The Optimal Inflation Tax and Structural Reform

Tiago V. de V. Cavalcanti Anne P. Villamil *

September 21, 2000

Abstract

This paper analyzes the robustness of the Friedman Rule to structural imperfections in labor, commodity, and currency markets. The Friedman Rule is a classic result in economics which claims that only monetary policies that generate a zero nominal interest rate will lead to optimal resource allocation (cf., Friedman (1969)). This "Ramsey equilibrium" result is robust in a wide range of dynamic, constrained, general equilibrium environments with distortionary taxes and commitment, but without imperfections in input, output, or financial markets. In many developing countries, a large fraction of activity takes place in the "informal" sector. Roughly speaking, the informal sector is the untaxed and unregulated market sometimes referred to as the underground economy. We obtain three results. First, we show that when structural imperfections such as an informal sector exist, the optimal inflation tax is positive. Second, we show that structural imperfections introduce an important asymmetry into the welfare cost function. Third, we provide quantitative results.

JEL Classification Numbers: E31; E52; H26

Keywords: Inflation; Informal sector; Ramsey policy; Friedman Rule

---

*Department of Economics, University of Illinois, 1206 S. 6th Street, Champaign, IL 61820 USA, cavalcan@students.uiuc.edu and avillami@uiuc.edu

We thank Stephen Parente for helpful comments. We also gratefully acknowledge financial support from the University of Illinois.

Correspondence to: T. Cavalcanti and A. Villamil

1 Introduction

The Friedman Rule is a classic result in economics which claims that only monetary policies that generate a zero nominal interest rate will lead to optimal resource allocation (cf., Friedman (1969)). In practice this means that the economy should have either no inflation or deflation at a constant rate. Based on this result, some economists and policy-makers have argued that price level stability should be the main goal of a Central Bank. This "Ramsey equilibrium" result is robust in a wide range of dynamic, constrained, general equilibrium environments with distortionary taxes and commitment, but without imperfections in input, output, or financial markets.[1] In particular, the result holds in environments with heterogeneous agents, endogenous growth, an open economy, and alternative demographic structures. In all cases markets are complete. In many developing countries a large fraction of economic activity takes place in the "informal" sector (e.g., 20% to 50%). Roughly speaking, the informal sector is the untaxed and unregulated sector sometimes referred to as the underground economy. In this sector it is difficult for the government to measure economic activity or enforce contracts (cf., Stone and Paredes (1996)).

This paper addresses the following questions: Does a Ramsey analysis of inflation as a tax support the Friedman Rule when the government's ability to tax input, output, and financial markets is limited? Further, when such structural distortions occur, are they important quantitatively? To answer these questions, we study the problem of a government that wishes to finance optimally a given expenditure sequence. The economy has two types of markets distinguished solely by the government's ability to observe economic activity in them: All transactions in the formal sector are observable by the government. As a consequence, the government can monitor and tax the income from these transactions. In contrast, in the informal sector the government cannot observe or tax labor income, commodities, or financial transactions. The key friction in the economy is this structural imperfection.

We follow the "Bewley approach" to incompleteness (cf., Ljungqvist and

[1]The "Ramsey problem" is to choose the optimal tax structure consistent with a competitive equilibrium when only distortionary taxes are available. Agents respond to tax distortions, and the government takes agents' responses into account when choosing its policy (cf., Chari and Kehoe (2000)). When direct taxation is not possible, inflation can serve as an indirect tax on consumption (cf., Bryant and Wallace (1984), Villamil (1988), and Smith and Villamil (1998)).

Sargent (2000)): Markets are exogenously incomplete by assumption and we investigate the implications of varying the degree of incompleteness. In our setting this means that the government is unable to tax input, output, and monetary transactions perfectly. In numerical experiments we treat the size of the structural friction as exogenous and determine the quantitative impact of varying the distortion.[2] Our results are reminiscent of Sargent and Wallace (1981), who showed that exogenous fiscal policy is a key determinant of optimal monetary policy. In a similar spirit we take structural conditions as given (i.e., limitations on the government's ability to tax), and characterize the link between exogenous structural conditions and monetary policy.

We obtain three main results. First, we prove that when structural imperfections exist the Friedman Rule is not optimal. Second, we show that the optimal inflation tax is positive, ranging between 6% and 14% for alternative calibrations. Third, we show how structural imperfections alter the welfare cost of inflation. A key finding is that structural imperfections "flatten out" the welfare cost function, leading to an important welfare asymmetry: The gain from reducing moderate inflation to the optimal (lower) level is small when structural imperfections are large. However, reducing inflation below the optimal level to an arbitrarily low level (e.g., zero) leads to large welfare losses. This result highlights the importance of structural reform for monetary policy.

2 The Model

Consider a production economy with a single input (labor), a single output (consumption), and $t = 0, 1, ...,$ time periods. Assume there is a formal sector and an informal sector for both the input and the output. The government:

- can observe and tax transactions in the formal sector; and
- cannot observe and tax transactions in the informal sector.

For simplicity, assume that commodity taxes are unavailable, $\tau_c^F = \tau_c^I = 0$. We consider incomplete commodity taxes in Section 3.2, $\tau_c^F > 0$ and $\tau_c^I = 0$.

[2] Recent computational work in Bewley Models has focused on the effect of incomplete loan markets (i.e., borrowing constraints) on agents' ability to self-insure when there are no external insurance opportunities (cf., Huggett (1993), Akyol (2000) and Erosa and Ventura (2000)). In contrast, our focus is on a classic public finance problem – incompleteness in the government's ability to tax agents.

Households

The economy has infinitely lived representative agent with preferences:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{1}$$

Let $\beta \in (0,1)$, $u(\cdot,\cdot)$ be a strictly concave, twice continuously differentiable function such that the INADA conditions are satisfied, and c_t and h_t denote consumption and leisure in period t, respectively. The representative household is endowed with one unit of time in each period that can be used as leisure, h_t, to make transactions, s_t, or allocated to production in either the formal sector n_t^F, or in the informal sector n_t^I, with $1 = n_t^F + n_t^I + s_t + h_t$. By working in the informal sector the agent can evade the taxes associated with formal job contracts. The labor tax in the formal sector is τ_n^F.

Let M_t denote fiat currency, B_t denote government bonds with return i_t, and w_t^F and w_t^I denote the wage rates in each sector. The representative agent's one period budget constraint is:

$$\begin{aligned} c_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \leq \frac{M_{t-1}}{P_t} + (1+i_{t-1})\frac{B_{t-1}}{P_t} + (1-\tau_n^F) w_t^F n_t^F \\ + w_t^I (1 - n_t^F - s_t - h_t) \end{aligned} \tag{2}$$

In present value form the budget constraint, with no-Ponzi game conditions for bonds and money, and with initial conditions, $B_{-1} = M_{-1} = 0$, is

$$\sum_{t=0}^{\infty} d_t c_t + \sum_{t=0}^{\infty} d_t I_t m_t \leq \sum_{t=0}^{\infty} d_t [(1-\tau_n^F) w_t^F n_t^F + w_t^I (1 - n_t^F - s_t - h_t)] \tag{3}$$

where $m_t = \frac{M_t}{P_t}$, $1 + r_t = (1+i_t)\frac{P_t}{P_{t+1}}$, $I_t = \frac{i_t}{1+i_t}$ and $d_t = \frac{1}{\prod_{s=0}^{t-1}(1+r_s)}$.

Production Technology:

There is a representative firm whose technology exhibits constant returns to scale. Assume there is a single factor of production, labor services:

$$F(N) = N \tag{4}$$

N is a CES aggregator of the labor employed with formal contracts, N^F, and the labor employed without informal contracts, N^I. Let

$$N = [\lambda (N^F)^\rho + (1-\lambda)(N^I)^\rho]^{\frac{1}{\rho}}$$

The parameter $0 < \lambda < 1$ measures the relative importance of formal and informal labor in production, and is a key determinant of the marginal product of labor in each sector. Workers allocate some fraction of their time to the formal sector, N^F, and some to the informal sector, N^I. The parameter $0 < \rho < 1$ denotes the elasticity of substitution between the two types of employment. We interpret ρ as a proxy for enforcement, since

- When $\rho \to 1$, firms can freely substitute workers. There is implicitly no "production penalty" for substituting labor between the two sectors.
- When $\rho < 1$, if the firm substitutes labor from the informal sector it faces a production penalty.[3]

The key distinction is that across countries, λ measures differences in *worker* productivity in each sector and ρ measures differences in institutions that affect *firm* employment practices. In the computational experiments λ and ρ will jointly determine the level of informality in the economy.

In every period the firm takes price as given and maximizes profit. The first order conditions are

$$w_t^F = F_{n^F}(t) \tag{5}$$

$$w_t^I = F_{n^I}(t) \tag{6}$$

Transaction Technology

As is standard, assume that real money balances are costless to produce. Real balances are useful for transaction purposes because they decrease the amount of time agents spend shopping. The transaction technology is represented by[4]

$$s_t \geq l(c_t, \tfrac{M_t}{P_t}) \equiv l(c_t, m_t) \tag{7}$$

and it satisfies the following properties:

[3] For example, this production penalty corresponds to the inefficiency a firm incurs from "hiding" an undocumented worker. Loyaza (1996) shows that informal status entails many disadvantages. Because of its illegal status, informal activities are subject to fines; workers may not enjoy mandated benefits; and firms scale down their size to avoid detection.

[4] Guidotti and Végh (1993) and Correia and Teles (1996) also use this specification. The Appendix shows that the results are qualitatively similar for CIA and MIUF specifications of money demand.

a. $l(c,m) \geq 0$ and $l(0,m) = 0$,

b. $l_c \geq 0$,

c. $l_{cc} \geq 0$, and $l_{mm} \geq 0$,

d. $l_m(c,m) = 0$ defines $m^\star = m(c)$ such that $l_m < 0$ when $m < m^\star$,

e. $l(\cdot,\cdot)$ is a convex function,[5]

f. $l(\cdot,\cdot)$ is homogeneous of degree k.

Property (d) implies that for a given level of consumption there is a point of "satiation" in real money balances. That is, for each level of consumption there is a level of real balances such that an additional dollar does not decrease the amount of time agents spend shopping. According to Friedman (1969), "cash balances ... are held to satiate, so that the real return from an extra dollar is zero." Assume that this "satiation level" of real money balances is finite, $m^\star < \infty$.[6] Otherwise, for each level of consumption, an extra dollar will always decrease shopping time. This is stated formally as:

L1. $m^\star = m(c) < \infty$

From property (f) we can write $l(c,m) = L(\frac{m}{c})c^k$, where $L' \leq 0$ and $L'' \geq 0$.

Household's Problem

The household's problem is to choose $\{c_t, h_t, m_t, b_t, s_t, n_t^F\}_{t=0}^{\infty}$ to maximize (1) subject to (2), (7) and the usual non-negativity constraints, where $b_t = \frac{B_t}{P_t}$. The necessary conditions for an interior solution are (2), (7) and:

$$\frac{u_h(t)}{u_c(t) - u_h(t)l_c(t)} = (1-\tau_n^F)w_t^F \tag{8}$$

$$w_t^I = (1-\tau_n^F)w_t^F \tag{9}$$

$$-l_m(t) = \frac{1}{(1-\tau_n^F)w_t^F} I_t \tag{10}$$

$$\frac{u_c(t) - u_h(t)l_c(t)}{u_c(t+1) - u_h(t+1)l_c(t+1)} = \beta(1+r_t) \tag{11}$$

[5]This assumption ensures the existence of a unique interior optimum solution.

[6]Friedman (1969), Phelps (1973), Brock (1975) and Mulligan and Sala-i-Martin (1997) use a finite "satiation level" of real money balances.

Equation (9) and the firm's first order conditions show that

$$\frac{N^I}{N^F} = [\frac{1-\lambda}{\lambda}\frac{1}{1-\tau_n^F}]^{\frac{1}{1-\rho}}$$

This ratio of labor inputs shows how the distortionary labor tax τ_n^F induces more informal contracts in the economy. The elasticity of substitution $\frac{1}{1-\rho}$ determines how strongly this ratio responds to changes in the tax. This is in accordance with Lemieux, Fortin and Frechette (1994) who show that while taxes affect labor-leisure choices (see our equation (8)), they also stimulate labor market activities in the untaxed sector of the economy (equation (9)).[7] Thus there is an additional margin on which agents will substitute.

Equation (10) implicitly defines a money demand function $\hat{m}_t(c_t, I_t, \tau_n^F, w_t^F)$. It is straightforward to show that the money demand function is increasing in consumption if and only if $l_{cm} < 0$.[8] As a consequence, we assume:

L2. $l_{cm} < 0$

L2 implies a positive elasticity of this money demand function with respect to c (scale elasticity).[9] Estimates by Lucas (1990) and Mulligan and Sala-i-Martin (1997) suggest that this elasticity is close to one.

The Friedman Rule is equivalent to setting $I_t = 0$. The household chooses the bliss point in real balances, $\hat{m} = m^\star = m(c)$, so that given the consumption level, no more resources can be saved by increasing the amount of real money per unit of transaction.

Government

The government finances its expenditure stream $\{g_t\}_{t=0}^\infty$ by levying a tax on labor earnings in the formal sector at rate τ_n^F, by printing money and by borrowing. The government's budget constraint is

$$g_t = \tau_n^F w_t^F n_t^F + \frac{B_t^g}{P_t} - (1+i_t)\frac{B_{t-1}^g}{P_t} + \frac{M_t^g}{P_t} - \frac{M_{t-1}^g}{P_t} \tag{12}$$

[7] Schneider and Enste (2000) claim that the most important cause of increases in informal activities is the rise of tax and social security burdens. The other variables that affect the equilibrium conditions which determine the level of informal activity are penalty rates and tax evasion detection probabilities, which are proxies for government institutions. In our economy the elasticity of substitution $\frac{1}{1-\rho}$ reflects the strength of these institutions.

[8] A negative cross derivative of the transaction function means that the marginal cost of transaction due to an additional unit of consumption decreases with money real balances.

[9] Using the implicit function theorem we can show that the scale elasticity and the interest rate elasticity are defined by $\epsilon_c = -\frac{c}{m}\frac{l_{cm}}{l_{mm}}$ and $\epsilon_I = \frac{I}{m}\frac{1}{l_{mm}(1-\tau_n^F)w^F}$, respectively.

Competitive Equilibrium

A competitive equilibrium is a sequence of prices $\{w_t^F, w_t^I, P_t\}_{t=0}^{\infty}$, a sequence of government policies[10] $\{\tau_n^F, i_t, \pi_t\}_{t=0}^{\infty}$, a sequence of consumer choices $\{c_t, n_t^F, n_t^I, s_t, m_t, b_t\}_{t=0}^{\infty}$ and a sequence of firm choices $\{n_t^F, n_t^I\}_{t=0}^{\infty}$ such that

- Given the sequence of prices and government policies, the consumer's allocation solves the representative household's problem.
- Given the sequence of prices and government policies, the firm's allocation maximizes profit.
- Market clearing:[11]

$$\begin{aligned} c_t + g_t &= F(n_t^F, n_t^I) \\ n_t^{s,j} &= n_t^{d,j} \text{ for } j = F, I \\ M_t &= M_t^g \\ B_t + B_t^g &= 0 \end{aligned} \tag{13}$$

3 The Ramsey Problem

Assume that the government can finance its expenditure only by levying a distortionary tax on labor income, by issuing bonds or by printing money.[12] As a consequence, the second best tax policy is found by choosing the allocation that maximizes the representative agent's welfare subject to the resource and implementability constraints. Formally, choose $\{c_t, h_t, m_t, n_t^F\}_{t=0}^{\infty}$ to

$$\max U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{14}$$

subject to

$$c_t + g_t \leq F(n_t^F, n_t^I) \tag{15}$$

and the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t - u_h(t)(1 - h_t) + u_h(t)l(t)(1 - k)] = 0 \tag{16}$$

[10] Where π_t is the rate of money creation.

[11] Walras' Law assures that the government's budget constraint is satisfied.

[12] Our results are robust to the introduction of a consumption tax as long as this tax is incomplete (i.e., the consumption tax can be levied on the formal sector but not on the informal sector). We consider this case in detail at the end of the section.

The first constraint is the standard resource constraint. The second constraint is the implementability constraint, which is a restriction that must be satisfied in order for an allocation from the centralized problem to be implemented as a competitive equilibrium. This implementability constraint eliminates all prices and taxes in the representative agent's present value budget constraint (equation (3)) to obtain quantities consistent with optimal firm and household behavior. Lucas and Stokey (1983) note that (14) and (15) provide a complete description of the set of *competitive* allocations that can be attained through feasible government policies. When the government cannot monitor transactions in the informal sector, an additional restriction is required to *implement* market allocations. To see this, maximize this second best problem with respect to c_t and h_t.[13] It follows that

$$\frac{v_h(t)}{v_c(t) - v_h(t)l_c(t)} = F_n^I(t) \tag{17}$$

where

$$v_h(t) = u_h(t) + \psi\{-u_{hh}(t)[(1-h_t) - l(t)(1-k)] + u_h(t)\}$$
$$v_c(t) = u_c(t) + \psi[u_{cc}(t)c_t + u_c(t) + u_h(t)l_c(t)(1-k)]$$

Note that ψ is the Lagrange multiplier on the implementability constraint. If this constraint does not bind ($\psi = 0$), then (17) becomes

$$\frac{u_h(t)}{u_c(t) - u_h(t)l_c(t)} = F_n^I(t) \tag{18}$$

This is the same condition found in the competitive equilibrium. This implies that when the implementability constraint binds ($\psi > 0$), the solution to the second best problem is not the solution to the competitive equilibrium. Since the Ramsey exercise characterizes tax patterns consistent with a competitive equilibrium, (18) must be imposed as an additional constraint. The main theoretical result follows directly from the Ramsey problem.

3.1 The Ramsey Result

The Ramsey problem in our economy is to choose $\{c_t, h_t, m_t, n_t^F\}_{t=0}^{\infty}$ to maximize the welfare of the representative agent subject to equation (18), the implementability constraint and the resource constraint.

[13]For simplicity, assume that the utility function is strongly separable between c and h. This assumption is not essential, but it simplifies the analysis.

Proposition 1: Assume that contracts cannot be monitored by the government in the informal sector and assumptions L1 and L2 are satisfied. Then the Friedman Rule is not the optimal monetary policy when the transaction cost function is homogeneous of degree one or greater.

Proof: By maximizing the Ramsey problem with respect to m_t it follows that

$$-l_m(t)[\theta_t F_n^I(t) - \psi u_h(t)(1-k) - \gamma_t F_{nn}^I(t)] = \frac{-\gamma_t u_h(t)^2 l_{cm}(t)}{[u_c(t) - u_h(t) l_c(t)]^2} \quad (19)$$

where ψ, θ_t and γ_t are the Lagrange multipliers on the implementability constraint, the resource constraint and constraint (18). Given the assumptions on the utility and production functions it is easy to verify that the term in brackets on the left hand side is positive for any $k \geq 1$. Since (18) must hold in any competitive equilibrium, γ_t is always strictly positive. In addition, $l_{cm}(t)$ is negative by assumption.[14] Thus the right hand side is positive, which implies that $-l_m(t) > 0$. This solution can be decentralized through a positive nominal interest rate (see equation (10)). As a consequence, the Friedman Rule is not optimal for any transaction technology with a finite "satiation level" that is homogeneous of degree one or greater.

This result does not hold for homogeneous functions with a "satiation level" of real money balances that is not finite.[15] In this case, the bliss point of real money balances is independent of the consumption level. Thus, assumptions L1 and L2 are essential for proving *Proposition 1.* L1 imposes directly that for each consumption level there is a finite satiation level of real money balances. This standard "Friedman assumption" implies that after some finite level of money holdings $m*$, the return from holding an extra dollar in terms of decreased shopping time is zero. L2 implies that money demand increases with consumption at a decreasing rate. It is not possible to ensure that *Proposition 1* holds when the transaction cost technology is homogeneous of degree less than one because the sign of the term in brackets on the left hand side of (19) cannot be determined.[16]

[14] When the transaction function is homogeneous of degree one, $l_{cm} < 0$ follows from $l_{mm} > 0$.

[15] The reason is that when $l(.,.)$ is a homogeneous function and $l_m(c,m) = 0$ this defines $m^\star = \infty$, then $l_{cm}(c, m^\star) = 0$.

[16] By assumption $l_{cm} < 0$. Thus, the right hand side of (19) is positive. This implies that $l_m = 0$ cannot satisfy (19). Since a utility maximizing agent will not hold additional cash beyond the "satiation level" of real balances, $l_m < 0$, thus $-l_m > 0$. In this case

Proposition 1 stands in contrast with Correia and Teles (1996) who show that, in the absence of an informal sector, the Friedman Rule is the optimal solution in monetary models with any type of homogeneous transaction cost function.[17] The intuition that motivates their result is that money is a free primary input (i.e., its production cost is negligible) and the inflation tax is a unit tax. At the point of satiation in real balances the marginal benefit of an extra dollar is zero. Moreover, Correia and Teles (1996) show that under certain assumptions the marginal impact of an additional dollar of government revenue is also zero, at that satiation level. This explains why the Friedman Rule is optimal in their model.

In our economy the government can reduce the distortion caused by the informal sector by setting a positive nominal interest rate. The reason is that a positive nominal interest rate, which leads to a positive inflation tax, acts as a consumption tax. The inflation tax affects the traded consumption good in both sectors, regardless of the sector in which it is produced. Thus the government tries to mitigate the distortion arising from labor tax evasion in the informal sector by imposing an inflation tax.

3.2 The Role of Consumption Taxes

We briefly consider the role of consumption taxes. We assumed at the outset that consumption taxes were not available. Assume now, in order to clarify our previous result, that the government can tax consumption at a uniform rate $\tau_c = \tau_c^F = \tau_c^I > 0$. For simplicity, assume that the transaction cost technology is not affected by the consumption tax. As in the previous analysis, the Ramsey problem is to choose $\{c_t, h_t, m_t, n_t^F\}_{t=0}^{\infty}$ to maximize the welfare of the representative agent (14), subject to the resource constraint (15), and the implementability condition (16).[18] When a uniform consumption tax is available, equations (15) and (16) describe completely the set of

(19) will hold only if the term in brackets on the left is positive. This solution can be decentralized through a positive nominal interest rate.

[17] In the absence of an informal sector, (19) is $-l_m(t)[\theta_t F_n^F(t) - \psi u_h(t)(1-k)] = 0$. This is the same equation found by Correia and Teles (1996). It can be shown that the term in brackets is different from zero, which implies that $-l_m(t) = 0$. This solution can be decentralized through a zero nominal interest rate. Thus the Friedman Rule is the optimal monetary policy for any homogeneous transaction function in the absence of an informal sector. Notice that this result does not depend on the term l_{cm}. This is why it was necessary to characterize this term to establish *Proposition 1.*

[18] Notice that in this case we do not need equation (18) to implement Ramsey allocations.

all competitive allocations that can be attained through government policies.

The solution of this problem with respect to m_t is

$$-l_m(t)[\theta_t F_n^I(t) - \psi(1-k)u_h(t)] = 0 \tag{20}$$

The term in brackets is positive for any degree of homogeneity of the transaction function,[19] which in turn implies that l_m must be zero. This Ramsey solution can be decentralized through a zero nominal interest rate, implying no inflation tax when a uniform consumption tax is possible. This result verifies the claim that when the government cannot levy a complete tax on consumption, because an informal sector is present, the inflation tax serves as an imperfect proxy. That is, inflation proxies for the government's inability to tax consumption in the informal sector.[20]

This consumption tax result indicates that in principle, the government can replace the inflation tax by a uniform tax on consumption, τ_c. In practice, we believe that it is difficult for governments to monitor transactions of final goods in the informal sector, for the same reasons that the government cannot monitor labor contracts in the informal sector. In our economy, there is only one consumption good. Thus, we assumed at the outset that the government cannot tax consumption. Instead we could have considered an economy with two goods c^F and c^I, preferences $u(c)$ with $c = [\mu(c^F)^\rho + (1-\mu)(c^I)^\rho]^{\frac{1}{\rho}}$, and assumed that the government can levy tax $\tau_c^F > 0$ on the good in the formal sector, but not on the good in the informal sector. The important part of both the single and two consumption good specifications is that taxation of consumption is not possible in the informal sector, $\tau_c^I = 0$. Thus the Diamond and Mirrlees (1971) conditions for production efficiency

[19] By maximizing this Ramsey problem with respect to h_t we can show that

$$\theta_t F_n^I(t) = u_h(t) + \psi[u_h(t) - u_{hh}(t)(1 - h_t - (1-k)l(t))]$$

Substituting this result into (19) yields:

$$-l_m(t)\{u_h(t) + \psi[ku_h(t) - u_{hh}(t)(1 - h_t - (1-k)l(t))]\} = 0$$

Given the assumptions on utility, it is easy to verify that the term in brackets is positive.

[20] The derivation of (19) uses the assumption that the consumption tax does not affect the transaction cost function. In practice, the amount of time spent shopping depends on the level of consumption expenditure (inclusive of consumption taxes) i.e., $l((1+\tau_c)c_t, m)$ instead of $l(c, m)$. In this case, Guidotti and Végh (1993) show that even without an informal sector, the government should resort to inflationary finance.

are not met. Effectively, money is an intermediate good in the model. As a consequence, it is optimal to tax it when an economy has an informal sector.

4 Quantitative Experiments

In our economy inflation serves as a proxy for the inability to tax directly labor and consumption in the informal sector. In an economy without distortions, Lucas (1994) has argued that the cost of reducing moderate inflation of 4 to 5% to the rate prescribed by the Friedman Rule could be substantial: 1 to 3% of gdp in the U.S. These results were calculated under the assumption that the lost revenues would be replaced by revenues from a lump sum tax.[21] Is this policy advice appropriate for economies with labor and goods market distortions of the type we have considered?

Table 1 shows labor force participation rates in the informal sector for eleven countries. Clearly, the informal sector is significant in many developing countries (see Schneider and Enste (2000) for additional data).

Table 1: Informal Labor Force in Urban Areas: Selected Countries

Country	Year	Percentage of Total Employment		
		Total	Men	Women
Bolivia	1996	57%	53%	62%
Brazil	1996	30%	27%	33%
Chile	1997	30%	32%	27%
Colombia	1996	53%	54%	53%
Ecuador	1997	40%	39%	42%
Gambia	1993	72%	66%	83%
Mexico	1996	35%	36%	34%
Peru	1996	51%	50%	52%
Philippines	1995	17%	16%	19%
South Africa	1995	17%	11%	26%
Uganda	1993	84%	68%	81%

Source: ILO (1999).

Figure 1 shows a positive relationship between the inflation rate and the

[21] Lucas (2000) claims that these results are robust to certain labor supply specifications.

informal labor force for the countries in Table 1.[22]

Insert **Figure 1** here

4.1 Commodity and Labor Market Distortions

The purpose of our quantitative analysis is to provide a numerical assessment of the welfare costs associated with informal labor and commodity market distortions. We calculate the welfare effects of alternative levels of inflation in the "distorted" and the "undistorted" economies. That is, we obtain a benchmark number for the welfare gains from reducing inflation in each economy. Implicitly, this provides a rough measure of the increase in aggregate welfare that would accrue from structural reform. The quantitative experiments require us to calibrate the theoretical model. Thus, we must determine functional forms for the (i) transaction technology; (ii) preferences; (iii) production technology; and (iv) government policy.

For the shopping time transaction technology, we follow Mulligan and Sala-i-Martin (1997) and assume the following parametric form:

$$l(c,m) = cL(\frac{c}{m}) + \gamma c \tag{21}$$

where $L(z) = A\frac{(z-\bar{z})^2}{z}$ is defined over $z \geq \bar{z}$. This functional form satisfies assumptions L1 and L2. Mulligan and Sala-i-Martin (1997) note that it implies that the interest elasticity of money demand approaches zero as the nominal interest rate approaches zero in a way that conforms closely to empirical evidence. This transaction cost technology was calibrated such that, at a 4% inflation rate, shopping time as a fraction of GNP was 0.02 and the interest elasticity was 0.45.

Preferences are represented by the following CES utility function:

$$u(c,h) = \frac{\sigma}{\sigma-1}[c^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}] \tag{22}$$

The parameters σ and α are chosen so that one-third of the time endowment is spent in market activities. We also assume that households discount future utility at the rate of 2 percent per year, which implies that $\beta = 0.98$.

[22]Inflation was calculated by averaging the inflation rate in each country for the period from 1995 to 1999.

Recall that the production technology is represented by

$$F(N^F, N^I) = [\lambda(N^F)^\rho + (1-\lambda)(N^I)^\rho]^{\frac{1}{\rho}} \tag{23}$$

where $0 < \lambda < 1$ and $0 < \rho < 1$. The parameters λ and ρ pin down the "size" and the "type" of informality in the economy. λ greater that 0.5 implies that workers employed with formal contracts are more productive than workers employed informally. In developed countries informality is associated with small scale activities, such as baby sitting and house repairs, and λ is close to 1. In developing countries informal jobs are present in almost all economic sectors, and λ is closer to 0.5. The parameter ρ measures the elasticity of substitution between the informal and formal sectors. When ρ is close to 1, this implies that firms can freely substitute workers. We interpret this as a proxy for weak enforcement of firm labor market regulations, a structural problem in many developing countries. In contrast, when ρ is less than one it is costly for firms in terms of foregone production to substitute workers in the formal sector with those from the informal sector. We interpret this as a proxy for strong enforcement.

The government policy parameters are calibrated so that government purchases are between 16% and 23% of GNP, which is close to the U.S. data. The lower bound corresponds to an economy with a "big" informal sector and the upper bound to an economy with a "small" informal sector.

We summarize the baseline economies in Table 2.

- α and σ are standard calibrations of preference parameters in (22)
- γ, A, and $\bar{z}$ are standard calibrations of money demand parameters in (21) (cf., Mulligan and Sala-i-Martin)
- π is a baseline inflation rate of 4% standard in the literature
- τ_n^F is the labor income tax in the formal sector, varied from 22 to 26%
- The "informality parameters" are λ and ρ. Measures of ρ, the elasticity of substitution by firms between labor in the two sectors, are obtained from labor market data.[23] The parameter λ, which is a key determinant

[23]For example, Lemieux, Fortin and Frechette (1994) estimate $\rho = 0.714$ for Canada. Given Canadian wage and tax data, and the estimate by Schneider and Enste (2000) that $N^I/N^F + N^I$ is 11% in Canada, (24) implies that $\lambda = 0.69$ These magnitudes are roughly consistent with Economy (1) in Table 2 where $\rho = .75$, $\lambda = .7$.

of worker productivity, follows from the first order condition:

$$\frac{N^I}{N^F} = [\frac{1-\lambda}{\lambda}\frac{1}{1-\tau_n^F}]^{\frac{1}{1-\rho}} \tag{24}$$

Table 2: Baseline Economies

Econ-mies	α	σ	γ	A	$\bar{z}$	λ	ρ	π	τ_n^F
(1)	1	1.7	0.01	0.009	1	0.7	0.75	4%	0.26
(2)	1	1.7	0.01	0.013	1	0.61	0.8	4%	0.25
(3)	1	1.7	0.01	0.019	1	0.577	0.85	4%	0.23

Table 3 summarizes selected statistics for these 3 benchmark economies. Note that transaction costs increase with inflation, which implies that inflation causes agents to devote productive time to activities that enable them to economize on cash balances.[24] The first column in Table 3 varies the left hand side of (24). That is, values of the parameters λ and ρ, which determine the level of informality, are chosen such that $\frac{N^I}{N^F+N^I}$ is 9%, 30% and 42%. Columns two, three and four correspond to the parameters in Mulligan and Sala-i-Martin (1997). Finally, columns five and six show that the optimal inflation and nominal interest rates are quite different from the Friedman policy recommendation. In an economy with a "small" informal sector, the optimal monetary policy, i=1.02%, is close to the Friedman Rule, $i = 0\%$. However, in an economy with a large informal sector, the optimal inflation and interest rate ($i = 16.30\%$) are significantly higher.

Figures 2a, 2b and 2c summarize the behavior of the economy with low, medium and high structural distortion. Observe that GNP is a concave function of inflation when structural problems are low, but is lower and "flattens out" when structural problems are high. Further, money holdings are qualitatively similar in all three economies. This suggests that substitution in the real side of the economy is important (i.e., between N^F and N^I to evade τ_n^F).

[24]This result was also found by Cooley and Hansen (1989, 1991), Dotsey and Ireland (1996), Lucas (2000), and Yoshimo (1993). They show that inflation and employment in banking have been positively correlated over time in the U.S. and other countries.

Table 3: Quantitative Results

Economies	Informal sector	Shop. time/GNP	Inter. elastic.	g/GNP	Optimal Infl. rate	Optimal Int. rate
(1)	9.23%	0.023	0.463	0.23	-0.01%	1.02%
(2)	30%	0.026	0.46	0.2	6%	8.1%
(3)	42%	0.026	0.44	0.16	14%	16.30%

Insert **Figures 2a, 2b and 2c** here

Figures 2a, 2b and 2c indicate:

(1) Leisure is a convex function of π when structural problems are low, but "flattens out" when structural problems are high.
(2) The level of N^I is higher when structural problems are high, as indicated by equation (24), but Figure 2 indicates that N^I decreases as π rises (given structural problems).
(3) The level of N^F decreases as π increases *and* the function changes its shape from a concave to a positive function of π.
(4) The increase in the level of N^I and the decrease in the level of N^F cause the level $N^I/N^I + N^F$ to rise for every level of π when structural problems increase. Further, the function becomes flatter due to the changes in the shape of N^F noted above.

In summary, Figures 2a, 2b and 2c and observations (1)-(4) indicate that our optimal taxation results are driven by substitution on the real side of the economy. (1) indicates that when distortions are high, leisure "flattens out." As a consequence, substitution occurs between the two sectors. N^I decreases with π for all levels of distortion. Interestingly, N^F is a concave function of π when distortions are low, but increases with π when distortions are high. The change in the shape of this function occurs for the following reasons:

- When structural distortions are low, it is optimal to keep π low and tax N^F directly via τ_n^F. However, if π were to become high, the agent would reduce N^F. The concavity of formal labor contracts as a function of π in this case is directly reflected in the sharp concavity of GNP as a function of π and in the sharp convexity of the welfare cost function.[25]

[25] The welfare cost of inflation is discussed in more detail below.

- When structural distortions are high, the optimal inflation rate is positive and the relative inefficiencies of the two imperfect taxes π and τ_n^F must be balanced. Observe that h, N^F and N^I all "flatten out" as π increases. This causes both the GNP and the welfare cost functions to become flat. Since N^I and N^F are now both taxed, the agent can no longer evade taxes by substituting between N^I and N^F.

The welfare gains from adopting the optimal inflation tax policy are computed as follows. Compute the welfare benefit of changing the inflation rate from the baseline rate of 4% to the optimal value. Figure 3 shows how the welfare cost of inflation changes with the inflation rate for these 3 baseline economies. As in Lucas (1994, 2000), welfare is expressed as a percentage increase or decrease relative to consumption under baseline parameters. Given a baseline economy with inflation rate π_b, the welfare cost of inflation is calculated by finding the value of $w(\pi_j)$ that satisfies:

$$u(c(\pi_j)(1 + w(\pi_j)), h(\pi_j)) - u(c(\pi_b), h(\pi_b)) = 0 \tag{25}$$

We calculate the steady-state of the economy for different values of the inflation rate, holding government purchases fixed. The government's budget constraint is balanced by adjusting the labor tax rate, as in Braun (1994), Mulligan and Sala-i-Martin (1997) and Lucas (2000).

Insert **Figure 3** here

Figure 3 has some notable features. In economy (1), welfare has the same shape as in Braun (1994) and Lucas (2000). As inflation falls from its optimal level, the welfare cost of inflation increases sharply. However, when inflation is above its optimal rate, the welfare cost increases only gradually. The shape of the welfare function is very important for policy purposes, particularly for a country considering a low inflation target. Figure 3 shows that the welfare cost of implementing the Friedman Rule (loosely a zero inflation rate) would be substantial for economies with serious structural problems. The optimal inflation rate is positive when structural problems are severe, and it is costly to drive inflation below its optimal level. Further, severe structural distortions "flattens out" the welfare cost function, as discussed after Figure 2. As a consequence, the cost of having inflation above the optimal level is small for economies with big structural problems.

Figure 3 indicates that the welfare gain of changing inflation from its baseline value of 4% to the optimal rate varies with the level of structural problems. The welfare gain from adopting the optimal inflation policy is 1.47, 0.6 and 0.05 percent of baseline consumption for economies with "low," "moderate" and "high" structural problems, respectively. Although these welfare gains are not substantial, the welfare cost of adopting high inflation policies is also not quantitatively significant for economies with structural problems. For instance, for an economy with a big informal sector, the welfare gain from reducing inflation from 25% to its optimal rate of 14% is only 0.21 percent of baseline consumption.

Finally, Figure 4 indicates how structural problems affect government revenue. The first figure indicates that high structural problems cause the inflation revenue function to shift up somewhat and become flatter. However, the labor revenue function shifts down significantly. As a consequence, the total revenue function shifts down as structural problems increase. Figure 4 provides further evidence that structural distortions are important because they provide an additional margin on which agents can substitute in order to evade taxes.

Insert **Figure 4** here

We summarize our findings for economies with limitations on the government's ability to tax labor and commodity markets: (1) The welfare cost of inflation increases sharply when inflation falls from its optimal level; (2) the welfare cost associated with high inflation is small for economies with structural problems that impede uniform taxation, but substantial for economies without such structural problems; and (3) structural distortions provide additional margins on which agents can substitute in order to evade taxes. These quantitative exercises are, therefore, an alternative explanation for the persistence of high inflation in economies with structural problems of the type we describe.

4.2 Currency Market Distortions

From the preceding analysis it is clear that, depending on the size of the informal sector, the optimal inflation policy requires not only analytical qualifications, but also raises important quantitative considerations. We now consider Currency Substitution (CS), a market distortion that is present in many developing countries. CS describes the replacement of domestic currency, that loses value in the presence of inflation, by a stronger and more reliable foreign money (frequently the U.S. dollar).[26] According to Giovannini and Turtelboom (1994), this phenomenon can be seen as the Gresham's Law in reverse, since the "good" currency drives out the "bad."

In many developing countries people hold U.S. dollars or another foreign money even when foreign currency is not legally acceptable.[27] In doing so consumers insure their wealth against the instability of the domestic currency and they evade the inflation tax imposed by the home government.[28] CS is an informal arrangement in transaction activities that has the same "tax evasion" effect as informal labor contracts. Informal contracts decrease the tax base from which labor income revenue can be raised, and the presence of foreign currency decreases the monetary base from which domestic seigniorage can be raised by printing money. The higher the elasticity of substitution between domestic and foreign money, the more difficult it is for the government to finance deficits by an inflation tax.

In this section we evaluate the effects of CS on our economic model. Assume now that agents can transact with both domestic currency, M, and foreign money, F. Modify the transaction technology as follows. Let

$$s_t \geq l(c_t, \frac{M_t}{P_t}, \frac{\xi_t F_t}{P_t}) \tag{26}$$

where ξ_t represents the exchange rate. Since prices in the model are flexible, Purchasing Power Parity (PPP) holds, i.e., $P_t = \xi_t P_t^*$, where P_t^* denotes the foreign currency price of the consumption good. Using the PPP equation in the transaction technology, it follows that $s_t \geq l(c_t, m_t, f_t)$, with $m_t = M_t/P_t$ and $f_t = F_t/P_t^*$.

The Appendix shows that CS does not change *Proposition 1*. As a consequence, the Friedman Rule is not the optimal monetary policy when labor markets are incomplete and both domestic and foreign money circulate as

[26]Porter and Judson (1996) estimate that 55 to 70% of U.S. dollars are held abroad, mainly in $100 bills.

[27]According to the JEC (2000) staff report, in developing economies where foreign money is present, wages, taxes and every day expenses such as groceries and electric bills are paid in domestic currency, but expensive items such as automobiles and houses are often paid in foreign currency.

[28]They, however, pay the seigniorage tax on the foreign currency.

media of exchange.[29] Although qualitatively CS does not change the main results, it is important to verify how it affects the quantitative experiments. Thus, let the shopping time technology take the following form:[30]

$$l(c, m, f) = cL(\frac{c}{m}, \frac{c}{f}) + \gamma c \tag{27}$$

where $L(z_1, z_2) = A((z_1 + \phi z_2) - \bar{z})^2/(z_1 + \phi z_2)$. Under this specification, the demand for foreign money relative to domestic currency is given by

$$\frac{f_t}{m_t} = (\phi \frac{I_t}{I_t^*})^{1/2} \text{ with } I_t = \frac{i_t}{1+i_t} \text{ and } I_t^* = 1 - \frac{1}{1+i_t}\frac{P_{t+1}}{P_t}\frac{P_t^*}{P_{t+1}^*} \tag{28}$$

If inflation in domestic currency increases, I_t^* decreases and agents replace domestic currency by foreign money. This is consistent with CS.

The parameter ϕ is the key determinant of the elasticity of substitution between domestic and foreign money.[31] We calibrate ϕ so that the demand for foreign currency relative to domestic is consistent with empirical evidence. According to the JEC (2000) report, countries are classified as "highly dollarized" if foreign currency deposits exceed 30% of a broad measure of the money supply. Let ϕ take the following values: 0, 0.01, 0.1 and 0.3.[32] Notice that when $\phi = 0.3$ and home and foreign inflation rates[33] are the same, 35.4% of total money holdings are in foreign currency.

Figures 5a and 5b show the effects of CS on the welfare cost of inflation for economies with low and high structural problems, respectively. In both economies there is a trade off between the distortion introduced by informal labor contracts and by CS. As before, the welfare cost increases sharply when inflation falls from its optimal rate and it increases only gradually when inflation is above its optimal level. Notice in Figure 5a that for an economy with low structural distortion: (i) the optimal inflation rate increases as the degree of CS increases; and (ii) the welfare gains of adopting the optimal policy decrease with CS. On the other hand, Figure 5b shows that for an economy with high structural problems: (i) the optimal inflation rate decreases with CS; and (ii) the welfare gains of adopting the optimal inflation policy also

[29]Végh (1989) shows in a similar model, but abstracting from the informal sector, that the Friedman Rule is not the optimal monetary policy in the presence of CS.

[30]We keep the previous forms for preferences, production and government policy.

[31]Notice that $\phi = 0$ yields an economy without the CS phenomenon.

[32]Countries with $\phi = 0.01$ correspond to low CS, and high CS corresponds to $\phi = 0.3$.

[33]We assume an inflation rate of 4% in foreign currency.

decrease with CS. CS leads the optimal inflation policy to be closer to the foreign inflation rate and it decreases the welfare gains from adopting the optimal inflation policy.

Insert **Figures 5a and 5b** here

Although CS decreases the optimal inflation rate in economies with structural problems, these quantitative exercises reinforce the idea that the welfare benefits from reducing inflation from high levels, such as 25%, to an optimal level of 10% is very small in such economies. In addition, reducing inflation further leads to significant welfare losses, as Figures 5a and 5b show.

5 Concluding Remarks

Proposition 1 shows that under some reasonable conditions the Friedman Rule is not optimal in an economy with structural problems in labor, goods, and financial markets. Since the inflation tax acts as a uniform consumption tax, it enables the government to reduce the distortion introduced in the economy by these imperfections. We also investigated whether the optimal inflation rate is quantitatively different from the Friedman policy. We found that it is, and calculated the welfare gains from adopting the optimal policy. In addition, we found that the welfare effect is highly asymmetric when structural problems are severe. The informal sector thus provides an alternative explanation for positive inflation rates, especially in developing economies.

Our results indicate that if developing economies wish to adopt a low inflation policy they must also improve their institutional framework for enforcing formal contracts and collecting tax revenue. In this case the tax base and welfare will increase, and taxation via inflation will be unnecessary. Simply put, the structural distortions introduce additional margins on which agents can substitute in order to evade taxes. The quantitative results indicate that these standard microeconomic tax evasion activities can have important implications for optimal monetary policy.

References

Akyol, A. (2000). Optimal Monetary Policy in an Economy with Incomplete Markets and Idiosyncratic Risk. University of Iowa.

Braun, R. Anton (1994). Another Attempt to Quantify the Benefits of Reducing Inflation. *Federal Reserve Bank of Minneapolis, Quarterly Review* 18, Fall, 17-25.

Bryant, J. and Wallace, N. (1984). A Price Discrimination Analysis of Monetary Policy. *Review of Economic Studies* 51, 279-288.

Chari, V., L. J. Christiano, P. Kehoe (1996). Optimality of the Friedman Rule in Economies with Distorting Taxes. *Journal of Monetary Economics* 37, 203-223.

Chari, V. and P. Kehoe (2000). Optimal Fiscal and Monetary Policy. In: *Handbook of Macroeconomics* vol. 1c, Elsevier.

Cooley, T. and Hansen, G. (1989). The Inflation Tax in a Real Business Cycle Model. *American Economic Review* 79, 733-748.

Cooley, T. and Hansen, G. (1991). The Welfare Cost of Moderate Inflation. *Journal of Money, Credit and Banking*23, 483-503.

Correia, Isabel and Pedro Teles (1996). Is the Friedman Rule Optimal when Money is an Intermediate Good? *Journal of Monetary Economics* 38, 223-244.

Diamond, Peter A. and James A. Mirrlees (1971). Optimal Taxation and Public Production. *American Economic Review* 61, 8-27, 261-268.

Dotsey, M. and Ireland, P. (1996). The Welfare Cost of Inflation in General Equilibrium. *Journal of Monetary Economics* 37, 29-47.

Erosa, A. and Ventura, G. (2000). Inflation, Heterogeneity and Costly Credit: How Regressive is the Inflation Tax? University of Western Ontario.

Faig, Miguel (1988). Characterization of the Optimal Tax on Money when it Functions as a Medium of Exchange, *Journal of Monetary Economics* 22, 137-148.

Friedman, Milton (1969). The Optimum Quantity of Money. In: M. Friedman, ed.: *The Quantity of Money and Other Essays* (Aldine, Chicago, IL).

Guidotti, Pablo and Carlos Végh (1993). The Optimal Inflation Tax when Money Reduces Transaction Costs. *Journal of Monetary Economics* 31, 189-205.

Huggett, M. (1993). The Risk Free Rate in Heterogeneous-Agent Incomplete Insurance Economies. *Journal of Economic Dynamics and Control* 17(5), 953-969.

Kimbrough, K. (1986). The Optimum Quantity of Money Rule in the Theory of Public Finance, *Journal of Monetary Economics* 18, 277-284.

Joint Economic Committee (JEC) (2000). Basics of Dollarization. *U.S. Congress, JEC, Office of the Chairman, Staff Report*, Jan. Transcript available online at http://www.senate.gov/~jec.

Lemieux, T., Fortin, B., and Frechette, P. (1994). The Effect of Taxes on Labor Supply in the Underground Economy. *American Economic Review* 84(1), 231-254.

Ljungqvist, L. and Sargent, T. S. (2000). *Recursive Macroeconomic Theory*, MIT Press.

Lucas, Robert E., Jr. (1990). Supply-side Economics: an Analytical Review. *Oxford Economic Papers* 42, 293-316.

Lucas, Robert E., Jr. (1994). On the Welfare Cost of Inflation. *CEPR Publication 394*. Center for Economic Policy Research, Stanford University.

Lucas, Robert E., Jr. (2000). Inflation and Welfare. *Econometrica* 68(2), 247-274.

Lucas, Robert E., Jr. and Nancy Stokey (1983). Optimal Fiscal and Monetary Theory in World without Capital. *Journal of Monetary Economics* 12, 55-93.

Mulligan, Casey B. and Xavier Sala-I-Martin (1997). The Optimum Quantity of Money: Theory and Evidence. *Journal of Money, Credit, and Banking* 29, 687-724.

Phelps, Edmund (1973). Inflation in the Theory of Public Finance. *Swedish Journal of Economics* 75, 37-54.

Porter, Richard and Ruth Judson (1996). The Location of U.S. Currency: How Much Is Abroad? *Federal Reserve Bulletin*, v. 82, no. 10, October, pp. 883-903.

Ramsey, Frank (1927). A Contribution to the Theory of Taxation. *Economic Journal* 37, 47-61.

Sargent, T. and Wallace, N. (1981). Some Unpleasant Monetarist Arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review* 5(3), 1-17.

Schneider, F. and Enste, D. (2000). Shadow Economies: Size, Cause and Consequences. *Journal of Economic Literature* XXXVIII, 77-114.

Smith, B. and Villamil A. (1998). Government Borrowing using Bonds with Randomly Determined Returns: Welfare Improving Randomization in the Context of Deficit Finance. *Journal of Monetary Economics* 41, 351-370.

Stone, A. Brian Levy and Ricardo Paredes (1996). Public Institutions and Private Transaction: a Comparative Analysis of the Legal and Regulatory Environment for Business Transactions in Brazil and Chile. In: L. Alston, T. Eggertsson and D. North, Eds.: *Empirical Studies in Institutional Change*, Cambridge University Press.

Végh, Carlos (1988). The Optimal Inflation Tax in the Presence of Currency Substitution. *Journal of Monetary Economics* 24, 139-146.

Villamil, A. (1988). Price Discriminating Monetary Policy: a Non-uniforming Pricing Approach. *Journal of Public Economics* 51, 378-386.

6 Appendix

We consider two alternative models of money to show that our results are robust to the specification of money demand. We then show that Proposition 1 is robust to currency substitution.

6.1 CIA Money Demand

Consider a production economy similar to the one in section 2. In each period there are two consumption goods: a cash good and a production good. The representative household is endowed with one unit of time that can be used as leisure, h_t, or allocated to production in the formal sector, n^F, or the informal sector, n^I. Preferences are:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, h_t) \tag{29}$$

where c_{1t} and c_{2t} are the cash and the credit good, respectively. The one-period budget constraint is given by

$$c_{1t} + c_{2t} + \frac{M_t}{P_t} + \frac{B_t}{P_t} \leq \frac{M_{t-1}}{P_t} + (1 + i_{t-1})\frac{B_{t-1}}{P_t} + (1 - \tau_n^F) w_t^F n_t^F + w_t^I (1 - n_t^F - h_t) \tag{30}$$

In present value form the budget constraint, with no-Ponzi game conditions for bonds and money, and with initial conditions, $B_{-1} = M_{-1} = 0$ is

$$\sum_{t=0}^{\infty} d_t c_{1t} + \sum_{t=0}^{\infty} d_t c_{2t} + \sum_{t=0}^{\infty} d_t I_t m_t \leq \sum_{t=0}^{\infty} d_t [(1 - \tau_n^F) w_t^F n_t^F + w_t^I (1 - n_t^F - h_t)] \tag{31}$$

Assume that the purchase of cash goods must satisfy the following cash-in-advance (CIA) constraint:

$$c_{1t} \leq m_t \tag{32}$$

The production function is as before and output can now be used for private consumption of either the cash good, c_{1t}, the credit good, c_{2t}, or for government consumption, g_t. The resource constraint is given by

$$c_{1t} + c_{2t} + g_t \leq F(N_t^F, N_t^I) \tag{33}$$

The household's equilibrium conditions are

$$\frac{u_h(t)}{u_{c2}(t)} = w_t^I \tag{34}$$

$$w_t^I = (1-\tau_n^F)w_t^F \tag{35}$$

$$\frac{u_h(t)}{u_{c1}(t)} = \frac{(1-\tau_n^F)w_t^F}{(1+I_t)} \tag{36}$$

$$\frac{u_{c2}(t)}{u_{c2}(t+1)} = \beta(1+r_t) \tag{37}$$

$$\frac{u_{c1}(t)}{u_{c2}(t)} = (1+I_t) \tag{38}$$

Notice that (38) implies that $u_{c1}(t) \geq u_{c2}(t)$.

Chari, Christiano and Kehoe (CCK) (1996) show that, in the absence of an informal sector and under the assumption that the utility function $u(c_1, c_2, h) = V(w(c_1, c_2), h)$ where w is a homothetic function,[34] the optimal Friedman rule is the Ramsey equilibrium.

Now consider the Ramsey equilibrium with informal labor contracts. The Ramsey problem is now to choose allocations that maximize utility subject to the CIA constraint, the resource constraint, the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t [u_{c1}(t)c_{1t} + u_{c2}(t)c_{2t} - u_h(t)(1-h_t)] = 0 \tag{39}$$

and the following market equilibrium conditions:

$$\frac{u_h(t)}{u_{c2}(t)} = F_{n^F}(t) \tag{40}$$

$$u_{c1}(t) \geq u_{c2}(t) \tag{41}$$

Proposition A1: Assume that contracts cannot be monitored by the government in the informal sector and the utility function is the same as in CCK (1996). Then the Friedman Rule is not necessarily the optimal monetary policy.

Proof: The utility function satisfies the following property:[35]

$$\sum_{j=1}^{2} u_{cj,c1}(t)cj/u_{c1}(t) = \sum_{j=1}^{2} u_{cj,c2}(t)cj/u_{c2}(t) \tag{42}$$

[34] In order to simplify the algebra, assume further that $u(c_1, c_2, h) = V(w(c_1, c_2)) + u(h)$.

[35] See CCK (1996).

Maximize the Ramsey problem with respect to c_1 and c_2 and use (42) to show that

$$\frac{\theta_t}{u_{c1}(t)} - \frac{\theta_t}{u_{c2}(t)} = \frac{u_h(t)u_{c2c2}(t)\gamma_t}{u_{c2}(t)^2 u_{c2}(t)} \tag{43}$$

where θ_t and γ_t are the Lagrange multipliers on the resource constraint and market equilibrium condition (40), respectively.[36] After rearranging this equation, it follows that

$$\frac{u_{c2}(t)}{u_{c1}(t)} = 1 + \frac{u_h(t)u_{c2c2}(t)\gamma_t}{u_{c2}(t)^2\theta_t} \tag{44}$$

Notice that the RHS is not equal to 1 as in CCK (1996). There are two cases:

a) If $|\frac{u_h(t)u_{c2c2}(t)\gamma_t}{u_{c2}(t)^2\theta_t}| < 1$, then $u_{c1}(t) > u_{c2}(t)$ and this solution can be decentralized by a positive nominal interest rate. See equation (38);

b) If $|\frac{u_h(t)u_{c2c2}(t)\gamma_t}{u_{c2}(t)^2\theta_t}| \geq 1$, then constraint (41) will bind and $u_{c1}(t) = u_{c2}(t)$ and the Friedman Rule is the optimal monetary policy.

6.2 Money in the Utility Function

Consider an economy with preferences represented by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t, h_t) \tag{45}$$

where $m_t = M_t/P_t$ is real money balances. The resource constraint is given by

$$c_t + g_t \leq F(N_t^F, N_t^I) \tag{46}$$

In equilibrium

$$\frac{u_h(t)}{u_c(t)} = F_{N^I}(t) \tag{47}$$

$$\frac{u_m(t)}{u_c(t)} = I_t \tag{48}$$

[36] Without an informal sector the RHS is zero and thus $u_{c1}(t) = u_{c2}(t)$. This solution can be decentralized through a zero nominal interest rate (compare with equation (38)).

The Ramsey allocations maximize preferences subject to the resource constraint, market equilibrium condition (47) and the following implementability constraint

$$\sum_{t=0}^{\infty} \beta^t u[u_c(t)c_t + u_m(t)m(t) - u_h(t)(1 - h_t)] = 0 \tag{49}$$

As before, assume that $u(c, m, h) = V(w(c, m), h)$, where w is a homothetic function. CCK (1996) show that under this assumption and in the absence of an informal sector, the Friedman Rule is the optimal monetary policy.[37] Moreover, as in CCK, assume that $m \leq \bar{m}$.

Proposition A2: Assume that contracts cannot be monitored by the government in the informal sector and that the utility function is the same as in CCK (1996). Then the Friedman Rule is not the optimal monetary policy.

Proof: Maximizing the Ramsey problem with respect to c and m and using the properties of the utility function, it can be shown that

$$\frac{u_m(t)}{u_c(t)} > 0 \tag{50}$$

This solution can be decentralized by a positive nominal interest rate.[38] See equation (48).

6.3 The Model with Currency Substitution

In this section we augment the model in Section 2 to include currency substitution. Preferences, production technology and the government sector are unchanged. Consumers may hold either domestic, M, or foreign, F, currency for transaction purposes. We consider the case of a small open economy. The transaction cost technology is represented by:

$$s_t \geq l(c_t, \frac{M_t}{P_t}, \frac{\xi_t F_t}{P_t}) \tag{51}$$

[37]To simplify the algebra, let $u(c, m, h) = V(w(c, m)) + u(h)$. This will not drive the result. In fact, it can be shown that under this assumption that the Friedman Rule is optimal in the absence of an informal sector.

[38]In CCK (1996) $\frac{u_m(t)}{u_c(t)} = 0$.

Since prices are flexible in the economy, Purchasing Power Parity holds (PPP), i.e., $P_t = \xi_t P_t^*$. Therefore, the transaction technology can be rewritten as[39]

$$s_t \geq l(c_t, m_t, f_t) \text{ where } m_t = \frac{M_t}{P_t} \text{ and } f_t = \frac{F_t}{P_t^*} \tag{52}$$

The representative household's one-period budget constraint is

$$c_t + \frac{M_t}{P_t} + \frac{\xi_t F_t}{P_t} + \frac{B_t}{P_t} \leq \frac{M_{t-1}}{P_t} + \frac{\xi_t F_{t-1}}{P_t} + (1 + i_{t-1})\frac{B_{t-1}}{P_t} + (1 - \tau_t^n) w_t^F n_t^F + w_t^I (1 - n_t^F - s_t - h_t) \tag{53}$$

The consumer's budget constraint in present value form is:

$$\sum_{t=0}^{\infty} d_t c_t + \sum_{t=0}^{\infty} d_t I_t m_t + \sum_{t=0}^{\infty} d_t I_t^* f_t \leq \sum_{t=0}^{\infty} d_t (1 - \tau^n) w_t^F n_t^F + \sum_{t=0}^{\infty} d_t w_t^I (1 - h_t - s_t - n_t^F) \tag{54}$$

where $d_t = \frac{1}{\prod_{s=0}^{t-1}(1+r_s)}$, $(1 + r_s) = (1 + i_s)\frac{P_s}{P_{s+1}}$, $I_t = \frac{i_t}{1+i_t}$ and $I_t^* = 1 - \frac{1}{1+i_t}\frac{P_{t+1}}{P_t}\frac{P_t^*}{P_{t+1}^*}$. The household's conditions for an interior optimum are the same as in the case without foreign money plus the following condition:

$$-l_f(t) = \frac{1}{(1 - \tau^n) w_t^F} I_t^* \tag{55}$$

I_t^* depends on i_t, on the domestic inflation rate and on the foreign inflation rate, which is given and cannot be controlled by the home government. The Ramsey problem has the same constraints as before, but the transaction technology now depends on foreign currency. Given that, it is straightforward to show that *Proposition 1* remains unchanged.

[39] The transaction technology satisfies the same assumptions as before. Let $l_{ff} \geq 0$ and $l_{mf} > 0$. In addition, $l_m = 0$ and $l_f = 0$ define $m^*(c, f)$ and $f^*(c, m)$ such that, $l_m < 0$ and $l_f < 0$ for any $m < m^*$ and $f < f^*$, and m^* and f^* are finite numbers.

Figure 1: Inflation vs. Informal Sector − Selected Countries

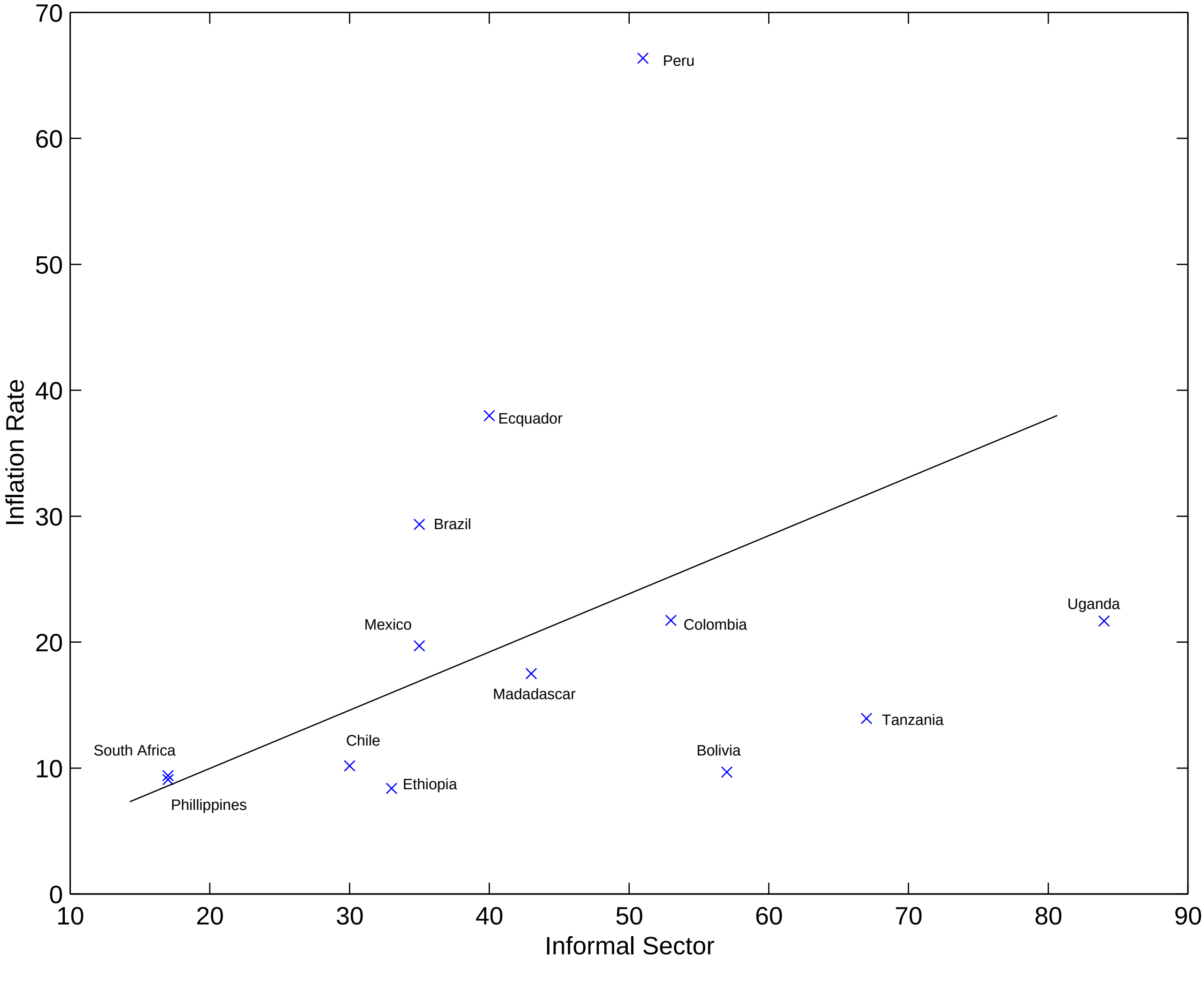

Figure 2a: Economy with Low Structural Problems

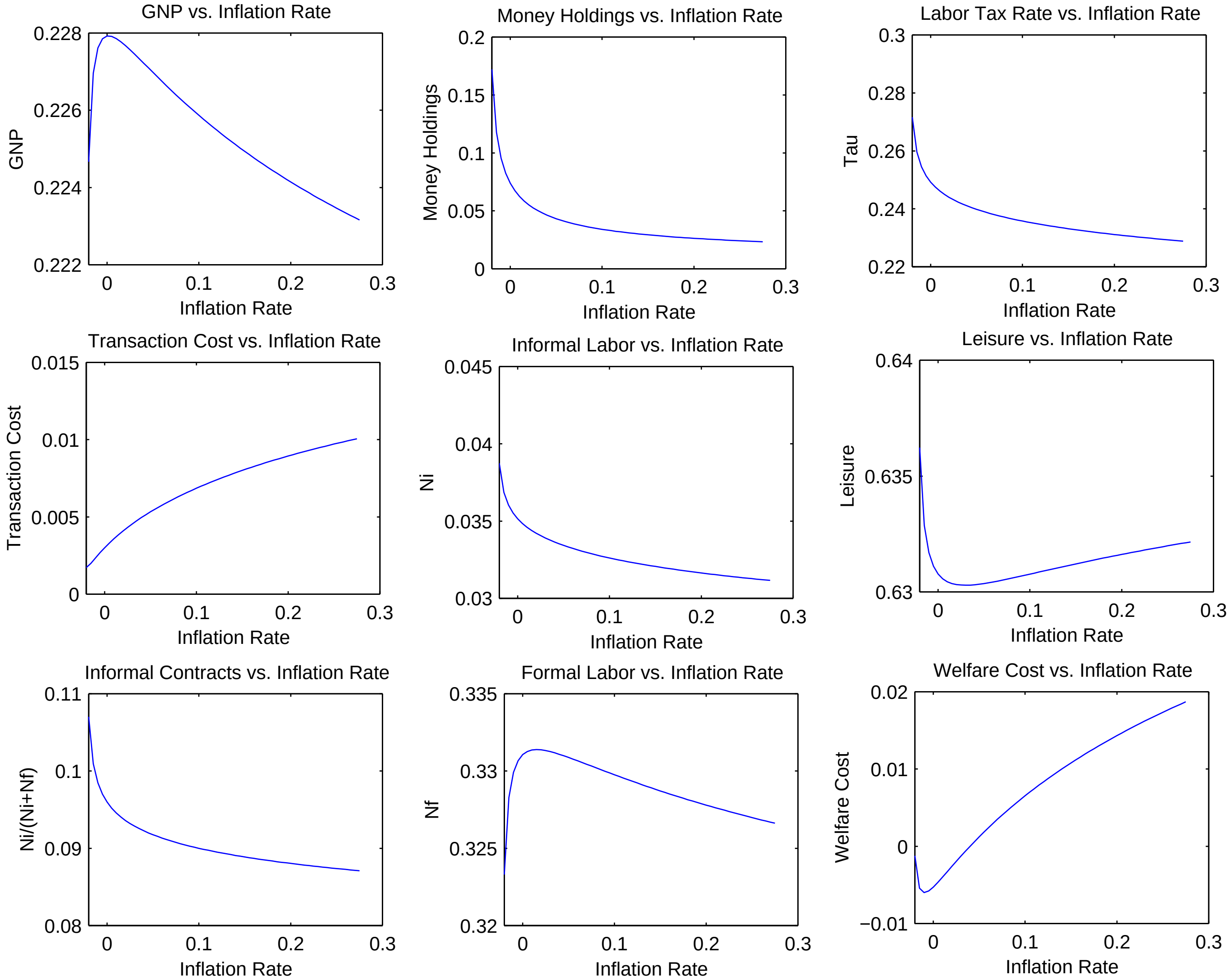

Figure 2b: Economy with Moderate Structural Problems

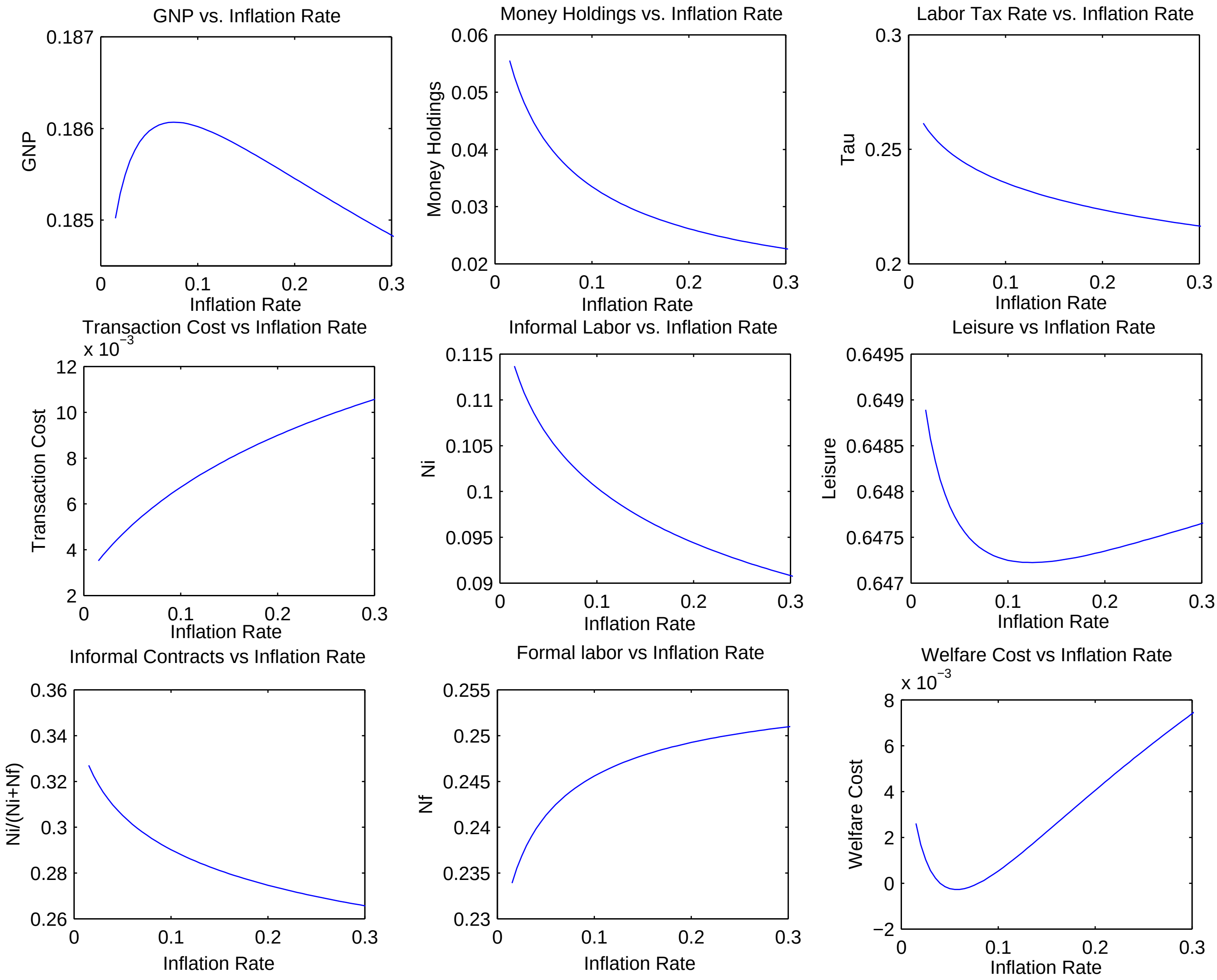

Figure 2c: Economy with High Structural Problems

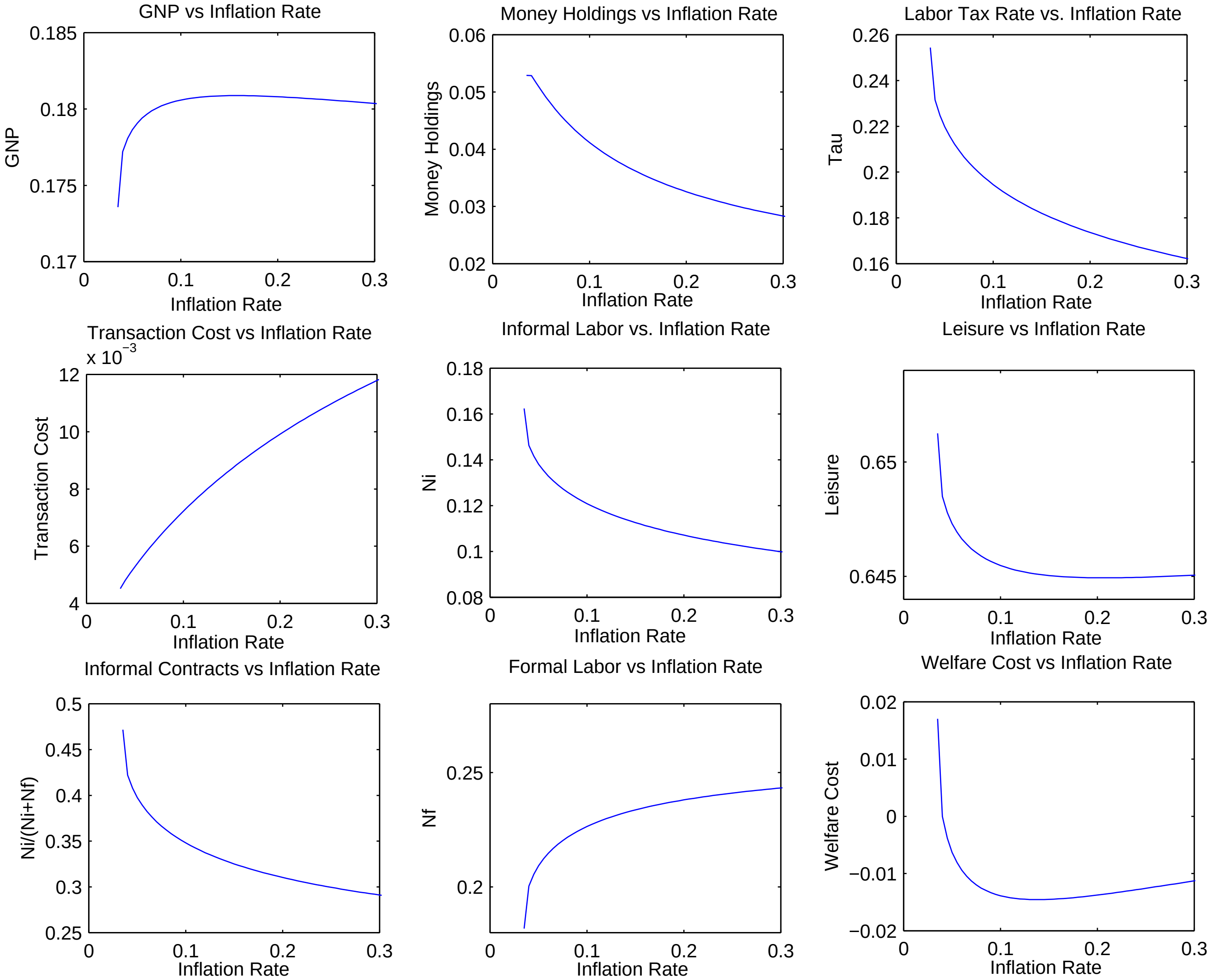

Figure 3: Welfare Cost of Inflation and Structural Problems

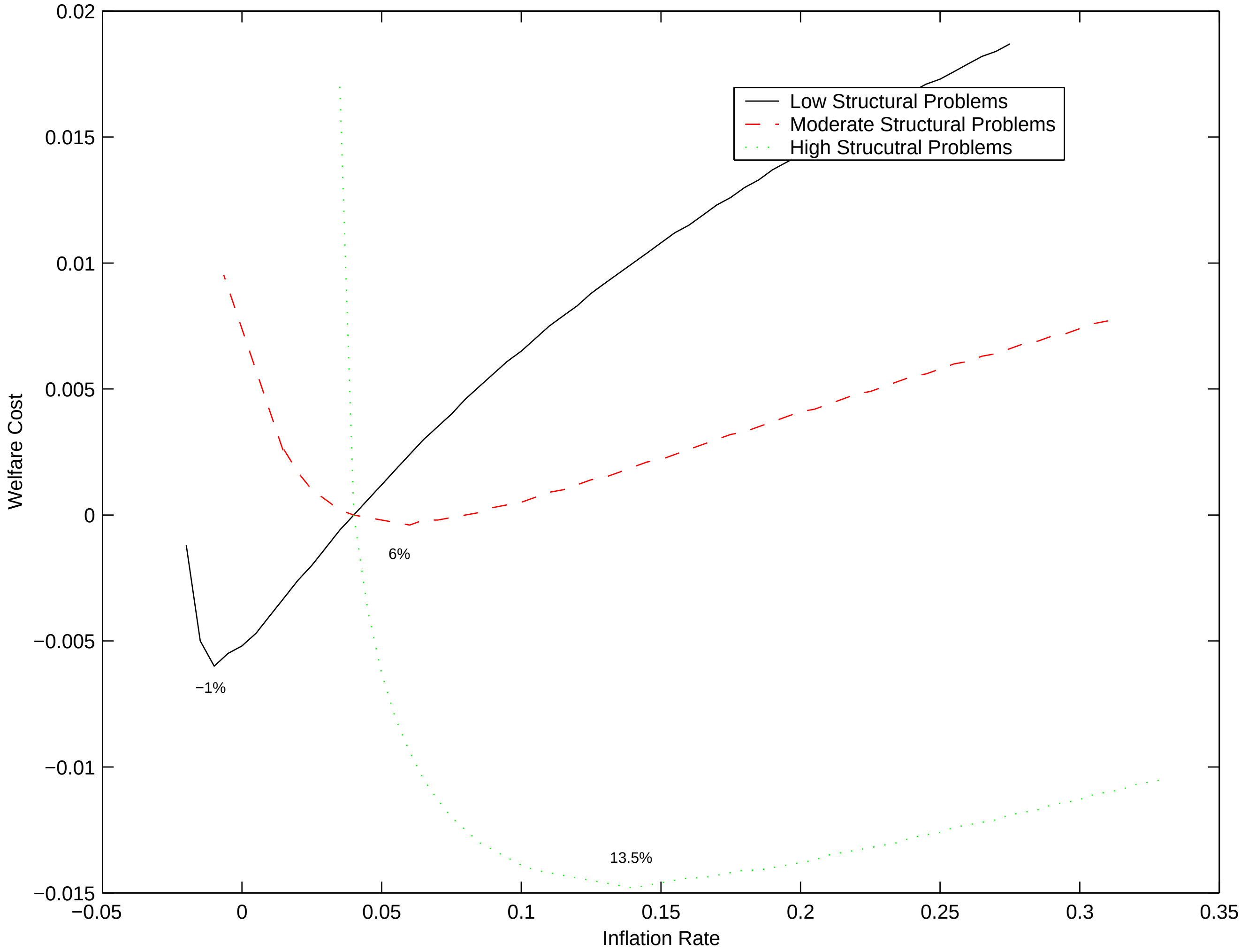

Figure 4: Government Revenue and Structural Problems

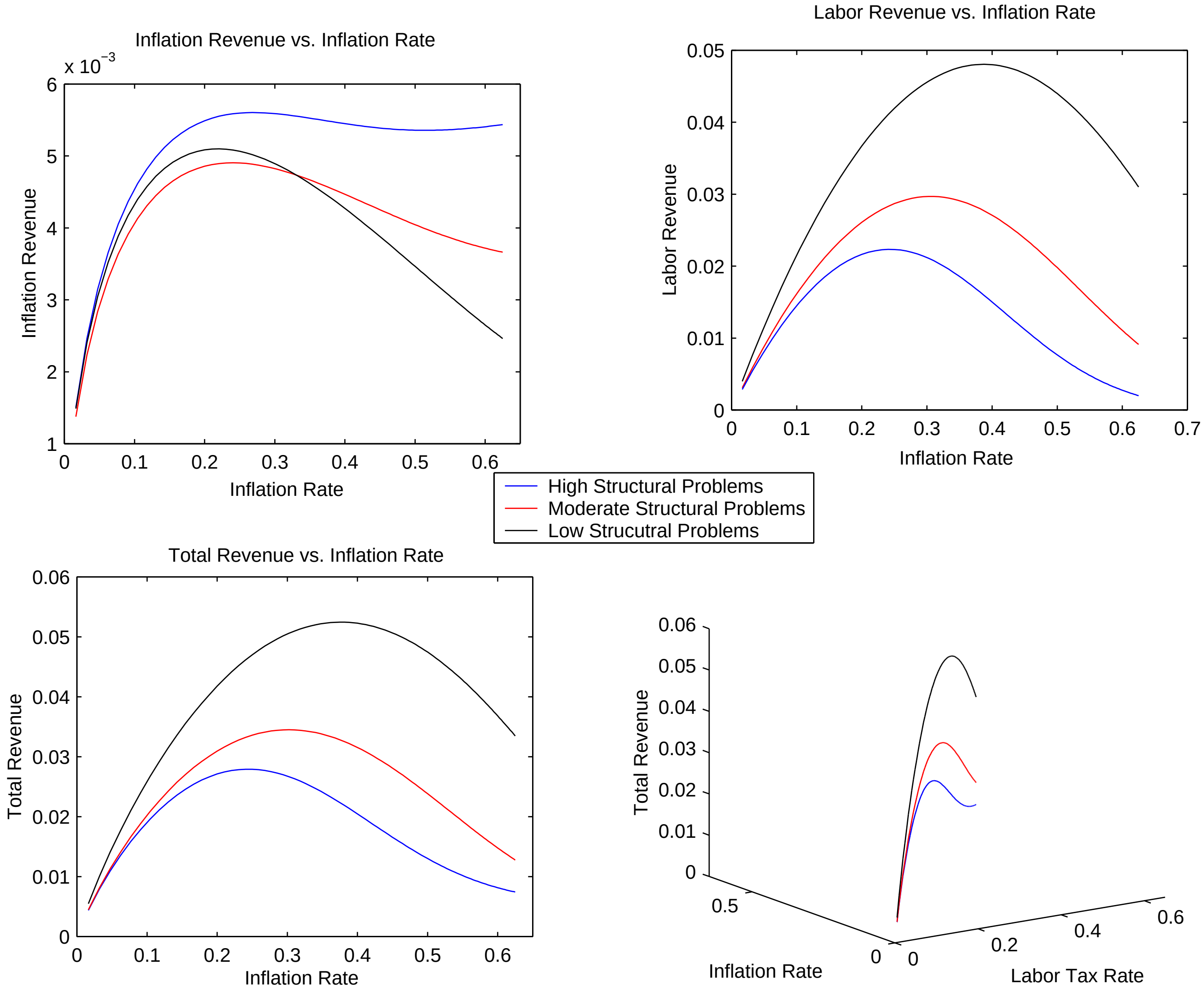

Figure 5a: Welfare Cost of Inflation for Different Degrees of Currency Substitution
Low Structural Problems

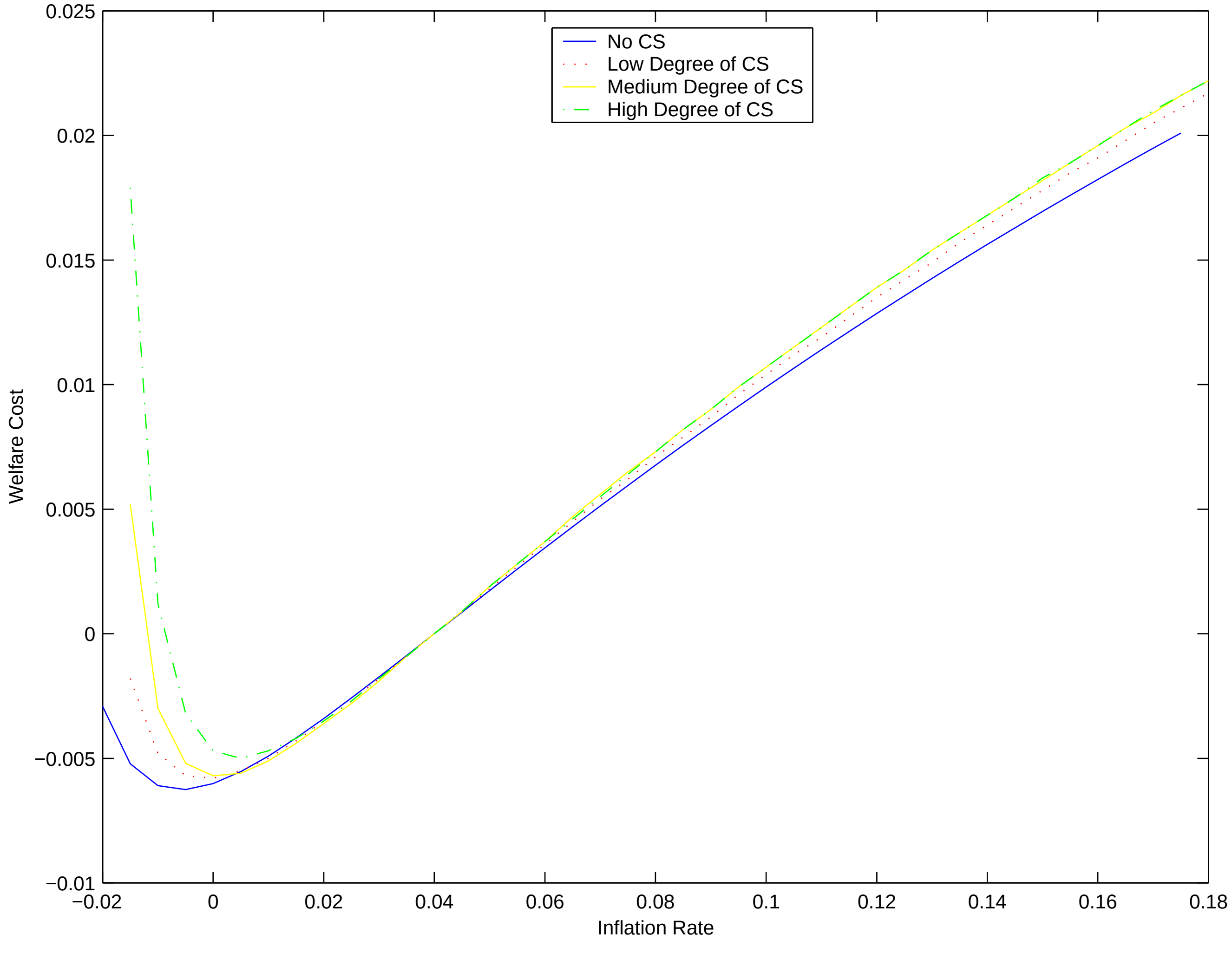

Figure 5b: Welfare Cost of Inflation for Different Degrees of Currency Substitution
High Structural Problems