GOVERNMENT SPENDING AND GROWTH IN A SMALL OPEN ECONOMY WITH NONTRADEABLE GOODS

by

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ABSTRACT

We study the relation between government spending and growth in an endogenous growth model with tradeable and nontradeable goods. There are two production sectors, export and nontraded. We assume that the export sector is the learning sector. Knowledge produced in the export sector can be used in the nontraded sector. There are two consumption goods, importable and nontradable. We found that when the government only spends on nontradable (importable) goods, the steady state growth rate decreases (increases). Also, when government spend on importable and nontradable goods, we found a combination of these goods where government spending does not affect the steady state growth rate. Thus, the composition of government spending between importable and nontradable goods is an important factor in the relation between government spending and growth.

Keywords: export sector, learning by doing, government spending, growth.

JEL Classification: O41.
1. INTRODUCTION

In this paper we study the relation between government expenditure and economic growth in an economy led by the export sector. Thus, in order to model an export-sector-led economy, we assume that the export sector is the sector that produces more technological change. Furthermore, to fully understand the impact of government expenditure on the growth rate, we consider nontradeable goods. Thus, we develop an endogenous growth model of a small open economy with tradeable and nontradeable goods where we have introduced government spending.

In endogenous growth models, the amount of government expenditure can affect the growth rate of the economy. Thus, in the AK model, when the government finances all its spending through lump-sum taxes and people have infinite horizons, the amount of public expenditure does not affect the long run growth rate. In contrast, when people have finite horizons and have no intergenerational bequest motive, the amount of government expenditure affects negatively the long run growth rate (see Alogoskoufis and van der Ploeg 1990, 1994 and Saint-Paul 1992). Furthermore, when government expenditure is financed by an income tax and it is spent in public goods, the relation between the growth rate and government expenditure is an inverted U curve (see Barro 1990). Therefore, in the theoretical literature, the relation between growth and government spending can be negative, neutral or positive. Durlauf and Quah (1998) resume the empirical literature, they show growth regressions where the relation between government consumption and growth is negative or positive. Also, they show that growth in consumption is positively correlated with growth and that government investment is positively correlated with growth.

The existence of nontradeable goods gives interesting results with respect to the relation between public expenditure and growth. Thus, for simplicity, we assume that the exportable good is produced and accumulated but not consumed, that the importable good is consumed but neither produced nor accumulated, and that the nontradeable good is produced, accumulated and consumed. Thus, there are two production sectors, export and nontraded, and there are two consumption goods, importable and nontradeable. Then, we assume that the export sector (the learning sector) is the only sector that generates technological knowledge through learning by doing. The knowledge produced in the export sector becomes available to the nontraded sector (the nonlearning sector). The relative price of nontradeable goods is endogenously determined by changes in supply and demand for the nontradeable goods. The total amount of public expenditure
is spent on importable and nontradeable goods. There is international trade in goods, but, for simplicity, there are no international capital flows. People have infinite horizons. This model is based on Casares (1997). In this paper we have introduced government expenditure. Casares (2000) studies public expenditure policy in an endogenous growth model with overlapping generations.

We study how government spending affects the steady state growth rate of the economy. The principal result is that the composition of public expenditure between importable and nontradeable goods is an important element in order to understand how government spending affects the growth rate of the economy. We found that when the government only spends on nontradeable goods, the growth rate of the economy decreases, and when the government only spends on importable goods, the growth rate of the economy increases. Furthermore, when the government spends on importable and nontradeable goods, there is a combination of these goods where government spending does not affect the growth rate of the economy.

The paper is organized as follows. In section 2, we develop a model of a competitive market economy, where we introduce government expenditure. In section 3, we construct a system of differential equations describing the state-like and the control-like variables. In section 4, we study how the amount of public expenditure affects the steady state growth rate of the economy. Section 5 presents our conclusions.

2. THE COMPETITIVE MARKET ECONOMY

The economy is a small open economy with international trade in goods, but, for simplicity, there is no international capital mobility. Thus, the economy takes the prices of tradeable goods as given and the value of exports is equal to the value of imports. There are three goods, exportable, importable and nontradeable. The exportable good is produced and accumulated but not consumed. The importable good is consumed but neither produced nor accumulated. The nontradeable good is produced, accumulated and consumed (see Gavin 1991). Thus, there are two production sectors, export and nontraded. The output in each production sector is produced through physical capital, labour and technological knowledge. The total labour supply is constant. Labour is freely mobile between the productive sectors. The consumption basket is formed by importable and nontradeable goods. The government collects lump-sum taxes and spends on importable and nontradeable goods.

2.1 THE EXPORT SECTOR

There are a large number of $N_X$ competitive export firms with the same production function. We drop the time index for legibility. We assume that the production functions for the $i$th export firm is Cobb-Douglas:

$$Y_{X_i} = K_{X_i}^{\alpha} L_{X_i}^{1-\alpha} T_1$$

where $T_1 = K_X^{1-\alpha}$

where $Y_{X_i}$ is the output of the $i$th export firm, $K_{X_i}$ the stock of physical capital accumulated from the exportable good of the $i$th export firm, $L_{X_i}$ the quantity of labour employed in the $i$th export firm, and $\alpha$ and $1 - \alpha$ the shares of $K_{X_i}$ and $L_{X_i}$, respectively. The production function of the $i$th export firm is assumed to use only $K_{X_i}$.

Let $K_X$ be the aggregate stock of physical capital accumulated from the exportable good in the export sector. Technological knowledge is created through learning by doing in the export sector, thus knowledge is a by-product of investment (Arrow 1962). Then, as cumulative investment is the same as the current stock of physical capital (with zero depreciation), knowledge of the $i$th export firm increases in a parallel manner with $K_{X_i}$. Since knowledge is a public good, there are spillover effects of each firm’s knowledge across firms, that is knowledge of the $i$th export firm is available to all the export firms. Thus knowledge of the $i$th export firm is a function of the total learning in the export sector, that is, $K_X$ is the index of the stock of knowledge. These intra-industry benefits of knowledge are completely external to the individual export firm, thus the existence of competitive prices is assured. Therefore, $T_1$ is the external effect of $K_X$ on the $i$th export production function.

Since we are assuming that the exponent on $K_X$ in the externality $T_1$ is $1 - \alpha$, then the production function of the $i$th export firm has constant returns with respect to $K_{X_i}$ and $K_X$ (to a broad measure of capital), thus the economy exhibits endogenous growth (Romer 1986 and 1989). The production function of the $i$th export firm has increasing returns to scale if we consider all the
inputs at the same time.

In equilibrium all the export firms make the same choices. By aggregating across firms, we have that \( Y_X = N_X Y_{XI} \), \( K_X = N_X K_{XI} \) and \( L_X = N_X L_{XI} \), where \( Y_X \) is the aggregate output in the export sector, \( K_X \) is the aggregate stock of physical capital formed from exportable goods and \( L_X \) is the aggregate labour employed in the export sector. Then, the aggregate production function of the export sector is:

\[
Y_X = K_X^{\alpha} L_X^{1-\alpha} T_1 = K_X^{\alpha} L_X^{1-\alpha}
\]  \( (1) \)

On the right-hand side of the second equality, we have explicitly taken into account the value of the externality, thus the aggregate production function has constant returns to a broad measure of capital and increasing returns to scale if we consider all inputs at the same time.

The ith export firm has perfect foresight, thus the ith export firm takes into account current and expected future value of the world price of the exportable good, \( P_X^* \) (that remains constant at all time), the time paths for wage, \( w \), and the interest rate, \( r \), that is, the ith firm knows the sequence \( \{P_X^*, w, r\}_{t=0}^{\infty} \). The decision problem of the ith export firm at time zero is to chooses the investment and employment time paths that maximizes the present value of its cash flow:

\[
\max V = \int_0^\infty [P_X^* K_X^{\alpha} L_X^{1-\alpha} T_1 - w L_X - P_X^* I_X] e^{-(r^* \alpha)} dt
\]

subject to \( \dot{K}_{XI} = I_{XI} \). Where \( I_{XI} \) is the investment in \( K_{XI} \). The Hamiltonian is:

\[
H = [P_X^* K_X^{\alpha} L_X^{1-\alpha} T_1 - w L_X - P_X^* I_X + \lambda_X (I_{XI})] e^{-(r^* \alpha)} dt
\]

where \( \lambda_X \) is the shadow price, as of time \( t \), of an additional unit of \( K_{XI} \) at time \( t \). The decision variables are \( L_X \) and \( I_X \). Taking the externality as given, and considering that in equilibrium all the export firms make the same choices, the first order conditions are:

\[
w = P_X^* (1-\alpha) K_X L_X^{1-\alpha}
\]  \( (2) \)
\[ r = \alpha t^{-\alpha} \]  

(3)

\[ \lim_{t \to \infty} e^{-(t^{\alpha}e^{\alpha})} \lambda_x K_x = 0 \]  

(4)

where \( \lambda_x = P_x^* \). Equation (2) states the wage rate is equal to the value of the marginal product of labour in the export sector. Equation (3) states that the interest rate is equal to the total return of \( K_x \), given that \( P_x \) is constant, so capital gains are zero, and so interest rate is equal to the marginal product of \( K_x \). Thus equation (3) is the equilibrium condition for \( K_x \). Equation (4) is the transversality condition.

It is assumed that the ith export firm issues bonds for financing net investment. The issue of aggregate export bonds, \( B_x \), is given by:

\[ \dot{B}_x = P_x^* I_x \]  

(5)

The ith export firm distributes dividends to the households. The aggregate dividends, \( \pi_x \), are:

\[ \pi_x = P_x^* Q_x - wL_x - rB_x \]  

(6)

2.2 THE NONTRADED GOODS SECTOR

There are a large number of \( N_N \) competitive nontraded firms with the same production function. We assume that the production functions of the ith nontraded firm is Cobb-Douglas:

\[ Y_{ni} = K_{ni}^{\beta} L_{ni}^{1-\beta} T_2 \quad \text{where} \quad T_2 = K_x^{1-\beta} \]

where \( Y_{ni} \) is the output of the ith nontraded firm, \( K_{ni} \) is the stock of physical capital accumulated from nontraded goods in the ith nontraded firm, \( L_{ni} \) is the quantity of labour employed in the ith nontraded firm, and \( \beta \) and \( 1 - \beta \) the shares of \( K_{ni} \) and \( L_{ni} \), respectively. We assume that the production function of the ith nontraded firm uses only \( K_{ni} \). Since, there are spillover effects of knowledge between the sectors, then \( T_2 \) is the contribution of technological knowledge, generated
in the export sector but used in the nontraded sector (Succar 1987, Boldrin and Scheinkman 1988 and Young 1991). We consider that these inter-industry benefits of knowledge are purely external to the ith nontraded firms, thus competitive prices exist in the nontraded sector. Since we are assuming that the exponent on $K_N$ in the externality $T_2$ is $1 - \beta$, then the ith nontraded firm production function has constant returns to a broad measure of capital and increasing returns to scale if we consider all the inputs at the same time. Note that with this condition, and the corresponding condition in the export sector, the model has solution and displays endogenous growth.

In equilibrium all the nontraded firms make the same choices. By aggregating across firms, we have that $Y_N = N_N Y_{N_i}$, $K_N = N_N K_{N_i}$ and $L_N = N_N L_{N_i}$, where $Y_N$ is the aggregate output in the nontraded sector, $K_N$ is the aggregate stock of physical capital formed from nontradeable goods, $L_N$ is the total labour employed in the nontraded goods sector. Then, the aggregate production function in the nontraded sector is:

$$Y_N = K_N^{1-\beta} T_2 = K_N^{1-\beta} L_N^{1-\beta}$$

(7)

The aggregate production function has constant returns to a broad measure of capital and increasing returns to scale if we consider all the inputs at the same time.

Given that the ith nontraded goods firm has perfect foresight, it knows current and future prices and takes them as given, that is, the firm faces the time paths of the price of the nontradeable goods, $P_N$, wage, $w$, and interest rate, $r$, that is, $\{P_N, w, r\}_{t=0}^{\infty}$. The decision problem of the ith nontraded goods firm is to select the time paths of investment and employment that maximizes the present value of its cash flow:

$$\max V = \int_0^{\infty} [P_N K_N^{1-\beta} T_2 - w L_N - P_N J_N] e^{-\rho t} dt$$

subject to $\dot{K}_N = I_N$, where $I_N$ is investment in $K_N$. The Hamiltonian is standard:

$$H = [P_N K_N^{1-\beta} T_2 - w L_N - P_N J_N + \lambda_N (J_N)] e^{-\rho t}$$

where $\lambda_N$ is the shadow price as of time $t$ of an additional unit of $K_N$ at time $t$. Taking the
externality as given, and considering that in equilibrium all the nontraded goods firms make the same choices, the first order conditions are:

$$w = P_N K_N^\beta L_N^{1-\beta} (1-\beta) L_N^{-\beta}$$  \hspace{1cm} (8)

$$r = \beta K_N^(\beta-1) L_N^{1-\beta} L_N + \frac{\dot{P}_N}{P_N}$$  \hspace{1cm} (9)

and the transversality condition \( \lim_{t \to \infty} e^{(\gamma M - d) N} \lambda_N K_N = 0 \), where \( \lambda_N = P_N \). Equation (8) states that the wage rate is equal to the value of the marginal product of labour in the nontraded sector. Equation (9) is the dynamic equilibrium condition for \( K_N \). It states that interest rate is equal to the total return of \( K_N \), that is, interest rate is equal to the marginal product of \( K_N \) plus capital gains.

The \( i \)th nontraded goods firm issues bonds for financing net investment. The issue of aggregate nontraded bonds, \( B_N \), is:

$$\dot{B}_N = P_N J_N$$  \hspace{1cm} (10)

The \( i \)th nontraded goods firm distributes dividends. The aggregate dividends, \( \pi_N \), are:

$$\pi_N = P_N Q_N - w L_N - r B_N$$  \hspace{1cm} (11)

2.3 THE REPRESENTATIVE HOUSEHOLD

The decision problem of the representative household is to choose a path of aggregated consumption that maximizes the present value of an instantaneous utility function subject to the dynamic budget constraint. The optimal consumption basket, which is defined as consumption of the importable good and the nontradeable good, is determined by a static utility maximimization. We define the world price of the importable good as \( P_M \). Since the representative household is forward looking with perfect foresight, the consumer knows current and future values of \( w, r, P_M, \) and \( P_N \) and takes them as given. Thus the household's problem is:
\[ \max U(0) = \int_0^\infty \frac{C^{1-1/\sigma}}{1-1/\sigma} e^{-(\rho t)} \, dt \] (12)

subject to the household's flow budget constraint:
\[ \dot{A} = rA + w[L_X + L_N] + \pi_X + \pi_N - T - P_M^* C_M - P_N^* C_N \] (13)

and to the solvency condition \( \lim_{t \to +\infty} e^{-\rho t} \dot{A} = 0 \). Then, \( C \) is aggregated consumption, \( \sigma \) is the elasticity of intertemporal substitution and \( \rho \) is the subjective discount factor. We note that in the equation (13) the assets are \( A = B_X^* + B_N^* \) and the household income is the sum of interest on bond holding, \( rA \), wages and dividends from the firms. \( T \) is a lump-sum tax. The disposable income is allocated to consumption or saving. Total consumption is the demand for consumption of the importable good, \( C_M \), and consumption of the nontradeable good, \( C_N \). Saving is the demand for new bonds, thus we have that saving is \( \dot{A} = B_X^* + B_N^* \).

Next, we can define \( C \) as a homothetic index of \( C_M \) and \( C_N \): \( C = DC_M^\gamma C_N^{1-\gamma} \), where \( D = 1/[\gamma^\gamma(1-\gamma)^1\gamma] \) is a parameter, and \( \gamma \) and \( 1 - \gamma \) are the shares of \( C_M \) and \( C_N \) with respect to total expenditure on consumption. The consumer price index, \( P_C \), should be defined as \( P_C = P_M^\gamma P_N^{1-\gamma} \). The total expenditure on consumption is:
\[ P_C C = P_M^* C_M + P_N^* C_N \] (14)

Note that when the total expenditure on total consumption is divided by the consumer price index the resultant \( C \) is a measure of the representative household's utility.

With equation (14), we can rewrite the household's flow budget constraint as:
\[ \dot{A} = rA + w[L_X^* + L_N^*] + \pi_X + \pi_N - T - P_C C \] (15)

Then, the problem of the representative household is to maximize (12) subject to (15). The corresponding Hamiltonian is:
\[ H = \left[ \frac{C^{1-1/\sigma}}{1-1/\sigma} + \lambda_c [rA + w(L_x + L_N) + \pi_x + \pi_N - T - P_c C] \right] e^{-\rho t} \]

Where \( \lambda_c \) is the shadow price, as of time \( t \), of \( A \) at time \( t \). The first order conditions are:

\[ \frac{\dot{\lambda}_c}{\lambda_c} = \rho - \frac{r}{\lambda_c} \]  \hspace{1cm} (16)

and the condition \( \lim_{t \to 0} \lambda_c e^{(-\rho)A} = 0 \). Next, considering that \( P_N \) is variable with the time, and \( P_c \) also, we can differentiate equation (17) with respect to time and obtain:

\[ \frac{\dot{\lambda}_c}{\lambda_c} = -(1/\sigma) \frac{\dot{C}}{C} \frac{\dot{P}_c}{P_c} \] \hspace{1cm} (18)

Considering that \( P_M^* \) will be constant most of the time, we can differentiate the consumer price index and get:

\[ \frac{\dot{P}_c}{P_c} = (1-\gamma) \frac{\dot{P}_N}{P_N} \] \hspace{1cm} (19)

Substituting equations (18) and (19) in (16), we obtain the dynamic allocation condition for aggregated consumption:

\[ \frac{\dot{C}}{C} = \sigma [r - (1-\gamma) \frac{\dot{P}_N}{P_N} - \rho] \] \hspace{1cm} (20)

The consumption basket of \( C_M \) and \( C_N \) results from maximization of the utility function \( u = C_M^\gamma C_N^{1-\gamma} \) subject to the total expenditure on consumption, equation (14), where \( P_c C \) is given by the household’s flow budget constraint, (15). The static first order condition is:
\[
\frac{U_{C_M}}{U_{C_N}} = \frac{\gamma C_N}{(1-\gamma)C_M} = \frac{P_M^*}{P_N} \tag{21}
\]

where \( U_{C_M} \) and \( U_{C_N} \) are the marginal utilities of the importable good and the nontradeable good, respectively, thus the marginal rate of substitution of \( C_M \) for \( C_N \) is equal to the price ratio.

Using the total expenditure on consumption, equation (14), and the previous condition (21), then the level of \( C_M \) is:

\[
C_M = \gamma \frac{P_C^C}{P_M^*} \tag{22}
\]

and the level of \( C_N \) is:

\[
C_N = (1-\gamma) \frac{P_C^C}{P_N} \tag{23}
\]

Using the household's flow budget constraint (15) with the condition (22) and (23), we could obtain the consumption demands for \( C_M \) and \( C_N \).

### 2.4 THE GOVERNMENT

The government purchases goods, \( G \). The government finances its spending through lump-sum taxes, \( T \). The balanced government budget constraint is:

\[
T = G \tag{24}
\]

The total amount of public expenditure is expressed by the following rule:

\[
G = \phi (P_X^*Y_X + P_M^*Y_M) \tag{25}
\]

Equation (25) says that the total amount of public expenditure is a fixed proportion of the value of total output at world prices, where \( \phi \) is a parameter (\( \phi < 1 \)). We assume that the total amount of public expenditure is spent on importable goods, \( G_M \), and on nontradeable goods, \( G_N \), that is:
\[ G = G_M + G_N \]  

(26)

The amount of public expenditure on importable goods is a fixed fraction \( \theta \) of the total amount of public expenditure and the amount of public expenditure on nontradeable goods is a fixed fraction \( 1 - \theta \) of the total amount of public expenditure, where \( \theta \) is a parameter and \( 0 \leq \theta \leq 1 \). Therefore, the amount of public expenditure on importable goods and the amount of public expenditure on nontradeable goods are expressed by the following rules:

\[ G_M = \theta \left[ \phi \left( P^*_X Y_X + P^*_M Y_M \right) \right] \]  

(27)

\[ G_N = (1 - \theta) \left[ \phi \left( P^*_X Y_X + P^*_M Y_M \right) \right] \]  

(28)

2.5 EQUILIBRIUM IN GOODS AND LABOUR MARKETS

We can now proceed to obtain the aggregate equilibrium condition for the goods market at world prices. Considering that \( \dot{A} = \dot{B}_X + \dot{B}_N \), we can consolidate the household's flow budget constraint, (15), with the issue bonds' identities, (5) and (10), and the definition of aggregate dividends, (6) and (11), in order to obtain:

\[ P^*_X Y_X + P^*_N Y_N - T = P^*_M C_M + P^*_N C_N + P^*_X I_X + P^*_N I_N \]  

(29)

Substituting the balanced government budget constraint, (24), and equation (26), in the previous equation, we get:

\[ P^*_X Y_X + P^*_N Y_N = P^*_M C_M + P^*_N C_N + P^*_X I_X + P^*_N I_N + G_M + G_N \]  

(30)

This is the economy resources constraint at world prices, or the aggregated equilibrium condition for the goods market at world prices, where the value of the output of the two produced goods, \( Y \), is \( Y = P^*_X Y_X + P^*_N Y_N \). Next, we define the equilibrium condition for the nontradeable goods market:

\[ Y_N = C_N + I_N + G_N \]  

(31)
The price of the nontradeable good is flexible, ensuring that this market is always balanced, thus \( P_N \) is determined by the market clearing condition. With the equilibrium condition for the nontradeable goods market, equation (30) becomes:

\[
P_X Y X = P_M C_M + P_X I_M + G_M
\]  

(32)

The size of the total population is constant and equal to total labour \( L \), so that the total labour supply is also constant and normalized to one. Thus, the labour market equilibrium condition is:

\[
L_M + L_N = L = n + (1 - n) = 1
\]  

(33)

where \( n \) is the fraction of labour employed in the export sector and \( (1 - n) \) is the fraction of labour employed in the nontraded goods sector. The values of \( n \) and \( (1 - n) \) are less than one.

2.6 SUMMARY

Since \( C, K_X, K_N \) and \( Q \) are growing at all times, in order to solve the model, it will be convenient to define the variables of the model in terms of state-like and control-like variables. The characteristic of these variables is that they remain constant and finite in the steady state (see Mulligan and Sala-i-Martin 1991 and 1993 and Barro and Sala-i-Martin 1995). Thus, let \( z = K_x/K_X \) be the state-like variable and let \( \alpha = C/K_N \) be the first control-like variable. As \( n \) is constant and finite in the steady state, we can use it as the second control-like variable. Using the definition of \( z \) and the full employment condition for labour, (33), we can rewrite the first order conditions (2), (3), (8) and (9) as:

\[
w = P_X(1 - \alpha)K_n n^{-\alpha}
\]  

(34)

\[r = \alpha n^{1 - \alpha}
\]  

(35)
\[ w = P_N z^\beta K_X (1-\beta)(1-n)^{-\beta} \]  
(36)

\[ r = \beta z^{\beta-1}(1-n)^{1-\beta} \left( \frac{\dot{P}_N}{P_N} \right) \]  
(37)

Equating (34) and (36), we get:

\[ P_X (1-\alpha)n^{-\alpha} = P_N z^\beta (1-\beta)(1-n)^{-\beta} \]  
(38)

This is the static efficient allocation condition for labour between the sectors, where the value of the marginal product of labour in both sectors must be equal.

With equations (35) and (37), we obtain:

\[ \alpha n^{1-\alpha} = \beta z^{\beta-1}(1-n)^{1-\beta} \left( \frac{\dot{P}_N}{P_N} \right) \]  
(39)

This is the dynamic arbitrage condition for the two capital goods, where the total private returns for both capital must be the same. Thus, equation (39) states that the private marginal product of \( K_X \) is equal to the private marginal product of \( K_N \) plus capital gains on \( K_N \).

Using equations (20) and (35), we can define the growth rate of consumption as:

\[ \frac{\dot{C}}{C} = \sigma \left( \alpha n^{1-\alpha} - (1-\gamma) \left( \frac{\dot{P}_N}{P_N} \right) - \rho \right) \]  
(40)

or alternatively with equations (20) and (37) as:

\[ \frac{\dot{C}}{C} = \sigma \left( \beta z^{\beta-1}(1-n)^{1-\beta} + \left( \frac{\dot{P}_N}{P_N} \right) - (1-\gamma) \left( \frac{\dot{P}_N}{P_N} \right) - \rho \right) \]  
(41)

Finally, we rewrite the constraints (31) and (32) in terms of the state-like and control-like variables. Using the definition of \( z \) and the full employment condition for labour, we rewrite the aggregate production functions as:
\[ Y_X = K_X n^{1-\alpha} \]  
\[ Y_N = z^\beta K_N (1-n)^{1-\beta} \]  

(42)  
(43)

Considering the previous aggregate production function of the export sector, (42), the definition of \( a = C/K_N \), the level of \( C_M \), (22), the identity \( I_X = \dot{K}_X \), we can now rewrite equation (32) as:

\[
\frac{\dot{K}_X}{K_X} = n^{1-\alpha} - \frac{M^\star Y P_C a \alpha}{p_M^\star p_M} - \frac{G_M}{K_X} \]

(44)

where \( \dot{K}_X/K_X \) is the growth rate of \( K_X \), denoted with \( g_{K_X} \) and \( G_M/K_X \) is given by:

\[
\frac{G_M}{K_X} = \theta \left[ \phi \left( P_X n^{1-\alpha} + P_M^\star (1-n)^{1-\beta} \right) \right] 
\]

(45)

Alternatively, with the aggregate production function of the nontraded goods sector, (43), the condition for the level of \( C_M \), (23), and the identity \( I_N = \dot{K}_N \), the equilibrium condition for the nontradeable goods market, (31), can be rewritten as:

\[
\frac{\dot{K}_N}{K_N} = (1-n)^{1-\beta} - \frac{(1-\gamma) P_c a}{p_N^\star} - \frac{G_N}{K_N} \]

(46)

where \( \dot{K}_N/K_N \) is the growth rate of \( K_N \), denoted with \( g_{K_N} \) and \( G_N/K_N \) is given by:

\[
\frac{G_N}{K_N} = (1-\theta) \left[ \phi \left( P_X \frac{1}{z} n^{1-\alpha} + P_M^\star \frac{1}{z^\beta} (1-n)^{1-\beta} \right) \right]. 
\]

(47)

3 THE DYNAMIC SYSTEM

We have a system with one state-like variable, \( z \), and two control-like variables, \( a \) and \( n \). We now proceed to form a dynamic system in terms of these variables, that is:
\[ \dot{z} = f_1(z(t), n(t), a(t)) \]
\[ \dot{n} = f_2(z(t), n(t), a(t)) \]
\[ \dot{a} = f_3(z(t), n(t), a(t)) \]  \hspace{1cm} (48)

where \( f_i \) are nonlinear functions.

Using the definition of \( z \), the growth rate of \( z \) is:

\[ \frac{\dot{z}}{z} = \frac{\dot{K}_N}{K_N} - \frac{\dot{K}_X}{K_X} \]  \hspace{1cm} (49)

Next, we can obtain the growth rates of \( K_X \) and \( K_N \) in terms of \( z, n, a \) and parameters. From the efficient allocation condition for labour market, (38), we can obtain the value of \( P_N \) in terms of state-like and control-like variables (it is repeated here for convenience):

\[ P_N = \frac{P_x^*(1-a)(1-n)^\beta}{z^\beta(1-\beta)n^\alpha} \]  \hspace{1cm} (50)

Using the previous definition of \( P_N \), the definition of \( P_c \), and equations (45) and (47), we can rewrite equations (44) and (46) as:

\[ \frac{\dot{K}_X}{K_X} = n^{1-\alpha} - \left( \frac{\gamma P_M^{\gamma \alpha}}{P_x^*(1+\tau)^{1-\gamma}} \left[ \frac{P_x^*(1-a)(1-n)^\beta}{z^\beta(1-\beta)n^\alpha} \right]^{1-\gamma} za \right) \]
\[ - \theta \left[ \phi(P_x^* n^{1-\alpha} + P_x^* z^\beta (1-n)^{1-\beta}) \right] \]  \hspace{1cm} (51)

\[ \frac{\dot{K}_N}{K_N} = \frac{(1-n)^{1-\beta}}{z^{1-\beta}} - \left[ (1-\gamma)P_M^{\gamma(1+\tau)} \left[ \frac{z^\beta(1-\beta)n^\alpha}{P_x^*(1-a)(1-n)^\beta} \right]^\gamma \right] a \]
\[ - (1-\theta) \left[ \phi(P_x^* n^{1-\alpha} + P_x^* z^{1-\beta} (1-n)^{1-\beta}) \right] \]  \hspace{1cm} (52)

Thus, the growth rate of \( z \), (49), is defined by equations (51) and (52).

Next, we can obtain the growth rate of the control-like variable \( n \). Taking logs and
derivatives of both sides of the efficient allocation condition of labour, we get:

\[
\frac{\dot{n}}{n} = \frac{(1-n)}{[a(1-n)+\beta n]} \left[ -\frac{\dot{P}_N}{P_N} - \beta z \frac{\dot{z}}{z} \right]
\]  \quad (53)

Using the dynamic arbitrage condition for the two capital goods, (39), we can obtain (the equation is repeated here for convenience):

\[
\frac{\dot{P}_N}{P_N} = \alpha n^{1-a} \beta z^{\gamma-1}(1-n)^{1-\beta}
\]  \quad (54)

Thus, with the previous equation, the growth rate of \( n \) can be rewritten as:

\[
\frac{\dot{n}}{n} = \frac{(1-n)}{[a(1-n)+\beta n]} \left[ \beta z^{\gamma-1}(1-n)^{1-\beta} \alpha n^{1-a} \beta z^{\gamma-1}(1-n)^{1-\beta} \beta \left( \frac{\dot{K}_N}{K_N} - \frac{\dot{K}_X}{K_X} \right) \right]
\]  \quad (55)

where \( g_{K_N} \) and \( g_{K_X} \) are given by (52) and (51). Next, we know that the growth rate of the control-like variable \( a = C/K_N \) is given by:

\[
\frac{\dot{a}}{a} = \frac{\dot{C}}{C} - \frac{\dot{K}_N}{K_N}
\]  \quad (56)

Where the growth rate of consumption is obtained substituting the previous equation for \( \dot{P}_N/P_N \), equation (54), in equations (40) or (41), thus getting:

\[
\frac{\dot{C}}{C} = \sigma \left[ \alpha n^{1-a} \gamma + (1-\gamma) \beta z^{\beta-1}(1-n)^{1-\beta} \right]
\]  \quad (57)

then the growth rate of \( a = C/K_N \) is given by equations (57) and (52).

Therefore, our dynamic system (48) is formed by equations (49), (51) and (52), by equation (55), and by equations (56) and (57). We can see that the system only depends on \( z, n, a \) and parameters.
Finally, it can be shown that the growth rate of the value of total output is:

\[
\frac{\dot{Y}}{Y} = \frac{P_X^*Y_X}{Y} \frac{\dot{Y}_X}{Y_X} + \frac{P_N^*Y_N}{Y} \frac{\dot{Y}_N}{Y_N} + \frac{\dot{P}_N}{P_N}
\]

where \( P_X^*Y_X/Y = 1/(1 + [(P_N^*n^{\beta(1-n)^{1-\beta}})/(P_X^*n^{1-\alpha})]) \) is the share of \( P_X^*Y_X \) in the value of total output, and \( P_N^*Y_N/Y = 1/[(P_X^*n^{1-\alpha})/(P_X^*z(1-n)^{1-\beta})+1] \) is the share of \( P_N^*Y_N \) in the value of total output. The growth rate of \( Y_X \) and \( Y_N \) are given by:

\[
\frac{\dot{Y}_X}{Y_X} = \frac{K_X}{K_X} + (1-\alpha)\frac{\dot{n}}{n}
\]

\[
\frac{\dot{Y}_N}{Y_N} = \frac{\dot{K}_X}{z} + \frac{\dot{K}_X}{K_X} - (1-\beta)\frac{\dot{n}}{n} \frac{n}{1-n}
\]

In the next section, we analyze the steady state property of this system when government expenditure is introduced.

4 GOVERNMENT SPENDING AND THE STEADY STATE GROWTH RATE

We study in this section, in the steady state, how the amount of government expenditure affects the growth rate of the economy. In order to understand the effect of government expenditure in the model, it is indispensable to know the proportion of public expenditure spent on importable and nontradeable goods. Given that when the government only spends on nontradeable goods, the growth rate of the economy decreases, and when the government only spends on importable goods,

\[\text{The growth rate of the economy refers, in this model, to the total output growth rate.}\]
the growth rate of the economy increases. In addition, when the government spends on importable and nontradeable goods, there is a combination of these goods where the growth rate is unaffected. Therefore, we stress the importance of the composition of public expenditure between importable and nontradeable goods in order to fully understand the relation between government expenditure and growth. Before to study how the amount of government expenditure affects economic growth, we explain the steady state properties of the model.

In the steady state the growth rate of the state-like and control-like variables are zero, thus with equations (49) and (56) we have that \( \dot{K}_N / K_N = \dot{K}_X / K_X \) and \( \dot{C} / C = \dot{K}_N / K_N \). So consumption and capital goods grow at the same rate in the steady state. Given that \( P_N \) depends on \( z, n \) and parameters, see equation (50), we also have that \( P_N \) is constant in the steady state. Since the value of the output of the two goods is \( Y = P_X X + P_N Y_N \), and given that \( P_N \) is constant in the steady state, it is easy to show that \( Y \) grows at the same rate as \( C, K_X \) and \( K_N \). Thus, in the steady state, the long term growth rate of \( Y \), denoted with \( g \), is:

\[
g = \sigma (\alpha n^{1-\alpha} - \rho) \tag{61}
\]

or alternatively:

\[
g = \sigma (\beta z^{\beta-1} (1-n)^{1-\beta} - \rho) \tag{62}
\]

We study the relation between public expenditure and growth through numerical simulation. We use the following parameter values: \( \alpha = 0.6, \beta = 0.3, P_X^* = 1, P_M^* = 1.2, \sigma = 0.10, \rho = 0.03, \gamma = 0.7 \) and \( \phi = 0.10 \). These parameter values are only for illustrative purposes.

Using the time-elimination method, Casares (1997) studies the transitional dynamics of the dynamic system (48). He solves the dynamic system (48) when government spending is zero. He obtains a system of differential equations describing policy functions for the control-like variables (see Mulligan and Sala-i-Martin 1991 and 1993). Policy function consists of a functional relationship between the control-like variable and the state-like variable, where the time component has been eliminated. He obtains the following two policy function \( n = f_1(z) \) and \( \alpha = f_2(z) \), where \( f_i \) are nonlinear functions. He finds that the policy function \( n = f_1(z) \) has positive slope and
that the policy function \( a = f_t(z) \) has negative slope. We use the results of Casares (1997) in order to explain how government expenditure affects the variables of the model.

We now solve the dynamic system (48) in the steady state, \( ( z = 0, \dot{n} = 0, \dot{a} = 0 ) \). When government expenditure is zero, we have that:

\[
\begin{align*}
n &= 0.578, \quad z = 0.214, \quad a = 4.492, \\
P_N &= 0.972, \quad g = 0.0452
\end{align*}
\]

(1st case)

We note that we can obtain feasible solutions for the private and social levels of \( n \) \( (n < 1) \) with \( \alpha > \beta \) or \( \beta > \alpha \). We can see that the growth rate is 4.52% per year. Now, we solve, in the steady state, the dynamic system (48) when government expenditure is positive. We have experimented with different values of \( \theta \) (the fraction of the total amount of public expenditure spent on importable goods). When \( \theta = 0 \), the values of the variables are:

\[
\begin{align*}
n &= 0.488, \quad z = 0.285, \quad a = 3.077, \\
P_N &= 1.045, \quad g = 0.0421
\end{align*}
\]

(2nd case)

We compare this second case with the first case where government expenditure is zero. We can see that the steady state level of \( n \) decreases, thus the long run private rate of return on capital in the economy decreases, equation (35), in turn producing a decrease in the long run growth rate of the economy from 4.52% to 4.21% per year, equation (61). In addition, the value of \( z \) is higher than that achieved by a market economy with zero government expenditure. Thus, the growth rate of \( z \) must be positive in the transition. Then, immediately after the introduction of government expenditure \( (\text{with } \theta = 0) \), the growth rate of \( K_x \) decreases and the growth rate of \( K_N \) increases. Thus, \( P_N \) increases instantaneously and \( n \) decreases instantaneously. We can say that the nontradeable goods market is in a situation of excess demand immediately after the introduction of government expenditure. After that, the economy slowly moves to the new steady state. In the long run, the export sector is damaged and the steady state growth rate of the economy decreases.

Now, we consider the case when \( \theta = 1 \). We get:
\[ n = 0.618, \ z = 0.186, \ a = 4.522, \]
\[ P_N = 0.944, \ g = 0.0465 \]  \hspace{1cm} (3rd case)

We compare this third case with the first case where government expenditure is zero. We can see that the steady state level of \( n \) increases and thus the long run growth rate of the economy increases from 4.52 % to 4.65 % per year. Also, the value of \( z \) is lower. Thus, the growth rate of \( z \) must be negative in the transition. Then, immediately after the introduction of government expenditure (with \( \theta = 1 \)), the growth rate of \( K_x \) increases and the growth rate of \( K_n \) decreases. Thus, \( P_N \) decreases instantaneously and \( n \) increases instantaneously. Therefore, the nontradeable goods market is in a situation of excess supply immediately after the introduction of public expenditure. After that the value of the variables slowly moves to the new steady state. In the long run, the export sector is stimulated and the steady state growth rate of the economy increases.

In figure 1, we show the relation between the total amount of public expenditure (for different values of \( \theta \)) and growth. Notice that when \( \theta = 0.69 \) (when 69 % of the total amount of government expenditure is spent on importable goods), we have a point where government spending does not affect the steady state growth rate of the economy. To the left of \( \theta = 0.69 \), the amount of government expenditure affects negatively the steady state growth rate. To the right of \( \theta = 0.69 \), the amount of public expenditure affects positively the steady state growth rate. The values of the variables when \( \theta = 0.69 \) are:

\[ n = 0.578, \ z = 0.214, \ a = 3.986, \]
\[ P_N = 0.972, \ g = 0.0452 \]  \hspace{1cm} (4th case)

When we compare this fourth case with the first case where government expenditure is zero, we can see that the values of \( n, z, P_N \) and \( g \) are the same and the value of \( a = C/K_n \) is lower. The presence of government expenditure leads to a crowding out of the quantity of \( a = C/K_n \).

Therefore, the composition of public expenditure between importable and nontradeable goods is an important element in order to fully understand how government expenditure affects the growth rate of the economy.

5 CONCLUSIONS
We have developed an endogenous growth model of a small open economy with tradeable and nontradeable goods where we have introduced government expenditure. We have assumed that the export sector is the learning sector, thus it is the leading sector in technological terms. We have used the model in order to study the relation between government expenditure and growth. We have found that the composition of government spending between importable and nontradeable goods is an important element in order to understand how government expenditure can affect the steady state growth rate of the economy. We have compared the model when government expenditure is zero with the model when public expenditure is positive. We have considered different compositions of public expenditure.

First, we have assumed that the government only spends on nontradeable goods. In this case, we have shown that the steady state growth rate with positive government expenditure is lower than the steady state growth rate with zero public expenditure. We have explained that immediately after the introduction of public expenditure, the growth rate of $K_X$ decreases and the growth rate of $K_N$ increases. This implies that $P_N$ increases instantaneously. In the steady state, the fraction of labour employed in the export sector decreases. Thus, the export sector, that is the learning sector, is damaged and the steady state growth rate decreases.

Also, we have assumed that the government only spends on importable goods. In this case, the steady state growth rate increases with respect to the case with zero public expenditure. We have explained that instantaneously after the introduction of government expenditure, the growth rate of $K_X$ increases and the growth rate of $K_N$ decreases. This implies that $P_N$ decreases instantaneously. In the steady state, the fraction of labour employed in the export sector increases. Thus, the export sector is stimulated and the steady state growth rate increases.

Finally, when the government spends on importable and nontradeable goods, we have found a combination of these goods where government expenditure does not affect the steady state growth rate of the economy. Thus, in this paper, we stress that the composition of government spending between importable and nontradeable goods is an important factor in order to fully understand the relation between government expenditure and economic growth.
REFERENCES


Figure 1 Government spending and growth.