

Credit Spread Arbitrage in Emerging Eurobond Markets

Caio Ibsen Rodrigues de Almeida ^{*}
Antonio Marcos Duarte, Jr. ^{**}
Cristiano Augusto Coelho Fernandes ^{***}

Abstract

Simulating the movements of term structures of interest rates plays an important role when optimally allocating portfolios in fixed income markets. These movements allow the generation of scenarios which, on their turn provide the assets' sensibility to the fluctuation of interest rates. The problem becomes even more interesting when the portfolio is international. In this case, there is a need to synchronize the different scenarios for the movements of the interest rate curves in each country. An important factor to consider, in this context, is credit risk. For instance, in the corporate Emerging Eurobond fixed income market there are two main sources of credit risk: sovereign risk and the relative credit among the companies issuers of the eurobonds. This paper presents a model to estimate, in a one step procedure, both the term structure of interest rates and the credit spread function of a diversified international portfolio of eurobonds, with different credit ratings. The estimated term structures can be used to analyze credit spread arbitrage opportunities in Eurobond markets. Numerical examples taken from the Argentinean, Brazilian and Mexican Eurobond markets are presented to illustrate the practical use of the methodology.

Please address all correspondence to:

Cristiano Augusto Coelho Fernandes
Rua Marquês de São Vicente, 225 - Gávea
22453-900, Rio de Janeiro, RJ, Brazil
Electrical Engineering Department
Phones: 55-21-5299415 or 55-21-5375905
E-mail: Cris@ele.puc-rio.br

* Pontifícia Universidade Católica do Rio de Janeiro, Brazil. E-mail: caio@ele.puc-rio.br

** Unibanco S.A., Brazil. E-mail: antonio.duarte@unibanco.com.br

*** Pontifícia Universidade Católica do Rio de Janeiro, Brazil.

1. Introduction

The prices of fixed income assets depend on three components (Litterman and Iben [1988]): the risk free term structure of interest rates, embedded options values and credit risk. Optimally allocating portfolios in fixed income markets demands a detailed analysis of each of these components.

Several authors have already considered the risk free term structure estimation problem. For example, Vasicek and Fong [1982] suggest a statistical model based on exponential splines. Litterman and Scheinkman [1991] verified that there are three orthogonal factors which explain the majority of the movements of the US term structure of interest rates. These three factors form the basis for many fixed income pricing and hedging applications. For instance, these factors are used in Singh [1997] to suggest optimal hedges.

Some bonds present embedded options. In general, the price of an embedded option is a nonlinear function of its underlying bond price on all dates before the option maturity date. An embedded option depends not only on the actual term structure of interest rates, but also on the evolution of this term structure during the life of the option. Several models have been proposed for the evolution of the term structure of interest rates. These models are classified in two major groups (Duffie [1992]): equilibrium models (Vasicek [1977], Cox et al. [1985], among others) and arbitrage free models (Ho and Lee [1986], Black et al. [1990], Heath et al. [1992], among others). At this point in time, the pricing of embedded options using arbitrage free models is perceived as the most appropriate because the parameters can be chosen to be consistent with the actual term structure of interest rates and, consequently, to the actual prices of bonds (Heath et al. [1992]). The process modeled can be the short-term interest rate, the whole term structure of interest rates, or the

forward rates curve. No matter what the process is, when it is Markovian, it is usually implemented using binomial trees (Black et al. [1990]) or trinomial trees (Hull and White [1993]).

Almeida et al. [1998] presented a model to decompose the credit risk of term structures of interest rates using orthogonal factors (Sansone [1959]), such as Legendre [1785] polynomials. In this model, the term structure of interest rates is decomposed in two curves: a benchmark curve and a credit spread function. The last one is modeled using a linear combination of Legendre polynomials.

In this article we present a model to estimate, in a single step, both the term structure of interest rate and the credit spread function of an international portfolio of bonds with different credit ratings. This model extends the approach proposed in Almeida et al [1998]. It allows the joint estimation of the credit spread function of any international portfolio with different credit ratings. This extension is crucial when analyzing credit spread arbitrage opportunities in fixed income markets. For the purpose of illustration, we concentrate on the Emerging Corporate Eurobond market, studying the three most important in Latin America: Argentina, Brazil and Mexico. However, the methodology is quite general, and can be applied to any fixed income portfolio composed by bonds with different credit ratings.

This article is organized as follows. Section 2 explains the model. Section 3 presents the estimation process for its parameters. Section 4 explains the methodology used for optimally allocating portfolios using the model. Section 5 presents three practical examples of detection and exploitation of arbitrage opportunities in the Latin American Eurobond market. Section 6 presents a summary of the article, and the conclusions.

2. The Model

We want to analyze a portfolio in the Emerging Eurobond market. Assets with the same cash flow and embedded option structures, but different credit ratings, ought to have different prices. For this reason, when structuring fixed income portfolios, it is fundamental to estimate and simulate the movements of different term structures of interest rates, one for each credit rating in the portfolio. One possibility would be to estimate a term structure, for each credit rating. There is a statistical problem with the amount of data available when relying on this approach: in the emerging eurobond market there are usually very few liquid bonds for each credit rating. A joint estimation procedure is necessary.

An interesting possibility is to capture the difference in risk between credit ratings using different credit spread functions. Using this approach, it is possible to estimate in a single step the term structures for different credit ratings by modifying the proposal in Almeida et al. [1998].

The equation that describes the term structure of an Emerging Eurobond market (that is, fixed income instruments of one country, issued in a same currency, with the same credit rating) can be written as

$$R(t) = B(t) + \sum_{n \geq 0} c_n P_n \left(\frac{2t}{\ell} - 1 \right), \forall t \in [0, \ell]. \quad (1)$$

where t denotes time, $B(t)$ is a benchmark (for example, the US term structure), P_n is the Legendre polynomial of degree n , c_n is a parameter to be estimated, and ℓ is the largest maturity of a bond in the Emerging Eurobond market under consideration.

The price of a bond (P_A) is related to the term structure of interest rates as

$$P_A = \sum_{i=1}^{n_A} C_i \exp(-t_i R(t_i)) \quad (2)$$

where C_i denotes the i^{th} cash flow paid by the bond at time t_i , and n_A denotes the total number of cash flows paid by the bond.

Setting up the notation, variables $r_j, j = 1, \dots, J$, denote different credit ratings. For instance, a credit rating such as AAA may be associated with r_1 , a credit rating such as AA1 may be associated with r_2 , and so on.

An extension of (1) is to consider the spread function depending on the different credit ratings, such as

$$R(t, r_1, \dots, r_J) = B(t) + C(t, r_1, \dots, r_J), \forall t \in [0, \ell]. \quad (3)$$

The spread function $C(t, r_1, \dots, r_J)$ can be modeled as a linear combination of orthogonal polynomials in order to exploit the modeling and estimation advantages illustrated in Almeida et al. [1998].

An application of this equation that captures the difference in risk between credit ratings using only a translation factor is given by

$$R^{(j)}(t) = B(t) + \sum_{i=1}^j S_i + \sum_{n \geq 1} c_n P_n\left(\frac{2t}{\ell} - 1\right), \forall t \in [0, \ell], \forall j = 1, 2, \dots, J. \quad (4)$$

where S_i is a nonnegative spread variable (that is, $S_i \geq 0 \forall i = 1, 2, \dots, j$) that measures the difference in risk between the $(i-1)^{\text{th}}$ and the i^{th} credit ratings, and J represents the total number of credit ratings.

A limitation of Model (4) is that all J estimated term structures are parallel. Although very limited, Model (4) captures the fact that bonds with higher ratings ought to have smaller prices (everything else being equal). In other words: the higher the rating, the higher the interest rates used to price bonds with that particular rating.

Exhibit 1 depicts a possible output for Model (4). It is possible to exhibit more general models than that given in (4) (that is, a model which allows the term structures obtained for different credit ratings to differ not only by a translation factor). Exhibit 2 presents a schematic drawing of possible term structures of interest rates for different credit ratings, in a more general model.

3. Joint Estimation of Term Structures

Let us consider the simplest case first (that is, Model (4)). The objective is to estimate the variables $S_i, i = 1, \dots, J$ and the coefficients c_1, c_2, c_3, \dots . The final results of this estimation process are J different term structures of interest rates, each related to a different rating.

Let us define the discount function $D^{(j)}(t)$ for rating r_j to be

$$D^{(j)}(t) = \exp(-R^{(j)}(t)t), \forall t \in [0, \ell], j = 1, \dots, J \quad (5)$$

We assume that m eurobonds are available to estimate the coefficients in Model (4). We assume that m_j eurobonds possess a rating r_j . The residual term e_k of the statistical fit obtained for the price of the k^{th} eurobond satisfies

$$p_k + a_k + 1_k^{put} o^p - 1_k^{call} o^c = \sum_{l=1}^{f_k} u_{kl} D^{(j_k)}(t_{kl}) + e_k, \forall k = 1, 2, \dots, m \quad (6)$$

where p_k denotes the price of the k^{th} eurobond, a_k denotes the accrued interest of the k^{th} eurobond, 1_k^{put} and 1_k^{call} are dummy variables (Draper and Smith [1966]) indicating the existence of embedded put and call options in the eurobond, o^p and o^c are unknown parameters related to the prices of the embedded put and call options, f_k denotes the number of remaining cash flows of the k^{th} eurobond, t_{kl} the time

remaining for payment of the l^{th} cash flow u_{kl} of the k^{th} eurobond, and j_k denotes the rating of the k^{th} eurobond (for instance, if the rating of the k^{th} eurobond is r_3 , then $j_k = 3$).

The estimation process is based in a two step procedure:

1. Identify influential observations (Rousseeuw and Leroy [1987]) using an extension of Cook's statistics (Atkinson [1988]). This first step is important because in the Emerging Eurobond market there are many illiquid or "badly" priced bonds. If these bonds are not appropriately handled during the estimation phase, they may distort the term structures estimated (as illustrated in Almeida et. al. [1998]).
2. Use a duration weighted estimation process after removing all the influential observations detected in the first step. The estimation should preferably use robust techniques, such as the Least Sum of Absolute Deviation or the Least Median of Squares (Rousseeuw and Leroy [1987]). The use of duration weights incorporates heterocedasticity in the nonlinear regression model by allowing the volatility of the eurobond prices to be proportional to its duration (as suggested in Vasicek and Fong [1982]).

A numerical example illustrating the practical use of this methodology is presented next.

4. A Numerical Example of the Estimation Process

As an example we consider the joint estimation of Brazilian and Mexican eurobonds term structures. Fifty-two eurobonds are used: twenty-five Brazilian;

twenty-seven Mexican. The eurobonds are classified in seven different credit ratings¹: BB1, BB2, BB3, B1, B2, B3 and NR. Exhibit 3 presents the main characteristics of the fifty-two eurobonds.

Three leverage points were detected in the first step of the estimation process: one Brazilian (Iochpe); two Mexican (Bufete and Grupo Minero). Exhibit 4 and Exhibit 5 provide the results obtained.

Note that for the Brazilian term structures, the translation factor varies just a few basis points when different ratings are compared: for instance, the difference between the B1 and the B3 translation factors is only 34 basis points. On the other hand there is a difference of 130 basis points between the B1 and B3 Mexican translation factors. The next sections illustrate how the term structures in Exhibit 5 can be used to exploit arbitrage in the Emerging Eurobond market.

5. Detection and Exploitation of Arbitrage Opportunities

The following five steps are proposed to detect and exploit arbitrage in Latin American Eurobond markets:

1. Choose a set of eurobonds with a common rating.
2. Estimate the term structures of interest rates for each country.
3. Based on the estimated term structures, consider possible future scenarios for their relative movement.
4. Analyze the sensibility of different eurobond portfolios to these scenarios generated.
5. Suggest a portfolio that better adjusts to the scenarios generated.

Three numerical examples are presented:

1. Brazil and Mexico: B1 Eurobonds.
2. Brazil and Mexico: B3 Eurobonds.
3. Argentina and Mexico: BB2 Eurobonds.

All the data were collected in June the third of 1998, on J.P. Morgan's web site⁴.

5.1 Brazil and Mexico: B1 Eurobonds

Exhibit 5 depicts the Brazilian and Mexican B1 term structures. The Mexican term structure lies below the Brazilian term structure, indicating that the Brazilian B1 eurobonds are cheap when compared to Mexican B1 eurobonds. The large difference between the translation, rotation and torsion factors of the two term structures suggests as the most probable future scenarios the ones where the curves become closer. That is, if there are no economic conditions leading these countries to behave radically different, we could expect a convergence between term structures of assets of the same rating. Exhibit 6 depicts a scenario representing the convergence of the term structures. The arrows indicate the direction of the movements that would be realized by each term structure in this situation. However, in spite of the suggested convergence, we generate for the analysis of portfolio's sensibility, unbiased future scenarios. There are 63 possibilities of generating future scenarios changing a subset of the term structures factors: Brazilian translation factor, Brazilian rotation factor, Brazilian torsion factor, Mexican translation factor, Mexican rotation factor and Mexican torsion factor. In what follows, without loosing generality, we fix as possible future scenarios the ones where just the Brazilian translation factor and the Mexican rotation factor change their values. A set of twelve scenarios for each term structure

is generated and we calculate the prices of the eurobonds on each of these scenarios. Exhibit 7 depicts the future scenarios for the Brazilian and the Mexican term structures. Exhibit 8 presents the prices of the Mexican eurobonds on each future scenario for the Mexican term structure and Exhibit 9 presents the prices of the Brazilian eurobonds on each future scenario for the Brazilian term structure.

For instance, suppose the occurrence of the scenario depicted in Exhibit 6, that is, a diminution in both the Brazilian translation factor and the Mexican rotation factor, with all the other factors remaining the same. This could mean a decrease in the external long term lend rate for emerging markets. We note in Exhibit 5 that the 9 years maturity is the largest one in both B1 eurobonds markets. The Legendre polynomials symmetry suggests an analysis of the term structures based in two regions: region I, for maturities less than 4.5 years, and region II, for maturities greater than 4.5 years. In the occurrence of such a scenario both term structures attain a decrease of interest rates in region II. Exhibit 8 and Exhibit 9 show that in this situation, all the Brazilian bonds would increase their values, short-term Mexican bonds would decrease their values and long-term Mexican bonds would increase their values. A good strategy would be buy Brazilian bonds and Mexican long-term bonds, and sell Mexican short-term bonds.

Exhibit 10 presents the percent variation of a proposed portfolio long US\$ 1.5 million in the Mexican eurobond Azteca (2007), short US\$ 1 million in the Mexican eurobond Vicap (2002), and long US\$ 1 million in the Brazilian eurobond Votorantin (2005). Note that in the worst case scenario the investor of such a portfolio would loose 9% of its initial wealth. On the other hand, he would profit only 5.6% in the best-proposed situation, which would occur when the Mexican rotation factor goes down 94 basis points (sixty percent of its value) and simultaneously the Brazilian

translation factor goes down 132 basis points (thirty percent of its value). The question here is why invest in this portfolio if its profit and loss distribution is asymmetric with higher extreme loss values than extreme profit values? The answer depends on the probabilities assigned to each scenario. In the uniform case, where all the scenarios receive the same probability, this portfolio would not be a good choice because in average the investor would loose money. However, if the investor perceives greater chances for occurrence of scenarios favorable to the portfolio, that is, the ones where the Mexican rotation factor and the Brazilian translation factor fall, than it would worth investing in such a portfolio. Exhibit 11 presents the portfolio sensibility to the Brazilian translation factor and the Mexican rotation factor. The portfolio value is more sensible to changes in the Brazilian translation factor than to changes in the Mexican rotation factor. For instance, take a variation of 10 percent in the Brazilian translation factor (44 bps). We observe, in Exhibit 10, that this scenario would generate around 1.3 percent variation in the portfolio value. Nevertheless, if we had taken the same percent variation in the Mexican rotation factor (16 bps) it would generate around 0.6 percent variation in the portfolio value. We also observe that the portfolio value varies almost linearly with respect to the Brazilian translation factor but non-linearly with respect to the Mexican rotation factor.

5.2 Brazil and Mexico: B3 Eurobonds

Exhibit 5 also depicts the Brazilian and Mexican B3 term structures of interest rates. These curves cross each other at the 3.2 years maturity. Due to the curves disposition and to the Legendre polynomials symmetry, we split the analysis of the term structures in three regions: region I, for maturities less than 3.2 years, region II,

for maturities between 3.2 and 4.5 years, and, region III, for maturities greater than 4.5 years. On region I, the Brazilian term structure of interest rates presents values greater than the Mexican ones, with the difference approaching 150 basis points, for short maturities. On regions II and III, there is an inversion of the situation: the Mexican curve presents values greater than the Brazilian curve values, with the difference approaching 100 basis points, for maturities around 9 years. The major cause for this difference between the term structures relies in the fact that the Mexican term structure rotation factor (156 basis points) is much superior to the Brazilian one (57 basis points).

Just looking at the term structures picture, we could suggest, as a possible future scenario, an approximation of the curves slopes, that is, the convergence of the rotation factors.

The B3 eurobonds characteristics are listed in Exhibit 3. Note that the three eurobonds have maturities less than or equal to two years, meaning that their pricing is based on region I values of the term structures. According to the supposed future scenario, the curves approximate each other, on region I, implying a depreciation of the Mexican eurobond, and an appreciation of the Brazilian eurobonds. In this context, a good strategy would be buy the Brazilian bonds and sell the Mexican one.

5.3 Argentina and Mexico: BB2 Eurobonds

In this example we consider the joint estimation of Argentinean and Mexican eurobonds term structures. Exhibit 12 depicts the Argentinean and Mexican BB2 term structures of interest rates. Exhibit 13 presents the factor values for both curves. We observe that the Mexican rotation factor is much higher than the Argentinean rotation

factor. Suppose a fixed income manager is positioned in these markets on a portfolio composed by the eurobonds listed in Exhibit 14. The manager is long US\$ 1 million in the Argentinean eurobond Multicanal (2007), long US\$ 1 million in the Argentinean eurobond Perez (2007), long US\$ 1 million in the Mexican eurobond Televisa (2006) and short US\$ 2 million in the Mexican eurobond Cemex (2006). Suppose also the economic committee is expecting a scenario of reduction on long-term Mexican interest rates combined to a reduction on Argentinean interest rates. How could the fixed income manager generate scenarios based on the described expectations to analyze the portfolio exposure? He could begin decreasing the Argentinean translation factor to get the effect of reducing Argentinean interest rates. On the other hand, a combination of changes in the Mexican rotation and torsion factors would provide the desired results for the Mexican term structure. As a matter of fact, if he reduced the Mexican rotation factor he would decrease long-term interest rates but also increase short-term interest rates (an undesired effect). Next, increasing the absolute value of the Mexican torsion factor he would decrease both the short-term rates and long-term rates, compensating the increase in short-term rates generated by the rotation factor movement. Exhibit 15 depicts different scenarios incorporating the economic committee expectations, that is, scenarios of reduction of the Argentinean translation factor, reduction of the Mexican rotation factor and reduction of the Mexican torsion factor. We note that the most extreme scenario for the Mexican term structure really reflects a reduction of long-term Mexican interest rates. In this scenario it would happen a reduction around a hundred basis points in the Mexican long-term interest rate, an increase around thirty basis points in the medium-term interest rate and an increase around ten basis points in the short-term interest rates. Exhibit 16 presents the prices for the Argentinean eurobonds in the manager's

portfolio for the different scenarios for the Argentinean Term Structure. Exhibit 17 presents the prices for the Mexican eurobonds in the manager's portfolio for the different scenarios for the Mexican Term Structure. Exhibit 18 presents, for each scenario observed in Exhibit 15, the percent variation of the portfolio. We note, in Exhibit 16 that the reduction of Argentinean interest rates contributes positively to the portfolio value due to the long position in the BB2 Argentinean market. However, without the help of Exhibit 17 and Exhibit 18, it would not be a simple task to identify the exact effects in the portfolio value caused by the expected changes in the Mexican interest rates. In Exhibit 18, we note that the reduction on Mexican long-term interest rates decreases the portfolio value (in 1.79%) in the worst case scenario, where the Argentinean Interest rates do not change. This behavior is attributed to the short position in the Mexican eurobond Cemex which pursues a very high coupon (12.5%) responsible for increasing its sensibility to interest rates movements. We also note in Exhibit 18 that the portfolio would profit in the major portion of the scenarios proposed. We conclude that translating abstract interest rates expectations to possible factor movements is a very good analysis tool, because it allows an easy interpretation of the portfolio sensibility to interest rates movements.

6. Conclusion

This article presents a methodology for jointly estimating term structures of interest rates for clusters of bonds with different credit ratings. The model is based on an optimization procedure, which assumes that the term structures movements are driven by orthogonal factors. The estimated curves are useful for risk analysis, derivatives pricing and portfolio selection. The methodology is efficient from the

computational point of view and is particularly useful when analyzing markets with few liquid bonds such as Emerging Eurobond markets.

The methodology is completely compatible with scenario analysis models which have become a very interesting alternative to single point forecasting methodologies such as the mean-variance model (Markovitz [1959]). The decomposition of the term structures movements in polynomial orthogonal factors allows easy generation of interest rates scenarios, even turning it possible to apply the methodology to dynamic asset-liability models (Cariño et al. [1994]). However, an important problem involving scenario analysis concerns the choice of the probability distribution that generates the future scenarios. Although this is not a simple problem, it could be addressed by applying historical simulation (Jorion [1997]), using the historical distribution of the term structure factors, or by applying Monte Carlo simulation (Jorion [1997]) by adjusting a model for the dynamic of the term structures.

Latin America Eurobond markets were chosen to illustrate the practical use of the methodology. We explore some simple examples of arbitrage between international term structures for the same rating, using scenario analysis to select portfolios. Although the joint estimations realized in the article involve just pairs of countries (such as Mexico, Brazil and Mexico, Argentina) the joint estimation process could involve many countries, not necessarily the same number of countries used in each arbitrage analysis.

Endnotes

¹ The ratings were made public by Bloomberg Agency. They were created as a combination of Moody's Investors Service ratings and Standard and Poors ratings. The rating "NR" denotes eurobonds not rated by both rating agencies. For more details see the web page www.bloomberg.com.

² The eurobonds in Exhibit 3 whose names are marked by a star are step-up bonds, that is, coupon increasing bonds. For each of these bonds, the coupon shown in Exhibit 3 is the current coupon on June the third of 1998.

³ We constrain the translation factor for the rating NR to be the greatest among all the translation factors for the different ratings.

⁴ www.jpmorgan.com.

⁵ The prices shown in bold on Exhibit 8 were obtained by discounting the Mexican eurobonds cash flows using the Mexican B1 estimated term structure.

⁶ The prices shown in bold on Exhibit 9 were obtained by discounting the Brazilian eurobonds cash flows using the Brazilian B1 estimated term structure.

⁷ The prices shown in bold on Exhibit 16 were obtained by discounting the Argentinean eurobonds cash flows using the Argentinean BB2 estimated term structure.

⁸ The prices shown in bold on Exhibit 17 were obtained by discounting the Mexican eurobonds cash flows using the Mexican BB2 estimated term structure.

⁹ In this context, we consider as a scenario for the BB2 Mexican term structure, a combination of simultaneous changes in the Mexican rotation and torsion factors.

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Exhibit 1. An Example of Term Structures for Different Ratings in Model (4)

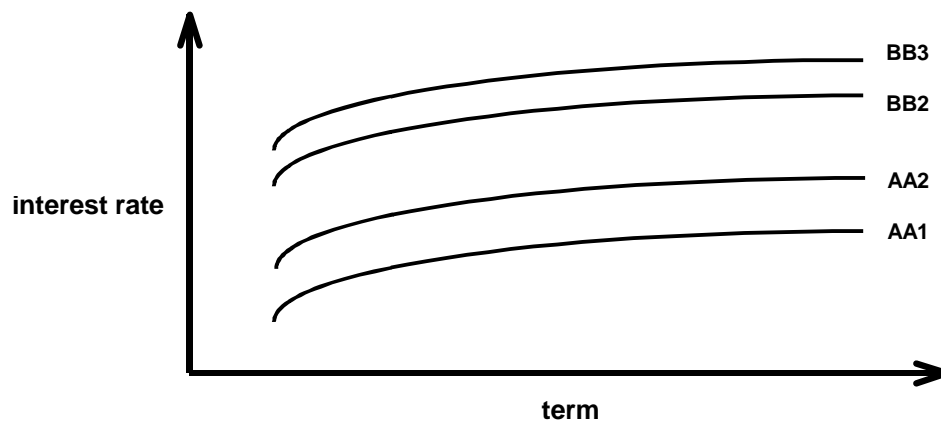


Exhibit 2. An Example of Term Structures for Different Ratings in a General Model

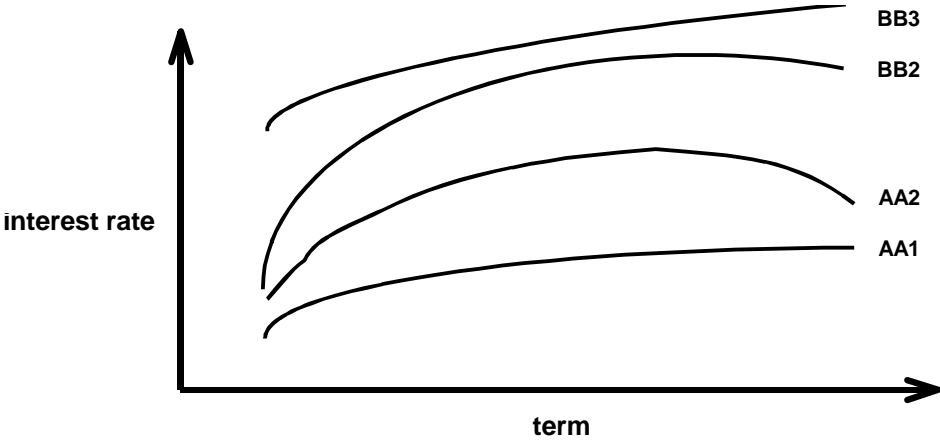


Exhibit 3. Eurobonds Used to Illustrate the Estimation Process

Eurobond ²	Rating	Country	Coupon (%)	Maturity
Bco Bradeco	B2	Brazil	8.000	28-Jan-2000
Bco Excel	B2	Brazil	10.750	08-Nov-2004
Bco Itau	B2	Brazil	7.500	11-Jul-2000
Bco Safra	B2	Brazil	8.125	10-Nov-2000
Bco Safra	B2	Brazil	8.750	28-Oct-2002
Bco Safra	B2	Brazil	10.375	28-Oct-2002
CEMIG	NR	Brazil	9.125	18-Nov-2004
CESP ³	NR	Brazil	9.125	28-Jun-2007
Copel	NR	Brazil	9.750	02-May-2005
CSN Iron	B2	Brazil	9.125	01-Jun-2007
CVRD	NR	Brazil	10.000	02-Apr-2004
Ford	B2	Brazil	9.250	22-Jan-2007
Ford Ltd	B2	Brazil	9.125	08-Nov-2004
Gerdau	NR	Brazil	11.125	24-May-2004
Iochpe	NR	Brazil	12.375	08-Nov-2002
Ipiranga ⁴	NR	Brazil	10.625	25-Feb-2002
Klabin	NR	Brazil	10.000	20-Dec-2001
Klabin ⁵	NR	Brazil	12.750	28-Dec-2002
Lojas	NR	Brazil	11.000	04-Jun-2004
Minas X WR-A	B3	Brazil	7.875	10-Feb-1999
Minas X WR-B	B3	Brazil	8.250	10-Feb-2002
Parmalat ⁶	NR	Brazil	9.125	02-Jan-2005
RBS	B1	Brazil	11.000	01-Apr-2007
Unibanco	B2	Brazil	8.000	06-Mar-2000
Votorantim	B1	Brazil	8.500	27-Jun-2005
Altos Hornos	B2	Mexico	11.375	30-Apr-2002
Altos Hornos	B2	Mexico	11.875	30-Apr-2004
Azteca	B1	Mexico	10.125	15-Feb-2004
Azteca	B1	Mexico	10.500	15-Feb-2007
Banamex	BB2	Mexico	9.125	06-Apr-2000
Bufete	B3	Mexico	11.375	15-Jul-1999
Cemex	BB2	Mexico	8.500	31-Aug-2000
Cemex	BB2	Mexico	9.500	20-Sep-2001
Cemex	BB2	Mexico	10.000	05-Nov-1999
Cemex	BB2	Mexico	10.750	15-Jul-2000
Cemex	BB2	Mexico	12.750	15-Jul-2006
Coke FEMSA	BB2	Mexico	8.950	01-Nov-2006
Cydsa	NR	Mexico	9.375	25-Jun-2002
DESC	BB3	Mexico	8.750	15-Oct-2007
ELM	NR	Mexico	11.375	25-Jan-1999
Empresas ICA	B1	Mexico	11.875	30-May-2001
Gruma	BB1	Mexico	7.625	15-Oct-2007
Grupo IMSA	BB2	Mexico	8.930	30-Sep-2004
Grupo Minero	BB1	Mexico	8.250	01-Apr-2008
Hylsa	BB3	Mexico	9.250	15-Sep-2007
Pepsi-Gemex	BB3	Mexico	9.750	30-Mar-2004
Televisa	BB2	Mexico	0	15-May-2008
Televisa	BB2	Mexico	11.375	15-May-2003
Televisa	BB2	Mexico	11.875	15-May-2006
Tolmex	BB2	Mexico	8.375	01-Nov-2003
Vicap	B1	Mexico	10.250	15-May-2002
Vicap	B1	Mexico	11.375	15-May-2007

Exhibit 4. Values of Factors for the Brazilian and Mexican Term
Structures for Different Ratings

Factor³	Value (bps)
Brazilian B1 Translation	441
Brazilian B2 Translation	451
Brazilian B3 Translation	475
Brazilian NR Translation	485
Brazilian Rotation	57
Brazilian Torsion	-56
Mexican BB1 Translation	249
Mexican BB2 Translation	322
Mexican BB3 Translation	322
Mexican B1 Translation	379
Mexican B2 Translation	509
Mexican B3 Translation	509
Mexican NR Translation	532
Mexican Rotation	156
Mexican Torsion	-114

Exhibit 5. A Comparison of Mexican and Brazilian Term Structures of Interest Rates for Different Credit Ratings

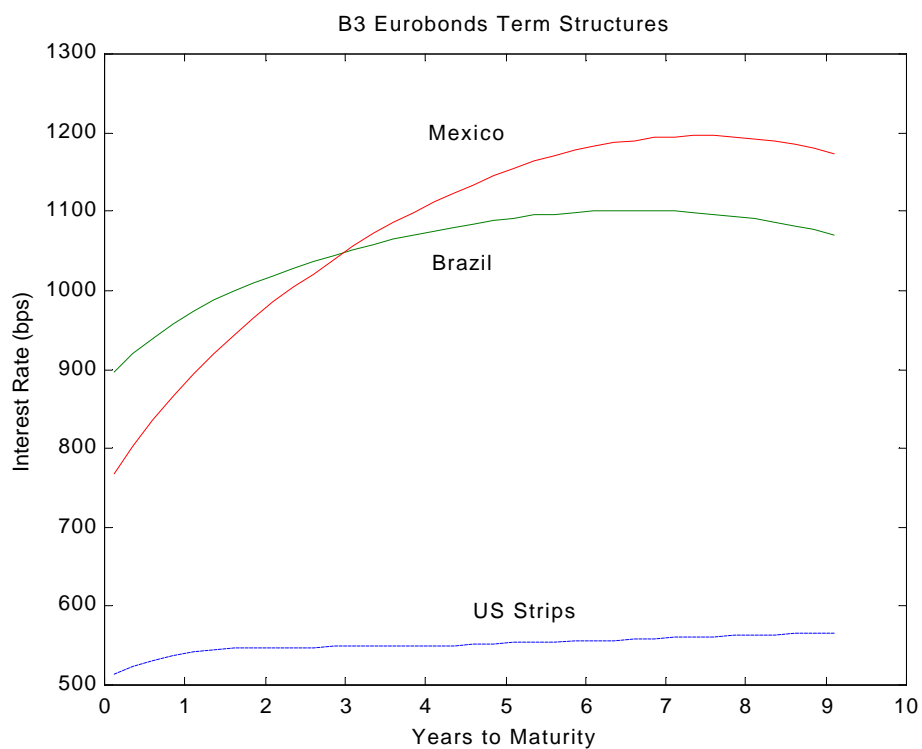
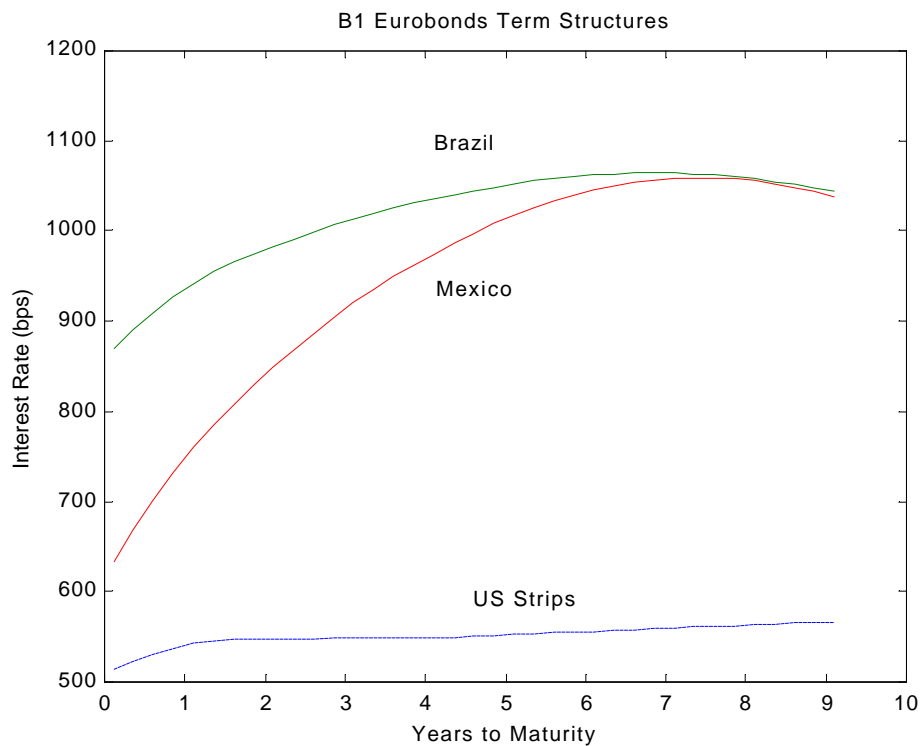


Exhibit 6. Possible Convergence Scenario

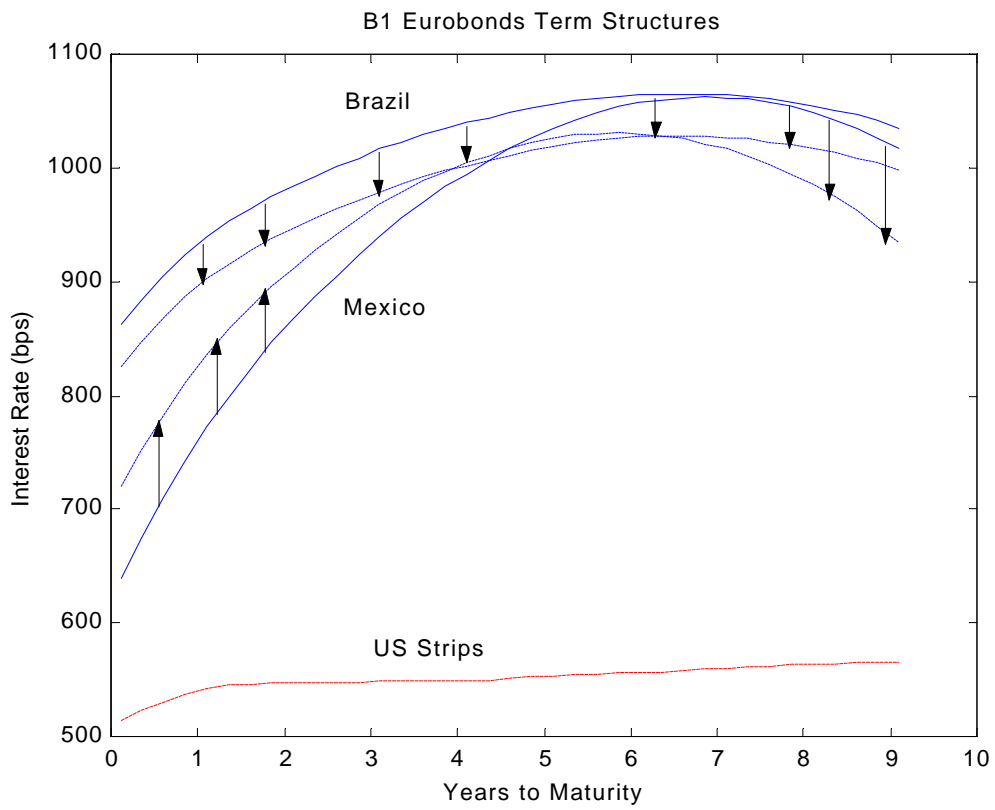


Exhibit 7. Different Scenarios for the B1 Term Structures

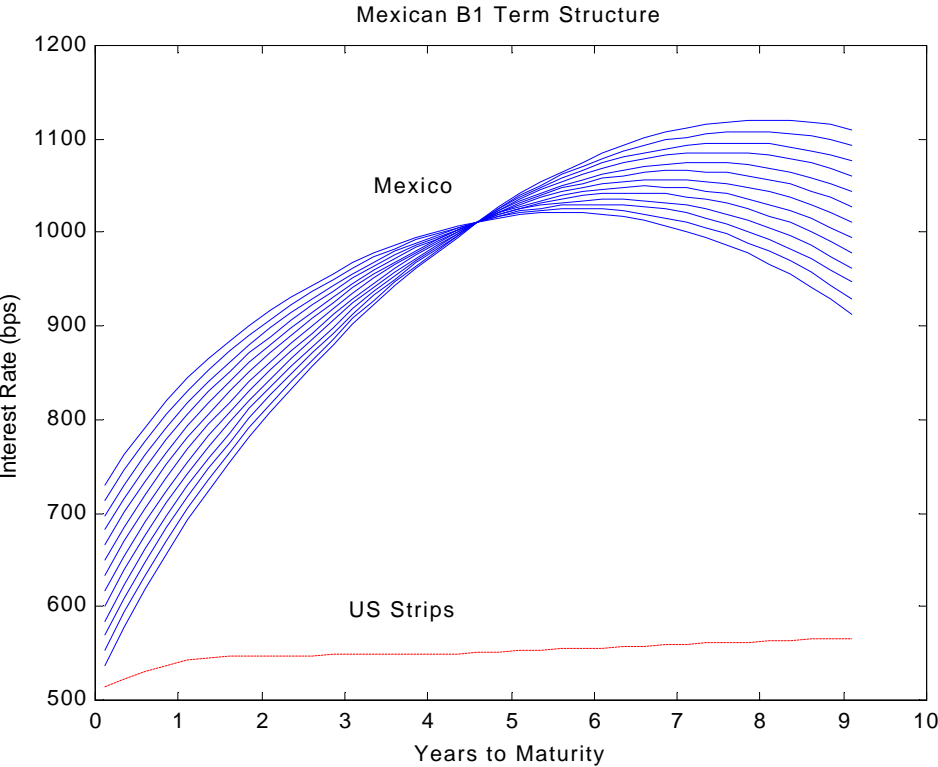
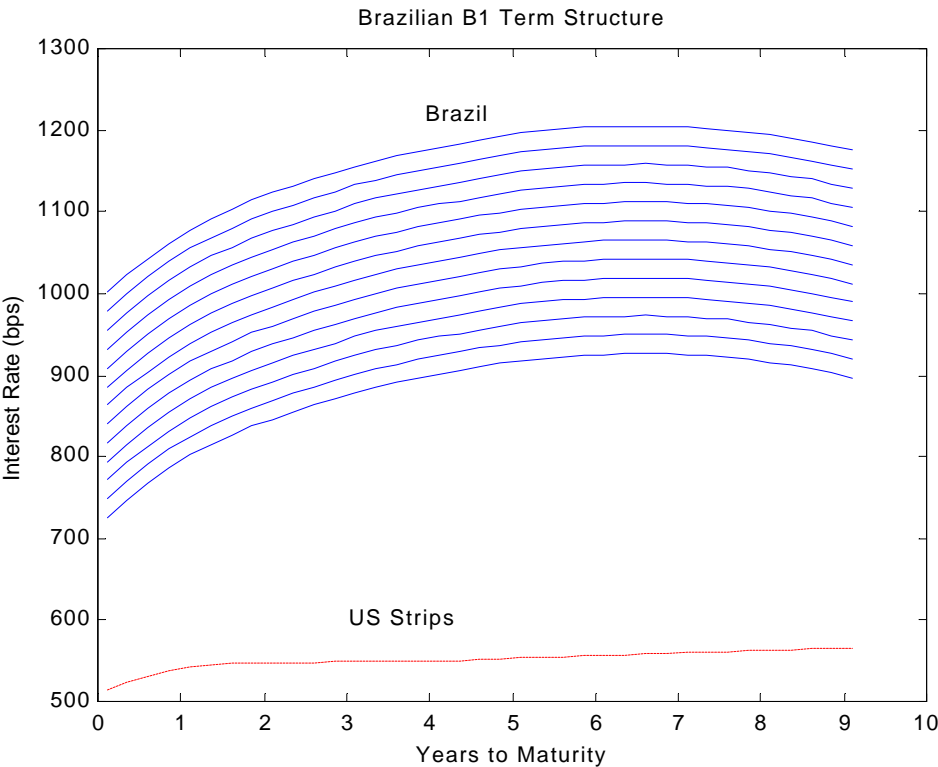


Exhibit 8. Price Variation⁵ of Mexican B1 Eurobonds for Different Scenarios for the Mexican Term Structure

Rotation Factor (variation in bps)	ICA 2001 (US\$)	Vicap 2002 (US\$)	Azteca 2004 (US\$)	Azteca 2007 (US\$)	Vicap 2007 (US\$)
94	108.6660	110.7541	102.5180	98.4830	79.0877
78	108.4775	108.6085	102.7050	99.7355	84.5002
62	108.2892	106.8840	102.8769	100.9473	90.3572
47	108.1013	105.5128	103.0322	102.1112	95.9951
31	107.9138	104.4372	103.1698	103.2205	100.9502
16	107.7265	103.6072	103.2897	104.2685	104.9842
0	107.5396	102.9799	103.3919	105.2489	108.0515
-16	107.3530	102.5176	103.4772	106.1557	110.2376
-31	107.1668	102.1876	103.5465	106.9832	111.6975
-47	106.9809	101.9613	103.6011	107.7264	112.6066
-62	106.7953	101.8140	103.6425	108.3804	113.1286
-78	106.6101	101.7247	103.6724	108.9411	113.3996
-94	106.4252	101.6756	103.6925	109.4048	113.5225

Exhibit 9. Price Variation⁶ of Brazilian B1 Eurobonds for Different Scenarios for the Brazilian Term Structure

Translation Factor (variation in bps)	Votorantin 2005 (US\$)	RBS 2007 (US\$)
132	80.8641	98.4083
110	83.1444	99.6497
88	85.4123	100.9116
66	87.6610	102.1943
44	89.8838	103.4982
22	92.0737	104.8238
0	94.2236	106.1714
-22	96.3265	107.5414
-44	98.3751	108.9342
-66	100.3624	110.3502
-88	102.2811	111.7899
-110	104.1245	113.2537
-132	105.8856	114.7419

Exhibit 10. Percent Variation on the B1 Portfolio Value for Different Scenarios for the Mexican and Brazilian Term Structures

	94	78	62	47	31	16	0	-16	-31	-47	-62	-78	-94
132	-8.96	-7.86	-6.89	-6.03	-5.28	-4.62	-4.05	-3.55	-3.12	-2.76	-2.45	-2.20	-2.00
110	-8.27	-7.17	-6.19	-5.34	-4.59	-3.93	-3.36	-2.86	-2.43	-2.07	-1.76	-1.51	-1.31
88	-7.58	-6.48	-5.51	-4.65	-3.90	-3.25	-2.67	-2.17	-1.75	-1.38	-1.07	-0.82	-0.62
66	-6.90	-5.80	-4.82	-3.97	-3.22	-2.56	-1.99	-1.49	-1.06	-0.70	-0.39	-0.14	0.06
44	-6.23	-5.12	-4.15	-3.30	-2.55	-1.89	-1.32	-0.82	-0.39	-0.02	0.28	0.54	0.74
22	-5.56	-4.46	-3.49	-2.63	-1.88	-1.23	-0.65	-0.15	0.27	0.64	0.95	1.20	1.40
0	-4.91	-3.81	-2.83	-1.98	-1.23	-0.57	0.00	0.50	0.93	1.29	1.60	1.85	2.05
-22	-4.27	-3.17	-2.20	-1.34	-0.59	0.06	0.64	1.14	1.56	1.93	2.24	2.49	2.69
-44	-3.65	-2.55	-1.58	-0.72	0.03	0.69	1.26	1.76	2.18	2.55	2.86	3.11	3.31
-66	-3.05	-1.95	-0.97	-0.12	0.63	1.29	1.86	2.36	2.79	3.15	3.46	3.71	3.92
-88	-2.47	-1.36	-0.39	0.46	1.21	1.87	2.44	2.94	3.37	3.73	4.04	4.29	4.50
-110	-1.91	-0.80	0.17	1.02	1.77	2.43	3.00	3.50	3.93	4.29	4.60	4.85	5.06
-132	-1.38	-0.27	0.70	1.56	2.31	2.96	3.54	4.03	4.46	4.83	5.13	5.39	5.59

Horizontal - Changes in the Mexican Rotation Factor in Basis Points

Vertical - Changes in the Brazilian Translation Factor in Basis Points

Exhibit 11. B1 Portfolio Sensibility for Different Scenarios

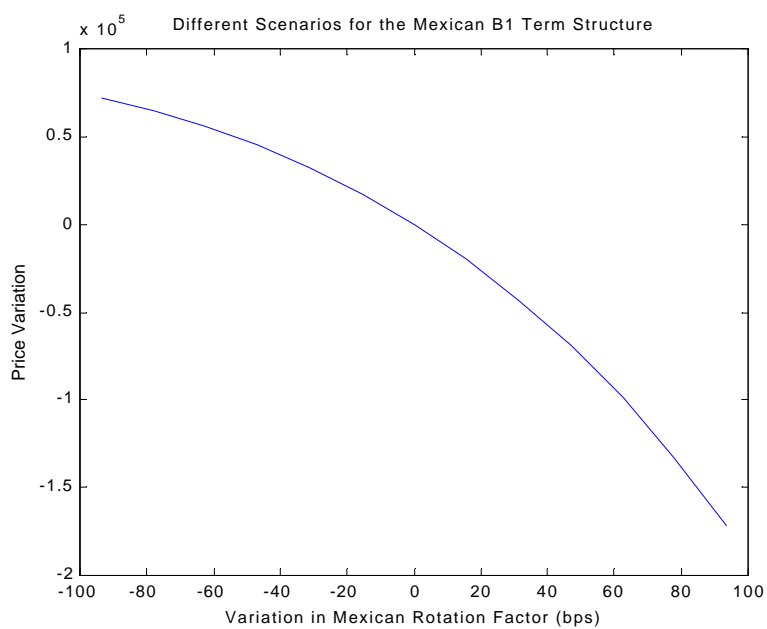
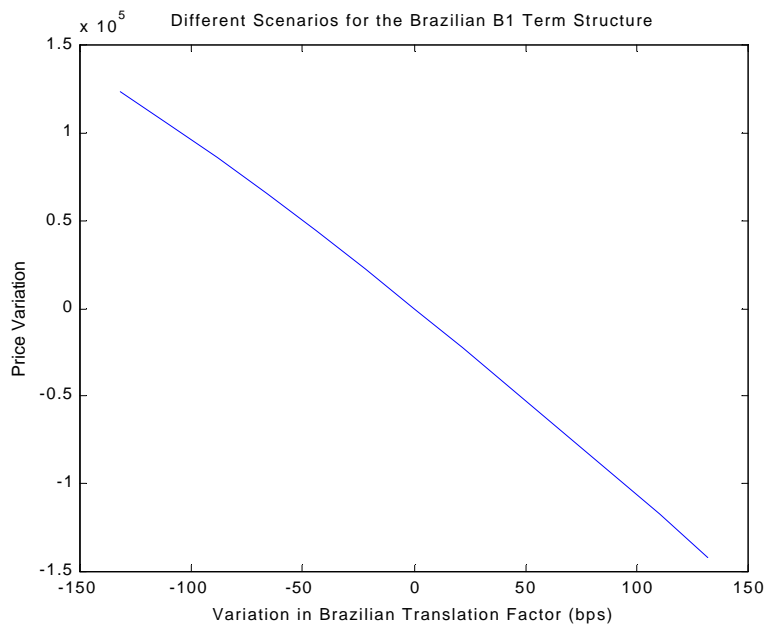


Exhibit 12. Detecting Arbitrage Opportunities in the BB2 Argentinean and Mexican Eurobond Markets

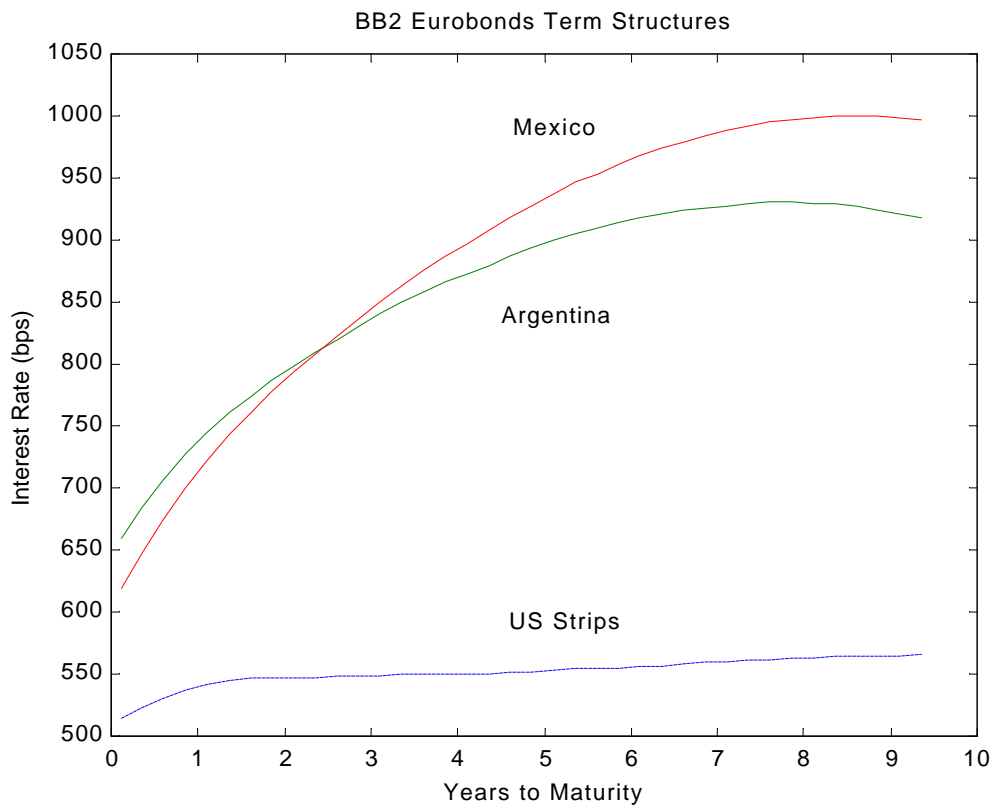


Exhibit 13. Values of Factors for the Argentinean and Mexican BB2

Term Structures

Factor	Value (bps)
Argentinean Translation	291
Argentinean Rotation	98
Argentinean Torsion	-57
Mexican Translation	322
Mexican Rotation	153
Mexican Torsion	-74

Exhibit 14. BB2 Eurobond Portfolio Used in Example 5.3

Eurobond	Country	Coupon (%)	Maturity
Multicanal	Argentina	10.500	01-Feb-2007
Perez	Argentina	8.125	15-Jul-2007
Cemex	Mexico	12.750	15-Jul-2006
Televisa	Mexico	11.875	15-May-2006

Exhibit 15. Different Scenarios for the BB2 Term Structures

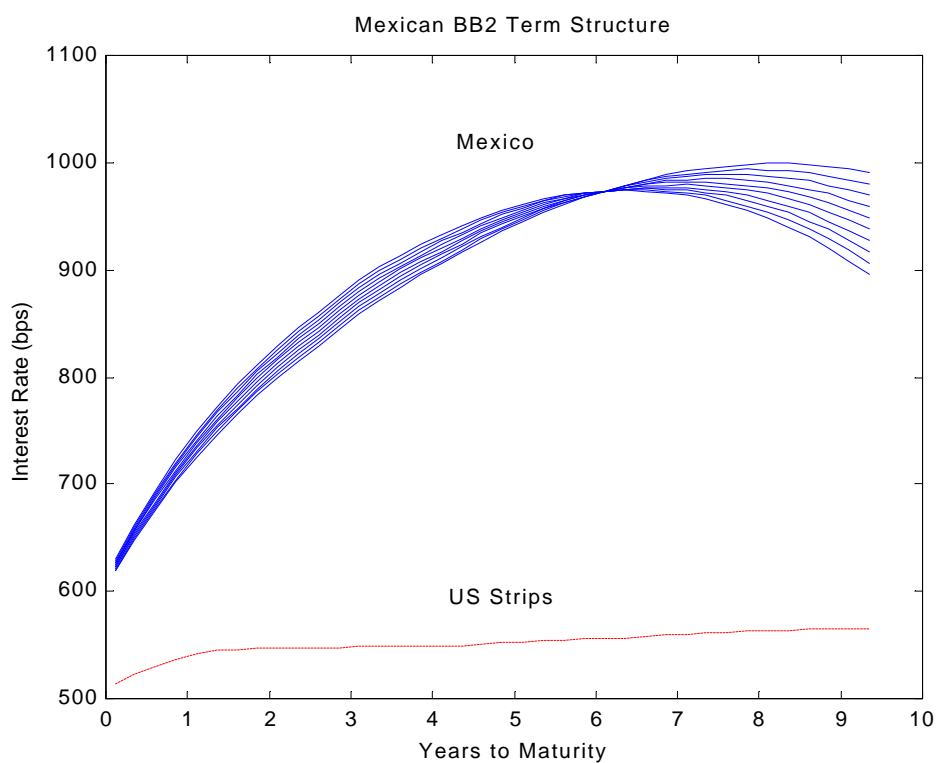
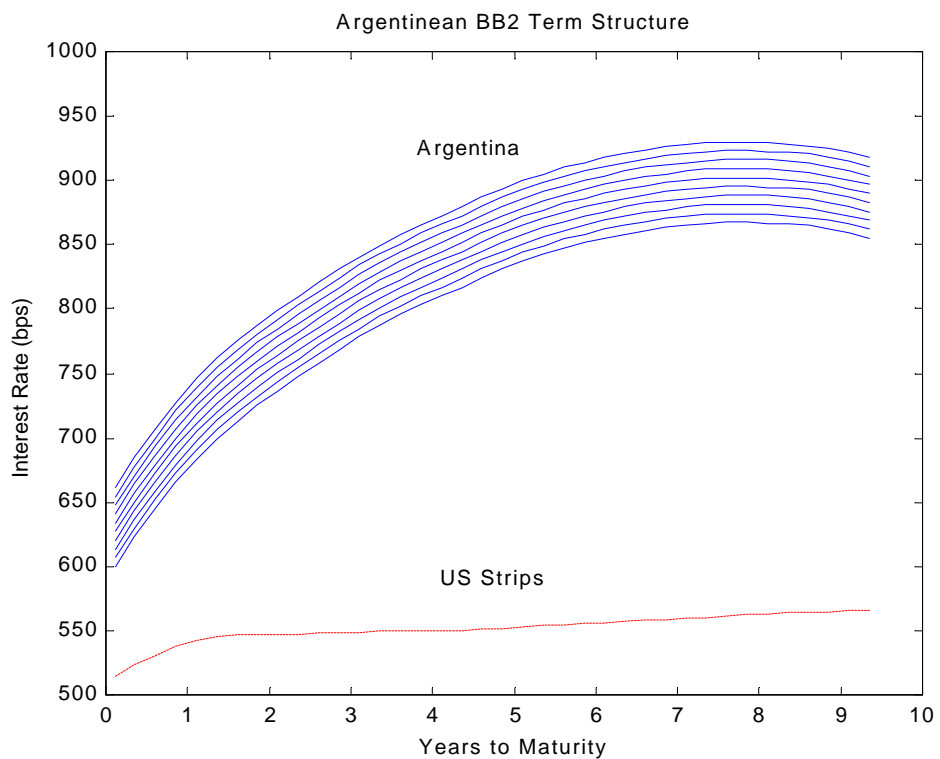


Exhibit 16. Price Variation⁷ of the Argentinean Eurobonds in the BB2 Portfolio, for Different Scenarios for the Argentinean Term Structure

Translation Factor (variation in bps)	Multicanal 2007 (US\$)	Perez 2007 (US\$)
0	112.5599	97.9793
-7	112.9957	98.3912
-13	113.4337	98.8054
-20	113.8739	99.2217
-27	114.3163	99.6403
-33	114.7608	100.0612
-40	115.2076	100.4842
-47	115.6566	100.9096
-53	116.1079	101.3372
-60	116.5614	101.7670

Exhibit 17. Price Variation⁸ of the Mexican Eurobonds in the BB2 Portfolio, for Different Scenarios⁹ for the Mexican Term Structure

Rotation Factor (variation in bps)	Torsion Factor (variation in bps)	Televisa 2006 (US\$)	Cemex 2006 (US\$)
0	0	113.4530	122.8172
-6	-4	113.5656	122.9501
-11	-9	113.6788	123.0836
-17	-13	113.7924	123.2177
-22	-18	113.9065	123.3524
-28	-22	114.0210	123.4877
-33	-27	114.1360	123.6236
-39	-31	114.2515	123.7602
-44	-36	114.3675	123.8974
-50	-40	114.4840	124.0352

Exhibit 18. Percent Variation on the BB2 Portfolio Value for Different Scenarios for the Argentinean and Mexican Term Structures

Rotation	Torsion	0	-7	-13	-20	-27	-33	-40	-47	-53	-60
0	0	0.00	1.08	2.17	3.26	4.36	5.47	6.58	7.69	8.81	9.94
-6	-4	-0.20	0.89	1.97	3.07	4.17	5.27	6.38	7.50	8.62	9.75
-11	-9	-0.39	0.69	1.78	2.87	3.97	5.07	6.18	7.30	8.42	9.55
-17	-13	-0.59	0.49	1.58	2.67	3.77	4.88	5.99	7.10	8.22	9.35
-22	-18	-0.79	0.29	1.38	2.48	3.57	4.68	5.79	6.90	8.03	9.15
-28	-22	-0.99	0.10	1.18	2.28	3.37	4.48	5.59	6.71	7.83	8.95
-33	-27	-1.19	-0.10	0.98	2.08	3.17	4.28	5.39	6.51	7.63	8.75
-39	-31	-1.39	-0.31	0.78	1.87	2.97	4.08	5.19	6.30	7.43	8.55
-44	-36	-1.59	-0.51	0.58	1.67	2.77	3.88	4.99	6.10	7.22	8.35
-50	-40	-1.79	-0.71	0.38	1.47	2.57	3.67	4.78	5.90	7.02	8.15

Horizontal - Changes in the Argentinean Translation Factor in Basis Points

Vertical – Changes in the Mexican Rotation and Torsion Factors in Basis Points