The NAIRU, Unemployment and the Rate of Inflation in Brazil

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Abstract

*This paper estimates the Brazilian NAIRU (**N**on-**A**ccelerating **I**nflation **R**ate of **U**nemployment) and investigates several empirical questions: the behavior of NAIRU along time, error bands for NAIRU and the usefulness of NAIRU to the conduct of monetary policy in Brazil.*

There are many recent studies about the NAIRU ¾ Staiger, Stock and Watson (1997), Blanchard and Katz (1997) and Portugal, Madalozzo and Hillbrecht (1999). This article innovates with respect to previous ones because it adopts an econometric model that, in our judgment, is more adequate to deal with the still recent instability of Brazilian economy. We estimate two different state-space models: one with ARCH residuals and another with a Markov- switching regime.

The article presents some new evidence on several questions. It shows that the NAIRU has been increasing since 1995. It concludes that there is a statistically significant relationship, with correct sign, between deviations of unemployment from the NAIRU and inflation. It also shows that the usefulness of the NAIRU to the conduct of monetary policy is very limited because its error bands are too wide.

1. Introduction

In this article we estimate the ***N***on- ***A***ccelerating ***I***nflation ***R***ate of ***U***nemployment (Nairu) for Brazil. We investigate the relationship between the rate of inflation and the deviation, from NAIRU, of the observed rate of unemployment. We intend to determine to what degree the observed rate of unemployment, which summarizes what happens in the labor market, can be used to forecast the rate of inflation and to what extent the Central Bank should observe the rate of unemployment before taking decisions about how to conduct monetary policy (i.g., fixing the short-run interest rate, controlling domestic credit, establishing targets for monetary aggregates, etc..)

There are many recent studies about the NAIRU, such as Staiger, Stock and Watson (1997), Blanchard and Katz (1997) for the United States and Portugal, Madalozzo and Hillbrecht (1999) for Brazil. Our research innovates when compared to others by adopting an econometric procedure that, in our view, is more adequate to deal with structural breaks in the model ' s equation. We also estimate error bands for the Brazilian NAIRU.

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Estimating the relationship between nominal and real variables in Brazil is a very hard task due to the instability experienced by the Brazilian economy in its recent past. Macroeconomic models are estimated using information provided by time series data and the number of required observations outnumber what is available for the most recent period of stability, after the Real Plan. This is a challenge for anyone who works with Brazilian macroeconomic data. Fortunately, since the beginning of the 90's there has been a variety of theoretical developments in time series analysis that enable us to extract information from data even when instabilities, like the ones observed for Brazilian data, are present [Kim and Nelson (1999)]. These developments allow us to deal with a larger amount of data. It is also true that the greater stability shown by the Brazilian economy since the Real Plan at the middle of 1994, despite the recent change in the exchange rate policy, gives greater confidence that the investment in econometric models is going to payoff in the future.

Two different models are used to calculate the natural rate of unemployment and its confidence interval: a state-space model with ARCH residuals (TVP model) and a state-space model with a Markov-switching regime (MSR model). A detailed description of both models and of the estimation methods can be found in Nelson and Kim (1999). The models are estimated using quarterly data for the average rate of open unemployment and for the rate of inflation measured by the national consumer price index (INPC) both collected by IBGE , respectively, in six and eleven Metropolitan Regions of Brazil, for the period 1982:1 - 1999:3.

The article is organized as follows: in Section 2 we describe the data used, how the monthly data were transformed into quarterly data and the estimated models; in Section 3 we present the estimation procedures adopted; in Section 4 we show the estimation results and discuss some statistical tests; in Section 5 we conclude.

2 . Data and Estimated Models

The basic data comprise the national consumer price index (INPC) of IBGE from 1981:12 to 1999:9 and the average rate of open unemployment of IBGE from 1982:1 to 1999:9. These monthly data were averaged into quarterly data and the model was estimated using only quarterly data, from the first quarter of 1982 to the third quarter of 1999. The monthly data were transformed into quarterly data as follows:

$\pi_t = (1/3).\log (P_{t,f} / P_{t-1,f})$ = monthly geometric average of the quarterly rate of inflation;
$P_{t,f}$ = centered INPC (national consumer price index) of IBGE for the last month of quarter t;
U_t = quarterly average of the monthly average rate of open unemployment of IBGE.

2.1 The Basic Model

Our basic model, for the relationship between the change of the rate of inflation and the rate of unemployment, can be represented by the following equations:

$$\Delta\pi_t = \mu_t + \sum_{s=1}^{3} b_s (u_{t-s} - \bar{u}) + Z_t \gamma + + \varepsilon_t \quad , \qquad (1)$$

and

$$\sum_{s=0}^{3} m_{t+s} = 0 \quad \text{(restriction that allows for the identification of } \bar{u}) \qquad (2)$$

where:

$$\mu_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} \qquad (3)$$

D_{it} = seasonal dummy of quarter i;

$\bar{u}$ = NAIRU ;

Z_t = is a row vector of control variables with the two first lags of $\Delta\pi_t$;

$\varepsilon_t \sim N(0,\sigma^2)$

In this version the NAIRU is an unknown parameter. The model, with this representation, is not linear in its parameters. Nevertheless, the above equation can be presented in a different form that makes it linear in its parameters:

$$\Delta\pi_t = \beta_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \sum_{s=1}^{3} b_s u_{t-s} + Z_t\gamma + \varepsilon_t \quad , \qquad (4)$$

where:

$$\beta_0 = \alpha_0 - \sum_{s=1}^{3} b_s \bar{u} \qquad (5)$$

Estimating the above model and considering the hypothesis that $\sum_{s=0}^{3} m_{t+s} = 0$, we obtain an estimate of $\bar{u}$ and α_0 . We explain below how to compute this estimate:

$$\sum_{s=0}^{3} m_{t+s} = 0 \quad \Rightarrow \quad \sum_{s=0}^{3} (\beta_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t}) = -4 \, . \sum_{s=1}^{3} b_s \bar{u}$$

therefore,

$$\bar{u} = - (\beta_0 + \alpha_1/4 + \alpha_2/4 + \alpha_3/4) / (\sum_{s=1}^{3} b_s) \qquad (6)$$

The inflation stabilization plans adopted by Brazil [Cruzado (1986:1 and 1986:2), Bresser (1987:3), Verão (1989:1), Collor (1990:2) and Real (1994:3)] have produced, in the quarters of their implementation, an abrupt reduction of the rate of inflation. To deal with these shocks, interventions were made in the model at each quarter of implementation of each stabilization plan. The basic model with interventions at each stabilization plan is described by the following equation:

$$\Delta\pi_t = \beta_0(1+\theta_{t\tau}) + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \sum_{s=1}^{3} b_s u_{t-s} + Z_t\gamma + \varepsilon_t \quad , \qquad (7)$$

$\theta_{t\tau}$ = 0, if there was no stabilization plan at quarter t;

$\theta_{t\tau} = \theta\tau$ = nonlinear intercept intervention parameters when stabilization plan "τ" happens at period t [τ =1 (Cruzado Plan), τ =2 (Bresser and Verão Plans), τ=3 (Collor and Real Plans)]

To simplify the description of the model, while changing notation to allow for time varying parameters, we summarize the representation of the model as follows:

Let,

$$y_t = \Delta\pi_t \; ; \; X_t = [(1+\theta_{t\tau})\ D_{1t}\ D_{2t}\ D_{3t}\ U_{t-1}\ U_{t-2}\ U_{t-3}\ \ Z_t] \;, \; \beta_t^* = [\beta_{0t}\ \alpha_{1t}\ \alpha_{2t}\ \alpha_{3t}\ \beta_{1t}\ \ \beta_{2t}\ \ \beta_{3t}\ \gamma_t] \;,$$

therefore the model can be represented, in a compact form, by

$$y_t = X_t\, \beta_t^* + \varepsilon_t \qquad (8)$$

The Brazilian economy, in its recent past, has experienced a period of great economic instability and the above equation may show structural breaks in its parameters. To deal with possible structural breaks we estimate two different versions of the above model: a) the TVP model (that allows for parameter change over time and ARCH residuals); b) the MSR model (that allows for parameter change over time and Markov-switching regimes).

2.2 The TVP and the MSR Models

The TVP Model

The TVP model is a state-space model with ARCH residuals and is a simplified version of the model proposed by Harvey, Ruiz and Sentana (1992):

$$y_t = X_t\, \beta_t^* + \Lambda\, \varepsilon_t^* + \varepsilon_t \quad , \quad \text{measurement equation,} \qquad (9)$$

$$\beta_t^* = \beta_{t-1}^* + \omega_t \quad , \quad \text{transition equation ,} \qquad (10)$$

$$\varepsilon_t \sim N(0,\sigma^2) \quad , \quad \omega_t \sim N(0,Q) \quad \text{and} \quad \varepsilon_t^* / \psi_{t-1} \sim N(0,h_{1t}).$$

Where: Λ, ε_t^* and σ are scalars and Q is, by hypothesis, diagonal and 9 x 9. The ARCH effect is introduced through the scalar residual ε_t^*. The variance of ε_t^* is given by:

$$h_{1t} = 1+\gamma_0\, {\varepsilon_{t-1}^*}^2 \qquad (11)$$

The MSR Model

The MSR model is a state-space model with a Markov-switching regime. It is a simplified version of the model suggested by Kim and Nelson (1999):

$$y_t \;=\; x_t \, \beta_t^* + \varepsilon_t \quad , \text{ measurement equation;} \qquad (12)$$

$$\beta_t^* = \beta_{t-1}^* + \omega_t \quad , \text{ transition equation;} \qquad (13)$$

$$\varepsilon_t \sim N(0, \sigma_{St}^2) \quad , \quad \omega_t \sim N(0, Q_{St})$$

The subscript St denotes that σ^2 and the parameters at the diagonal of matrix Q take values that depend on a discrete, non-observed variable, that follows a Markov-switching process with 2 different states (regimes). The transition probabilities are given by:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \qquad (14)$$

where, p_{iJ} = probability of state J, at period t, given state i at period t-1 and

$$\sum_{J=1}^{2} p_{iJ} = 1, \quad i=1,2 \qquad (15)$$

3. Estimation Procedures

3.1 The TVP Model Estimation Procedure

Harvey, Ruiz and Sentana (1992) substitute the $\varepsilon_{t-1}^{*\,2}$ variable in equation (11), which is non-observed, by its conditional expectation, $h_{1t} = 1 + \gamma\, E[\varepsilon_{t-1}^{*\,2} / \psi_{t-1}]$. Therefore, the algorithm is an approximation. To get $E[\varepsilon_{t-1}^{*\,2} / \psi_{t-1}]$, Harvey, Ruiz and Sentana augmented the original state vector, in the transition equation (10), in the following way:

$$\begin{bmatrix} b_t^* \\ e_t^* \end{bmatrix} = \begin{bmatrix} I_9 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} b_{t-1}^* \\ e_{t-1}^* \end{bmatrix} + \begin{bmatrix} I_9 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_t \\ e_t^* \end{bmatrix}$$

where I_9 is the identity matrix with 9x9 dimension. Therefore, the measurement equation (9) is replaced by:

$$y_t = \begin{bmatrix} x_t & \Lambda \end{bmatrix} \begin{bmatrix} b_t^* \\ e_t^* \end{bmatrix} + \varepsilon_t$$

It is not hard to show that,

$$E[\varepsilon_{t-1}^{*\,2} / I_{t-1}] = E[\varepsilon_{t-1}^* / I_{t-1}]^2 + E[(\varepsilon_{t-1}^* - E(\varepsilon_{t-1}^* / I_{t-1}))^2]$$

The two expectations at the right hand side of the last equation can be computed, recursively, using the Kalman filter. The first expectation is equal to the recursive estimation of the second element of the state vector (ε_{t-1}^*) squared and the second expectation is equal to its recursively estimated covariance matrix.

Given the intercept intervention parameters $\theta\tau$ [τ =1,2 and 3], σ, Λ, γ_0 and Q it is possible to obtain the model's likelihood value and β_t^* through the Kalman Filter recursions. That is, we can concentrate the likelihood with respect to β_t^* , using the Kalman filter and, with the help of a numerical optimization routine, estimate the value of the other parameters [$\theta\tau$ (τ=1,2 and 3), σ, Λ, γ_0 and the elements at the diagonal of matrix Q] that maximize the likelihood.

To arrive at a more parsimonious model the following restrictions were imposed a priori:

Q(5,5)=Q(6,6)=Q(7,7), equality between parameters, at the diagonal of matrix Q, that controls for the time change of lag unemployment coefficients.

Q(8,8)=Q(9,9) , equality between parameters, at the diagonal of matrix Q, that controls for the time change of lag $\Delta\pi_t$ coefficients.

With these restrictions matrix Q has only 4 unknown parameters. Therefore the TVP model has, if we exclude the parameters of β_t^* from the counting, a total of 10 parameters to be estimated. In the next section we describe how, departing from this more general model and using a few statistical tests, we are able to reduce, from 10 to 5, the number of parameters to be estimated .

3.2 The MSR Model Estimation Procedure

In order to estimate the MSR model it is necessary to make inferences about both the unobserved states and the latent Markov state. The regime shifts of the MSR model imposes a nonlinearity that precluded the estimation of this type of model until the estimation methods by Albert and Chib(1993), Shephard (1994) and Kim (1994) were developed. I use Kim's method to estimate the MSR model. The Kim filter is an optimal estimator in the sense that no other estimator, based on a linear function of the information set, yields a smaller mean squared error [Smith and Makov (1980)].

Let,

$I_{t-1} = [\, y_{t-1}\,,\; y_{t-2}\,, \ldots ,\, y_1]$, information available up to time t-1;

$\beta^{*(i,j)}_{t|t-1} = E(\beta^*_t \,/\, I_{t-1}, S_t = j\ , S_{t-1} = i)$;

$\Sigma^{(i,j)}_{t|t-1} = E[(\beta^*_t - \beta^*_{t/t-1})(\beta^*_t - \beta^*_{t/t-1})' \,/\, I_{t-1}, S_t = j\ , S_{t-1} = i]$;

$\beta^{*(i)}_{t-1|t-1} = E(\beta^*_{t-1} / I_{t-1}, S_{t-1} = i)$;

$\Sigma^{(i)}_{t-1|t-1} = E[(\beta^*_{t-1} - \beta^*_{t-1/t-1})(\beta^*_t - \beta^*_{t/t-1})' \,/\, I_{t-1}, S_{t-1} = i]$.

$\Pr[S_t = j\ ,\ S_{t-1} = i] = p_{iJ}$

$P = [\, p_{ij}\,] =$ matrix of transition probabilities;

$\Pr[S_t = i \,|\, I_t]$ = probability of state i, at period t, given information up to time t.

The objective of Kim's nonlinear filter is to obtain estimates of the unobserved state vector (β^*_t) and its associated mean squared error matrix (Σ_t), and of the marginal probability of the latent Markov state variable S_t , at each date t. The estimates are based on information available up to time t. That is, at each date t , given $\beta^{*(i)}_{t-1|t-1}$, $\Sigma^{(i)}_{t-1|t-1}$, $\Pr[S_{t-1} = i \,|\, I_{t-1}]$, y_t, Q_j, σ_j, (for i=1,2 and j=1,2), $\theta\tau$ (τ =1,2 and 3) and the matrix of transition probabilities (P) Kim's filter allows us to obtain approximate estimations of $\beta^{*(i)}_{t|t}$, $\Sigma^{(i)}_{t|t}$, and $\Pr[S_t = j \,|\, I_t]$ (j= 1,2). To start the filter we need the initial values of $\beta^{*(i)}_{0\,|\,0}$, $\Sigma^{(i)}_{0\,|\,0}$ and $\Pr[\, S_0 = i \,/\, I_0]$ (i=1,2) . We set $\beta^{*(i)}_{0\,|\,0} = 0$, $\Sigma^{(i)}_{0\,|\,0} = I \times 100.000$ and $\Pr[\, S_0 = I \,/\, I_0]$ (i=1,2) equal to the ergodic distribution of the Markov chain (the steady-state marginal distribution of the states).

What we have called Kim's filter is actually a nonlinear version of the Kalman filter combined with Hamilton's nonlinear filter. We describe both filters below[1].

The Nonliner Kalman Filter – Part I

Given $\beta^{*(i)}_{t-1|t-1}$, $\Sigma^{(i)}_{t-1|t-1}$, y_t, Q_i , σ_i , (for i=1,2) and $\theta\tau$ (τ =1,2, 3), the nonlinear Kalman Filter, in its first part, obtains $\beta^{*(i,j)}_{t|t}$, $\Sigma^{(i,j)}_{t|t}$, the conditional one step prediction error ($\eta^{(i,j)}_{t|t-1}$), and the conditional variance of the one step prediction error, $f^{(i,j)}_{t|t-1}$ (for i=1,2 and j=1,2) . The algorithm estimates 2 (number of different possible regimes) state vectors for each possible value of $S_{t-1.}$ Therefore, at each date t, the algorithm estimates 4 state vectors. The first part of Kalman filter recursions are presented below:

[1] A detailed discussion of the Kim´s filter can be found in Kim (1994) and Kim & Nelson (1999).

Prediction equations:

$$\beta^{*(i,j)}_{t|t-1} = \beta^{*(i)}_{t-1|t-1}$$
$$\Sigma^{(i,j)}_{t|t-1} = \Sigma^{(i)}_{t-1|t-1} + Q_j$$

Updating equations:

$$\eta^{(i,j)}_{t|t-1} = y_t - \beta^{*(i,j)}_{t|t-1}$$
$$f^{(i,j)}_{t|t-1} = x_t \ \Sigma^{(i,j)}_{t|t-1} \ x_t' + \sigma^2_j$$
$$\beta^{*(i,j)}_{t|t} = \beta^{*(i,j)}_{t|t-1} + \Sigma^{(i,j)}_{t|t-1} \ x'_t \left[\ f^{(i,j)}_{t|t-1} \ \right]^{-1} \eta^{(i,j)}_{t|t-1}$$
$$\Sigma^{(i,j)}_{t|t} = (I - \Sigma^{(i,j)}_{t|t-1} \ x_t' \left[\ f^{(i,j)}_{t|t-1} \ \right]^{-1} x_t \ ) \ \Sigma^{(i,j)}_{t|t-1}$$

The Hamilton Filter

Given $\Pr[S_{t-1} = i \,|\, I_{t-1}]$, P, $\eta^{(i,j)}_{t|t-1}$, $f^{(i,j)}_{t|t-1}$, the Hamilton Filter obtains $\Pr[S_{t-1} = i \ , \ S_t = j \ | \ I_t\ ]$ and $\Pr[S_t = i \,|\, I_t]$. The Hamilton Filter is presented below:

$$\Pr[S_t = j, S_{t-1} = i \,|\, I_{t-1}] = \Pr[S_t = j \ , \ S_{t-1} = i] \Pr[S_{t-1} = i \,|\, I_{t-1}] \quad , \quad (i, j = 1,2)$$
$$\Pr[S_{t-1} = i \ , \ S_1 = j \,|\, I_t] = \frac{f(y_t \,|\, S_{t-1} = i \ , \ S_t = j \ , \ I_{t-1}) \ \Pr[S_{t-1} = i \ , \ S_t = j \,|\, I_{t-1}]}{f(y_t \,|\, I_{t-1})}$$
$$\Pr[S_t = j \,|\, I_t] = \sum_{i=1}^{2} \Pr[S_{t-1} = i \ , \ S_t = j \,|\, I_t]$$

Where,

$$f(y_t \,|\, S_{t-1} = i, S_t = j, I_{t-1}) = (2\pi)^{-\frac{1}{2}} \left| \ f^{(i,j)}_{t|t-1} \ \right|^{\frac{1}{2}} \exp\{-\frac{1}{2} \eta^{(i,j)}_{t|t-1}{}' f^{(i,j)\,-1}_{t|t-1} \eta^{(i,j)}_{t|t-1}\} \ (\ i \ , \ j = 1, \ 2)$$

and

$$f(y_t \,|\, I_{t-1}) = \sum_{j=1}^{2} \sum_{i=1}^{2} f(y_t \,|\, S_t = j \ , \ S_{t-1} = i, \ I_{t-1}) \Pr[S_t = j \ , S_{t-1} = i \,|\, I_{t-1}]$$

The Nonlinear Kalman Filter – Part II

Given $\beta^{*(i,j)}_{t|t}$, $\Sigma^{(i,j)}_{t|t}$ (both estimated by the first part of the nonlinear Kalman filter) and $\Pr[S_{t-1} = i \ , S_t = j \ | \ I_t\ ]$, $\Pr[S_t = i \,|\, I_t]$ (both estimated by the Hamilton filter) the nonlinear Kalman filter, in its second part, collapses $\beta_{t|t}^{*(i,j)}$ into $\beta_{t|t}^{*(j)}$ and $\Sigma_{t|t}^{(i,j)}$ into $\Sigma_{t|t}^{(j)}$. The collapsing is of crucial importance because without it the number of filter evaluations is multiplied by 2, at each date t, and it is computationally unfeasible to estimate the model. The collapsing is an approximation based on the work of Harrison and Stevens (1975). The approximation consists of a weighted average of the updating procedures by the probabilities of the Markov state, in which the mixture of 4 Gaussian densities is collapsed, after each observation, into a mixture of 2 densities. It is done as follows:

Collapsing equations:

$$\beta_{t|t}^{*(j)} = \frac{\sum_{i=1}^{2} \Pr[S_{t-1} = i, S_t = j \,|\, I_t]\ \beta_{t|t}^{*(i,j)}}{\Pr[S_t = j \,|\, I_t]}$$

$$\Sigma_{t|t}^{(j)} = \frac{\sum_{i=1}^{2} \Pr[S_{t-1} = i, S_t = j \,|\, I_t]\{\Sigma_{t|t}^{(i,j)} + (\beta_{t|t}^{*(i)} - \beta_{t|t}^{*(i,j)})(\beta_{t|t}^{*(i)} - \beta_{t|t}^{*(i,j)})'\}}{\Pr[S_t = j \,|\, I_t]}$$

The Likelihood

The likelihood is given by : $L = \prod_{t=1}^{T} f(y_t / I_{t-1})$

For the MSR model we adopt the same restrictions on the parameters of the diagonal of matrix Q that were imposed on the TVP model in section 3.1. Furthermore equation (15) shows that we can estimate only p_{11} and p_{21} to obtain the entire matrix P. Therefore, given p_{11} , p_{22} , $\theta\tau$ (τ =1,2, 3) , σ_i (the measurement equation residual's standard deviation at state i, i=1,2), the parameter at the diagonal of matrix Q at states 1 and 2, the fixed initial values of the state vector and it's covariance matrix we can use Kim's filter to estimate β_t^* and calculate the value of model's likelihood. That is, the likelihood can be concentrated with respect to β_t^* making it easier to estimate p_{11} , p_{22} , $\theta\tau$ (τ =1,2, 3) , σ_1 , σ_2 and the parameters at the diagonal of matrix Q, at each state, with the help of a numerical optimization routine. The Kim's filter obtains the mapping from the hyperparameters – σ_i and Q_i - the parameters in P, and $\theta\tau$ (τ =1,2, 3) to the likelihood value. We use this mapping to estimate the likelihood maximizing values of these hyperparameters and parameters using a numerical optimization routine. The results of this estimation are presented in section 4.

If we exclude the parameters of β_t^* the MSR model has 15 parameters to be estimated. We show, in the next section, how departing from this more general model and using a few statistical tests we end up with a model with only 8 parameters to be estimated through the numerical optimization routine.

4 . The Model's Estimation Results

The two models were estimated using computer programs developed with the help of the Matlab software .

4. 1 The TVP Model Results

The TVP model was initially estimated with 10 parameters if we exclude, from the counting, the parameters that belong to β_t^* and that were concentrated out of the likelihood using the Kalman filter. We could not reject — at a significance level higher than 10% and using the likelihood ratio test - that Λ , γ_0 and the parameters of the diagonal of matrix Q which controls for the time variation of coefficients of the dummy variables, of unemployment rate and of the lags of the inflation rate, are all equal to zero. The log likelihood values with and without the restrictions are presented in table 4.

Therefore, we could not reject that the TVP model, without ARCH residual in the measurement equation, with time variation only in the intercept and with intercept intervention at the stabilization Plans fits the data well. With the above mentioned restrictions the TVP model shows only 5 parameters to be estimated through the numerical optimization routine. The estimated values of these 5 parameters can be found in Table 1. The one-step ahead forecast and fitted values of the TVP model are presented at the set of graphics II. At the set of graphics III we show the smoothed estimates of the NAIRU, as defined in section 2, and its confidence interval computed from the smoothed estimates of β^* and of Var(β^*) , at each period t. . The confidence intervals for the NAIRU were computed from 10,000 extractions of the distribution of the estimator of β^* and , in each extraction, the NAIRU was computed as explained in section 2.

Table 1
TVP Model

Log Likelihood = 80.91, Theil-U = 0.29

Hyperparameters	Parameter	S.D.
Q(1,1)	0.0088	0.0053
σ (ε_t standard deviation)	0.0189	0.0036
Intercept Intervention Parameters		
Cruzado Plan (θ_1)	-0.8282	0.2225
Bresser and Verão Plans (θ_2)	-1.4461	0.2549
Collor I and Real Plans (θ_3)	-3.7243	0.7364

4.2 The MSR Model Results

The MSR model was initially estimated in its more general version, with 15 parameters if we exclude, from the counting, the parameters that belong to β_t^* . At a significance level higher than 10% and using the likelihood ratio test it could not be rejected that all the parameters at the diagonal of matrix Q are equal to zero at state 2 and that these parameters – with the exception of the one that controls the time variability of the intercept – are also equal to zero at state 1. At table 4 we present the log likelihood for the MSR model with and without these restrictions.

The MSR model, with the above restrictions, has 8 parameters to be estimated through the numerical optimization routine. At Table 2 we present the estimated values of these 8 parameters. The estimated transition probability matrix (P) can be found in Table 3. The state 1 probability, at each period, can be viewed in graph **I**. The one-step-ahead forecast and fitted values of the MSR model are presented at the set of graphics **II**. At the set of graphics **III** we show the smoothed estimates of the NAIRU, and its confidence interval computed using the same procedure adopted for the TVP model.

Table 2
MSR Model

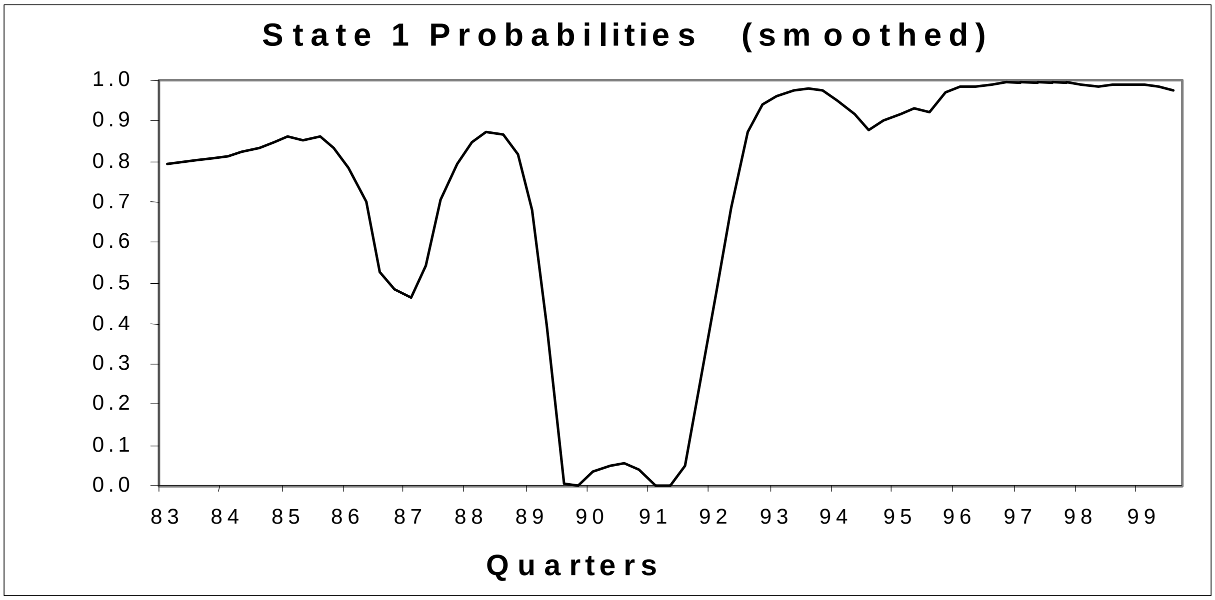
Log Likelihood = 88.58, Theil-U= 0.30

Hyperparameters	Parameter	S.D.
Q(1,1) at state 1	0.0059	0.0024
σ_1 (ε_t standard deviation at state 1)	0.0087	0.0043
σ_2 (ε_t standard deviation at state 2)	0.0389	0.0044
Transition Probabilities		
p_{11} [Pr (St=1 | St-1=1)]	0.9607	0.0009
p_{21} [Pr (St=1 | St-1=2)]	0.1325	0.0153
Intercept Intervention Parameters		
Cruzado Plan (θ_1)	-1.1472	0.3207
Bresser and Verão Plan (θ_2)	-1.9991	0.4604
Collor and Real Plan (θ_3)	-5.3728	1.3375

Table 3
Transition Probability Matrix (P)

	St=1	St=2
St-1=1	0.961	0.039
St-1=2	0.133	0.867
Steady –State probabilities	0.7713	0.2287

Graphic I



4.3 Comparison Between the PVT and MSR Models and Some Additional Statistical Tests

The maximizing value of log likelihood for the MSR model is 88.58 and the same value for the TVP model is 80.91. The log likelihood for the two models and for a variety of alternative hypothesis is presented at Table 4. The TVP model is more parsimonious than the MSR model since it has 3 parameters less. It is also true that the TVP model (without ARCH residuals) is nested in the MSR model. Nevertheless, as it has been pointed out by Engel and Hamilton (1990) in the region of the parameter space where the MSR model becomes the TVP model the information matrix is singular and the standard regularity conditions, needed to establish asymptotically valid hypothesis tests, are not satisfied. Therefore we cannot test, using the usual tests, which of the models better fits the data and the results for both models are going to be presented.

The MSR model, as can be seen in Table 5, passes satisfactorily two tests: one suggested by Engel and Hamilton (1990) designed to test if the measurement equation residuals are homoscedastic without regime switching ($p_{11} = 1-p_{22}$) and with a time-varying intercept in the measurement equation and another to test if there is no time-varying intercept at state 1 ($Q_1(1,1)=0$) when the hypothesis of Markov-switching, for the standard deviation of the residual of the measurement equation, is in place. When the hypothesis $p_{11} = 1-p_{22}$ is imposed to the MSR model there is no longer a Markov-switching process and the rate of inflation is an i.i.d. sequence with individual densities given by a mixture of two normals. We are able to reject the first hypothesis, at a significance level smaller than 1%, using the likelihood ratio and Wald tests. The second hypothesis is rejected at a significance level smaller than 1% if the likelihood ratio test is used or at a significance level smaller than 5% if the Wald test is used.

Table 4
Log Likelihood and Alternative Hypothesis

	TVP Model	
Initial Model (10 parameters)	80,91	
Simplified Models (Q (j,j) = 0, for j ≠ 1 , Λ= 0 and γ_0 = 0)		
Q(1,1) ≠ 0 (5 parameters)	80,91	**(selected model)**
Q(1,1) = 0 (4 parameters)	79,77	
	MSR Model	
Initial Model (15 parameters)	88,93	
Simplified Models (Q_i (j,j) = 0, for j ≠ 1 and i=1,2)		
$Q_1(1,1) \neq Q_2(1,1)$ (regime dependent) (9 par.)	88,64	
$Q_1(1,1) = Q_2(1,1)$ (regime independent) (8 par.)	88,46	
$Q_1(1,1) \neq 0$ and $Q_2(1,1) = 0$ (regime dependent) (8 par.)	88,58	**(selected model)**
Q(1,1) = 0 , both regimes (7 par.)	84,44	

Note : the parameter count, in this table, does not include the parameters of β_t^* (β_t^* has 9 other parameters)

Table 5
Additional Hypothesis Tests – MSR Model

	Ho*: $p_{11} = 1- p_{22}$	**Ho: $Q_1(1,1)=0$**
Wald test	1.42e+004	5,23
Likelihood Ratio test	8,76	8,28
Log Likelihood under Ho	84,20	84,44

* Suggested by Engel & Hamilton (1990)

We also tested both models (TVP and MSR) for serial correlation in the standardized forecast errors and in the squares of the standardized forecast errors. Tables 6 and 7 present these tests. Using the Q statistic, for the TVP (MSR) model, we do not reject at the 4.7% (4.4%) significance level the null of no serial correlation in the standardized forecast errors for lags smaller than 21 (19). For the MSR model, at all investigated lags and at the 5% significance level, we do not reject the hypothesis of no serial correlation in the squares of the standardized forecast errors. For the TVP model the last statement is true only at the 1% significance level.

5 . Conclusions

In our analysis of the results we will emphasize two main aspects: is there a statistically significant relationship between an upward (downward) deviation of the rate of unemployment from our estimated value of the NAIRU and a reduction (increase) in the rate of inflation? To what degree our measure of the NAIRU can be used as a guide to the conduct of monetary policy in Brazil?

The first question can be easily answered by testing if the sum of coefficients of lag unemployment is negative and significantly different from zero in the two models. This sum belongs, with a 95% degree of confidence, to the (-1.9, -1.2) interval for the TVP model and to the (-1.2,-0.7) interval for the MSR model. Neither of the models allows for time variation in the coefficients of lag unemployment. Therefore, it cannot be rejected that the deviations of the unemployment rate from the NAIRU can have a significant effect, with correct sign, on the inflation rate. Nevertheless, it is also true that the above estimates of the effect are not very precise. If we consider the uncertainty with respect to what is the right model then no value between (-1.9,-0.7) can be rejected.

The answer to the second question depends on the degree of precision with which the NAIRU is estimated. Observing the set of graphics III we can conclude that the estimates of the NAIRU, considering both models, are very imprecise and that from the second quarter of 1995 the estimated value of the NAIRU is contained within its error bands and therefore it cannot be rejected that the observed rate of unemployment was equal to the NAIRU. That is , from the second half of 1995 the error bands estimated for the NAIRU do not give a good guidance as to what should be the monetary policy followed by the Central Bank.

Despite what we have said one cannot conclude that our estimates of the NAIRU are useless. It should be pointed out that, during a good part of the period of high inflation faced by Brazil, the rate of unemployment was, systematically, below the error bands estimated for the NAIRU. Therefore there was a continuous up trend in the rate of inflation whose upward trajectory was only broken by the stabilization plans. This is true for the MSR model from the beginning of 1985 until the second half of 1995. For the TVP model this is true from the beginning of 1986 to the beginning of 1991. It is also interesting to note that both models indicate that there was a decrease in the NAIRU from 1994 until the middle of 1995 when it started increasing again until it reached values next to its previous 1994 level by the third quarter of 1999.

Unfortunately there is not a large number of articles with different models and with estimates of the NAIRU for Brazil. The degree of confidence in the results depends critically on the increase of research in this area.

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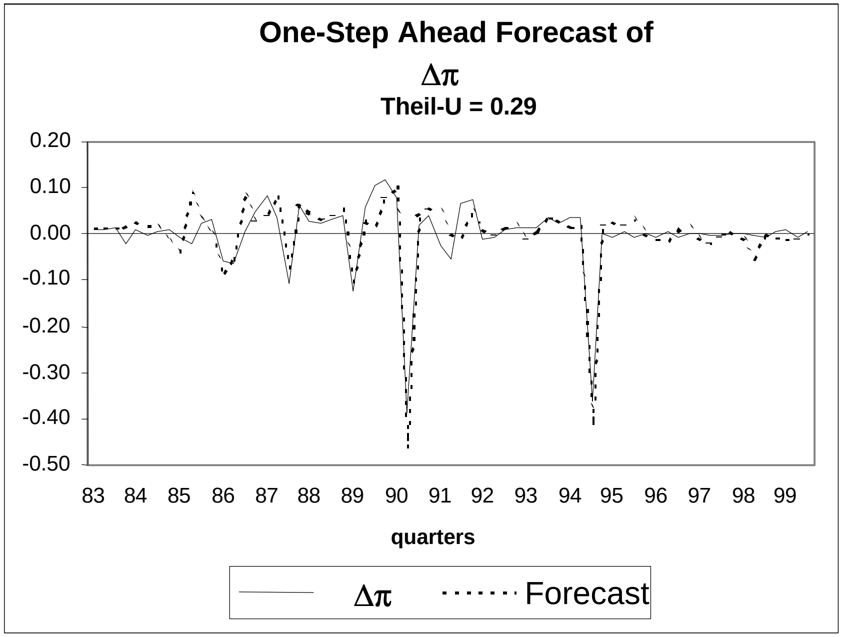
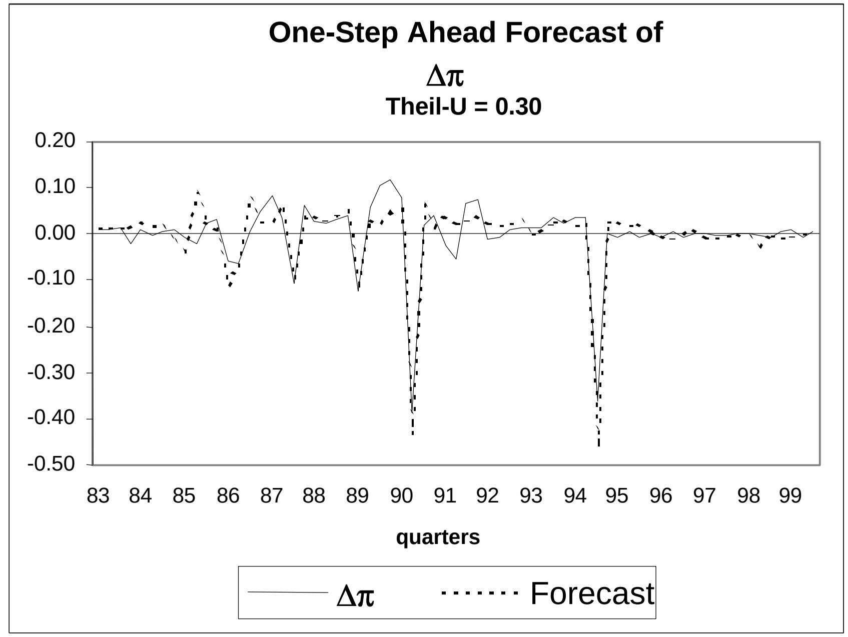
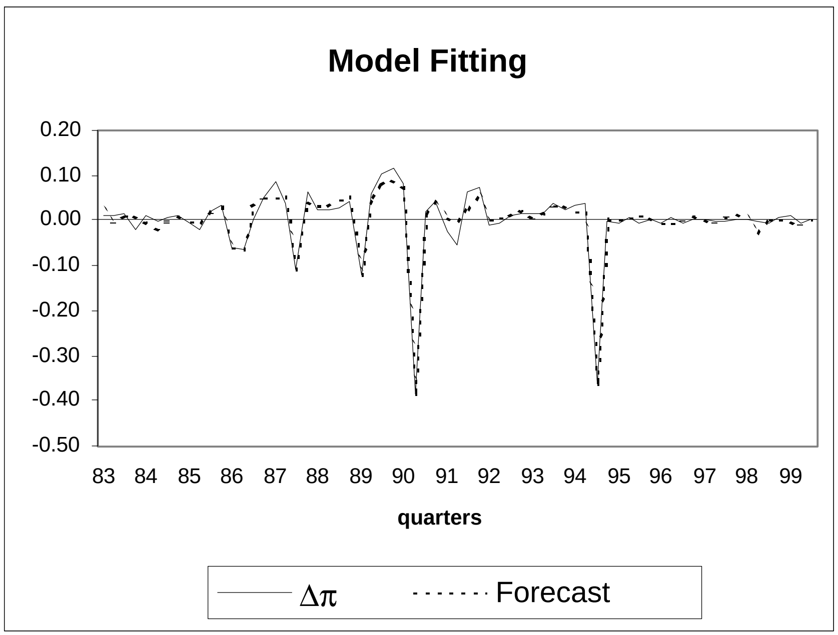
Graphics II

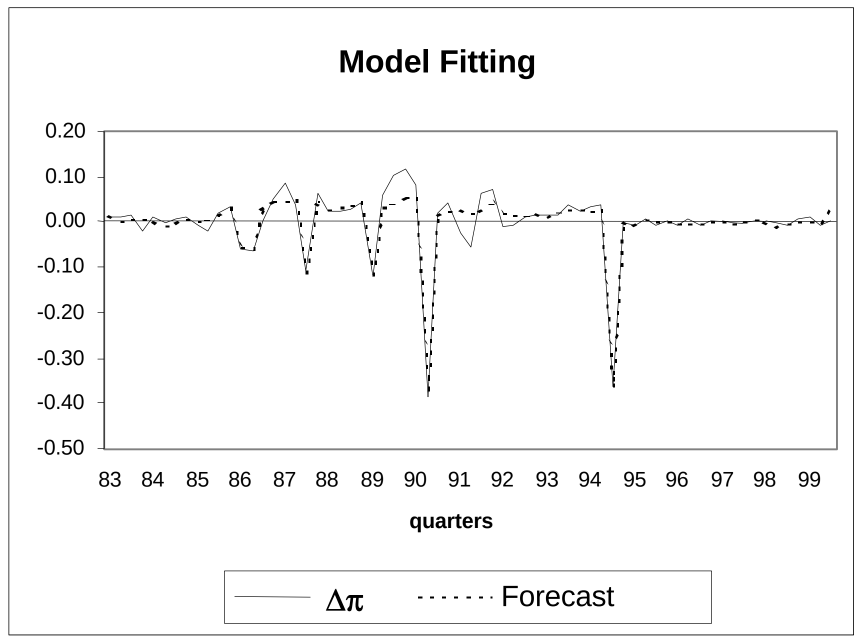
TVP Model



MSR Model

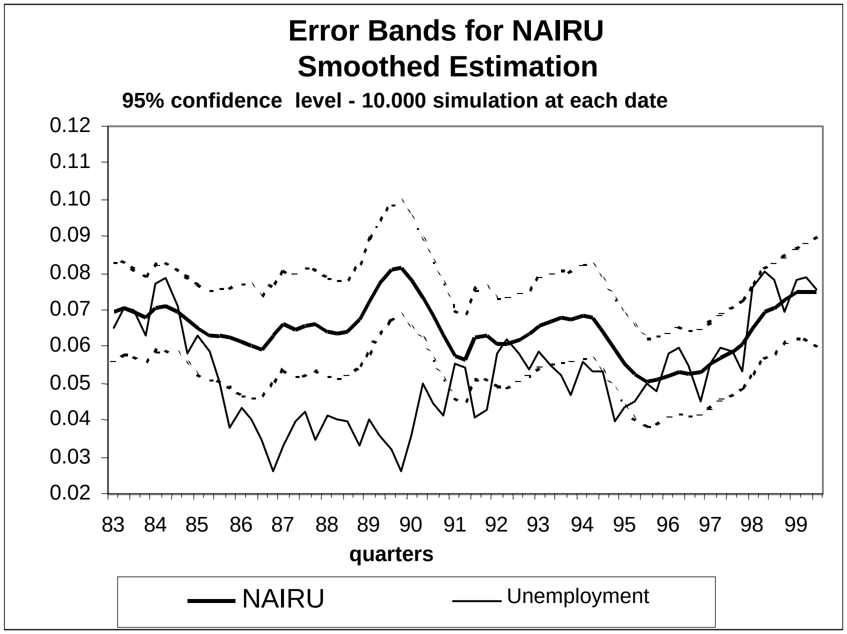


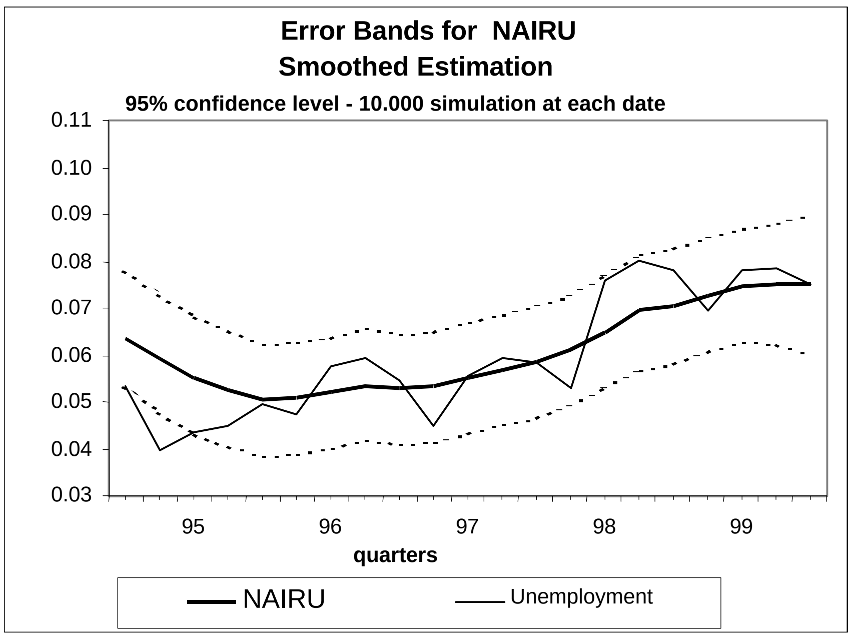




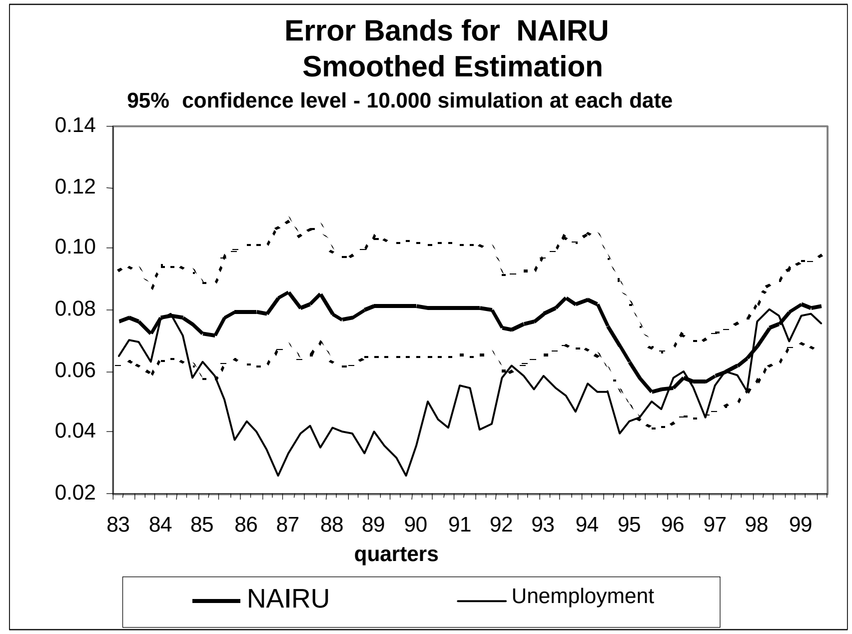
Graphics III

TVP Model





MSR Model



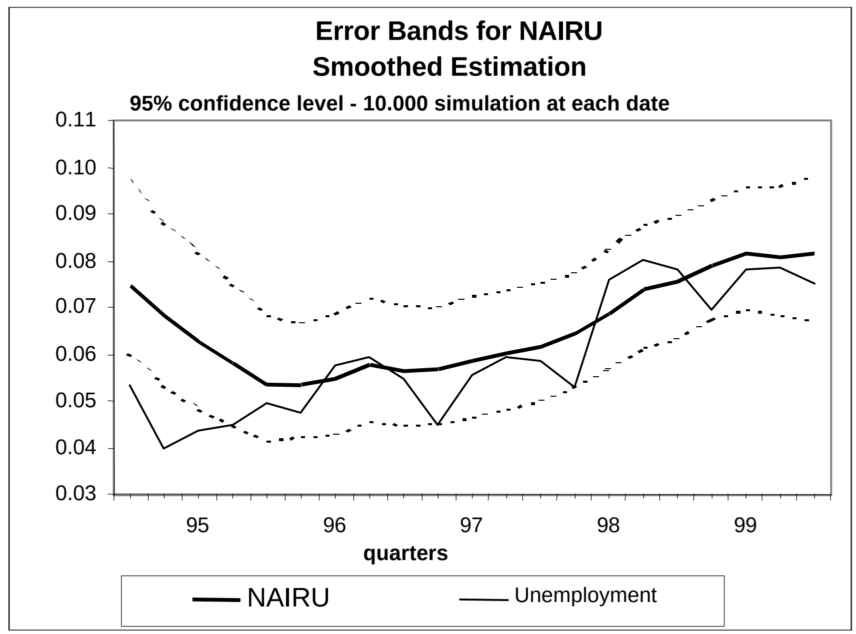


Table 6

Tests for Serial Correlation of Standardized Forecast Errors

	MRS MODEL				TVP MODEL			
	Autocorrelation	Partial Correlation	Q-Statistic	Probability	Autocorrelation	Partial Correlation	Q-Statistic	Probability
1	0.256	0.256	4.054	0.044	0.170	0.170	1.785	0.182
2	-0.110	-0.188	4.824	0.090	-0.262	-0.300	6.132	0.047
3	-0.050	0.036	4.982	0.173	-0.091	0.020	6.663	0.083
4	-0.055	-0.079	5.181	0.269	0.044	-0.021	6.789	0.147
5	-0.087	-0.060	5.687	0.338	0.017	-0.012	6.809	0.235
6	-0.256	-0.258	10.151	0.118	-0.210	-0.228	9.817	0.133
7	-0.155	-0.041	11.812	0.107	-0.145	-0.063	11.278	0.127
8	-0.045	-0.092	11.955	0.153	0.060	-0.019	11.531	0.173
9	-0.083	-0.116	12.447	0.189	-0.006	-0.111	11.534	0.241
10	0.015	0.007	12.464	0.255	0.030	0.063	11.599	0.313
11	0.065	-0.024	12.777	0.308	0.047	-0.004	11.764	0.382
12	-0.131	-0.273	14.100	0.294	-0.108	-0.171	12.650	0.395
13	-0.124	-0.121	15.306	0.289	-0.152	-0.159	14.462	0.342
14	-0.017	-0.110	15.329	0.356	-0.002	-0.016	14.462	0.416
15	0.231	0.165	19.699	0.184	0.164	0.062	16.660	0.340
16	0.048	-0.187	19.895	0.225	-0.047	-0.158	16.842	0.396
17	-0.048	0.010	20.090	0.270	-0.062	0.065	17.173	0.443
18	0.236	0.152	24.973	0.126	0.235	0.193	22.023	0.231
19	0.255	0.112	30.845	0.042	0.286	0.142	29.390	0.060
20	-0.030	-0.150	30.926	0.056	-0.126	-0.177	30.850	0.057
21	-0.199	-0.071	34.682	0.031	-0.188	0.058	34.207	0.034
22	-0.086	0.011	35.406	0.035	-0.019	-0.035	34.241	0.046
23	-0.065	-0.063	35.824	0.043	0.057	-0.015	34.564	0.057
24	-0.119	-0.040	37.279	0.041	-0.053	0.005	34.856	0.071

Table 7

Tests for Serial Correlation of Squared Standardized Forecast Errors

	MRS MODEL				TVP MODEL			
	Autocorrelation	Partial Correlation	Q-Statistic	Probability	Autocorrelation	Partial Correlation	Q-Statistic	Probability
1	0.244	0.244	3.707	0.054	0.285	0.285	5.032	0.025
2	-0.052	-0.119	3.879	0.144	0.222	0.153	8.135	0.017
3	-0.112	-0.074	4.685	0.196	-0.031	-0.143	8.197	0.042
4	-0.117	-0.081	5.585	0.232	-0.137	-0.147	9.419	0.051
5	-0.061	-0.028	5.831	0.323	-0.083	0.026	9.873	0.079
6	0.172	0.187	7.833	0.251	0.151	0.261	11.424	0.076
7	0.111	-0.001	8.685	0.276	0.064	-0.043	11.707	0.111
8	-0.112	-0.150	9.575	0.296	0.173	0.041	13.822	0.087
9	-0.141	-0.058	11.014	0.275	-0.105	-0.204	14.619	0.102
10	0.033	0.121	11.095	0.350	-0.123	-0.048	15.733	0.108
11	0.018	-0.016	11.119	0.433	-0.084	0.104	16.261	0.132
12	0.056	0.002	11.360	0.498	-0.050	-0.002	16.456	0.171
13	-0.032	-0.113	11.438	0.574	-0.115	-0.204	17.494	0.178
14	-0.072	0.003	11.855	0.618	-0.051	-0.125	17.706	0.221
15	-0.120	-0.036	13.030	0.600	-0.072	0.100	18.133	0.256
16	-0.069	-0.072	13.423	0.642	0.024	0.157	18.181	0.313
17	0.089	0.072	14.106	0.660	-0.040	-0.088	18.319	0.369
18	0.140	0.079	15.817	0.605	0.062	-0.044	18.655	0.413
19	0.055	0.020	16.087	0.651	-0.054	-0.102	18.913	0.462
20	0.059	0.070	16.406	0.691	-0.022	0.089	18.955	0.525
21	-0.023	-0.023	16.456	0.743	-0.077	0.066	19.516	0.552
22	-0.068	-0.025	16.912	0.768	-0.129	-0.228	21.139	0.512
23	-0.012	0.017	16.927	0.813	-0.082	-0.155	21.805	0.532
24	0.090	0.040	17.762	0.814	-0.086	-0.060	22.568	0.545