Two-Sided Search and Temporary Employment

Dimitri Paolini[*]
IRES, Université Catholique de Louvain
3 Place Montesquieu; B1348 Louvain La Neuve, Belgium
paolini@ires.ucl.ac.be

May 2000

Abstract

The objective of this paper is to model explicitly the possibility to form temporary matching in a model of two-sided search. The agents (workers and employers) differ by their human quality endowment. In a search equilibrium agents form subintervals and are only matched to agents within their class. The introduction of a temporary market can have a positive impact on unemployment, but it may have a negative impact on low-skilled agents' utility. When a delay cost is introduced, this negative impact can be reduced especially if an education policy is implemented to decrease the heterogeneity of the human capital.

Keywords: Two-Sided Search, Matching, Temporary Employment.
JEL Classification: C78, J41, J64.

[*]I would like to thank L. Artige, A. Gautier, R. Nicolini, G. Parodi, and especially F. Bloch for his very helpful comments and discussions. I am also grateful to seminar participants at DMQTE and YEC Annual Meeting Oxford. Remaining errors are mine. The financial support of the PAI program P4/01 is gratefully acknowledged.

1 Introduction

The share of temporary work in total employment has been increasing in Europe in recent years. As Cahuc and Postel-Vinay (1999) recall us, at the end of the seventies, labor market regulations required that temporary works were destined to specific tasks, characterized by large variations in productivity. Those regulations have changed somewhat since the eighties, and it is now possible in a number of European countries to hire workers on a temporary basis even on jobs which are not subject to large variations in productivity.

While in 1983 only 4% of the employees in the EC held temporary jobs, in 1991 10% did (see Table 1).

Country	Share (%)	Country	Share (%)
Belgium	5.1	Germany	10.5
UK	5.4	France	11
Italy	7.5	Portugal	11
Ireland	9	Denmark	11.5
Netherlands	10	Finland	13
Greece	10.5	Spain	33

Table 1: Share of temporary jobs in total employment (source: OECD (1996), quoted in Cahuc and Postel-Vinay (1999))

Using a panel of Spanish firms, Bentolila and Dolado (1994) and Bentolila and Saint-Paul (1992) generally show that the introduction of fixed-duration contracts is equivalent to a reduction in firing cost and that its impact on unemployment is ambiguous.

Wasmer (1999), Cahuc and Postel-Vinay (1999), in a theoretical model, have introduced temporary job in matching models. The former, in a model with exogenous job destruction, shows that in the periods of low growth the firms are more willing to make use of temporary contracts, which is favorable to employment. The latter, in a model with endogenous job destruction, show that the combination of temporary jobs and firing restriction may be both inefficient in terms of aggregate welfare and an inadequate weapon to fight unemployment. This result comes because the higher the firing cost, the lower the share of temporary jobs transformed into permanent jobs.

The objective of this paper is to model explicitly the possibility to form temporary matching in a model of two-sided search and to study the impact of the labor force heterogeneity, of education, and of the search process efficiency on the unemployment and on the formation of the matching equilibria. We impose conditions that guarantee the existence of a Steady State equilibrium and then characterize it.

In labor, marriage, and related markets, the central problem is the creation of cooperating coalitions composed of two or more agents of different types, e.g., worker and employer, man and woman, etc. In the labor market case, a cooperating coalition is a producing unit composed of a job-worker match. The job-worker match is formed when an unemployed worker and a vacancy meet, and they agree to sign a contract.

There is a growing literature examining non-cooperative matching models in which populations of heterogeneous agents are matched. The matching literature focuses on the ways in which the search strategies of agents in‡uence outcomes for all agents. Recent contributions to this literature are Bloch and Ryder (2000), MacNamara and Collins (1990), Morgan (1994), Burdett and Coles (1997) and Smith (1995).

In the former three articles the matching problem is simpli…ed by assuming that any pair who leaves the market is immediately replaced by clones. In the Burdett and Coles' model, on the contrary, there is an exogenous in‡ow of new singles into the market. Clearly, the Steady State Equilibrium requires that the distribution of types of those who exit equals the in‡ow distribution[1]. This extension is important because not only it best capture the characteristics of the labor market but, moreover it allows the matching decisions to in‡uence the Steady State distributions of types in the market. This extension is important because not only it captures the characteristics of the labor market in the most appropriate manner, but also it allows the matching decisions to in‡uence the Steady State distributions of types in the market.

Burdett and Coles provide a proof for why multiple class equilibria may arise in a marriage market with two-sided heterogeneity and with exogenous in‡ows of new agents, even when there are constant returns to matching. The equilibrium requires that all agents use utility-maximizing strategies, given the behavior of other agents, where a strategy is a list of people to whom a particular single will propose a matching. In such a framework, the optimal strategy for an agent i is to accept all of the partners whose human capital is above a critical level and to reject all of the agents whose human capital is below this reservation level. In their …rst proposition Burdett and Coles show that in equilibrium the participants partition themselves into n distinct classes and that a woman (employer) in class n will only propose to men (workers) who are in the same class or higher n, and will always reject a man from a lower class. Men do the same. In equilibrium only the men and women from the same class marry.

We consider a discrete version of Burdett and Coles (1997) and both for employers and the workers we introduce the possibility of forming a temporary match through a centralized temporary agency. The temporary agency will match the agents at random because we suppose that it can not distinguish the agents' quality.

Employers and workers have two options: they can look for a partner in the search market or look for a temporary match through a specialized agency (temporary agency).

Formally, both workers and employers are characterized by an index of human quality. We analyze the simple case of a binomial distribution of human capital.

[1] Smith (1995) assumes a nonsteady state matching dynamics through the time.

In the first part of the paper, we introduce the basic model of two-sided search, where agents meet randomly every period and simultaneously decide at each meeting whether to sign a contract. Search costs are captured by a discount factor, taking into account the agents' impatience to form a matching. We will show in the case of a binomial distribution of types there are two possible equilibria, and that these equilibria depend on the degree of human capital heterogeneity, on the search efficiency, and the proportion of educated.

In the second part of the paper, we introduce the possibility to form temporary matching, through a centralized temporary agency. We assume that if the agents consider that the agent they met in the search market is not acceptable or if they do not meet anybody, they can either decide to wait until the next matching or to match temporarily, in the temporary market. We will suppose that, if an agent signs a contract for temporary employment, he will remain committed for two periods. First of all, we will make the hypothesis that the agents can enter and exit the temporary market without delay. Then, we will introduce a delay cost to enter this market.

This paper shows that the introduction of a temporary market has the effect of reducing the key importance of the search efficiency and the proportion of educated for the determination of the equilibrium. Moreover, if on the one hand it has a positive impact on unemployment, and on the other hand it implies less opportunities for less skilled agents. The latter can just try to stay as long as possible in the temporary market in the hope of forming a temporary match with a high-skilled agent. The best way to improve low-skilled agents' welfare is to give all of them a higher quality education: as human capital heterogeneity decreases, skilled-agents will be more willing to accept less-educated agents as partners. This policy should always been accompanied by a temporary market regulation to avoid that the improvement of quality education loses its positive effect.

The rest of the paper is organized as follows: in the next section we present the model. In section 3 we look at a model without temporary market. In section 4 we introduce the temporary market. In section 5 we make some comparative statics and Section 6 provides some conclusions.

2 The Model

2.1 Workers and Employers

The model is a discrete time, two-sided search with heterogeneous agents.

The economy is composed of a binomial of identical agents differentiated by a real-valued index called the agent's type. Assume the economy is composed of two types of agents: high-skill (H) and low-skill (L) agents. H and L agents are endowed with human capital $x_h = 1 + \Delta$ and $x_l = 1 - \Delta$ where Δ measures the degree of human capital heterogeneity.

All employers (workers) have identical preferences over the set of the workers (employers), and the utility obtained corresponds to the index quality of the same worker (employer).

$$
\begin{aligned}
u_w(x_l) &= u_e(x_l) = x_l \\
u_w(x_h) &= u_e(x_h) = x_h
\end{aligned}
$$

2.2 Market

Let us suppose that a large and equal number of workers and employers is looking for a partner. We denote $N(t)$ the number of workers (employers) on the market at period t.

There exist two markets: a search market and a temporary market. The workers and the employers have two options: they can look for a partner in the search market or look for a temporary match through a specialized agency (temporary agency). The temporary agency matches the agents at random because we suppose that it cannot distinguish their qualities. Time is discrete and runs as $t = 0, 1, \ldots, +\infty$. At any date t, the worker and the employer are matched according to a simple random matching technology. Let be α the arrival rate of a worker (employer) faced by an employer (worker), where α is the parameter of a Poisson process. As α is assumed to be independent of the number of participants to the search market, the matching function exhibits constant returns.

Let $\beta > 0$ denote the probability that any individual dies in an interval of time. To simplify the turnover dynamics, assume that an agent never returns to this market once a match has been formed, including the case when the partner has died.

Let $\lambda_w = \lambda_e = \lambda$ denote the number of new workers and new employers who enter the market in any time interval, looking for a partner of production. Further, assume that the distribution of types among workers who flow into the market in any time interval is the same as that among employers; i.e., $F_w(\cdot) = F_e(\cdot) = F(\cdot)$. In particular, assume that the distribution of types who flow into the market in any time interval is $\eta_w = \eta_e = \eta$ for the high-skill worker (employer). Further suppose that the proportion of singles that have type x_h at time t be $\pi(t)$.

All agents have a common intertemporal discount factor denoted $\delta > 0$, and both worker and employer obtain zero utility when single. The discount factor captures the search cost.

2.3 Timing

At each meeting in the search market the worker and employer can observe the type of the other one. If both propose to sign a contract, they form a match and leave the market. On the other hand, if at least one of the two agents does not accept to sign the contract, they remain in the market and they can decide to find a temporary matching through the temporary agency or wait for a period in order to have a new possibility of matching.

We develop three models: the basic model without temporary employment (model I), a model with temporary employment and no delay cost in the temporary market (model II), and a model with temporary employment and with delay cost in the temporary market (model III).

The models have the following time sequence:

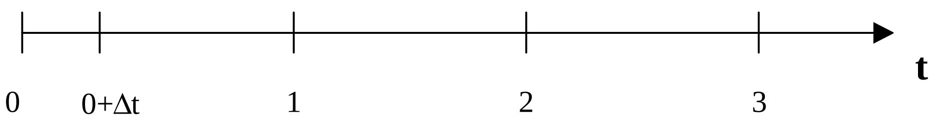

Figure 1: time sequence

Model I In the next section we will analyze the simple case without temporary agency.

In $t = 0$ the agent has a probability to meet somebody; if he meets somebody and finds the matching acceptable and the agent whom he meets does the same, they both leave the market for life. The agents start enjoying the utility when they sign the contract. On the contrary, if a worker (employer) finds the matching unacceptable or if he does not match with anybody (with a probability $(1 - \alpha)$), he can try to form a new match in $t = 1$;

Model II In the third section we give the agent the possibility to form a temporary matching through the temporary agency. We suppose that after the temporary matching the agent can come back directly in the market without delay.

In $t = 0$ if the agent matches with anybody, he can either decide to wait until $t = 1$ or to go directly to the temporary agency, in $t = 0 + \Delta t$, and look for a temporary matching. We will suppose that, if an agent sign a contract for temporary employment, he will remain committed for two periods, so he can go back to the market only at t = 2.

The agents are committed for two periods because, if they were not, all agents would always enter the temporary market after the matching has failed in the search market. This can be interpreted as an opportunity cost of entering the temporary market.

Model III In the last model we will introduce a cost in the temporary market: if the agent forms a temporary matching before coming back in to the market he has to wait for a period (delay cost). So he can come back only at $t = 3$.

3 The Search Equilibrium

Before introducing the possibility to form a temporary matching, we present the basic model without temporary agency (TA). The model is a discrete time two-sided search in the fashion of Burdett and Coles (1997).

In the case of a binomial distribution of types, without temporary agency, there are two possible types of equilibria, characterized by whether high-skill types are willing to work with low-skill types. We will refer to the first equilibrium as a Single Class Equilibrium (SCE), and to the second one as an Elitist Equilibrium (EE).

Definition 1 We define Single Class Equilibrium (SCE) the equilibrium in which high-skill types are willing to work with low-skill agents. We define Elitist Equilibrium (EE) the equilibrium in which the high-skill agents refuse low-skill agents.

Definition 2 Given $N(t)$, the number of agents (workers or employers) on the market at period t, and $\pi(t)$, the proportion of these agents that have type x_h on the market at period t, we define Steady State Equilibrium as $N(t+1) = N(t) = N^1$ and $\pi(t+1) = \pi(t) = \bar{\pi}$. Moreover the agent's strategies should be consistent with the considered equilibrium.

Single Class Equilibrium We first analyze the conditions under which high-skill (H) agents decide to work with the low-skilled (L).

The intertemporal utility accepting x_l is:

$$U = \sum_{t=0}^{\infty} \delta^t (1-\beta)^t x_l = \frac{x_l}{\delta(1-\beta)}$$

where δ denotes the discount factor, and $(1-\beta)$ the probability to be alive in the next period.

Let U_h denote the expected discounted utility of an H unemployed worker (employer). An agent i has a probability α to meet somebody in the search market. If this is the case, he will obtain a utility x_h, with probability $\bar{\pi}$, and utility x_l, with a probability $(1-\bar{\pi})$. If this is not the case, with the probability $(1-\alpha)$, he will try again to meet somebody in the next period.

Hence, U_h satisfies

$$U_h = \frac{\alpha[\bar{\pi} x_h + (1-\bar{\pi}) x_l]}{\delta(1-\beta)} + \delta(1-\beta)(1-\alpha)U_h$$

This equation can be rewritten as follows:

$$U_h = \frac{\alpha[\pi x_h + (1-\pi)x_l]}{[\delta(1-\beta)][1-(1-\beta)(1-\alpha)\delta]}$$

The condition of existence asks that the intertemporal utility of H agent accepting x_l is bigger than the utility of continuing to search. This condition require that:

$$U_h < \frac{x_l}{\delta(1-\beta)}\ , \quad \alpha < \alpha^*_{NS} = \frac{[1-(1-\beta)\delta]x_l}{\pi x_h - [\delta(1-\beta)-(1-\pi)]x_l} \tag{1}$$

In the SCE, average unemployment duration is the same for both types of agents and is equal to $1/\alpha$.

Let $N(t)\pi(t)$ describe the number of H agents at time t. Given the equilibrium, $N(t)$ and $N(t)\pi(t)$ are expected to evolve according to:

$$\begin{aligned} N(t+1) &= (1-\alpha-\beta)N(t)+\lambda \\ N(t+1)\pi(t+1) &= (1-\alpha-\beta)N(t)\pi(t)+\lambda\eta \end{aligned}$$

Hence, a stationary state occurs if and only if

$$N^{\infty} = \frac{\lambda}{\alpha+\beta} \text{ and } \pi = \eta \tag{2}$$

Elitist Equilibrium We now change the strategies of the agents. Suppose that H types will not sign a contract with the L types. If this is an equilibrium, L types must be willing to marry each other.

When the H worker refuses to work with the L type, the L type is obliged to work with the L agents. The average unemployment duration for the H is therefore $1/\alpha\pi$, whereas the average unemployment duration for the L is $1/\alpha(1-\pi)$.

We have an equilibrium if and only if H types prefer to reject L types, so that $U_h > \frac{x_l}{\delta(1-\beta)}$, where $U_h = \frac{\alpha[\pi x_h]}{\delta(1-\beta)} + [\alpha(1-\pi)+(1-\alpha)]U_h\delta(1-\beta)$, which requires that

$$\alpha > \alpha^*_S = \frac{[1-(1-\beta)\delta]x_l}{\pi(\eta)[(x_h - x_l) - [1-\delta(1-\beta)]x_l]} \tag{3}$$

Given these equilibria, $N(t)$ and $N(t)\pi(t)$ are now expected to evolve according to:

$$N(t+1) = N(t)[\alpha\pi(t)(1-\pi(t)) + \alpha(1-\pi(t))\pi(t) + 1 - \alpha - \beta] + \lambda$$

$$N(t+1)\pi(t+1) = N(t)\pi(t)[\alpha(1-\pi(t)) + 1 - \alpha - \beta] + \lambda\eta$$

In this case the stationary state is as follows:

$$N^{\infty} = \frac{\lambda}{\alpha + \beta - 2\alpha\pi(1-\pi)} \quad \text{and} \quad \eta = \frac{\pi[\alpha + \beta - \alpha\pi(1-\pi)]}{[\alpha + \beta - 2\alpha\pi(1-\pi)]} \tag{4}$$

where η is the exogenous income of H agents in any time interval and π is his endogenous value.

Straightforward algebra shows that a solution $\pi \in [0, 1]$ always exists and is unique. Furthermore, if $\eta < 0.5$ then $\pi \in [\eta, 0.5]$; while if $\eta > 0.5$ then $\pi \in [0.5, \eta]$.

There are parameter values where both steady-state equilibria exist.

Theorem 3 (Burdett-Coles): both a SCE and an EE exist if and only if $0 < \eta < 0.5$, and

$$\alpha \in \left[\frac{[1-(1-\beta)\delta]x_l}{\pi(\eta)x_h - [\delta(1-\beta) - (1-\pi(\eta))]x_l}, \frac{[1-(1-\beta)\delta]x_l}{[\pi(\eta)(x_h - x_l) - [1-\delta(1-\beta)]x_l]} \right] \tag{5}$$

where $\pi(\eta) > \eta$.

Proof. See Burdett and Coles (1997) ■

If the proportion of H (high type) agents goes beyond 0.5, the labor market becomes segmented since the optimal strategy for H is to refuse to sign a contract with L agent. Then, the probability of finding a partner decreases for both types of agents. Nevertheless, the rise in unemployment duration affects L more than H. In fact, the situation of L types worsens because of the smaller probability to match somebody and the lower level of human capital of their potential partner.

4 Search Equilibrium with Temporary Matching

4.1 Temporary Matching without Delay Cost

We first characterize the equilibria when the Agent, after the temporary employment, come back to the market immediately without waiting, so without cost in temporary market.

The set of possible equilibria becomes larger because we added the additional strategy "going into the temporary employment". Moreover, while in the previous section the L agents endure passively the choice of the H agents, here, on the contrary, they can react strategically.

As in the previous sections there are two classes of possible types of equilibria, characterized by whether H types are willing to work with L types (Single Class Equilibrium (SCE)) or are not willing to work with L types (Elitist Equilibrium (EE)).

Moreover, we also have to consider the case that the L agent prefers always to reject another L type in the search market, with the hope of meeting a H type in the temporary market.

Definition 4 We define Temporary Unskilled Equilibria (TUE) the equilibrium in which low-skill types match exclusively in the temporary market.

Within each class of possible equilibria we have to consider the possibility for the agents to go to the temporary agency (TA) if they do not match in the search market.

In this model we have the following possible equilibria:

SCE

1) L and H go to the temporary agency (TA);
2) L goes to the TA (not H);
3) H goes to the TA (not L);
4) nobody goes to the TA;

EE

1) L and H go to the temporary agency;
2) L goes to the TA (not H);
3) H goes to the TA (not L);
4) nobody goes to the TA;

TUE

1) H goes to the TA;
2) H does not go to the TA.

Theorem 5 In a labor market with a simultaneous presence of a search market and a temporary market, only two types of equilibrium exist. EE-2 exists iff $\Delta \geq \Delta_1^*$, and TUE-1 exists iff $\Delta \geq \Delta_2^*$ and $\Delta \leq \Delta_3^*$ where:

- $\Delta_1^* = \frac{(1 - c^2 + \alpha c^2 \sigma)}{[1 - c^2 + \alpha c\sigma(2 + c)]}$
- $\Delta_2^* = \frac{(1 - c)[(1 - x)(\alpha\pi + \alpha^2 c\sigma) + \alpha c^2 \sigma x(1 - \alpha) - (1 + c)(\alpha - x + cx - \alpha cx)]}{(1 + c)(-1 + c + \sigma - \alpha c\sigma) + 2c^2\sigma(1 + \alpha\sigma)}$
- $\Delta_3^* = \frac{\bar{x}(1 - c^2) + \alpha c\sigma(\bar{x} + c\bar{x} - 1)}{\alpha c\sigma}$. With $c = \delta(1 - \beta)$ and $\bar{x}$ is the expected value in the temporary employment.

Proof. See the Appendix 2 ■

Given the equilibria, we can calculate the Steady State value of the unemployed (vacancies) and of the unemployed (vacancies) that have type x_h.

Given the equilibria TUE-1 and EE-2, we have the following steady state equilibria

$$N^1_{TUE-1} = \frac{\lambda}{\alpha\pi^2 + \beta + \beta^2} \text{ and } \delta_{TUE-1} = \frac{\pi\left[\alpha\pi + \beta + \beta^2\right]}{\alpha\pi^2 + \beta + \beta^2}. \tag{6}$$

$$N^1_{EE-2} = \frac{\lambda}{(1-\pi)\left(\beta^2 - 2\alpha\pi + \alpha + \beta\right)} \text{ and } \delta_{EE-2} = \frac{\pi(\alpha\pi + \beta)}{(1-\pi)\left(\beta^2 - 2\alpha\pi + \alpha + \beta\right)} \tag{7}$$

Straightforward algebra shows that a solution $\pi_{TUE-1;EE-2} \in [0;1]$ always exists and is unique, furthermore, $\forall\delta$, $\pi_{TUE-1} \in [0;\delta]$ and $\pi_{EE-2} \in [\delta;1]$.

Proposition 6 If $\delta > 0.5$, the situation, of both types of agents is always improved in the model with TA and without delay cost (model II) compared to the model without TA (model I)

If $\delta < 0.5$, the situation of L type, from model I to model II, is worse off because of the lower level of human capital of their potential partner. In model II, the H type will always refuse the L type, for every δ. On the contrary, in the model I, the H agent always prefers to refuse a L type only if $\delta > 0.5$.

If $\delta > 0.5$, the situation of L type, from model I to model II, is unchanged because he is always rejected from the H types. Nevertheless, the number of worker (employers) looking for a job (worker) will be lower: both steady state equilibria (equations (6) and (7)) of model II will lead to a smaller N^1 (less unemployed/vacancies), than in the EE (equation (4)) of model I[2].

4.2 Temporary Matching with Delay Cost

Theorem 5 shows us that the introduction of the possibility to form temporary labor relations could have positive effects in the labor market because unemployment is lower. Nevertheless, this improved situation is exploited unilaterally from the more qualified agents that change their strategies refusing systematically to sign a contract with the less endowed agents. This comes about for the hypothesis that once the temporary matching is dissolved, the agents can form another (temporary or life) without delay. Why does an agent have to accept an unskilled agent if he can wait for meeting a preferred agent and can keep in the meantime his temporary employment?

In this section, we maintain the hypothesis that if the agent does not match in the search market, he can go to the temporary market. But now if he matches in the TA, he has to pay a cost of delay: after the temporary matching the agent has to wait for one period out of the market before searching again.

[2] Notice that if $\delta > 0.5$ (model I) only EE holds. See Theorem 3 for more details.

Theorem 7 in a labor market with a simultaneous presence of a search market and a temporary market, SCE-1 exist if and only if $\Delta \leq \Delta_1^{**}$ and $\Delta \leq \Delta_2^{**}$, EE-1 iff $\Delta \geq \Delta_1^{**}$, $\Delta \leq \Delta_3^{**}$ and $\Delta \leq \Delta_l^{**}$, EE-2 iff $\Delta \geq \Delta_4^{**}$, TUE-1 iff $\Delta \geq \Delta_1^{**}$ and $\Delta \leq \Delta_3^{**}$ and $\Delta \geq \Delta_l^{**}$. With

- $\Delta_1^{**} = \frac{(1-c^3)-\xi(1-c^2)}{(1-c^3)+2\alpha c^3\pi}$,
- $\Delta_2^{**} = \frac{[1-c(1-\alpha\pi)][\alpha\xi+(1-\alpha)(1-c^2)\xi]-\alpha[1-(1-\alpha)c^3]}{\alpha[1-(1-\alpha)c^3]\pi}$,
- $\Delta_3^{**} = \frac{\xi(1-c)-\alpha c\pi(1-\xi)}{\alpha c\pi}$,
- $\Delta_4^{**} = \frac{(1-c)}{(1-c+2\alpha c\pi)}$
- $\Delta_l^{**} = \frac{1+c+c^2-\xi(1+c)}{1+c+c^2}$. Where $c = \delta(1-\beta)$ and ξ is the expected value in the temporary employment.

Proof. See Appendix 3 ■

Given the equilibria, we can calculate the Steady State value of the unemployed (vacancies) and of the unemployed (vacancies) that have type H.

Given the equilibria SCE-1, EE-1, EE-2 and TUE-1, we have the following steady state equilibria:

$$N^1_{SCE-1} = \lambda/(\alpha+\beta+\beta^2+\beta^3) \text{ and } \eta = \pi. \tag{8}$$

$$N^1_{EE-1} = \frac{\lambda}{\alpha+\beta+\beta^2+\beta^3-2\alpha\pi(1-\pi)} \text{ and } \eta_{EE-1} = \frac{\pi\left[\alpha+\beta+\beta^2+\beta^3-\alpha\pi(1-\pi)\right]}{\alpha+\beta+\beta^2+\beta^3-2\alpha\pi(1-\pi)} \tag{9}$$

$$N^1_{EE-2} = \frac{\lambda}{(1-\pi)\left(\beta^3+\beta^2-2\alpha\pi\right)+\alpha+\beta} \text{ and } \eta_{EE-2} = \frac{\pi(\alpha\pi+\beta)}{(1-\pi)\left(\beta^3+\beta^2-2\alpha\pi\right)+\alpha+\beta} \tag{10}$$

$$N^1_{TUE-1} = \frac{\lambda}{\alpha\pi^2+\beta+\beta^2+\beta^3} \text{ and } \eta_{TUE-1} = \frac{\pi\left[\alpha\pi+\beta+\beta^2+\beta^3\right]}{\alpha\pi^2+\beta+\beta^2+\beta^3} \tag{11}$$

Straightforward algebra shows that a solution $\pi_{SCE-1;EE-1;EE-2;TUE-1} \in [0;1]$ always exists and is unique for each equilibria. Furthermore:

- $\forall \eta$, $\pi_{EE-2} \in [\eta; 1]$ and $\pi_{TUE-1} \in [0; \eta]$;
- if $\eta < 0.5$ then $\pi_{EE-1} \in [\eta; 0.5]$; while if $\eta > 0.5$ then $\pi_{EE-1} \in [0.5; \eta]$.

If we consider a cost in the TA (model III), the situation becomes more complex because of the presence of different equilibria. Now, if an agent decides to enter the temporary market, he has to pay a cost (delay cost).

The positive effect of the TA in N^∞ remains unchanged: indeed, in model III, EE-2 and TUE-1 will lead to a smaller N^∞ (less unemployed\vacancies), than the respective ones emerging from the model II (cfr. (10) with (7) and (11) with (6)).

Moreover, the H agent cannot exploit the improved situation in the market, because of the presence of the temporary agency, and the cost for entering the temporary market: an H agent accepts a L agent if π and δ are sufficiently low.

5 Comparative Statics

In the former sections we have characterized the equilibria of three models. In each model, the market is composed of either one or two groups, depending on the efficiency of the matching process (α), the proportion of qualified agents (π) and the degree of heterogeneity of the population (δ). This section will analyze the impact of the variations of these terms on unemployment and on the segmentation degree.

5.1 Search Efficiency

A common view about unemployment is that an increase in the efficiency of the matching process has always a positive impact on unemployment N^∞, where N^∞ denotes the Steady State number of workers (employers) searching for an employment (worker). Model I, without TA, suggests that a rise in efficiency matching, α, may have adverse effects on unemployment, because it modifies matching strategies. Therefore, a rise in α can have either a positive or a negative effect on the rate of unemployment.

In the SCE case, N^∞ (see equation (2)) decreases with α:

$$\frac{\partial N^\infty}{\partial \alpha} = \frac{-\lambda}{(\alpha + \beta)} < 0$$

In the EE case, N^∞ (see equation (4)) also decreases with α

$$\frac{\partial N^\infty}{\partial \alpha} = \frac{\lambda\,[2\pi(1-\pi) - 1]}{[\alpha + \beta - 2\alpha\pi(1-\pi)]^2} < 0$$

The interesting point is what happens if we pass from an equilibrium to the other. We know that the labor market will be divided (segregate) in two segments when $\alpha > \alpha^*_S$

(see condition (3)) and will stay unified (non segregate) if $\alpha < \alpha^*_{NS}$ (see condition (1)). An increase in α over α^*_S and α^*_{NS} augments the willingness of the H agents to match between them segregating the L types. It appears clearly that N^1 in EE is bigger than N^1 in SCE (see conditions (4) and (2))[3].

If we introduce a centralized TA, without delay cost (model II), the situation changes. An increase in α has a positive effect on unemployment both in EE-2 (see equation (7)) and in TUE-1 (see equation (6)):

$$\frac{\partial N^1_{EE-2}}{\partial \alpha} = \lambda\left[2\pi(1-\pi)-1\right] < 0$$

$$\frac{\partial N^1_{TUE-1}}{\partial \alpha} = -\lambda\left[\pi^2 + (1-\pi)\right] < 0$$

Moreover, this increase in the efficiency of the matching process tends to decrease the critical level of heterogeneity δ^*_1 beyond which a EE-2 exists

$$\frac{\partial \delta^*_1}{\partial \alpha} = -2c\left(1-c^2\right)\pi < 0$$

It is easy to demonstrate that, if β is small enough, N^1_{TUE-1} (6) is always greater than N^1_{EE-2} (7). Unemployment will be lower in the EE-2 (7) respect to TUE-1 (6).

In the search market, the matching strategies of the skilled agents will not change depending on α: this market will be always segmented. Instead, the strategies will depend on α in the temporary market: the skilled agents have to decide whether to enter or not this market (the less skilled agents always enter). If they do so, unemployment will be higher because of the greater instability of the market. The reason is that fixed-duration contracts are temporary by nature, so their lifetime is much shorter than that of the regular, long-term ones.

The differences between the two equilibria is not so important. We can just pass from an economy (TUE-1) where both the more and the less skilled agents enter the temporary market, to an economy (EE-2) where unemployment is lower. However, the situation of the less skilled agents is worse because they can not meet higher skilled agents (indeed the more skilled agents do not enter the temporary market).

With a delay cost in the TA (model III), the situation becomes more complex because we have four possible equilibria. We can easily show that an increase in α has always a positive effect on each steady state unemployment[4] as long as it does not force to change from an equilibrium to another. If this happens we could pass from an equilibrium where the steady state unemployment is low (like the SCE-1) to an equilibrium with high

[3] See also Rioux (1995) for general results in the basic model without temporary market.

[4] $\frac{\partial N^1_{TUE-1}}{\partial \alpha}, \frac{\partial N^1_{EE-2}}{\partial \alpha}, \frac{\partial N^1_{EE-1}}{\partial \alpha}; \frac{\partial N^1_{SCE-1}}{\partial \alpha} < 0$. See equations (11), (10), (9) and (8).

unemployment (like TUE-1). We can verify easily that, if β is small enough, $N^1_{TUE-1} > N^1_{EE-2} > N^1_{EE-1} > N^1_{SCE-1}$.

A better efficiency in the matching leads the more skilled to reject the less skilled agents in the search market[5], and to not enter the temporary market[6].

The less skilled agents respond by making matches between themselves[7] but they continue to stay in the temporary market with the hope to meet a more skilled agent[8].

5.2 Proportion of Educated

For some economists we can respond to the depression in the labor market by increasing the proportion of educated people.

As we pointed out in Theorem 3, if η the proportion of high-skill (H) agents who enter the market at each period goes beyond 0.5, the market becomes segmented since the optimal strategy for the H agents is to segregate from the rest of the economy. An increase of η, if this critic value is attained, has a negative effect on unemployment. On the contrary, if we do not go beyond this value, this increase has no effect on unemployment (see equation (2)).

If η is already bigger than 0.5, an increase in η (and so in π) has a positive effect (see equation (4)):

$$\frac{\partial N^1}{\partial \pi} = \frac{2\alpha\lambda(1-2\pi)}{[\alpha+\beta-2\alpha\pi(1-\pi)]^2} < 0$$

If we turn to the in a model with TA, the change in this proportion can have some effects on unemployment, the strategies will remain unchanged.

An increase in the proportion of the educated, π, has the same effect as the increasing in the search efficiency, α:

- either in a model with TA without delay cost (model II), decreasing the critical level of heterogeneity Δ_1^* beyond which a EE-2 exists, $\frac{\partial \Delta_1^*}{\partial \pi} = \frac{-2\alpha c(1-c)^2}{[1-c^2+\alpha c\pi(2+c)]^2} < 0$;
- or in the model with a delay cost (model III) if we pass from an equilibrium to another one.

In the opposite, the rise in π can have ambiguous effects on the rate of unemployment in each single equilibrium. In the SCE-1 this variation has no effect (equation (8)). In

[5] $\frac{\partial \Delta_1^{**}}{\partial \alpha} = -2c^5(1-c)\pi < 0$ and $\frac{\partial \Delta_2^{**}}{\partial \alpha} = -2(1-c)(1-\pi)\left[1-c^2(1+c(1-2\alpha+\alpha^2\pi))+(1-\alpha)^2\right] < 0$ (in the SCE-1).

[6] $\frac{\partial \Delta_4^{**}}{\partial \alpha} = -2c(1-c)\pi < 0$ (in the EE-2).

[7] $\frac{\partial \Delta_I^{**}}{\partial \alpha} = 2c^2(1+c)(1-\pi)\pi^2 > 0$ (in the EE-1).

[8] $\frac{\partial \Delta_I^{**}}{\partial \alpha} = -2c^2(1+c)(1-3\pi+2\pi^2)\pi < 0$ (in the TUE-1).

the EE-1 case (equation (9)), and in the TUE-1 case (both in model II and in model III, see equations (6) and (11)) the effect are always positive. Finally in EE-2 the effect is positive iff $\pi \leq \pi^*_{II} = \frac{\beta^2 + 2\alpha}{4\alpha}$, in the model II, and iff $\pi \leq \pi^*_{III} = \frac{\beta^2 + \beta^3 + 2\alpha}{4\alpha}$, in the model III (equations (7) and (10)).

5.3 Degree of Heterogeneity

We are now interested in the impact of a change in Δ where this parameter measures the difference in human capital endowment between high-skilled and low-skilled agents. In each model, this has no implications for the steady state unemployment in the single equilibrium. There can be some effect, like the variations of α and π, just if we pass from an equilibrium to another one.

In model I, the impact of an increase in inequality clearly appears from equations (1) and (3)

$$\frac{\partial \alpha^*_{NS}}{\partial \Delta} = \frac{-2(1-c)\pi}{(1-c-\Delta+c\Delta+2\pi\Delta)^2} < 0$$

$$\frac{\partial \alpha^*_{S}}{\partial \Delta} = \frac{-2(1-c)}{\pi(1-c-3\Delta+c\Delta)^2} < 0$$

That is, for given c and π, an increase in the level of heterogeneity tends to reduce the critical levels α^*_{NS}, under which the labor market does not segregate, and α^*_S, beyond which the labor market segregates. Since L agents become less desiderable than potential "partners", a more unequal distribution of human capital in the population raises the willingness of the good agents to segregate.

In model II, the agents strategies will change with the critical values of the degree of heterogeneity Δ^*_1, Δ^*_2 and Δ^*_3[9]. We have three possible situations:

- if $\Delta > \Delta^*_1$ and $\Delta < \Delta^*_2$ (or $\Delta > \Delta^*_3$), only the EE-2 exists;
- if $\Delta < \Delta^*_1$, $\Delta > \Delta^*_2$, and $\Delta < \Delta^*_3$; only the TUE-1 exists;
- if $\Delta > \Delta^*_1$, $\Delta > \Delta^*_2$, and $\Delta < \Delta^*_3$, both the EE-2 and TUE-1 exist.

An increase in Δ over $\Delta^*_{1;2;3}$ augments the willingness of the more skilled agents to match between them segregating the less skilled agents also in the temporary market (refusing to enter this market). Consequently, the willingness of the less skilled to match between them will augment.

[9] These values are been defined in the section 4.1.

In model III, the impact of an increase in inequality appears from Theorem 7. It is not easy to analyze it because there are too much critical values. Moreover it seems clear that an increase in ¢ over ¢$_{1}^{¤}$[10] augments the willingness of the more skilled to be more selective. Consequently the SCE-1 will be smaller and the EE-2 and the TUE-1 will be bigger.

In addition, an increase in ¢ over ¢$_{l}^{¤}$ augments the willingness of the low-skilled to match, even if temporarily, with the more qualified agents: the TUE-1 will be bigger and the EE-1 smaller.

6 Conclusions

The depression in the labor market in the developed countries is explained by the fact that there are too many unskilled workers and too less skilled workers for the existing technology at the time. To solve this disequilibrium the level of education should be augmented. If we increase sufficiently the education of the population, the unskilled workers could be employed because they can take advantage of the decreasing competition in the market where they search. Nevertheless, as the first model suggests, the higher proportion of skilled workers can lead them to reject the unskilled workers because they prefer to form more homogenous productive unities.

The introduction of a temporary market has the effect of reducing the key importance of the search efficiency and the proportion of educated for the determination of the equilibrium. Moreover, if on the one hand it could have a positive impact on unemployment, on the other hand it implies less opportunities for less skilled agents. The latter can just try to stay as long as possible in the temporary market in the hope of forming a temporary match with a high-skilled agent.

This policy could lose its positive impact on unemployment if additional appropriate policies were not carried out. The possibility of free entry in this market could produce a great instability in the labor market, because of the temporary nature of these kinds of contracts. If we introduce an entry cost, we can keep the positive effects of this policy and people will enter this market only if they really need.

Moreover, the best way to improve low-skilled agents' welfare, as Saint-Paul (92) and Rioux (95) suggest, is not necessarily to give a fraction of them education and change them into high-skilled: it can even lead to a perverse effect if this increase in the number of H agents is sufficient to encourage them to segregate. A better way to improve the situation of the poor would be to give all of them a higher quality education: as human capital heterogeneity decreases, H agents will be more willing to accept less educated agents as partners.

[10] The critical values have been defined in the section 4.2.

This policy should always been accompanied by a temporary market regulation to avoid that the improvement of quality education loses its positive effect.

Appendix 1: Proof of Theorem 5

We establish Theorem 5 by a series of lemmas.

Lemma 1 *The only candidate equilibria are SCE-1, EE-1, EE-2 and TUE-1.*

We can exclude all other equilibria configurations. SCE-2 and SCE-3 are excluded because with the assumption that a high-type worker will accept the low-type in the search market (SCE) and that the utility to be unmatched is zero, a H type will always prefer to go to the temporary market if he does not match in the search market: SCE-2 does not exist. We can use the same argument for the L type in SCE-3.

EE-3 is excluded because with the assumption that the temporary agency matches the agents at random, if the H types are in this market, a L agent will enter the temporary market too: the EE-3 never exist.

TUE-2 is excluded because if the H type does not go to the TA, the L agents will accept each other in contact in the search market: the TUE-2 never exist.

Finally, we can restrict the possible equilibria by eliminating the possible equilibrium SCE-4 and EE-4 because are dominated respectively by SCE-1, for both types of agents, and by EE-2 for L types agents.

SCE-1 is preferred to SCE-4 if and only if

$$U_{l;h}(\text{SCE-1}) \geq U_{l;h}(\text{SCE-4}) \Leftrightarrow (1-\alpha)[\pi x_h + (1-\pi)x_l] \geq 0 \tag{12}$$

where $U_{l;h}(\text{SCE-1}) = \frac{[\pi x_h + (1-\pi)x_l]}{(1-c)}$, $U_{l;h}(\text{SCE-4}) = \frac{\alpha[\pi x_h + (1-\pi)x_l]}{(1-c)[1-(1-a)c]}$ and $c = \delta(1-\beta)$[11].

EE-2 is preferred to EE-4 by the L types if and only if

$$U_l(\text{EE-2}) \geq U_l(\text{EE-4}) \Leftrightarrow (1-c)\left[1-c^2+\alpha c^2(1-\pi_{EE-2})\right][1-\alpha(1-\pi_{EE-4})] \geq 0 \tag{13}$$

where π_{EE-2} and π_{EE-4} are the Steady State proportion of H agents respectively in EE-2 and EE-4, $U_l(\text{EE-2}) = \frac{\alpha(1-\pi)x_l + x_l[1-\alpha(1-\pi)](1-c^2)}{(1-c)[1-c^2(1-\alpha(1-\pi))]}$, $U_l(\text{EE-4}) = \frac{\alpha(1-\pi)x_l}{(1-c)[1-c(1-\alpha(1-\pi))]}$ and $c = \delta(1-\beta)$.

The conditions (12) and (13) are always true because the terms on the left side of these equations are always positive.

Now, we will compute the equilibrium utility. Let us define U_{h1}, U_{h2}, U_{h3}, U_{h4} for the H type, and U_l, U_{ta} for the L type:

[11] The Steady State proportion of H agents, π, are exactly the same in the SCE-1 and SCE-4.

ut. of accepting x_l but going to the TA	$U_{h1} = \frac{\alpha[\pi x_h + (1-\pi)x_l]}{(1-c)[1-c^2(1-\alpha)]} + \frac{(1-\alpha)\bar{x}(1+c)}{[1-c^2(1-\alpha)]}$
ut. of rejecting x_l and going to the TA	$U_{h2} = \frac{\alpha\pi x_h}{(1-c)[1-c^2(1-\alpha\pi)]} + \frac{(1-\alpha\pi)\bar{x}(1+c^2)}{[1-c^2(1-\alpha\pi)]}$
ut. of accepting x_l but rejecting TA	$U_{h3} = \frac{\alpha[\pi x_h + (1-\pi)x_l]}{(1-c)[1-(1-\alpha)c]}$
ut. of rejecting x_l and rejecting TA	$U_{h4} = \frac{\alpha\pi x_h}{(1-c)[1-c(1-\alpha\pi)]}$
ut. of accepting x_l and going to the TA	$U_l = \frac{\alpha(1-\pi)x_l + (1-\alpha+\alpha\pi)(1-c^2)\bar{x}}{(1-c)[1-(1-\alpha+\alpha\pi)c^2]}$
ut. of rejecting x_l and going to the TA	$U_{ta} = \frac{\bar{x}(1+c)}{(1-c^2)}$

where $\bar{x}$ is the expected value in the TA and $c = \delta(1-\beta)$.

Lemma 2 SCE-1 (L and H go to the TA) never exists.

The utility of a H type in the SCE-1 is U_{h1}, where $\bar{x} = [\pi x_h + (1-\pi)x_l]$.

In the utility of the agents we consider the possibility to go to the temporary agency. After the matching in the search market the proportion of agents unmatched in SCE will be unchanged.

This is an equilibrium if and only if every possible deviation from the equilibrium is not profitable. This condition requires that U_{h1} has to be bigger than the following utilities of deviation: $U_{h2}; U_{h3}; U_{h4}$.

If we look just at the first of these three conditions, we find:

$$U_{h1} > U_{h2} \iff \alpha\pi[(1-\pi)(x_l - x_h)] > 0$$

This condition is never true.

Lemma 3 EE-1 (L and H go to the TA) never exists.

We have an equilibrium if and only if H refuses to sign a contract with L and wishes to go to the TA, and if and only if an L type accepts another L type in the search market.

This is never an equilibrium because the unemployed L prefers to deviate from the equilibrium by refusing the L vacancies in order to go directly to the TA. The equilibrium exists if and only if

$$U_l > U_{ta} \iff \alpha[(1-\pi)(x_l - \bar{x}) - 2c\pi x_l] \geq 0$$

where $\bar{x} = \frac{(1-\pi)(1-\alpha+\alpha\pi)x_l + \pi(1-\alpha\pi)x_h}{1-\alpha+2\alpha\pi-2\alpha\pi^2}$. This condition is never true.

Lemma 4 EE-2 (L go to TA, H go to the TA never) exists iff $\Delta > \Delta^*_1$, where $\Delta^*_1 = \frac{(1-c^2+\alpha c^2\pi)}{[1-c^2+\alpha c\pi(2+c)]}$.

We have an equilibrium if and only if $U_{h4} > U_{h3}; U_{h1}; U_{h2}$.

We can demonstrate easily that U_{h2} is always bigger than U_{h1}. In fact $U_{h2} \geq U_{h1} \iff 2\alpha^2 c^2 \Delta (1-\pi)\pi \geq 0$. So we can eliminate the second condition. The new condition will be: $U_{h4} > U_{h3}; U_{h2}$.

- The first condition is

$$U_{h4} > U_{h3} \iff \Delta \geq \Delta^* = \frac{(1-c)}{(1-c+2\alpha c\pi)}$$

- the second condition is

$$U_{h4} > U_{h2} \iff \Delta \geq \Delta_1^* = \frac{\left(1-c^2+\alpha c^2\pi\right)}{[1-c^2+\alpha c\pi(2+c)]} \tag{14}$$

where $\bar{x} = x_l$.

We can demonstrate that $\Delta^* \leq \Delta_1^*$. So, we can eliminate Δ^*. We have consider just the second condition.

Lemma 5 TUE-1 exists iff $\Delta > \Delta_2^*$ and $\Delta > \Delta_3^*$ where:

$$\Delta_2^* = \frac{(1-c)[(1-x)(\alpha\pi+\alpha^2 c\pi)+\alpha c^2\pi x(1-\alpha)-(1+c)(\alpha-x+cx-\alpha cx)]}{(1+c)(-1+c+\pi-\alpha c\pi)+2c^2\pi(1+\alpha\pi)} \text{ and } \Delta_3^* = \frac{\bar{x}(1-c^2)+\alpha c\pi(\bar{x}+c\bar{x}-1)}{\alpha c\pi}.$$

We have an equilibrium if and only if H refuses to sign a contract with L and wishes to go to the TA, and if and only if an L type does not accept another L type in the search market.

The utility of H is U_{h2}, with $\bar{x} = \frac{[x_h(1-\alpha\pi)\pi+x_l(1-\pi)]}{(1-\alpha\pi^2)}$. We have an equilibrium iff $U_{h2} > U_{h1}, U_{h3}, U_{h4}$.

- The first condition

$$U_{h2} > U_{h1} \iff (1-\pi)(1-c^2)(\bar{x}-x_l)+2\alpha c^2\Delta\pi \geq 0 \tag{15}$$

- The second condition

$$U_{h2} > U_{h3} \iff \Delta \geq \Delta_2^*$$

where $\Delta_2^* = \frac{(1-c)[(1-\bar{x})(\alpha\pi+\alpha^2 c\pi)+\alpha c^2\pi\bar{x}(1-\alpha)-(1+c)(\alpha-\bar{x}+c\bar{x}-\alpha c\bar{x})]}{(1+c)(-1+c+\pi-\alpha c\pi)+2c^2\pi(1+\alpha\pi)}$.

- The third condition

$$U_{h2} > U_{h4} \iff \Delta \geq \Delta_3^*$$

where $\Delta_3^* = \frac{\bar{x}(1-c^2)+\alpha c\pi(\bar{x}+c\bar{x}-1)}{\alpha c\bar{x}}$

For the L type we have an equilibrium iff:

$$U_{ta} \geq U_l \iff \alpha[(1-\pi)(x - x_l) + 2c\pi x_l] \geq 0 \qquad (16)$$

The conditions (15) and (16) are always true for any parameters value.

Appendix 2: proof of theorem 7

We establish the Theorem 7 by a series of lemmas.

Lemma 1 The only candidate equilibria are SCE-1, EE-1, EE-2 and TUE-1.

We can exclude all other equilibria configurations. SCE-2, SCE-3, EE-3 and TUE-2 are excluded for the same arguments of the previous model (see Appendix 2, Lemma 1).

Moreover, SCE-4 and EE-4 are excluded because they are always dominated respectively by SCE-1, for both types of agents, and by EE-2 for L types agents.

SCE-1 is preferred to SCE-4 if and only if

$$U_{l;h}(SCE\text{-}1) \geq U_{l;h}(SCE\text{-}4) \iff (1-\alpha)(1-c)^2[\pi x_h + (1-\pi)x_l] \geq 0 \qquad (17)$$

where $U_{l;h}(SCE\text{-}1) = \frac{[\bar{\pi} x_h + (1-\bar{\pi})x_l][1-c^2(1-\alpha)]}{(1-c)[1-c^3(1-\alpha)]}$, $U_{l;h}(SCE\text{-}4) = \frac{\alpha[\bar{\pi} x_h + (1-\bar{\pi})x_l]}{(1-c)[1-(1-a)c]}$ and $c = \delta(1-\beta)$[12]

EE-2 is preferred to EE-4 if and only if

$$U_l(EE\text{-}2) \geq U_l(EE\text{-}4) \iff \frac{(1-c)\left[(1-c^2)(1-\alpha)+\alpha(\pi_{EE-4} - c^2\pi_{EE-2})\right]}{[1-c(1-\alpha(1-\pi_{EE-4}))][1-c^3(1-\alpha(1-\pi_{EE-2}))]} \geq 0 \qquad (18)$$

where π_{EE-2} and π_{EE-4} (the Steady State proportion of H agents respectively in EE-2 and EE-4) are defined from equations (4) and (10), $U_l(EE\text{-}2) = \frac{\alpha(1-\bar{\pi}_{EE-2})x_l + x_l[1-\alpha(1-\bar{\pi}_{EE-2})](1-c^2)}{(1-c)[1-c^2(1-\alpha(1-\bar{\pi}_{EE-2}))]}$, $U_l(EE\text{-}4) = \frac{\alpha(1-\bar{\pi}_{EE-4})x_l}{(1-c)[1-c(1-\alpha(1-\bar{\pi}_{EE-4}))]}$:

The condition (18) is always true iff:

$$(\pi_{EE-4} - c^2\pi_{EE-2}) \geq \alpha(1-c^2)(1-2(1-\pi)\pi) + \beta(1-c^2+(1-\pi)\beta(1+\beta)) \geq 0 \qquad (19)$$

the conditions (17) and (19) are always true.

Now, we will compute the equilibrium utility. Let us define U_{h1}, U_{h2}, U_{h3}, U_{h4} for the H type, and U_l, U_{ta} for the L type:

the ut. of accepting x_l but going to the TA	$U_{h1} = \frac{\alpha[\bar{\pi} x_h + (1-\bar{\pi})x_l]}{(1-c)[1-c^3(1-\alpha)]} + \frac{(1-\alpha)\chi(1+c)}{[1-c^3(1-\alpha)]}$
the ut. of rejecting x_l and going to the TA	$U_{h2} = \frac{\alpha\bar{\pi} x_h}{(1-c)[1-c^3(1-\alpha\bar{\pi})]} + \frac{(1-\alpha\bar{\pi})\chi(1+c^2)}{[1-c^3(1-\alpha\bar{\pi})]}$
the ut. of accepting x_l but rejecting TA	$U_{h3} = \frac{\alpha[\bar{\pi} x_h + (1-\bar{\pi})x_l]}{(1-c)[1-(1-\alpha)c]}$
the ut. of rejecting x_l and rejecting TA	$U_{h4} = \frac{\alpha\bar{\pi} x_h}{(1-c)[1-c(1-\alpha\bar{\pi})]}$
the ut. of accepting x_l and going to the TA	$U_l = \frac{\alpha(1-\pi)x_l + (1-\alpha+\alpha\pi)(1-c^2)\chi}{(1-c)[1-(1-\alpha+\alpha\pi)c^3]}$
the ut. of rejecting x_l and going to the TA	$U_{ta} = \frac{\chi(1+c)}{(1-c^3)}$

[12] The Steady State proportion of H agents, $\bar{\pi}$, are exactly the same in the SCE-1 and SCE-4.

where $\bar{x}$ is the expected value in the TA and $c = \delta(1-\beta)$.

Lemma 2 SCE-1 (L and H go to the TA) exists if and only if $\Delta \leq \Delta_1^{**}$ and $\Delta \leq \Delta_2^{**}$, where $\Delta_1^{**} = \frac{(1-c^3)-\bar{x}(1-c^2)}{(1-c^3)+2\alpha c^3\pi}$ and $\Delta_2^{**} = \frac{[1-c(1-\alpha\pi)]\left[\alpha\bar{x}+(1-\alpha)(1-c^2)\bar{x}\right]-\alpha\left[1-(1-\alpha)c^3\right]}{\alpha[1-(1-\alpha)c^3]\pi}$.

The utility of a H type in the SCE-1 is U_{h1}; where $\bar{x} = [\pi x_h + (1-\pi)x_l]$.

SCE-1 is an equilibrium iff every possible deviation to the equilibrium is not profitable. This condition requires that $U_{h1} > U_{h2}; U_{h3}; U_{h4}$. The low type has no interest to deviate from the equilibrium.

- the first condition:

$$U_{h1} \geq U_{h2} \iff \Delta \leq \Delta_1^{**} = \frac{(1-c^3)-\bar{x}(1-c^2)}{(1-c^3)+2\alpha c^3\pi}$$

- the second condition:

$$U_{h1} \geq U_{h3} \iff \bar{x}(1-\alpha)(c-1)^2(1+c) \geq 0 \tag{20}$$

- the third condition:

$$U_{h1} \geq U_{h4} \iff$$

$$\Delta \leq \Delta_2^{**} = \frac{[1-c(1-\alpha\pi)]\left[\alpha\bar{x}+(1-\alpha)(1-c^2)\bar{x}\right]-\alpha\left[1-(1-\alpha)c^3\right]}{\alpha[1-(1-\alpha)c^3]\pi}$$

The condition (20) is always true for every parameters' value.

Lemma 3 EE-1 (L and H go to the TA) exists iff $\Delta \geq \Delta_1^{**}$[13] and $\Delta \leq \Delta_3^{**}$, and $\Delta \leq \Delta_l^{**}$ where $\Delta_3^{**} = \frac{\bar{x}(1-c)-\alpha c\pi(1-\bar{x})}{\alpha c\pi}$ and $\Delta_l^{**} = \frac{1+c-c^2-\bar{x}(1+c)}{1+c-c^2}$.

We have an equilibrium if and only if H refuses to sign a contract with L and wishes to go to the TA, and if and only if an L type accepts another L type in the search market. The utility of H agent is U_{h2}, where $\bar{x} = \frac{(1-\pi)(1-\alpha+\alpha\pi)x_l+\pi(1-\alpha\pi)x_h}{1-\alpha+2\alpha\pi-2\alpha\pi^2}$

We have an equilibrium if and only if $U_{h2} \geq U_{h1}; U_{h3}; U_{h4}$. We know by Lemma 2 that U_{h1} is always bigger than U_{h3} for every $\bar{x}$. So we can eliminate the second condition. The new condition will be: $U_{h2} > U_{h1}; U_{h4}$.

[13] The value of Δ_1^{**} is defined in Lemma 2.

- the first condition:

$$U_{h2} \geq U_{h1} \iff \Delta \geq \Delta_1^{**}$$

- the second condition

$$U_{h2} \geq U_{h4} \iff \Delta \leq \Delta_3^{**} = \frac{\bar{x}(1-c) - \alpha c\pi(1-\bar{x})}{\alpha c\pi}$$

The L type has just one possible deviation: he can refuse another L type and go in the TA:

$$U_l \geq U_{ta} \iff \Delta \leq \Delta_l^{**} = \frac{1+c+c^2-\bar{x}(1+c)}{1+c+c^2}$$

Lemma 4 EE-2 (L go to TA, H never go to TA) exists for $\Delta \geq \Delta_4^{**}$, where $\Delta_4^{**} = \frac{(1-c)}{1-c+2\alpha c\pi}$.

The utility of H in EE-2 is U_{h4}.

We have an equilibrium if and only if $U_{h4} > U_{h3}; U_{h1}; U_{h2}$, where $\bar{x} = x_l$.

$$U_{h4} > U_{h3}; U_{h1}; U_{h2} \iff \Delta \leq \Delta_4^{**} = \frac{(1-c)}{1-c+2\alpha c\pi}$$

Lemma 5 TUE-1 exists iff $\Delta \geq \Delta_1^{**}$, $\Delta \leq \Delta_3^{**}$ and $\Delta \leq \Delta_l^{**}$[14].

We have an equilibrium if and only if H refuses to sign a contract with L and wishes to go to the TA, and if and only if an L type refuses another L type in the search market.

The utility of H agent in Tue-1 is U_{h2}, with $\bar{x} = [x_h(1-\alpha\pi)\pi + x_l(1-\pi)] \backslash \left(1-\alpha\pi^2\right)$. We know by Lemma 2 that U_{h1} is always bigger than U_{h3} for every $\bar{x}$. So we can eliminate the second condition. The equilibrium condition is: $U_{h2} > U_{h1}; U_{h4}$.

- the first condition:

$$U_{h2} \geq U_{h1} \iff \Delta \geq \Delta_1^{**}$$

- the second condition

$$U_{h2} \geq U_{h4} \iff \Delta \leq \Delta_3^{**}$$

The L type has just one possible deviation: he can accepts another L type in the search market. We have an equilibrium iff:

$$U_{ta} \geq U_l \iff \Delta \geq \Delta_l^{**}$$

[14] The values Δ_1^{**}, Δ_3^{**} and Δ_l^{**} are defined in Lemma 2, the first one, and in Lemma 3, the two others.

References

[1] Bloch F. and H. Ryder (2000), "Two-Sided Search, Marriages and Matchmakers", International Economic Review, 41 (1), 93-115.

[2] Bentolila, S. and J. J. Dolado (1994), "Labour flexibility and wages: lessons from Spain", Economic Policy, 18, 55-85.

[3] Bentolila, S. and G. Saint-Paul (1992), "The Macroeconomic Impact of Flexible Labor Contracts, With An Application to Spain", European Economic Review, 36, 1013-53.

[4] Burdett K. and M. G. Coles (1997) "Marriage and Class", Quarterly Journal of Economics, 112, 141-168.

[5] Cahuc P. and F. Postel-Vinay (1999) "Temporary jobs, Employment Protection and Labor Market Performance", Mimeo.

[6] MacNamara J. and E. Collins (1990) "The Job Search Problem as an Employer-Candidate Game," Journal of Applied Probability, 28, 815-827.

[7] Morgan P. (1994) "A model of Search, Coordination and Market segmentation,", mimeo., Department of Economics, State University of New York at Buffalo.

[8] Rioux L. (1995) "Heterogeneity, Matching, and Endogenous Labor Market Segmentation", DELTA 95-35.

[9] Saint-Paul G. (1992) "Are the unemployed unemployable?", CEPR WP n.74.

[10] Smith L. (1993) "Cross Sectional Dynamics in a Model of Two Sided Search", mimeo., Department of Economics, Massachusetts Institute of Technology.

[11] Wasmer, E. (1999) "Competition for Jobs in a Growing Economy and the Emergence of Dualism in Employment", Economic Journal, 109, 349-371.