeness. The question is whether there exists a banking system that implements the first best. The following subsection explores this.

3.2. Implementation of the first-best: the case of a currency board.

Suppose the following banking system. It consists of a mutual fund bank owned by all consumers that polls resources and offers contracts to consumers. There is also a Central Bank that supplies local currency holdings. This institution acts as a currency board (at least in this subsection). At date 0, the private bank borrows d^1 dollars from abroad and invests this amount in either the long run or the short run asset. The commercial bank issues contracts to consumers, specifying the amount of pesos to be withdrawn at the corresponding period.

At the beginning of date 1, the state s is realized. However neither the commercial bank nor the Central Bank know this. Those who turn out to be impatient form a queue at the commercial bank. The commercial bank must learn s through the size of the queue. Since I assume a sequential service constraint, the financial intermediary does not know the true s until the first αp agents withdraw at date 1. If after this amount of people more consumers show up, then the bank interprets that $s = 2$: Otherwise $s = 1$: At the beginning of date 1 the commercial bank borrows from an international lender an exact amount to satisfy the pesos withdrawn by impatient agents. Immediately after this the commercial bank sells this amount plus the revenue from short run investment to the Central Bank at an exchange rate equal to one. Then the commercial bank pays off pesos to the agents withdrawing at date 1. These agents use this amount of pesos for purposes not modeled here. This use gives utility $v(m_1)$. After using this, agents exchange pesos for dollars at an exchange rate equal to one. Impatient consumers consume dollars and disappear.

At the beginning of date 2 the financial intermediary repeats the process with the remaining patient consumers. Before this the bank must repay total debt with the long run investment returns. After repaying the debt, the remaining is sold to the Central Bank in exchange for pesos (at an exchange rate equal to one). Patient consumers receive from the commercial bank these pesos to be used before being sold to the Central Bank. Finally, agents consume the dollars sold by the Central Bank and the economy disappears.

The commercial bank then solves a similar problem as in the first best. The extra-constraints that must be added are as follows.

$$u\left(c_1^1\right)+v\left(m_1^1\right) \geq \max\left\{u\left(c_2^2(2)\right)+v\left(m_2^2(2)\right);\max_s\left[u\left(c_2^1(s)\right)+v\left(m_2^1(s)\right)\right]\right\}$$

$$u\left(c_1^2(1)\right)+v\left(m_1^2(1)\right) \geq \max\left(u\left(c_2^2(2)\right)+v\left(m_2^2(2)\right);\max_s\left[u\left(c_2^1(s)\right)+v\left(m_2^1(s)\right)\right]\right)$$

$$p\alpha m_1^1+(1-p)\,m_1^2(1) \leq p\alpha c_1^1+(1-p)\,c_1^2(1)$$
$$m_1^1 \leq c_1^1$$
$$m_2^1(1) \leq c_2^1(1)$$
$$p(1-\alpha)\,m_2^1(2)+(1-p)\,m_2^2(2) \leq p(1-\alpha)\,c_2^1(2)+(1-p)\,c_2^2(2)$$

The first two inequalities correspond to the incentive compatibility constraints and the last one states that the amount of pesos used in every period and state cannot exceed the amount of pesos withdrawn from the commercial bank (which is equal to the amount of dollars consumed since those pesos are sold at the Central Bank after being used).

It is not difficult to show the conditions under which the planner's allocation presented above can be implemented in this currency board regime. The following proposition shows when this is possible.

Proposition 3.2. Under the stated assumptions the planner's solution (with partial suspension of convertibility) can be offered in a decentralized banking system equilibrium with a currency board as long as $\hat{m} \leq \hat{c}$; that is

$$R-1 \geq \frac{\hat{m}}{\bar{d}}$$

Otherwise implementation under these conditions is not possible. In this last case an equilibrium with perfect risk sharing (although strictly dominated by the first best allocation) in the currency board regime always exists.

The proof of this result is in the appendix. The intuition is not difficult to see. Given that the first best consumption of dollars is larger than the satiation level of pesos, all consumers use only a portion of the local currency withdrawn from banks and this amount is exactly the satiation level, $\hat{m}$. After using this agents sell all pesos to the Central Bank, equal to the first best amount of dollars $\hat{c}$: On the contrary, if the inequality is the reverse, the currency board is unable to implement the first best allocation since the amount of pesos sold by the Central Bank is below the satiation level. Note that the condition is directly referred to the availability of credit at date 0 vis-a-vis the satiation level of pesos.

The next subsection presents the decentralized economy in which the first best is always implementable.

3.3. A fixed-exchange rate banking system with a local lender of last resort

Assume a banking system similar to that in last subsection. However there is a crucial distinction. Even though the exchange rate between dollars and pesos remains always fixed to one, the Central Bank now acts as a local lender of last resort in periods 1 and 2. At the beginning of date 1, commercial banks sell all dollars to be paid to impatient consumers. Those dollars come from foreign borrowing and revenues from the short run investment, as before. On the other hand, the Central Bank may also lend an amount of local currency, so that every impatient consumer gets $\bar{m}$ pesos. After using these, they return an amount of pesos to the commercial banks equal to the amount borrowed from the Central Bank by the intermediaries. These institutions repay then the pesos lent by the Central Bank. The remaining is sold to the Central Bank in exchange for the dollars bought by the monetary authority to the commercial banks. Impatient agents consume those dollars at the end of period 1.

In period 2 a similar timing of event is observed. At the beginning of this period commercial banks obtain the results of the long term investment. The intermediaries pulls apart an amount equal to the dollars owed to the international lender. The remaining is sold to the Central Bank in exchange for pesos. Commercial banks potentially borrow from the Central Bank so that they pay $\bar{m}$ to the patient consumers. After using them they return the amount of pesos lent by the Central Bank to the commercial banks. They immediately repay their debt to the monetary authority. Patient consumers finally sell the remaining pesos to the Central Bank in exchange for $\bar{e}$ dollars and finally consume them.

This system is clearly able to implement the first best allocation. Let $w_t^j(s)$ denote the amount of pesos withdrawn at date t, state s, by agent in position $j = 1, 2$. The problem for the commercial bank now can be written as the maximization of 3.1 subject to the constraints 3.2,

$$p\alpha w_1^1 + (1-p)\, w_1^2(1) = \lambda(1) + h_1(1)$$

$$p\alpha c_1^1 + (1-p)\, c_1^2(1) \leq p\alpha w_1^1 + (1-p)\, w_1^2(1) - h_1(1)$$

$$p\alpha w_1^1 = \lambda(2) + h_1(2)$$

$$p\alpha c_1^1 + (1-p)\, c_1^2(1) \leq p\alpha w_1^1 - h_1(2)$$

$$p(1-\alpha)\, w_2(1) = Rx - \bar{d} + h_2(1)$$

$$p(1-\alpha)\, c_2(1) \leq p(1-\alpha)\, w_2(1) - h_2(1)$$

$$[p(1-\alpha)+(1-p)]\,w_2(2) = Rx - d^1 + h_2(2)$$

$$[p(1-\alpha)+(1-p)]\,c_2(2) \quad [p(1-\alpha)+(1-p)]\,w_2(2) - h_2(2)$$

and

$$m_t^j(s) \quad w_t^j(s)$$

together with the incentive compatibility and non-negativity constraints (including those for the variables $h_t(s)$). However it is obvious that this is equivalent to maximize 3.1 subject to the constraints 3.2, 3.3, 3.4, 3.5, 3.6 and the incentive compatibility constraints. Then the following results arises.

Proposition 3.3. The solution to the commercial banking problem coincides with the first best allocation solution. Hence a banking system with an international and a local lender of last resort (with fixed exchange rates) is able to implement the social efficient allocation.

The proof of this is in the appendix. The intuition is immediate. If the first best allocation satisfies $\bar{m}$ $\quad \bar{e}$ the local lender of last resort actually does not enter into action. This is the last subsection case. If the reverse inequality holds, then the Central Bank lends an amount equal to $\bar{m} - \lambda(s)$ in period 1, state s; and $\bar{m} - (Rx - d^1)$. In both cases the satiation level of local currency $\bar{m}$ is ensured to all agents. Given this, the optimal level of borrowing by the commercial banks ensure the first best consumption allocation of dollars, $\bar{e}$:

4. Liquidity-based bank runs, prevention and lenders of last resort

This section deals with the issue of fragility of banks. Obviously, the fact that the first best allocation can be implemented does not rule out the possibility that the same equilibrium contract is subject to panics due to asset illiquidity and expectation coordination.

4.1. Conditions for existence of a run equilibrium

The following result states the conditions under which the banking system within the currency board system, even when implementing the first best allocation, has another inefficient run equilibrium.

Proposition 4.1. Suppose that $\bar{m} \geq \bar{e}$. Assume that

$$p(1-\alpha)(R-1) > r\left(1-\frac{[p(1-\alpha)+1-p](R-1)}{R}\right)$$

Then there is another equilibrium in which all consumers withdraw at date 1 and commercial banks fail in period 1. The Central Bank does not fail although it is left with no resources at date 1.

This result just replicates the standard result in the literature. It demonstrates that with r sufficiently low the contract cannot prevent a satiation in which all consumers withdraw at date 1. This also extends the result by Chang and Velasco [4], who proved the fragility only of commercial banks without aggregate uncertainty.

If $\bar{e} < \bar{m}$ then by proposition 3.3 a local lender of last resort (coupled with the international lender) within a fixed exchange rate regime is able to implement the first best allocation as an equilibrium contract. However the same condition implies the existence of another equilibrium in which the Central Bank fails at date 1.

Proposition 4.2. Suppose $\bar{e} < \bar{m}$. Assume the same condition as in the last proposition. Then there is an equilibrium in which all consumers withdraw at date 1 and the Central Bank fails in period 1. Commercial banks do not fail.

The proof is similar to proposition 4.1 and thus omitted. The only difference is that now the Central Bank is the institution that fails, not the intermediary. This is so because at date 1, if all agents withdraw from the commercial banks, the Central Bank issues a total amount of $\bar{m} - \lambda(1) - r\left(\bar{x} - \left(\frac{\bar{d}+\lambda(1)}{R}\right)\right)$ pesos as a credit to the intermediaries. Now the Central Bank has $\lambda(1) + r\left(\bar{x} - \frac{\bar{d}+\lambda(1)}{R}\right)$ dollars. The total amount of pesos that consumers will try to sell to the monetary authority is equal to $(p\alpha + 1 - p + p(1-\alpha))\bar{e} = \bar{e}$, strictly greater than the amount of dollars in the Central Bank. Hence the monetary authority runs out of dollars at date 1.

These two results obviously pose a question. Is an international lender of last resort able to eliminate the run equilibria in both cases? The next subsection answers this.

4.2. The bank-panics preventing role of the international lender of last resort.

Assume that the international institution commits to lend at zero net interest rates any amount of dollars above $\lambda(1)$: The next result shows that this commitment is enough to eliminate the run equilibrium in both exchange rate systems.

Proposition 4.3. Suppose that the international lender lends any amount of dollars to the banking system at zero net interest rates. Hence the run equilibrium is eliminated, independently of the exchange rate system. (In the case of the fixed exchange regime the Central Bank may still act temporarily as a local lender of last resort).

The proof is in the Appendix. The intuition is standard. The fact that $R > 1$ implies that the long term investment is always able to honor all debt (including that corresponding to the patient consumer allocation who withdraw early). Hence no bank can fail (in the fixed exchange rate regime, the Central Bank always have enough dollars to be sold in exchange for pesos). Since the equilibrium consumption allocation of dollars satisfies the incentive compatibility constraint then all patient consumers prefer to wait (in the absence of bank failure). Thus the international lender of last resort acts as a coordinating device that ensures that the unique equilibrium is the one without runs. Hence this institution is able to implement the first best allocation as an equilibrium (for the situations in which either proposition 3.2 or 3.3 holds).

The result previously gotten assumes a zero net interest rate for the bank-run preventing credit line. The effectiveness of this credit line can also be shown for a range of interest rates lower than an upper bound imposed by the equilibrium contract. The next result specifies this accurately.

Proposition 4.4. An international lender of last resort eliminates the run equilibrium in both a currency board regime and a fixed exchange regime if the interest rate of this credit line is less than or equal to $c_2(1) = c_1^1(2)$[4].

The proof is in the Appendix. Note that the upper bound at the first best is equal to one. This implies that the maximum net interest rate that the international lender of last resort to be charged is still zero.

[4] In the case in which the local Central Bank acts as a local lender to prevent runs, the net rate of interest on these loans are assumed to be still zero.

5. The planner's problem with date-1 borrowing constraints.

The last section presents an economy with perfect international credit markets. This clearly contradicts evidence. This is seriously a problem since most of the recent crises were somehow caused by problems in lending. This section adds constraints in the ability of the social planner (or the commercial banks in the banking system) in period 1.

More formally, assume that the social planner maximizes 3.1 subject to the constraints 3.3, 3.5, 3.6 and the following equations.

$$x + y \le d$$
$$d + \lambda(s) \le \bar{d}^1; \quad s = 1; 2 \tag{5.1}$$

where now d is a decision variable, and $\bar{d}^1$ is the total availability of credit at date 1: Here it is assumed that the international lender of last resort learns about state s exactly as the social planner. Relaxations of this assumption seem important but it involves more complicated issues on contract design. This is left for future research.

This is clearly a second best problem, due to the presence of borrowing constraint at date 1. This implies that the optimal allocation under these constraints is non-deterministic. The next subsection characterizes more sharply the allocation.

5.1. Characterization of the second best allocation.

Given the new constraint, one may suspect that the consumption allocation of dollars may not be the same for the impatient consumers who withdraw first and those who withdraw in a second place. The next proposition shows that in fact those first in line get more than the second group.

Proposition 5.1. The second best allocation implies that $c_1^1 > c_1^2(1)$. In this allocation all agents consume the satiation level of pesos, $\bar{m}$:

The proof is again presented in the appendix. This shows that the partial suspension of convertibility result by Wallace [18] still holds here, even though (imperfect) credit markets are available for the planner in the interim period. This means that it is sufficient to impose an (exogenous) constraint to the lender period 1 to generate this result. Therefore, absence of partial suspension of convertibility is obtained only in the first best allocation. In the next subsection the implementation of the second best is discussed.

5.2. Implementation of the second best allocation

The allocation obtained above can be implemented through similar banking systems to those in section 3. This depends on the assumption about $\bar{m}$ and the optimal consumption of dollars.

First, if $\bar{m} \quad c_1^2(1)$; then a banking system within a currency board regime implements the second best allocation. The idea is that the commercial banks (as de…ned before) face the borrowing constraint (5:1) at date 1. Therefore …nancial intermediaries can only borrow up to d^1 dollars in total. However, given the inequality above, the liquidity needs in pesos are completely satis…ed for all agents. This is so since total withdrawals of pesos is always at least the satiation level. In this case then the currency board regime is enough so that the banking system implements the second best, as it was the case in the unrestricted borrowing model. This is summarized in the following proposition.

Proposition 5.2. Whenever $\bar{m} \quad c_1^2(1)$ the second best allocation is implemented as an equilibrium of a banking system similar to that of section 3.2 within a currency bard regime.

The formal proof is omitted, since it follows similar lines as in the proof of proposition 3.2. If the condition is not satis…ed, then this banking system with the currency board regime cannot have the second best allocation as an equilibrium. The reason is that if $c_1^2(1) < \bar{m}$ now implies that at least the impatient consumers in the second group in line would get less pesos than the satiation level. Hence whenever this happens the currency board imposes a too tight restriction on allocations.

In this case then a local lender of last resort, providing transitory liquidity in pesos for at least a group of agents, is needed to implement the second best. The banking system and the Central Bank face an economy as in section 3.3. The only di¤erence is the imposition of the date 1 borrowing constraint for the …nancial intermediaries. Although the proof of the next result is also omitted, the same argument as in proposition 3.3 is used to show that:

Proposition 5.3. Whenever $\bar{m} \quad c_1^2(1)$ the second best allocation is implemented as an equilibrium of a banking system similar to that of section 3.2 within a …xed exchange rate regime with a local lender of last resort.

The intuition is the same as in proposition 3.3. As long as the satiation level is at most equal to c_1^1; impatient consumers …rst in line gets enough pesos as in the second best solution. Otherwise the Central Bank must lend the difference to the commercial banks. This loan is repaid immediately before consumers sell pesos for dollars at the Central Bank. Similarly, if $m^1 \quad c_2(s)$ then patient consumers at date 2, state s; withdraw an amount pesos which is enough to reach the satiation level. Otherwise, a local lender of last resort must lend the difference. This loan is repaid before agents sell pesos for dollars, as before.

The next subsection analyzes financial fragility under the second best and how the lenders of last resort could eliminate that problem.

5.3. Bank Runs and Lenders of Last Resort

Conditions for bank runs are similar to those in the first best case. However since the consumption allocations are now random, the inequality must refer to one of those levels of dollar-consumption. In fact the next proposition is a trivial extension of proposition 4.1.

Proposition 5.4. Assume that

$$r < R\left(\frac{c_1^2(1)}{c_2(1)}\right)$$

$$p(1-\alpha)c_1^2(1) > r\left(x - \frac{\lambda(1)+d}{R}\right)$$

Then the contract that implements the second best (whether it is done under the currency board or the fixed exchange rate regime) is subject to runs. That is, there is another equilibrium in which either commercial banks (in the currency board case) or the Central Bank (in the fixed exchange rate regime case) fail in period 1.

The intuition is the same as before. Given the condition above the commercial banks need to liquidate all of the long term investment in the interim period. Recall that, if more than αp consumers withdraw at date 1 intermediaries and the Central Bank think that the true state is $s = 1$: But then each consumer should get $c_1^2(1)$: However the inequality states that total assets are lower than total dollars demanded in period 1, implying the failure of either the intermediary (in the currency board regime) or the monetary authority (in the fixed exchange rate regime, due to its role as local lender in pesos).

In terms of the international lender of last resort as a run-preventing device, its role is exactly the same as in the first best case. It is easy to show the following result (again, the proof is left to the reader since the argument must be repeated).

Proposition 5.5. Assume that an international lender of last resort is able to lend any amount whenever a threat of panic arises in the banking system. Then, as long as the interest rate is less than or equal to $c_2(1) = c_1^1(2)$ then the run equilibrium is eliminated.

The last proposition is a generalization of both propositions 4.3 and 4.4. The main conclusion that this international lending institution always eliminates the run equilibrium allowing for implementation of the second best allocation without panics, eliminating fragility. Therefore the international lender of last resort is always sufficient to prevent liquidity-based bank runs in a two-currency economy, where the local currency is used only intra-period.

To illustrate better these conditions I present a numerical example based on linear quadratic utility functions.

5.4. An example: linear quadratic case

Assume that

$$u(c) = -\frac{\gamma}{2}c^2 + \beta c + \delta$$

where all coefficients are strictly positive and $\beta > R d^1$. This is so to assume that, on the relevant domain, $u'(c) > 0$. Assume also that

$$v(m) = -\frac{1}{2}m^2 + \pi m + \nu$$

Then:

$$\hat{m} = \pi$$

Given the utility function u above we have that the first order conditions can be reduced to the following linear system.

$$\begin{aligned}
c_1^1 - q_1 c_1^2(1) - q_2 c_2(2) &= 0 \\
-\gamma c_1^1 + \gamma R q_1 c_2(1) + \gamma R q_2 c_2(2) &= \beta(R-1) \\
R\alpha p c_1^1 + R(1-p)c_1^2(1) + p(1-\alpha)c_2(1) &= (R-1)d^1 \\
R\alpha p c_1^1 + (R-1)(1-p)c_1^2(1) + (p(1-\alpha)+1-p)c_2(2) &= (R-1)d^1
\end{aligned}$$

which can be written in the following way

$$\begin{bmatrix} 1 & -q_1 & 0 & -q_2 \\ -\gamma & 0 & \gamma R q_1 & \gamma R q_1 \\ R\alpha p & R(1-p) & p(1-\alpha) & 0 \\ R\alpha p & (R-1)(1-p) & 0 & (p(1-\alpha)+1-p) \end{bmatrix} \begin{bmatrix} c_1^1 \\ c_1^2(1) \\ c_2(1) \\ c_2(2) \end{bmatrix} = \begin{bmatrix} 0 \\ \beta(R-1) \\ \bar{d}(R-1) \\ \bar{d}(R-1) \end{bmatrix}$$

The proof of this is directly derived from the first order conditions and left to the reader. As the explicit solution does not give a specially intuitive condition, I prefer to report the solutions to these problems for numerical examples. Assume the following values for the parameters.

p	α	q_1	R	β	γ
0.25	0.20	0.5	1.5	10	1

The following table shows the second best allocations for three values of $\bar{d}$:

allocation\value of $\bar{d}$	4	5	6
c_1^1	1.1742	1.5419	1.9097
$c_1^2(1)$	0.5548	0.9484	1.3419
$c_2(1)$	6.4387	6.5871	6.7355
$c_2(2)$	1.7935	2.1355	2.4774

The next table shows values of π that allows implementation by a commercial banking system with a currency board regime for each value of $\bar{d}$; given in fact by the second row $c_1^2(1)$. It also gives the upper bound for r so that illiquidity is verified. It finally has upper bounds for the interest rate charged by the international lender of last resort in order to prevent runs.

variables\value of $\bar{d}$	4	5	6
π	0.5548	0.9484	1.3419
r	0.1292	0.2159	0.2988
int. rate	11.605	6.9455	5.0194

Note that the maximum interest rate that can be charged by the international lender of last resort behaves as if it is decreasing in $\bar{d}$. A possible interpretation states that, as the availability of credit in the second best problem increases, the disparity between consumption assigned to (second group) impatient agents and the patient consumers at date 2 decreases. But the upper bound of the interest rate on the credit line provided by the international liquidity provider is lower the closer are these two consumption values. Basically, the greater c^1; the more generous is the payment to the impatient consumers in period 1 and the greater is the need of credit in this same period to prevent panics. Hence the interest rate must be less tight to implement this procedure.

6. Policy Implications

The results in this chapter allow for a discussion about how to implement contingent credit lines such as the one discussed in the introduction. Since the decision made by the G7 countries has been attacked from several points of view, it is useful to see what such propositions teach us about their effectiveness.

First, propositions 3.1 and 5.1 state that international institutions may provide funds when there are extra withdrawals in the short run, so that withdrawals do not have to be suspended. In terms of evidence, some facts from the banking distress situation in the case of Argentina, in 1995, suggest that the funds coming from the Inter American Development Bank and the World Bank had as one of the main purposes to enhance liquidity for the healthy banks of this country.

Also, both propositions 3.3 and 5.3 state that an international lender might need to be coupled with a suitable local lender of last resort to implement the optimal allocation (whether is first or second best). In a sense these results suggest that both institutions tend to complement, not to substitute, each other. However such a local lender cannot have a loose behavior. Its main purpose is to lend local currency to the financial institutions in the short run whenever it is proved that the banking system has extra liquidity needs. The main danger of using a local lender in this situation is to worsen the foreign reserves situation having more customers with local currency running against the Central Bank. Thus the purpose of the local institution must limited only to cover local currency liquidity needs.

An important remark is that the condition to have implementation by a currency board is that the net rate of return on the long term investment is at least equal to the ratio of satiation level of pesos divided by the total amount of dollars available at date 0. This gives a criterion to know whether a currency board is indeed optimal (from a banking point of view). If the net return on long term investments is high enough compared to monetization in pesos, then a currency board may be efficient. Clearly this result does not intend to answer questions on how optimal is a currency board compared to other exchange rate regimes. The scope of this paper is below that. However it suggests that, for economies with low level of demand for local currencies, which is the case in several countries such as Argentina, a currency board may not impose a too heavy burden on efficiency, at least from the point of view of banking deposit contracts. As long as either the return on long term investments is high enough (and certain) or the availability of credit in dollars is high enough, then a currency board is able to implement in a banking system an efficient deposit contract with perfect insurance and with a desired level of monetization in pesos.

A special comment about the local lender is the fact that it is active only when the transitory illiquidity in local currency arises. This assumes a large degree of commitment by the institution acting as a local lender (usually the Central Bank). This also has policy implications. Implementing such mechanism in this way implies the creation of institutions or legal systems that prevents irresponsible behavior by the acting local lender (creating liquidity when there is no need of it). Actually in the first best we know that the limit of this amount of credit is equal to the difference between the per-capita desired amount of pesos and the net return on the long term investment, multiplied by the credit available in the first period, in per capita terms. This is a measurable limit for credit in pesos by the local monetary authority. Hence the setting up of a local lender of last resort demands the creation of very solid laws and institutions to avoid local lender misbehavior. Another possible way is to have international institutions such as the IMF monitoring the functioning of such local lenders.

On the other hand, as long as the funds from this credit line are used to help transitorily illiquid financial systems, then propositions 4.3 and 5.4 specify that these credit lines, regardless of the exchange rate regime, always prevents runs. However, it is important to remark that, when the first best implies that a currency board cannot implement this as an equilibrium, then the local lender of last resort not only is needed to implement the allocation but also is useful to prevent runs appropiately. Otherwise the commercial banks could run out of pesos in period 1 even though the international lender is available. Also, propositions 4.4 and 5.4 suggest upper bounds for the interest rate that the international lender of last resort must charge in order to make repayment feasible. Once more the main problem here is to measure deposits with different horizons so that interest rates on these credit lines still allow its preventing role. I do not suggest to take these ratios literally, but they constitute a major guide for interest rate negotiations.

From the paragraphs above it is clear that implementing such institutions is not easy. Monitoring costs (in the sense of keeping track of deposits) and the problem of measuring the liquidity needs in each currency are difficult. This does not mean that they are infeasible in practice, but the implicit informational assumptions give a warning in terms of how to implement them. In any case all these regulatory regimes implied by the results deal with liquidity problems. It does not say anything in terms of solvency issues. The main challenge in practice is to discover whether certain ...nancial distress phenomena were caused by liquidity or solvency problems. This still remains an open question for the policy makers.

7. Concluding remarks and possible extensions

This paper has presented an extension of the Diamond - Dyvbig framework to a banking system with two currencies. I have done that with the two alternative assumptions of the proportion of impatient consumers known and unknown (ex-ante). The ...rst point is that a banking system within a currency board regime may not always implement a ...rst best allocation as an equilibrium. Therefore, a local lender of last resort may be needed to pursue that object. International lending institutions per se may not be able to implement optimal allocations.

A second message is that an international lender of last resort per se can always prevent runs regardless of implementation. The only caveat to this statement is the fact that, whenever the local lender of last resort is essential for implementation of the ...rst best allocation, it is also useful (not necessarily essential) to prevent runs too. Hence complementing the international institution with a local lender might be more e¤ective. Another message is that interest rates charged by the international lending institution cannot be too high. The results above indicates that in order to set these rates it is important to know the ratio of long term over short term deposits. This indicator can give an important guideline for policy makers to set suitable interest rates for these credit lines. Another remark is that these institutions do not have to know precisely the amount of withdrawals in the short run. Knowledge of distribution is enough. However other elements such as perfect commitment of repayment and common objectives between bank managers and depositors are key features for these institutions to work.

A possible direction for future work is the construction of a version of this model in a world integrated economy with two tradable currencies, following also similar ideas as in Allen and Gale [2]. There are several issues that can be addressed with this framework. Perhaps one of the most discussed issues is the incentive to constitute the reserves for the international lender of last resort. In the paper I have presented such problem could not be studied since the economy was of the small open type. A world integrated economy with well-de...ned participants could help to see when each country is willing to deposit funds in an international institution.

Fundamental shocks can be introduced, making either the short term rate (as in Chang and Velasco [5]) or the long investment return (as in Allen and Gale [1]) stochastic. This would allow to study solvency - based runs and the role of the lenders of last resort to prevent such runs, if these are not optimal. Nevertheless, problems of asymmetric information could worsen here. The reason is that, when returns are risky, adverse selection may not allow for availability of an international lender of last resort. This issue should be studied in combination with a world-integrated environment.

A related topic to the solvency problem is the explicit separation between managers and depositors. By study a version of this banking model in which managers do not have the same objective as the depositors the moral hazard considerations mentioned above could be seriously addressed. That is, moral hazard considerations are to be studied in settings where those objectives are discordant, since it is obvious that when they are the same hidden action problems cannot arise. There are several alternatives for modelling this. There is a vast literature on incomplete contracts in banking (see [9] and [10]). Chang and Velasco [5] also present a model in which the banking sector is monopolistic. Any of these frameworks could be helpful to study moral hazard and lenders of last resort. Finally, issues on insurance schemes (in the spirit of Druck [12], for example) can also be considered in the international setting.

A. Proofs

A.1. Proof of Proposition 3.1

The …rst order conditions are as follows:

$$R\left[\acute{A}_2(1) + \acute{A}_2(2)\right] = \acute{A}_0 \tag{A.1}$$

$$\left[\acute{A}_1(1) + \acute{A}_1(2)\right] \quad \acute{A}_0 \tag{A.2}$$

$$\acute{A}_1(1) \quad \acute{A}_2(1) \tag{A.3}$$

$$\acute{A}_1(2) \quad \acute{A}_2(2) \tag{A.4}$$

$$u'\left(c_1^1\right) = \acute{A}_1(1) + \acute{A}_1(2) \tag{A.5}$$

$$q_1 u'\left(c_1^2(1)\right) = \acute{A}_1(1) \tag{A.6}$$

$$q_1 u' \left(c_2^1(1) \right) = \psi_2(1) \quad (A.7)$$
$$q_2 u' \left(c_2^j(2) \right) = \psi_2(2) \quad (A.8)$$

where $j = 1; 2$ in the last inequality. This last equality implies that $c_2^1(2) = c_2^2(2) \equiv c_2(2)$: The first four expressions correspond to the first derivative of the Lagrangian with respect to $x; y; \lambda(1)$ and $\lambda(2)$ respectively. The last four expressions are the first order conditions with respect to $c_1^1; c_1^2(1); c_2^1(1)$ and $c_2(2)$:

I first show that $y^* = 0$: Using contradiction, assume that $y^* > 0$: Then from A.1 and A.2 we have

$$\begin{aligned}[\psi_1(1) + \psi_1(2)] &= R[\psi_2(1) + \psi_2(2)] \\ &> [\psi_2(1) + \psi_2(2)]\end{aligned}$$

On the other hand, we have

$$\begin{aligned}\psi_1(1) &\quad \psi_2(1) \\ \psi_1(2) &\quad \psi_2(2)\end{aligned}$$

and therefore

$$[\psi_1(1) + \psi_1(2)] \quad [\psi_2(1) + \psi_2(2)]$$

a contradiction. Hence $y^* > 0$:

Then this implies that $\psi_1(1) = \psi_2(1)$ and $\psi_1(2) = \psi_2(2)$: From the expressions A.7, A.8 and A.5 we get

$$\begin{aligned} u'\left(c_1^2(1)\right) &= u'\left(c_2^1(1)\right) \\ u'\left(c_1^1\right) &= q_1 u'\left(c_2^1(1)\right) + q_2 u'(c_2(2)) \end{aligned}$$

Obviously, from the first equation we get $c_1^2(1) = c_2^1(1)$: On thee other hand, from the constraints (holding with strict equality)

$$p\alpha c_1^1 + (1 - p)\, c_1^2(1) = \lambda(1)$$

$$p\alpha c_1^1 = \lambda(2)$$

$$p(1 - \alpha)\, c_2^1(1) + \left(d^1 + \lambda(1) \right) = Rd^1$$

$$p(1-\alpha)c_2^1(2) + (1-p)c_2^2(2) + \left(\bar{d} + \lambda(2)\right) = R\bar{d}$$

implies that

$$\begin{aligned} & p(1-\alpha)c_2^1(1) + p\alpha c_1^1 + (1-p)c_1^2(1) \\ = \; & (R-1)\bar{d} \\ = \; & [p(1-\alpha) + (1-p)]c_2^2(2) + p\alpha c_1^1 \end{aligned}$$

and therefore

$$p(1-\alpha)c_2^1(1) + (1-p)c_1^2(1) = [p(1-\alpha) + (1-p)]c_2^2(2)$$

But $c_1^2(1) = c_2^1(1)$. This implies that $c_1^2(1) = c_2^1(1) = c_2^2(2) \equiv \hat{c}$. But then, from

$$u'\left(c_1^1\right) = q_1 u'\left(c_2^1(1)\right) + q_2 u'(c_2(2))$$

we get that $c_1^1 = \hat{c}$, showing that perfect risk sharing is the only solution to the planner's problem.

Then, from the constraint

$$p(1-\alpha)c_2^1(2) + (1-p)c_2^2(2) + \left(\bar{d} + \lambda(2)\right) = R\bar{d}$$

we have that

$$[p(1-\alpha) + (1-p)]\hat{c} + \lambda(2) = (R-1)\bar{d}$$

But

$$\lambda(2) = p\alpha\hat{c}$$

so that

$$[p(1-\alpha) + (1-p)]\hat{c} + p\alpha\hat{c} = (R-1)\bar{d}$$

and then

$$\hat{c} = (R-1)\bar{d}$$

This ends the proof. ■

A.2. Proof of Proposition 3.2

Ignoring the incentive compatibility constraints for a moment, the first order conditions of the commercial banking problem can be written as follows.

$$R\left[\phi_2(1) + \phi_2(2)\right] = \phi_0 \tag{A.9}$$
$$\left[\phi_1(1) + \phi_1(2)\right] \quad \phi_0 \tag{A.10}$$
$$\phi_1(1) \quad \phi_2(1) \tag{A.11}$$
$$\phi_1(2) \quad \phi_2(2) \tag{A.12}$$

$$u'\left(c_1^1\right) = \phi_1(1) + \phi_1(2) - \psi_1(1) - \psi_1(2) \tag{A.13}$$
$$v'\left(m_1^1\right) = \psi_1(1) + \psi_1(2) \tag{A.14}$$
$$q_1 u'\left(c_1^2(1)\right) = \phi_1(1) - \psi_1(1) \tag{A.15}$$
$$q_1 v'\left(m_1^2(1)\right) = \psi_1(1) \tag{A.16}$$
$$q_1 u'\left(c_2^1(1)\right) = \phi_2(1) - \psi_2(1) \tag{A.17}$$
$$q_1 v'\left(m_2^1(1)\right) = \psi_2(1) \tag{A.18}$$
$$q_2 u'\left(c_2^j(2)\right) = \phi_2(2) - \psi_2(2) \tag{A.19}$$
$$q_2 v'\left(m_2^j(2)\right) = \psi_2(2) \tag{A.20}$$

where the multipliers $\psi_1(1);\ \psi_1(2);\ \psi_2(1)$ and $\psi_2(2)$ are the multipliers of the following restrictions

$$p\alpha m_1^1 + (1-p)\, m_1^2(1) \quad p\alpha c_1^1 + (1-p)\, c_1^2(1) \tag{A.21}$$
$$p\alpha m_1^1 \quad p\alpha c_1^1 \tag{A.22}$$
$$p(1-\alpha)\, m_2^1(1) \quad p(1-\alpha)\, c_2^1(1) \tag{A.23}$$
$$p(1-\alpha)\, m_2^1(2) + (1-p)\, m_2^2(2) \quad p(1-\alpha)\, c_2^1(2) + (1-p)\, c_2^2(2) \tag{A.24}$$

Note first that at the solution of this problem it must be that $y^* = 0$ here too. Then we must have again that:

$$\phi_1(1) = \phi_2(1)$$
$$\phi_1(2) = \phi_2(2)$$

and also it still true that $c_2^1(2) = c_2^2(2)$ and now $m_2^1(2) = m_2^2(2) \equiv m_2(2)$. Then:

$$
\begin{aligned}
q_2\left[u'(c_2(2)) + v'(m_2(2))\right] &= \phi_1(2) \\
\left[u'\left(c_1^2(1)\right) + v'\left(m_1^2(1)\right)\right] &= \left[u'\left(c_2^1(1)\right) + v'\left(m_2^1(1)\right)\right]
\end{aligned}
$$

And also it must happen that:

$$
\begin{aligned}
& \left[u'\left(c_1^1\right) + v'\left(m_1^1\right)\right] \\
=\; & \left[\phi_1(1) + \phi_1(2)\right] \\
& R\left[\phi_2(1) + \phi_2(2)\right] \\
=\; & R\left\{\left[u'\left(c_2^1(1)\right) + v'\left(m_2^1(1)\right)\right] + q_2\left[u'(c_2(2)) + v'(m_2(2))\right]\right\}
\end{aligned}
$$

On the other hand, from the date 2 budget constraints we need to have

$$
p(1-\alpha)\, c_2^1(1) + (1-p)\, c_1^2(1) = \left[p(1-\alpha) + (1-p)\right] c_2(2) \qquad \text{(A.25)}
$$

Consider first the case in which $\bar{m} \leq \hat{m}$. This implies that the solution to the bank problem is identical to the first best allocation. This is so because at $\bar{m}$ we have $v'(\bar{m}) = 0$. In this case we should have then $\psi_1(1) = \psi_1(2) = \psi_2(1) = \psi_2(2) = 0$. But then the consumption allocation that satisfies the first order conditions of the bank problem are identical to those of the social planner. But the first best allocation satisfies with equality the incentive compatibility constraint. Then whenever $\bar{m} < \hat{m}$ the first best is the solution to the commercial bank optimization problem. By strict concavity of the objective function this solution is unique.

Suppose then that $\hat{m} < \bar{m}$. Now the first best cannot be a solution to the commercial bank problem because otherwise the constraints A.21, A.22, A.23 and A.24 are violated. This shows the first part of the proposition.

Next I show the existence of an equilibrium in which $c_1^1 = c_1^2(1)$ and $m_t(s) < \bar{m}$ whenever $\hat{m} < \bar{m}$. Guessing that all four constraints A.21, A.22, A.23 and A.24 are binding it must be that $\psi_t(s) > 0$. Guessing also that $c_1^1 = c_1^2(1)$ we have that:

$$
\begin{aligned}
& \left[u'\left(c_1^1\right) + v'\left(m_1^1\right)\right] \\
=\; & \left[\phi_1(1) + \phi_1(2)\right] \\
=\; & q_1\left[u'\left(c_1^2(1)\right) + v'\left(m_1^2(1)\right)\right] + \phi_1(2) \\
=\; & q_1\left[u'\left(c_1^1\right) + v'\left(m_1^1\right)\right] + \phi_1(2)
\end{aligned}
$$

Then

$$\begin{aligned} q_2\left[u'\left(c_1^1\right)+v'\left(m_1^1\right)\right] &= \phi_1(2) \\ &= q_2\left[u'(c_2(2))+v'(m_2(2))\right] \end{aligned}$$

and then

$$\left[u'\left(c_1^1\right)+v'\left(m_1^1\right)\right] = \left[u'(c_2(2))+v'(m_2(2))\right]$$

But we also had

$$\begin{aligned} u'\left(c_2^1(1)\right)+v'\left(m_2^1(1)\right) &= u'\left(c_1^2(1)\right)+v'\left(m_1^2(1)\right) \\ &= u'\left(c_1^1\right)+v'\left(m_1^1\right) \\ &= u'(c_2(2))+v'(m_2(2)) \end{aligned}$$

But then it must be the case that $c_2^1(1) = c_2(2)$. Suppose otherwise. The first case is that $c_2^1(1) < c_2(2)$. Then $u'(c_2^1(1)) > u'(c_2(2))$. But then $v'(m_2^1(1)) < v'(m_2(2))$. This implies that $m_2^1(1) > m_2(2)$. But then $c_2^1(1) > c_2(2)$, a contradiction. The opposite inequality works in a similar way. Then it must be the case that $c_2^1(1) = c_2(2)$ and $m_2^1(1) = m_2(2)$. This also implies from the equality A.25 that

$$c_2^1(1) = c_2(2) = c_1^2(1)$$

but then $c_1^1 = c_2^1(1) \equiv c^{cb}$ and then also $m_2^1(1) = m_2(2) = m_1^2(1) = m_1^1 \equiv m^{cb}$. This implies perfect risk sharing but it is clearly less preferable than the first best since the level of local currency consumption m^{cb} is less than $\bar{m}$. ■

A.3. Proof of Proposition 3.3

Ignoring the incentive compatibility constraints, the problem can be written as the maximization of

$$\begin{aligned} p\Bigg[\alpha\left[u\left(c_1^1\right)+v\left(m_1^1\right)\right] + (1-\alpha)\left(\sum_{s=1}^{2} q_s\left(u\left(c_2^1(s)\right)+v\left(m_2^1(s)\right)\right)\right)\Bigg] \\ +(1-p)\left(q_1\left(u\left(c_1^2(1)\right)+v\left(m_1^2(1)\right)\right)+q_2\left(u\left(c_2^2(2)\right)+v\left(m_2^2(2)\right)\right)\right) \end{aligned}$$

subject to

$$x + y \leq \bar{d}$$

$$p\alpha c_1^1 + (1-p)\,c_1^2(1) \leq \lambda(1) + y$$
$$p\alpha c_1^1 + (1-p)\,c_1^2(1) \leq \lambda(2) + y$$
$$p(1-\alpha)\,c_2(1) \leq Rx - d^1$$
$$[p(1-\alpha) + (1-p)]\,c_2(2) \leq Rx - d^1$$

together with

$$\lambda(1) \leq p\alpha w_1^1 + (1-p)\,w_1^2(1) \tag{A.26}$$
$$\lambda(2) \leq p\alpha w_1^1 \tag{A.27}$$
$$Rx - d^1 \leq p(1-\alpha)\,w_2^1(1) \tag{A.28}$$
$$Rx - d^1 \leq p(1-\alpha)\,w_2^1(2) + (1-p)\,w_2^2(2) \tag{A.29}$$
$$m_t^j(s) \leq w_t^j(s) \tag{A.30}$$

The first order conditions with respect to x, y and $\lambda(s)$ are as follows.

$$\begin{aligned} R\,[\phi_2(1) + \phi_2(2) - \eta_2(1) - \eta_2(2)] &= \phi_0 \\ [\phi_1(1) + \phi_1(2)] &\leq \phi_0 \\ \phi_1(1) &\leq \phi_2(1) + \eta_1(1) \\ \phi_1(2) &\leq \phi_2(2) + \eta_1(2) \end{aligned}$$

where $\eta_1(1)$ is the multiplier of A.26, $\eta_1(2)$ that of A.27, $\eta_2(1)$ that of A.28 and $\eta_2(2)$ that of A.29. The rest of the first order conditions are as follows.

$$\begin{aligned} u'\left(c_1^1\right) &= \phi_1(1) + \phi_1(2) \\ q_1 u'\left(c_1^2(1)\right) &= \phi_1(1) \\ q_1 u'\left(c_2^1(1)\right) &= \phi_2(1) \\ q_2 u'\left(c_2^j(2)\right) &= \phi_2(2) \end{aligned}$$

$$\begin{aligned} p\alpha v'\left(m_1^1\right) &= \mu_1^1 \\ q_1(1-p)\,v'\left(m_1^2(1)\right) &= \mu_1^2(1) \\ q_1 p\alpha v'\left(m_2^1(1)\right) &= \mu_2(1) \\ q_2 v'(m_2(2)) &= \mu_2(2) \end{aligned}$$

and

$$\begin{aligned}
\left[\eta_1^1 + \eta_1^2(1)\right] p\alpha + \mu_1^1 &= 0 \\
\eta_1^2(1)\, p\alpha + \mu_1^2(1) &= 0 \\
p(1-\alpha)\,\eta_2(1) + \mu_2(1) &= 0 \\
[p(1-\alpha) + 1 - p]\,\eta_2(2) + \mu_2(2) &= 0
\end{aligned}$$

where $\mu_t^j(s)$ is the multiplier of the constraints A.30. However, since all multipliers are non-negative, then the last four equations imply that $\eta_1^1 = \eta_1^2(1) = \eta_2(1) = \eta_2(2) = 0$. This implies that the optimal local currency holding is equal to $\bar{m}^1$, the satiation level. But then the first order conditions above are exactly those of the socially efficient allocation. Then the optimal dollar consumption allocation is equal to $\bar{c}^1$. This completes the proof. ■

A.4. Proof of Proposition 4.1

Suppose that

$$p(1-\alpha)(R-1) > r\left(1 - \frac{[p(1-\alpha) + 1 - p](R-1)}{R}\right)$$

Multiplying each side by $\bar{d}^1$ gives

$$p(1-\alpha)\bar{c}^1 > r\left(\bar{x}^1 - \frac{\bar{d}^1 + \lambda(1)}{R}\right)$$

Then the following happens. Suppose that all patient consumers think that the others will withdraw from the commercial banks at date 1. The intermediaries pay $\bar{c}^1$ to the first $\alpha p + 1 - p$ consumers borrowing $\lambda(1)$ dollars from abroad. Note that even though the true state is $s = 2$, if $\alpha p + 1 - p$ agents show up then the commercial bank thinks that the true state is $s = 1$. If more consumers show up the intermediaries must still pay $\bar{c}^1$ pesos (which will be exchanged for dollars at the Central Bank) to each one. But the amount of resources left is equal to $r\left(\bar{x}^1 - \frac{\bar{d}^1+\lambda(1)}{R}\right)$. This is because the commercial bank liquidates $\left(\bar{x}^1 - \frac{\bar{d}^1+\lambda(1)}{R}\right)$ units of the long term investment at date 1 in order to satisfy withdrawals. Since this is strictly less than payments needed to be done if all the rest of agents withdraw then the bank fails (in the sense that not all consumers are satisfied, although debt at date 2 is perfectly honored).

All agents know that this happens if everybody runs against the bank. Then if every individual consumer thinks that the rest of the population runs, then it is optimal to withdraw early. This is because the expected utility of withdrawing early is strictly greater than $u(0) + v(0)$. But then in this case the commercial bank fails. This concludes the proof. ■

A.5. Proof of Proposition 4.3

Suppose that the proportion of consumers withdrawing at date 1 is $p\alpha + 1 - p + \pi$, with $\pi \le p(1-\alpha)$. Then the commercial banks borrow $\lambda(1) + \pi c_1^2(1)$. None of the long term investment is liquidated in period 1. This amount of dollars is sold to the Central Bank. Consumers withdrawing early get exactly the quantity of dollars corresponding to the contract. The first αp consumers get c_1^1 dollars and the rest $c_1^2(1)$. In period 2 there are remaining $p(1-\alpha) - \pi$ consumers. Intermediaries liquidate the long term investment, obtaining Rx dollars. After paying total debt (equal to $\bar d^1 + \lambda(1) + \pi c_1^2(1)$) the rest is sold to the Central Bank. At the end of this period the total amount of pesos to be exchanged for dollars at the Central Bank is equal to $[p(1-\alpha) - \pi]\, c_2(1)$. This is because the proportion who actually withdrew in period 1 is strictly higher than $p\alpha$. Then the intermediaries interpret that the state is $s = 1$. The amount of dollars that the Central Bank has is equal to:

$$Rx - \left(\bar d^1 + \lambda(1) + \pi c_1^2(1)\right)$$

On the other hand it must be true that

$$Rx - \bar d^1 - \lambda(1) = p(1-\alpha)\, c_2(1)$$

Then

$$Rx - \left(\bar d^1 + \lambda(1) + \pi c_1^2(1)\right) = p(1-\alpha)\, c_2(1) - \pi c_1^2(1)$$

However in equilibrium (for any exchange rate regime)

$$c_2(1) \ge c_1^2(1)$$

Then

$$\begin{aligned} & Rx - \left(\bar d^1 + \lambda(1) + \pi c_1^2(1)\right) \\ = \; & p(1-\alpha)\, c_2(1) - \pi c_1^2(1) \\ \ge \; & [p(1-\alpha) - \pi]\, c_2(1) \end{aligned}$$

Therefore all remaining consumers in period 2 get at least $c_2(1)$, greater than or equal to c_1^1 and $c_1^2(1)$. Hence it is (weakly) preferable for a patient consumer to wait until period 2 instead of withdrawing early. Assuming that under indifference all agents tell then truth, then all patient consumers choose not to withdraw early. Hence there is no run equilibrium.

In the case in which $(R-1)d^1 < \bar{m}$, then the local Central Bank also needs to lend temporarily (intra - period) pesos at date 1. In this case the local lender lends $\left(p(1-\alpha)-\pi\right)\bar{m}-(R-1)d^1$ extra to the intermediaries to satisfy all pesos withdrawals. However these consumers return this amount of pesos to the commercial banks and these honor this debt before the consumers sell the remaining amount of pesos. Therefore here is no failure of commercial banks in this case either. The proof is complete. ■

A.6. Proof of Proposition 4.4

The only change in the proof of proposition 4.3 is in period 2 resources. If the interest rate is equal to ρ then the amount of dollars available at date 2 is

$$Rx - \left(d^1 + \lambda(1)\right) - \rho\pi c_1^2(1)$$

By the same arguments as before this is equal to

$$p(1-\alpha)c_2(1) - \rho\pi c_1^2(1)$$

Since $\rho \leq c_2(1)/c_1^2(1)$ then the last expression is at least equal to $c_2(1) \geq c_1^2(1)$. This implies again that for any patient consumer it is best to wait until period 2. This ends the proof. ■

A.7. Proof of Proposition 5.1

Firstly, it is clear that the optimal amount of local currency is $\bar{m}$, since the cost of printing pesos is always zero. Next, the necessary and sufficient first-order conditions with respect of x, y and $\lambda(s)$ of the second best problem are the following.

$$\begin{aligned} R[\phi_2(1)+\phi_2(2)] &= \phi_0 \\ [\phi_1(1)+\phi_1(2)] &\quad \phi_0 \\ \phi_1(s) &\quad \phi_2(s)+\tau(s) \end{aligned}$$

where $\tau(s)$ is the multiplier of the constraint $d + \lambda(s) \; 1$. The FOC corresponding to the consumption allocations are as follows.

$$\begin{aligned}
u'\left(c_1^1\right) &= \phi_1(1) + \phi_1(2) \\
q_1 u'\left(c_1^2(1)\right) &= \phi_1(1) \\
q_1 u'\left(c_2^1(1)\right) &= \phi_2(1) \\
q_2 u'\left(c_2^j(2)\right) &= \phi_2(2)
\end{aligned}$$

which again implies that $c_2^1(2) = c_2^2(2) \equiv c_2(2)$. Finally, the FOC with respect to d is

$$\phi_0 = \sum_{s=1} \tau(s) + \sum_{s=1} \phi_2(s)$$

This condition must hold since it must be the case that $d > 0$. Otherwise $x = 0$ but then consumption is always zero. Hence this last FOC must hold with equality. Since the objective function is strictly concave, the solution to be characterized need to be unique. I show now that this equilibrium is characterized by $\lambda(1) > 0$, $\lambda(2) > 0$ and $y \geq 0$. Under these conditions it must be the case that

$$R[\phi_2(1) + \phi_2(2)] = \phi_0 \quad (A.31)$$

$$[\phi_1(1) + \phi_1(2)] \quad \phi_0 \quad (A.32)$$

$$\phi_1(1) = \phi_2(1) + \tau(s) \quad (A.33)$$

$$\phi_0 = \sum_{s=1} \tau(s) + \sum_{s=1} \phi_2(s) \quad (A.34)$$

and so

$$R[\phi_2(1) + \phi_2(2)] = \sum_{s=1} \tau(s) + \sum_{s=1} \phi_2(s)$$

This implies

$$\sum_{s=1} \tau(s) = (R - 1)[\phi_2(1) + \phi_2(2)] > 0$$

which means that for at least one s, $\tau(s) > 0$.

But then:

$$R[\phi_2(1) + \phi_2(2)] = [\phi_1(1) + \phi_1(2)]$$

Hence we have:

$$u'\left(c_1^1\right) = R[q_1 u'(c_2(1)) + q_2 u'(c_2(2))]$$

On the other hand we have that:

$$\begin{aligned} q_1 u'\left(c_1^2(1)\right) &= q_1 u'\left(c_2^2(1)\right) + \tau(1) \\ &\geq q_1 u'\left(c_2^2(1)\right) \end{aligned}$$

which implies $c_1^2(1) \leq c_2^2(1)$. Also, from the date 1 and 2 budget constraints:

$$p(1-\alpha)c_2(1) + (1-p)c_1^2(1) = [p(1-\alpha) + (1-p)]c_2(2)$$

since they hold with equality. Therefore it must be the case that

$$c_1^2(1) \leq c_2(2) \leq c_2^2(1)$$

However, the fact that the date 1 budget constraints hold with equality implies

$$\lambda(1) = (1-p)c_1^2(1) + \lambda(2)$$

but so $\lambda(1) > \lambda(2)$. But then it must be the case that $\tau(1) > 0$ and $\tau(2) = 0$. This implies that

$$\begin{aligned} u'\left(c_1^1\right) &= [\phi_1(1) + \phi_1(2)] \\ &= q_1 u'\left(c_1^2(1)\right) + \phi_1(2) \\ &= q_1 u'\left(c_1^2(1)\right) + \phi_2(2) \\ &= q_1 u'\left(c_1^2(1)\right) + q_2 u'(c_2(2)) \end{aligned}$$

and so

$$q_1\left[u'\left(c_1^1\right) - u'\left(c_1^2(1)\right)\right] = q_2\left[u'(c_2(2)) - u'\left(c_1^1\right)\right]$$

meaning that

$$\operatorname{sgn}\left[c_1^1 - c_1^2(1)\right] = \operatorname{sgn}\left[c_2(2) - c_1^1\right]$$

I show now that $\operatorname{sgn}[c_1^1 - c_1^2(1)] > 0$, which proves partial suspension of convertibility. Suppose that this is not the case, that is, $\operatorname{sgn}[c_1^1 - c_1^2(1)] < 0$. Then $c_2(2) < c_1^1$. But then $c_2(2) < c_1^2(1)$, contradicting the statement above. Therefore it must be that $c_1^1 \geq c_1^2(1)$. However, if $c_1^1 = c_1^2(1)$ then it must be true that

$$c_1^1 = c_1^2(1) = c_2(1) = c_2(2)$$

But this implies that $q_1 u'(c_1^2(1)) = q_1 u'(c_2^2(1))$. But then, from

$$q_1 u'\left(c_1^2(1)\right) = q_1 u'\left(c_2^2(1)\right) + \tau(1)$$

then $\tau(1) = 0$, contradicting the result above. This implies that $c_1^1 > c_1^2(1)$ showing that partial suspension of convertibility of deposits must hold. This ends the proof of this proposition. ■

A.8. Proof of proposition 5.4

The condition

$$r < R\left(\frac{c_1^2(1)}{c_2(1)}\right)$$

is equivalent to

$$r < \frac{p(1-\alpha)c_1^2(1)}{\left(\frac{p(1-\alpha)c_2(1)}{R}\right)}$$

which is true if and only if

$$r < \frac{p(1-\alpha)c_1^2(1)}{\left(\frac{p(1-\alpha)c_2(1)+\bar{d}-\bar{d}}{R}\right)}$$

However, the second best implies that

$$\bar{d} = \lambda(1) + d$$

and so

$$r < \frac{p(1-\alpha)c_1^2(1)}{\left(\frac{p(1-\alpha)c_2(1)+(\lambda(1)+d)-(\lambda(1)+d)}{R}\right)}$$

and also at the solution of the second best problem:

$$Rx - d - \lambda(1) = p(1-\alpha)c_2(1)$$

so that the last inequality is equivalent to

$$r\left(x - \frac{\lambda(1)+d}{R}\right) < p(1-\alpha)c_1^2(1)$$

But then the same argument as in proposition 4.1 is applied here. Suppose all patient consumers believe that the others withdraw from the commercial banks at date 1. The intermediaries pay c_1^1 to the first αp and $c_1^2(1)$ consumers, financing both by borrowing $\lambda(1)$ dollars from abroad and or with the total amount of short run investment y (if this is positive). Note again that although the true state may be $s = 2$, if $\alpha p + 1 - p$ agents show up then the commercial bank thinks that the true state is $s = 1$. If more consumers show up the intermediaries must still pay $c_1^2(1)$pesos to each one, which will be exchanged for dollars at the Central Bank. But the amount of resources left is equal to $r\left(x - \frac{d+\lambda(1)}{R}\right)$. This happens because the intermediary liquidates $\left(x_1 - \frac{d+\lambda(1)}{R}\right)$ units of the long term investment at date 1 in order to satisfy withdrawals. Since this is strictly less than payments needed to be done if all the rest of agents withdraw then the bank fails (in the sense that not all consumers are satisfied, although debt at date 2 is perfectly honored).

All agents know that this happens if everybody runs against the bank. Then if every individual consumer thinks that the rest of the population runs, then it is optimal to withdraw early. This is because the expected utility of withdrawing early is strictly greater than $u(0) + v(0)$: But then in this case the commercial bank fails. This concludes the proof. ■

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