

**The optimal currency composition  
of Uruguayan public debt.**

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May 24, 2000

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## **Abstract**

We use a optimal taxation model to analyse the optimal currency composition of Uruguayan public debt. Three possible instruments are examined: a nominal bond, an CPI indexed bond and a dollar denominated bond. In the model, the optimal decision arises from the resolution of a trade-off between considerations of costs and the requirements of optimal currency risk management of the government's budget. The model is calibrated using quarterly data for the period 1979-1998.

From the study of the stochastic properties of the variables affecting the government's budget constraint, we conclude that the current full dollarization of public debt is not optimal. In the Uruguayan case, this debt has two undesirable features. On the one hand, dollar instruments have associated a highly volatile real yield. On the other hand, the cost of dollar denominated debt is negatively correlated with the rate of GDP growth, and thus, with government revenues.

The paper makes a case for the inclusion of CPI indexed debt in the Uruguayan government's portfolio of liabilities. Indexed bonds provide government with a hedge against high ex-post real borrowing costs. The issue of this kind of instruments would help to reduce the variability of public deficit, and thus, it would be a way of diminishing fiscal vulnerability. Moreover, in the case of Uruguay, nominal debt should be discharged, not only because of its high cost, but also because of its undesirable stochastic properties.

## Resumen

En este documento se utiliza un modelo de nivelación impositiva para analizar la composición óptima por monedas de la deuda pública uruguaya. Tres instrumentos son considerados: un bono nominal, un bono indexado al IPC y un bono denominado en moneda extranjera (US dólares). El portafolio de deuda surge como resultado de un problema de optimización que tiene implícito un *tradeoff* entre la minimización de los costos financieros y del riesgo presupuestario. El modelo se calibra en base a datos trimestrales para el período 1979-1998.

A partir del estudio de las características estocásticas de las variables que afectan la restricción presupuestal del gobierno, se concluye que la proporción actual de deuda en moneda extranjera es inadecuada. Los papeles en dólares no solamente tiene asociado un rendimiento sumamente volátil, sino que además, su costo esta correlacionado negativamente con la tasa de crecimiento del producto.

Se argumenta que, en la actualidad, existe un lugar para los instrumentos indexados al IPC en el portafolio de deuda del Estado uruguayo. La introducción de este título ayudaría a disminuir la variabilidad del resultado de las cuentas públicas y sería, por tanto, una forma de disminuir la vulnerabilidad fiscal. La deuda nominal en moneda nacional debería ser descartada, no solamente por su elevado costo, sino también porque la misma no presenta las propiedades estocásticas deseables.

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## **I. Introduction**

In the 90's, after almost two decades where debt was strongly concentrated in the placement of foreign-currency variable-rate papers in the domestic market, the Uruguayan public debt policy entered into a different pattern, upon Uruguay's having gained access to the main international bond markets (Japan, Europe, and, mainly, the American market) and to the possibility of issuing fixed-rate papers. This change may be explained by the improvement in general funding conditions at the international level, a strong availability of capital for underdeveloped countries in the first half of the 1990s, and the opening of new funding options for the so-called "emerging markets"; but this change may be also mostly associated to a deliberate attempt by the last two Administrations to improve debt management on the basis of economic effectiveness criteria.

It should not be surprising to find that this change in trends encompasses the increasing importance assigned to public debt policy management by academics, national governments, and international financial organizations. Recent financial crises, in particular those in Mexico, Southeast Asia, and Brazil, are clear illustrations of the risks to which a country is exposed when debt is concentrated in very short horizons or one single currency.

In the late 1980's and early 1990's, academics started questioning the practice of holding one-currency debt portfolios, on the allegation that different kinds of debt have different cyclic properties. In particular, Bohn (1988) has built a justification for the placement of nominal local-currency public debt. Yet, in a later work (Bohn 1990b), he emphasizes the positive role of foreign-currency debt. On the other hand, the significance of indexed debt has been recently emphasized in papers written by Barro (1997), Blanchard and Missale (1997), and Missale (1998). The whole reference

literature on the above-mentioned optimal taxation issue indicates that, if governments are willing to facilitate an inter-temporal smoothing of consumption through a leveling of the tax burden, the debt portfolio must be chosen taking the stochastic properties of the yields of the available instruments into consideration.

This paper deals with the debt management problem from a very wide standpoint, contemplating not only issues referred to a minimization of the debt's financial cost for the country but also the basic allegations contained in the literature on optimal taxation. From this latter standpoint, the government should choose debt instruments associated to low costs in low-revenue periods (i.e., economic recession) and whenever the country faces a high public expenditure level (due, for instance, to the growth of real wages or a rise in international interest rates).

This paper examines the currency mix of the Uruguayan public debt in the light of the theoretical models and empirical evidences available on the stochastic behavior of several variables in the Uruguayan economy<sup>1</sup>. The core issue underlying this research is whether the bias towards foreign-currency debt chosen by Uruguay is warranted in the light of the budget risk inherent to US Dollar-denominated instruments.

Traditionally, an evaluation of the optimal mix of a debt portfolio has been based on the assumption that investors are neutral to risks. This implies that all instruments have the same anticipated cost (i.e. real interest rate). Within this context, the purpose of minimizing debt costs *ex ante* is equivalent to minimizing budget risks. In the Uruguayan case, financial instruments are associated to very different *ex ante* yields. A more realistic problem for the debt manager will be likely to imply a trade-off between the minimization of financial costs and budget risks. Unfortunately, the instruments that may be desirable in terms of their cyclic properties are not always the

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<sup>1</sup> This paper makes part of a wider project carried out by the Central Bank of Uruguay, designed to develop general guidelines for a more effective management of the Uruguayan public sector's debt.

“cheapest” for the government. One of the contributions of this paper is to explicitly introduce risk premiums in a standard debt-portfolio selection model. In addition, this paper also examines a higher number of stochastic variables having an impact on the governmental budget restraint and their correlation with the return of different debt instruments.

This paper has allowed us drawing the following main conclusions:

- The strong concentration of foreign-currency-denominated instruments in the current and past structure of the Uruguayan public debt may be only explained by the assumption that the government was trying to minimize the financial cost of debt but was not contemplating an *ex ante* minimization of the volatility of its budget.
- From an optimal taxation standpoint, the share of foreign currency debt is not adequate, as such debt is not only associated to an extremely volatile yield but also involves a cost which is negatively correlated to the GDP growth rate. Thus, a fall in revenues associated to an economic recession will be simultaneous to a rise in the service of foreign currency debt.
- Since at present the costs associated to the issue of indexed debt should not be extremely high as compared to those of US Dollar debt, we may conclude that the Uruguayan public debt portfolio might include CPI-indexed instruments. These instruments would help reducing the variability of public account results and, would be, thus, an appropriate means to reduce fiscal vulnerability.
- Local currency nominal debt should be disregarded under the present situation, not only because of its extremely high cost but also because these instruments have undesirable stochastic properties. The evidence available

for the last 20 years clearly shows that, in the Uruguayan case, there is no significant correlation between inflation and the exogenous variables having the most important impact on fiscal results.

The rest of the paper is organized as follows. Section II contains a description of the model, a description of the optimal mix of different debt instruments for several interesting cases, and an examination of the role of risk premiums. Section III includes an examination of the stochastic structure of the shocks having an impact on public accounts using an VAR methodology. In addition, the model is gauged, and an examination is made of the costs incurred by the government in connection with its having held a fully Dollarized debt over the 1978-1998 period. Section IV includes our findings and several recommendations of debt management policy for the next years.

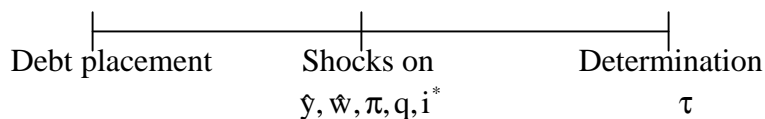
## II. A Theoretical Approach to the Optimal Currency Mix of Public Debt

### II.1 The Model

Our theoretical approach is mainly based on the papers by Calvo and Guidotti (1992), Goldfajn (1997), and Missale (1998). We used a two-period model similar to the one used by these authors. In Period 1, the government must make a decision on the mix of public debt that will be settled in full as at the close of the period. With a view to being able to focus on the currency selection problem we assume that the stock of debt to be funded in Period 1 is exogenously established. The debt manager may use three kinds of instruments: local-currency bonds, CPI-indexed bonds, and foreign-currency bonds. This model assumes that these instruments are similar in terms of both their horizon and the associated default risk.

The governmental budget restraint suffers the impact of stochastic shocks in several macroeconomic variables. In particular, the random variables having an impact on the consolidated fiscal results of the public sector (primary deficit plus debt service) are as follows: real devaluation rate, GDP growth rate, real wage growth rate, international interest rate, and inflation rate.

In Period 2, once the shocks have taken their toll, the government must pay the debt service and the primary expenditures accrued over Period 1. The only way to balance public accounts is to adjust the tax rate. The taxable base, i.e. the level reached by the GDP as at the close of Period 2, is also considered as a random component of the budget equation. Thus, the temporal sequence of the problem may be expressed by the following time line:



The budget restraint for the government in Period 2 is shown as:

$$t_2 Y_2 = G_2 + B_1 \left( q \frac{1+i_1}{1+p_2} + q^* \frac{(1+i_2^*)(1+e_2)}{1+p_2} + (1-q-q^*)(1+r_1) \right),$$

where  $\tau$  is the tax rate;  $\pi$ , the inflation rate;  $e$ , the devaluation rate;  $G$ , the public expenditures level;  $B$ , debt;  $Y$ , the GDP;  $i$ , the nominal rate of the local-currency bond;  $i^*$ , the foreign-currency rate; and  $r$ , the real rate of the indexed instrument. The variables showing sub-index 2 are considered to be random variables as at the time of decision-making on the debt mix. It is worth noting that the nominal interest rate in local currency,  $i$ , is a fixed rate, and thus it is known for Period 1. The same applies to the real interest rate of the indexed instrument,  $r$ , on the additional assumption that there is a daily indexing unit, whereby the yield of the indexed instrument precisely reflects inflation for the period on which such yield prevailed. As regards the rate of foreign-currency papers,  $i^*$ , we assume a floating rate, and thus that its value is unknown as at the time when the instrument is issued. The foreign-currency paper involves the commitment of paying the rate prevailing in the international market in Period 2, adjusted for a country-risk spread. Finally,  $\theta$  and  $\theta^*$  are the decision variables, representing the respective shares of local-currency bonds and foreign-currency bonds.

We are dealing with risk-averse investors, and thus we assume that there is no non-hedged parity of interest rates. The arbitrage condition for the different financial assets is shown as:

$$1+i = (1+i^*)(1+e^e)(1+p_1) = (1+r)(1+\pi^e)(1+p_2),$$

where  $p_1$  and  $p_2$  are the risk premiums required by agents to respectively invest in a local currency bond rather than in a foreign currency or an indexed bond. We assume that both  $p_1$  and  $p_2$  have a positive sign, and that  $p_1 > p_2$ . This means that, as at the time the

debt is incurred, the local currency instrument is more expensive and the foreign currency instrument is cheaper<sup>2</sup>.

All of Goldfajn (1997), Calvo and Guidotti (1992, 1994), and Missale (1998) have chosen to work on the assumption that investors are neutral to risk, (i.e., they assume that  $p_1 = p_2 = 0$ ). Thus, they emphasize the role of the debt currency choice as a means to smooth the tax burden over time<sup>3</sup>. In these models, the *ex ante* minimization of the debt service is not a relevant purpose for the debt manager, since he considers *a priori* that all instruments are associated to the same cost<sup>4</sup>.

Perfect assets substitution does not seem to be a realistic assumption, at least in the case of many countries having poorly developed capital markets. A quick glance to public debt management experiences throughout the globe clearly shows that governments are not indifferent to the financial cost of instruments but rather that they are reluctant to being indebted in “cheaper” instruments. In particular, in countries such as Uruguay and its partners in the Mercosur, the risk premiums paid, based on the currency choice, are extremely significant, and thus they may not be disregarded when a decision must be made regarding the selection of an optimal portfolio. These risk premiums are usually positively correlated to the duration of the underlying instrument. Although this paper does not take the term of the instrument into consideration, we are implicitly assuming a high share of long-horizon papers (as both theoretical models and

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<sup>2</sup> In general, based on currency risk considerations,  $p_1$  will obviously have a positive sign in a country having a floating rate of exchange or a poor reputation as regards monetary policy control. This is not so obvious in the case of  $(p_1 - p_2)$ . In a stable country such as the USA, the UK, or the countries in the Euro area, investors will be likely to prefer indexed debt to foreign currency debt, in which case  $(p_1 - p_2)$  would have a negative sign. In countries having long-lasting inflation, the US Dollar is a refuge asset and most financial assets are denominated in this currency; thus any new assets, even if indexed, might be likely to pay a risk premium over similar US Dollars instruments. In addition, the existence of an index lag creates the risk of a mismatch in the index when the economy negotiates an inflation process, and this might also result in  $(p_1 - p_2)$ 's having a positive sign.

<sup>3</sup> Both Calvo and Guidotti (1992) and Goldfajn (1997) include also other considerations related to the temporal consistency literature, whereunder it would not be appropriate to issue local currency papers. This paper only deals with the aspects involving smoothing fiscal cycles since this is, in our opinion, the most important issue for Uruguay.

<sup>4</sup> The *ex post* minimization of the debt service cost is implicitly considered in the loss function.

empirical evidence show that long-term instruments represent a significant share of the optimal debt structure), whereby the existence of risk premiums becomes a particularly relevant issue.

To the effects of further simplifying the model and being able to only work with growth rates, we must make an additional assumption on the behavior of primary public expenditures. Since we are mainly interested in the endogenous component of expenditures, i.e. the portion of primary expenditures which may not be affected by the government's discretionary decisions, we have chosen to establish a link between the development of expenditures and the behavior of an exogenous macroeconomic variable. We are, thus, assuming that:

$$G_2 = G_1(1 + \hat{w}_2),$$

where  $\hat{w}_2$  is the (stochastic) growth rate of the real wage in Period 2 and  $G_1$  is the expenditure level in Period 1, a known datum for the problem. This assumption is based on the findings of Borchardt, Rial, and Sarmiento (1998) for Uruguay, i.e. that the payment of social security allowances is the only endogenous component of Uruguayan public expenditures, apart from interest payments. In recent years, these allowances have proved to be highly sensitive to the real wage<sup>5</sup>.

The problem faced by the government in Period 1 may be expressed as follows:

$$\text{Min } V(\theta, \theta^*) = E_1 \left[ \frac{\tau_2^2}{2} \right],$$

subject to the following restraints:

$$(i) \tau_2 = g_1 \frac{1 + \hat{w}_2}{1 + \hat{y}_2} + b_1 \frac{1}{1 + \hat{y}_2} \left( \theta \frac{1 + i_1}{1 + \pi_2} + \theta^* \frac{(1 + i_2^*)(1 + e_2)}{1 + \pi_2} + (1 - \theta - \theta^*)(1 + r_1) \right),$$

$$(ii) 1 + i = (1 + i^*)(1 + e^c)(1 + p_1) = (1 + r)(1 + \pi^e)(1 + p_2),$$

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<sup>5</sup> Their study considers the 1989-1996 period.

(iii)  $0 \leq \theta \leq 1$ , and

(iv)  $0 \leq \theta^* \leq 1$ ,

where  $\hat{y}_2$  is the (stochastic) GDP growth rate in Period 2,  $g_1$  is the expenditures/GDP ratio for Period 1, and  $b_1$  is the debt/GDP ratio for Period 1. The above restraints (iii) and (iv) eliminate the possibility of active positions in governmental instruments, and thus allow us to concentrate on the cases which are relevant for the Uruguayan economy. In this sense, it is worth noting that, as in the optimal taxation literature, we are working with a quadratic cost function. We are thus assuming that a change in the tax rate will cause the government to face growing marginal costs. The existence of a growing marginal dis-profit allows debt management to be aimed at stabilizing the budget equation.

By dividing restraint (ii) into  $(1 + \pi)$  and carrying out a linearization operation, the real financial cost of each of the instruments after the shocks may be approximated using the following expressions:

nominal bond:  $r_1 + (\pi^e - \pi) + p_2$ ,

foreign currency bond:  $r_1 + (q - q^e) + (p_2 - p_1)$ ,

indexed bond:  $r_1$ ,

where  $q_t = e_t - \pi_t$  is the real devaluation rate. The real cost of the indexed debt is constant, and independent of the actual development of random variables, and in particular of the inflation rate. The debt cost in Peso terms will depend not only on the risk premium,  $p_2$ , but also on the inflation prediction error,  $\pi^e - \pi$ . Thus, the cost of local currency nominal debt will increase when anticipated inflation is higher than actual inflation. In turn, the US Dollar-denominated instrument will be more expensive for the government when the real devaluation rate actually recorded in Period 2 is higher than the devaluation rate that had been anticipated by investors. It is worth

noting that if  $q = q^e$ , US Dollar-denominated debt will be cheaper than indexed debt, since we have assumed that  $p_1 - p_2 < 0$ .

The quadratic form of the loss function ensures that the government is interested in tax smoothing at all nature states. If all instruments had similar *ex ante* financial costs ( $p_1 = p_2 = 0$ ), in Period 1 it would be advisable to use those papers whose yield involves the desirable stochastic properties. The minimization of the variability in the tax rate would lead to including in the debt portfolio those instruments which are associated to lower *ex post* costs (in the case of local currency bonds, when  $\pi > \pi^e$ , and in the case of US Dollar debt when  $q < q^e$ ) in the nature states where tax revenues fall under the anticipated level; i.e., in terms of the model, when the GDP is lower than anticipated ( $y < y^e$ ), when the endogenous component of primary expenditures is higher than anticipated ( $w > w^e$ ), or when the debt cost is higher than anticipated ( $i^* > i^{*e}$ ). In other words, local currency instruments would be particularly attractive when the stochastic structure of the economy is such that:

$$\begin{aligned}\sigma_{\pi,y} &= E(\pi - \pi^e)(y - y^e) < 0 \\ \sigma_{\pi,i^*} &= E(\pi - \pi^e)(i^* - i^{*e}) > 0 \\ \sigma_{\pi,w} &= E(\pi - \pi^e)(w - w^e) > 0\end{aligned}$$

On the other hand, foreign currency papers would be useful to mitigate the volatility of the government's budget if:

$$\begin{aligned}\sigma_{q,y} &= E(q - q^e)(y - y^e) > 0 \\ \sigma_{q,i^*} &= E(q - q^e)(i^* - i^{*e}) < 0 \\ \sigma_{q,w} &= E(q - q^e)(w - w^e) < 0\end{aligned}$$

This result, which seems to concur with our intuition, may be more formally obtained by solving the above-described problem. By expanding the

$\tau_2(\pi_2, \pi_2^e, e_2, e_2^e, \hat{y}_2, \hat{w}_2, r_2, p_1, p_2)$  function using Taylor,<sup>6</sup> we can obtain the final expression of the budgetary equation:

$$\tau_2 = [g_1(1 + \hat{w}_2 - \hat{y}_2) + b_1 \{1 - \hat{y}_2 + i_2^* + q_2^e + \theta(\pi_2^e - \pi_2) + \theta^*(q_2 - q_2^e) + \theta p_1 + (1 - \theta - \theta^*)(p_1 - p_2)\}]$$

A reading of the governmental budget restraint shows that, in terms of the GDP for Period 2, public expenditures will depend on the difference between the real wage growth rate and the GDP growth rate. On the other hand, the debt service level in terms of the GDP for Period 2 will negatively depend on the growth of the GDP and will positively depend on the cost of debt. The measurement of this cost will be based on the yield of US Dollar papers. The cost of credit may be decomposed into several factors. In the first place we have the anticipated cost associated to foreign currency debt ( $i^{*e} + q^e$ ). As regards local currency debt, a surplus will be always paid whenever inflation expectations exceed actual inflation. The same will apply to US Dollar debt when real devaluation (US Dollar deflation) exceeds its anticipated level. The last two terms represent the additional costs associated to the existence of risk premiums. Local currency debt will pay a  $p_1$  premium while indexed debt will pay a  $(p_1 - p_2)$  premium.

The above Lagrange problem is shown as:

$$L = E_1 [g_1(1 + \hat{w}_2 - \hat{y}_2) + b_1 \{1 - \hat{y}_2 + i_2^* + q_2^e - \theta(\pi_2 - \pi_2^e) + \theta^*(q_2 - q_2^e) + \theta p_1 + (1 - \theta - \theta^*)(p_1 - p_2)\}]^2 + \lambda_1 \theta + \lambda_2 \theta^* + \lambda_3 (1 - \theta) + \lambda_4 (1 - \theta^*)$$

The first-order restraints underlying the problem are as follows:

$$(5) \frac{\partial L}{\partial \theta} = E_1 [b_1 (p_2 - (\pi_2 - \pi_2^e)) [\tau_2]] + \lambda_1 - \lambda_3 = 0$$

$$(6) \frac{\partial L}{\partial \theta^*} = E_1 [b_1 ((q_2 - q_2^e) - (p_1 - p_2)) [\tau_2]] + \lambda_2 - \lambda_4 = 0$$

<sup>6</sup> Taylor's expansion lies around (0,0,0,0,0,0,0,0), which proves to be reasonable if we recall we are working with variation rates.

$$(7) \quad \frac{\partial L}{\partial \lambda_1} = \theta \geq 0, \quad \lambda_1 > 0, \quad \theta \lambda_1 = 0$$

$$(8) \quad \frac{\partial L}{\partial \lambda_2} = \theta^* \geq 0, \quad \lambda_2 > 0, \quad \theta^* \lambda_2 = 0$$

$$(9) \quad \frac{\partial L}{\partial \lambda_3} = 1 - \theta \geq 0, \quad \lambda_3 > 0, \quad (1 - \theta) \lambda_3 = 0$$

$$(10) \quad \frac{\partial L}{\partial \lambda_4} = 1 - \theta^* \geq 0, \quad \lambda_4 > 0, \quad (1 - \theta^*) \lambda_4 = 0$$

Our interest is focused on two particular solutions of this general Kuhn Tucker problem. In the first place, we will examine the case where all three kinds of bonds are issued ( $\theta > 0, \theta^* > 0, 1 - \theta - \theta^* > 0$ ). In the second place, we will examine the case when the cost of the local currency debt is so high that such debt has no positive share in the debt portfolio and thus the left-hand side of restraint (iii) is operative; in other words, when

$$\begin{aligned} \theta &= 0, \\ \theta^* &> 0, \\ 1 - \theta^* &> 0 \end{aligned}$$

## II.2 The Optimal Portfolio, with the Three Kinds of Debt

We shall try to establish under which conditions it will be possible for a debt portfolio to include the three kinds of instruments under consideration. In terms of the above-described problem, this holds when  $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$ . The first-order restraints underlying the problem may be summarized using the following two equations:

$$(12)$$

$$\theta(\sigma_\pi^2 + p_2^2) = \theta^* b_1 (\sigma_{\pi q} + p_2 (p_1 - p_2)) + g_1 ((\sigma_{w,\pi} - \sigma_{y,\pi}) + b_1 (\sigma_{\pi,i^*} - \sigma_{\pi,y}) - p_2 k)$$

(13)

$$\theta^*(\sigma_q^2 + (p_1 - p_2)^2) = \theta b_1(\sigma_{\pi q} + p_2(p_1 - p_2)) + g_1((\sigma_{y,q} - \sigma_{w,q}) + b_1(\sigma_{q,y} - \sigma_{q,i^*})) + (p_1 - p_2)k,$$

where

$$k = g_1(1 + w_2^c - y_2^c) + b_1(1 + i_2^{*c} + q_2^c - y_2^c + p_1 - p_2).$$

Although the system might be theoretically solved for two unknowns,  $\theta$  and  $\theta^*$ , this effort would not yield any significant additional result. The impact of the correlation on the optimal values of  $\theta$  and  $\theta^*$  may be directly examined departing from the above two expressions.

Restraint (11) indicates, in the first place, that the share of local currency debt is a decreasing function of the variability of inflation. This derives from the budget risk aversion inherent to the government's loss function. The more volatile the inflation rate, the wider the fluctuation in the debt service in Peso terms; whereby, the government will be more reluctant to incur in debt in these instruments.

On the other hand, local currency debt may operate as a risk insurer if the debt's *ex post* yield is associated to unexpected values in other macroeconomic variables. If the correlation between inflation and these variables is zero, local currency debt would not operate as a fiscal risk insurer. Among the factors favoring local currency debt it is worth noting the following:

- (i) the existence of a negative correlation between the inflation rate and the GDP growth rate - In this model, when the GDP falls below the anticipated level, so does tax revenue, thus resulting in a fall in the endogenous funding of the government. If there is a high negative correlation between inflation and the GDP, actual inflation will exceed anticipated inflation, and thus the service of local currency debt will be

lower. In other words, when  $\sigma_{\pi,y} < 0$ , a local currency instrument will operate following an anti-cyclic pattern.

- (ii) Similarly, a positive correlation between inflation and the international interest rate, where  $\sigma_{\pi,i^*} > 0$ , will ensure that when the actual cost of interest payments rises, non-anticipated inflation will tend to reduce the service of local currency debt.
- (iii) When we find a positive correlation between the wage growth rate and inflation, i.e.  $\sigma_{\pi,w} > 0$ , the local currency instrument will become attractive, insofar as it allows mitigating the unexpected rise in expenditures through an unexpected fall in debt service payments. This is an optimal alternative from the tax-smoothing standpoint: as the primary surplus falls, there will also be an endogenous fall in debt interest payments.

In summary, local currency debt may be included in a debt portfolio if the premium payable in connection with the issue of the corresponding instruments is small, if inflation variability is low, and if inflation shows desirable stochastic properties, i.e. if inflation is anti-cyclic as regards the GDP and pro-cyclic as regards the international interest rate and the real wage.

In the literature, several temporal consistency allegations have been used to dismiss local currency debt. Barro (1997), for instance, states that the issue of local currency papers creates a moral hazard problem. The government may be tempted to liquefy the real value of the Pesos debt through monetary expansion or an unexpected devaluation. This element, which has not been taken into account in the theoretical formulation of this paper, increases the cost of local currency debt, since agents will internalize the government's incentives and require a higher interest rate in local

currency. This problem will exist when the central bank is free to introduce changes to the monetary/exchange policy, and it is particularly significant in countries with a long tradition of monetary/exchange instability. Experience shows, however, that the governments of most developed countries issue debt in their respective local currencies, which suggests that once the economy is stable, once the country has established a reputation in terms of inflation control, it is likely, and even desirable, to issue this kind of instruments.

As in the case of debt denominated in Pesos, the use of foreign currency debt will negatively depend on its yield variability, determined in this case by the volatility of the devaluation rate. Based on the allegation of tax smoothing through nature states, we may conclude that issuing debt denominated in US Dollars will be desirable in the following events:

- (i) If there is a negative correlation between the unanticipated evolution of the real price of the US Dollar and the real wage, i.e.  $\sigma_{q,w} < 0$ , so that if the cost of foreign currency debt suffers an unanticipated rise, the endogenous component of public expenditures will tend to fall;
- (ii) If we find a positive covariance between the real devaluation rate and the GDP growth rate, i.e.  $\sigma_{q,y} > 0$ , in which case foreign currency debt will be more attractive, since the above correlation implies that in the periods when revenues are low this debt will be cheaper and the contrary will hold in the periods where revenues are high;
- (iii) If the covariance between the real price of the US Dollar and the real interest rate is negative, i.e.  $\sigma_{q,i^*} < 0$ , since a negative covariance between these two variables implies that the cost of foreign currency debt will increase when international interest rates are falling.

Given the complexity of equations (11) and (12), it is impossible to find an explicit solution for the impact of risk premiums on the optimal values of  $\theta$  and  $\theta^*$ . Thus, we have chosen to reduce the problem to a choice between two different instruments. In the case of Uruguay, given the prohibitive cost of Peso-denominated medium- and long-term instruments, and considering, in addition, that local currency debt does not have the desirable ensuring properties<sup>7</sup>, our comparison involves foreign currency debt and indexed debt.

### II.3 US Dollar-Denominated versus CPI-Indexed Debt

If the cost of local currency debt is too high, its share in the debt portfolio will disappear. In this case, the tax-smoothing role through nature states will be only played by foreign currency debt.

In terms of the Kuhn Tucker problem, we are dealing with the following case:

$$\begin{aligned} \theta &= 0, \\ \lambda_1 &> 0, \\ \lambda_2 = \lambda_3 = \lambda_4 &= 0 \end{aligned}$$

The first-order restraints of the problem described in the preceding section are reduced to the following:

$$(14) \quad \theta^* = \frac{(\sigma_{q,y} - \sigma_{q,w})g_1 + (\sigma_{q,y} - \sigma_{q,i^*})b_1 + (p_1 - p_2)k}{b_1(\sigma_q^2 + (p_1 - p_2)^2)}$$

The above expression shows the keys for the selection of either debt indexed to the evolution of the CPI or debt denominated in foreign currency. As already stated, the variability in the real price of the US Dollar is an allegation against this kind of debt and in favor of indexed bonds. This feature has already been noted in the papers by Missale (1998), Barro (1997), and Bohn (1990).

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<sup>7</sup> Please refer to the empirical section of the paper.

The pro-cyclic nature of foreign currency debt service cost, resulting from a negative covariance between the real devaluation rate and the GDP growth rate, is another strong allegation in favor of indexed debt.

On the other side, the existence of a positive premium in the interest rates of indexed bonds would operate in favor of foreign exchange debt; this premium is mainly explained by the poor development of instruments denominated in indexed units. In addition, the existence of a negative correlation between the real wage growth rate and inflation in terms of the US Dollar is a property favoring the issue of debt denominated in US Dollars.

#### II.4 The Role of Risk Premiums

Based on Equation (14), we may proceed to make an analytical derivation of the relationship between the optimal debt mix and the size of the risk premium. In the second half of the paper, the model is gauged and attempts are made to quantitatively establish the range of premiums for each kind of instrument.

To the effects of this analysis, we must regroup the terms in (14) as follows:

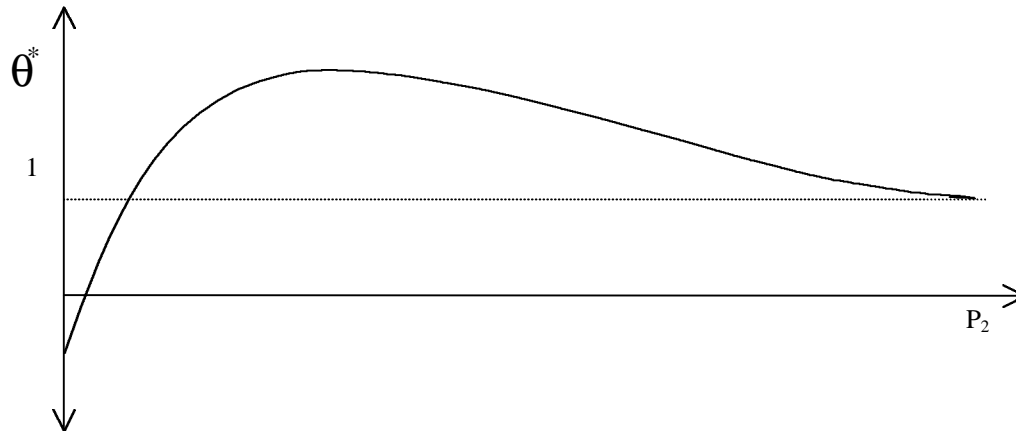
$$\theta^*(\phi) = \frac{b_1\phi^2 + \Delta\phi + \Psi}{b_1(\sigma_q^2 + \phi^2)}$$

where  $\phi = p_1 - p_2$  is the premium payable on the indexed instrument as compared to the US Dollar instrument, and

$$\begin{aligned} \Delta &= b_1(1+i_2^{*e} + q_2^e - \hat{y}_2^e) + g_1(1+\hat{w}_2^e - \hat{y}_2^e) \\ \Psi &= (\sigma_{q,y} - \sigma_{q,w})g_1 + (\sigma_{q,y} - \sigma_{q,r})b_1 \end{aligned}$$

are the model parameter functions.

Exhibit 1 shows that function  $\theta^*$  behaves as follows:



We may notice that when the *ex ante* cost of indexed debt is low, a rise in the premium will result in a rise in the desired amount of foreign currency debt. As from a certain point, the government will start incurring in US Dollar debt to grant loans in indexed money. In spite of the above, since we will hardly find supply of indexed papers at these interest rates and considering that no government will incur in debt in one currency to grant loans in another<sup>8</sup>, we have eliminated this theoretical possibility through the imposition of sign restraints.

We can thus conclude that, in the relevant section of the cost function (i.e., when  $0 \leq \theta^* \leq 1$ ), the higher the premium paid on indexed debt, the higher the optimal share of foreign currency debt. This is a particularly significant result, as it shows that the development of capital markets, and in particular those operating with indexed instruments, is likely to result in a fall in the premium associated to such instruments, and this, in turn, would create the necessary conditions for such instruments to be included in the optimal debt portfolio of Uruguay.

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<sup>8</sup> It is worth noting that any reference in this paper to the government's debt implies in fact a reference to the consolidated debt of the non-financial public sector and the Central Bank of Uruguay, leaving fully aside the positions of government-owned banks (BROU and BHU), since these entities operate as financial intermediation agents and are fully alien to the Uruguayan public debt's management.

### **III. Using the Debt Model in the Uruguayan Case**

This section of the paper examines the stochastic features of the shocks having an impact on the budget equation of the government. The core issue dealt with in this paper is that both the correlation between these shocks and the yields of the different financial instruments and the volatility inherent to the returns of such instruments should be contemplated by Uruguayan public debt managers when making a decision on the optimal currency mix of the government's liabilities.

Under certain circumstances, a trade-off may be the case between instruments having a lower associated financial cost and those having the desirable properties in terms of a lower volatility and an appropriate correlation between their yields and the relevant variables having an impact on public accounts.

Thus, once the corresponding variance and covariance estimates have been obtained, we shall proceed to gauge the model, using to such ends reasonable values of the risk premiums that will be obtained from an examination of the structural characteristics of the Uruguayan economy and the specific features of its capital market. These values will also allow making an evaluation of the losses the Uruguayan government suffered in the past as a consequence of its holding a fully Dollarized debt portfolio.

#### **III.1 The Stochastic Structure of the Uruguayan Budget: Empirical Evidence**

##### **III.1.a The Statistical Base Used**

To the effects of obtaining the strongest possible results, we chose to work with a relatively long period, i.e. the last 20 years (1979-1998). We were particularly interested in covering the period when the stabilization program known as the "Tablita" collapsed (1982-83), in the understanding that, at the time, the evolution of the

macroeconomic variables considered in this paper tended to dramatically worsen fiscal results.

Uruguayan series are all comprised of official data, as they were obtained from the Economic Statistics Department of the Central Bank of Uruguay and the National Statistics Institute. As regards the international interest rate, we used the 6-month LIBO rate (taken from International Financial Statistics, as published by the IMF).

We chose to work with quarterly data, as this allows having an adequate number of data to obtain econometric estimates. With the only exception of the interest rate, all the variables taken into consideration are expressed as a variation rate as compared to the preceding quarter. The LIBO rate used is the monthly simple average for each quarter.

### III.1.b Simple Correlation Among the Variables

In the first place, as a first general step, we estimated the variance and covariance matrix of the variables included in the theoretical model, i.e. the inflation rate ( $\pi$ ), the real devaluation rate ( $q = e - \pi$ ), the international interest rate ( $i^*$ ), the real wage variation rate ( $w = W - \pi$ ), and the GDP growth rate ( $y$ ). As regards the latter, given the highly seasonal nature of the Uruguayan GDP, we de-seasonalized the series prior to computing variation rates.

The following chart shows the covariance and correlation matrixes found for the 1979-1998 period<sup>9</sup>.

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<sup>9</sup> The temporal series moment analysis is meaningful insofar as we are dealing with stationary series. Thus, prior to examining results, we proceeded to test the existence of unit roots in the series. Using a 3-lag Phillips-Perron test, with no trend (except in the case of the LIBO Rate, where a trend was indeed included) and with a 5% significance level, we concluded that all series are stationary.

**Chart 1**

**Simple Moments**

**A. Variance and Covariance Matrix**

	<i>LIBO Rate</i>	<i>GDP</i>	<i>Real Wage</i>	<i>Real Devaluation</i>	<i>Inflation</i>
<i>LIBO Rate</i>	0.00100				
<i>GDP</i>	-0.00020	0.00090			
<i>Real Wage</i>	-0.00007	0.00003	0.0022		
<i>Real Deval.</i>	0.00008	-0.00210	-0.0016	0.03080	
<i>Inflation</i>	0.00007	0.00007	-0.0002	-0.00180	0.00310

**B. Correlation Matrix**

	<i>LIBO Rate</i>	<i>GDP</i>	<i>Real Wage</i>	<i>Real Devaluation</i>	<i>Inflation</i>
<i>LIBO Rate</i>	1.000				
<i>GDP</i>	-0.133	1.000			
<i>Real Wage</i>	-0.039	0.017	1.000		
<i>Real Deval.</i>	0.012	-0.382	-0.199	1.000	
<i>Inflation</i>	0.003	0.039	-0.094	-0.191	1.000

To the effects of this analysis, we are particularly interested in examining the sample values of parameters  $\sigma_q^2$ ,  $\sigma_\pi^2$ ,  $\sigma_{q,y}$ ,  $\sigma_{q,i^*}$ ,  $\sigma_{q,w}$ ,  $\sigma_{\pi,y}$ ,  $\sigma_{\pi,i^*}$ , and  $\sigma_{\pi,w}$ . The examination of the above charts allows identifying three aspects deserving special consideration:

- a) The real devaluation rate variance is extremely higher than the inflation rate variance.
- b) The real devaluation rate is strongly negatively correlated with the GDP variation.
- c) The inflation rate has no statistically significant correlation with any of the variables in the system.

Although these results may be considered to be a starting reference point, it is evident that they should be supplemented with other more detailed studies. The second moments, estimated above using the full sample period, may prove not to be an appropriate approximation of the parameters considered in the theoretical model. In fact, using the variance and covariance matrix of the sample implicitly assumes that the expected values of the variables at each moment will be constant and equal to their average value for the whole of the period. This assumption may prove not to be satisfactory, bearing in mind that the agents' expectations are not static but are rather adjusted on ongoing bases as a function of the events taking place in the economy. It does not seem reasonable to state, for instance, that the expected value of the inflation rate by the close of the period (when the annual real inflation rate has only one digit) will be the value that was anticipated early in the decade (when the annual inflation rate was a three-digit figure).

This called for the need to consider an alternative procedure that would allow us, in the first place, to estimate the expected values of the variables at each moment and, in the second place, to compute the correlations corresponding to the above-described theoretical model.

### III.1.c Procedure to Obtain the Expected Values of the Variables

As we all know, there is no single way to estimate the expectations of agents as regards a given variable. This paper follows a technique similar to those used by Missale (1998) and Goldfajn(1997), designed to obtain the expected values of each of the variables departing from the estimation of sliding auto-regressive vectors (VARs)<sup>10</sup>.

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<sup>10</sup> This procedure is, doubtless, richer than the alternative of adjusting a monovariant auto-regressive process for each of the series. Anyway, it is worth emphasizing that, by its mere construction, the procedure used implies an adaptive or "looking backward" expectation-formation process. Although this may prove to be appropriate for certain variables such as the inflation rate or the real wage variation (having a major inertial component), the looking-backward expectation assumption is more questionable for the real devaluation rate or the GDP growth rate.

Yet, conversely to the developments of the above authors, the innovation vector  $[X_t - X_t^e]$  (where  $X_t$  represents the observed values and  $X_t^e$ , the expected values of the variables) is not directly obtained from the last residues of each VAR but is rather obtained as the difference between the values of the variables actually observed in the following quarter and the values projected by the model for such quarter ( $X_{t+1} - X_{t+1}^p$ ).

In other words, our work considers that agents use the information available for the most recent five years to build their projections of the values of the variables in the following quarter. The model includes the five variables that were considered in the theoretical model<sup>11</sup>. A new VAR is estimated for each quarter, departing from a sample including the last 20 observations. In other words, each new VAR incorporates the most recent observations available and excludes the oldest data, corresponding to the first observation in the preceding period.

Once the VAR for the  $[t-19, t]$  period has been estimated, this figure is used to project the variables for the  $t+1$  quarter. By subtracting this projection from the value actually observed in  $t+1$  we obtain the innovation corresponding to such quarter. This procedure allows building innovation series for the five variables. Once all values have been obtained, in a second stage we proceed to compute the variances and covariances that are significant to the effects of choosing the debt portfolio mix.

It is worth noting that the international interest rate equation does not include any domestic variable, in the understanding that Uruguay is a small country and thus no Uruguayan variable has an impact on the behavior of the LIBO Rate. In this sense, we are estimating a model known in the literature as the “Near VAR” model<sup>12</sup>.

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<sup>11</sup> The model also includes seasonal dummy variables in the equations corresponding to domestic variables.

<sup>12</sup> A Near-VAR model allows eliminating redundant variables from the reduced VAR equations, thus restricting certain coefficients to being equal to zero.

Each equation includes only one lag. Although working with quarterly data might theoretically result in the introduction of four lags per equation, this would deprive us from a significant number of freedom degrees, which is not advisable given the limited number of observations included in each VAR. On the other hand, the incorporation of dummy variables allows contemplating the seasonal nature of some of the variables under consideration without the need to incorporate additional lags.

The period from I-1979 to IV-1983 was used to estimate the first VAR. The estimated innovation series start, thus, in the first quarter of 1984. These series are presented in Figures 1-5.

### III.1.d Results

The following charts include the covariance and correlation matrixes of the innovation series, estimated as described above, using data for 1979-1998:

#### Chart 2

#### Moments Derived from the VAR Model

##### A. Variance and Covariance Matrix

	<i>LIBO Rate</i>	<i>GDP</i>	<i>Real Wage</i>	<i>Real Devaluation</i>	<i>Inflation</i>
<i>LIBO Rate</i>	0.00007				
<i>GDP</i>	0.00002	0.00220			
<i>Real Wage</i>	-0.00020	0.00007	0.00290		
<i>Real Deval.</i>	-0.00006	-0.00250	-0.00130	0.01740	
<i>Inflation</i>	-0.00002	-0.00010	0.00000	-0.00001	0.00080

##### B. Correlation Matrix

	<i>LIBO Rate</i>	<i>GDP</i>	<i>Real Wage</i>	<i>Real Devaluation</i>	<i>Inflation</i>
<i>LIBO Rate</i>	1.000				
<i>GDP</i>	0.045	1.000			
<i>Real Wage</i>	-0.410	0.028	1.000		

<i>Real Deval.</i>	-0.049	-0.408	-0.183	1.000	
<i>Inflation</i>	-0.092	-0.110	0.002	-0.002	1.000

Although certain differences may be identified as compared to the results obtained using the full sample period (a logical outcome, since we are measuring different things), the main results obtained from the analysis of simple correlations are still valid. It is worth noting that when this technique is used, the difference in the real devaluation rate variability and the inflation rate variability is even higher than in the preceding case. In addition, among the covariances contained in the theoretical model, the only covariance that is still significant at a 1% level is the covariance between the real devaluation rate and the GDP variation rate. In general, the signs found with this procedure coincide with those that might be expected under economic theory (see Exhibit 2), although in most cases the estimates are not statistically significant.

It may be thus concluded that, from a tax-smoothing standpoint, given the stochastic nature of the shocks having an impact on the budgetary structure of the Uruguayan government, foreign currency debt does not offer desirable properties. This debt not only introduces “noise” in the government’s budget equation (given the high variability of its *ex post* cost) but also involves a pro-cyclic behavior (a fall in revenues associated to a recession will be simultaneous to a rise in debt service resulting from the real devaluation of the currency) and cannot operate as an insurance against shocks in the international interest rate or an unexpected variation in the real wage (although  $\sigma_{q,w}$  has the desirable sign, it is not significant at a 5% level). These results seem to be quite strong, and hold for different sub-periods and for different VAR specifications.

On the other hand, this study shows that issuing nominal debt in local currency would be meaningless as regards a minimization of the global fiscal risk. The *ex post* yield of this kind of debt does not seem to be correlated with the macroeconomic

variables having an impact on the national budget. We might even state that in the case of Uruguay, nominal instruments denominated in Pesos do not operate as an insurance against unexpected shocks in the international interest rate, the GDP, or the real wage.

As a corollary of the empirical analysis, we see that indexed debt would be in a position to play a major role in Uruguay, given the stochastic structure of the shocks having an impact on the government's budget equation. Such debt would allow stabilizing the real debt service independently of the evolution of the inflation rate and the real devaluation rate. Using these instruments would allow achieving a more predictable pattern of real expenditures, and this would contribute to stabilizing the aliquot of taxes over time.

## **III.2 Gauging the Model**

### **III.2.a Risk Premium Values**

Departing from the above results, we must find to what extent indexed debt will be desirable if the different *ex ante* costs of these instruments are incorporated to the analysis. To such ends, we must make certain assumptions on the premium associated to local currency and indexed papers.

As regards the additional cost of the Peso-denominated debt, it is impossible to make an estimation for Uruguay, given the absence of nominal long-term instruments<sup>13</sup>. In this sense it is worth recalling that, even if not directly incorporated to the model, we are implicitly assuming that public debt involves a high share of long-term instruments.

A likely procedure would be to refer to empirical studies conducted in other countries. Thus, for instance, in the United Kingdom, Deacon and Derry (1994) estimated that the inflation risk premium resulting from the rate spread between conventional debt and indexed debt amounts to some 300 to 500 base points. Similar

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<sup>13</sup> At present, the longest-term local currency instruments in the Uruguayan financial market are one-year time deposits.

evidences seem to have been obtained for Sweden (see Penati, 1995). But we should be extremely careful when extrapolating to Uruguay results obtained in other very different economies. In countries having recorded long-lasting chronic and volatile inflation, as Uruguay, the premium would be likely to be quite higher, especially when long horizons are considered. Thus, we worked with a 400 base points premium, assuming that this value may represent a minimum reference floor ( $p_1 = 0.04$ ).

As regards the indexed instrument, this instrument would not be likely to be associated, in the long term, to a significant risk premium, since it offers a constant real yield to the investor. In other words, these assets are free from the inflation risk. In spite of the above, in the short run an additional cost would indeed be likely to be paid upon the issue of indexed debt, mainly due to two factors. In the first place, there is no development of this kind of instruments in the Uruguayan capital market, and thus a premium will necessarily result from the lack of liquidity of these instruments. In the second place, given the extremely high Dollarization of the Uruguayan economy, we might say that foreign exchange is the natural habitat of investors. In Uruguay, the US Dollar may be considered to be the account unit, and thus, from this standpoint, indexed instruments would continue to involve a risk associated to an uncertainty as to the future evolution of the real rate of exchange. Based on the above, we have chosen to work with a 100 base points premium for the indexed instrument over a similar instrument denominated in US Dollars ( $\phi = 0.01$ )<sup>14</sup>.

### III.2.b Simulation Results

By way of summary, the following chart includes the values of the parameters used as a basis of the simulations:

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<sup>14</sup> This amount seems to be in line with the preferences suggested by private agents in mid 1998 when the Central Bank of Uruguay contemplated the possibility of launching Indexed Bonds in the marketplace.

**Chart 3**

<b>Gauging Base Parameters</b>			
$P_I$	0.040	$S_p^2$	0.001
$\phi$	0.010	$S_q^2$	0.018
		$S_{p,i^*}$	-0.092
$D/GDP$	0.450	$S_{p,y}$	-0.110
$y^e$	0.035	$S_{p,w}$	0.002
$W^e$	0.025	$S_{p,q}$	-0.002
$G/GDP$	0.325	$S_{q,i^*}$	-0.049
$I^*$	0.050	$S_{q,y}$	-0.408
$Q^e$	-0.010	$S_{q,w}$	-0.183

For purposes of being able to draw a conclusion on the currency structure of the Uruguayan public debt in coming years, we have considered the anticipated values of  $b$ ,  $g$ ,  $w$ ,  $i^*$ , and  $q$  rather than their historical values for 1979-1998. In addition, we assumed that the stochastic structure of the economy will be unchanged in the future, and thus we used the variance and covariance values that were estimated in the preceding section.

The optimal values of  $\theta$  and  $\theta^*$  are obtained by substituting the above estimates of the parameters and risk premiums in equations 12 and 13. Given the values used, we find that:

$$\theta = -30.596,$$

$$\theta^* = 0,284$$

The no-restraint solution shows that it would be optimal for the government to incur in indexed debt (and to a lower extent in US Dollar-denominated debt) and grant loans in local currency. This is a logical outcome if we take the high yield of local currency papers into consideration. If the non-financial public sector could grant “expensive” loans (at a real  $r + p_2$  rate) and incur in “cheap” borrowings (at a  $r$  rate), it would be meaningful to have a strong active position in local currency instruments, as

this would allow obtaining financial gains that would in turn allow minimizing the target function. In other words, the benefits obtained through financial intermediation would allow the government to reduce taxes.

To the effects of examining the variation of these results if the values of the premiums were changed, several simulations were conducted. In the first place, we changed the cost of the Peso-denominated paper and kept the cost of the indexed instrument constant (100 base points above the return of the US Dollar security); in a second stage, we introduced a variation in the cost of the indexed paper but kept the yield of the local currency paper constant (400 base points above the return of the US Dollar security). By changing  $p_1$  and  $\phi$ , we obtain:

**Chart 4**

<b>OPTIMAL SOLUTION WITH ALL THREE INSTRUMENTS</b>			
<b><math>q</math></b>	<b><math>p_1</math></b>	<b><math>q^*</math></b>	<b><math>\phi</math></b>
0.282	0.01	0.016	0.005
-1.907	0.011	0.063	0.006
-4.079	0.012	0.113	0.007
-6.219	0.013	0.167	0.008
-8.312	0.014	0.224	0.009
-10.344	0.015	<b>0.284</b>	<b>0.01</b>
-19.194	0.02	0.349	0.011
-28.902	0.03	0.417	0.012
<b>-30.596</b>	<b>0.04</b>	0.490	0.013

It is enough for the premium of the local currency paper to be only slightly higher than the premium of the indexed instrument to fully dismiss the Peso security as a means of indebtedness. The logic underlying this result lies in the fact that, as noted in the previous section, local currency debt cannot operate as an insurance as desirable, and thus if local currency papers become more expensive than indexed bonds, they will fully disappear from the debt portfolio and will hold a key position in the government's

assets portfolio. Although this is a theoretically possible solution, it is not viable in real life, since governments do not obtain resources from financial intermediation.

Turning now to the relevant corner solution, i.e. considering only a choice between the indexed instrument and the US Dollar instrument, given the estimated parameters and the value of the risk premium associated to the indexed instrument, the model suggests that it would be optimal for the government to hold some 20% of its debt in indexed instruments:

$$\theta^* = 0.791,$$

$$1 - \theta^* = 0.209$$

This result is, obviously, sensitive to the additional cost payable on indexed debt.

By introducing changes to  $\phi$  we obtain the following chart:

**Chart 5**

<b>Risk Premium Role: Two Instruments</b>	
<b><math>q^*</math></b>	<b><math>\phi</math></b>
1.276	0.0150
1.035	0.0125
<b>0.791</b>	<b>0.0100</b>
0.546	0.0075
0.301	0.0050
-0.190	0.0000

In this case, we clearly see that as the indexed debt cost falls, there is a rise in its weight in the optimal debt mix. In the extreme case where no premium was paid on this debt, it would be optimal for the whole debt to be held in this kind of instruments. As in the previous case, this is explained by the lack of desirable stochastic properties in the yields of US Dollar debt.

To the effects of examining the changes in the optimal values of  $\theta$  and  $\theta^*$  upon a change in the values of the variances and covariances, several simulations were conducted, which allowed building the following charts:

Chart 5

Sensitivity of the Optimal Portfolio to Certain Selected Moments

$q^*$	$s_q^2$	$q^*$	$s_{qi^*}$
0.622	0.0225	0.771	0.35
0.700	0.02	0.784	0.15
<b>0.791</b>	<b>0.0175</b>	<b>0.791</b>	<b>-0.05</b>
0.931	0.015	0.803	-0.15
1.116	0.0125	0.816	-0.35

$q^*$	$s_{qy}$	$q^*$	$s_{qw}$
0.672	-0.6	0.887	-0.49
0.732	-0.5	0.837	-0.32
<b>0.791</b>	<b>-0.4</b>	<b>0.791</b>	<b>-0.18</b>
0.867	-0.3	0.768	-0.08
0.925	-0.2	0.692	0.18

These were the results that had been anticipated, to which reference was already made in the previous section. It is interesting to emphasize the high sensitivity of the optimal foreign currency share,  $\theta^*$ , to a change in the variance of the real devaluation rate,  $\sigma_q^2$ , and in the covariance between the GDP growth rate and the real devaluation rate,  $\sigma_{qy}$ .

**III.3 Costs Associated to a Dollarized Portfolio**

We have established that, given the properties of the stochastic processes governing the determination of the Uruguayan fiscal deficit, there is at present room for indexed local currency debt. This section is devoted to examining whether holding a fully Dollarized portfolio has been, in the past, the optimal strategy. Even if it is not our intent to precisely quantify the costs associated to holding debt fully denominated in US Dollars, we shall try to make a comparison of debt policy performances in 1979-1985 and 1986-1998. These two periods were chosen due to the different characteristics they show. The first one involves the adoption of the stabilization program, its subsequent

collapse, and the management of the ensuing crisis; the second is rather a sustained growth phase, with gradual stabilization as from 1991.

The most intuitive way to compute the additional cost in this model is comparing the costs associated to the Dollarized portfolio to those that would have been incurred if the optimal portfolio had been held in each period. Building an index of the excess losses per period may give us an indication of the period where the portfolio mix proved to be particularly damaging.

We define index I, in algebraic terms, as:

$$I_t = \frac{V_t(\theta^* = 1) - V_t(\theta^* = \theta_t^*)}{V_t(\theta^* = \theta_t^*)} * 100$$

where  $V_t$  is the loss suffered in period t as a function of the portfolio's parameters.  $\theta_t^*$  is the optimal value of  $\theta^*$  in period t. This index will result in positive values when the losses associated to the Dollarized portfolio are higher than those that would have been associated to the optimal portfolio, and will be nil if the Dollarized portfolio proved to be optimal in the period<sup>15</sup>.

For purposes of computing the index for the different periods, we shall use the average values of the involved variables and we shall consider the way in which these variables interacted over each period. In other words, for purposes of gauging losses we shall use the estimate variance and covariance matrix for each period. Since there is no information available on this parameter for the period under consideration, we assumed that the wider variability of the real exchange rate was linked to higher risk premiums<sup>16</sup>. We assumed the risk premium amounted to 3% in 1979-1985, 1.5% in 1986-1998, and 2.25% in 1979-1998.

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<sup>15</sup> In this period we find a corner solution.

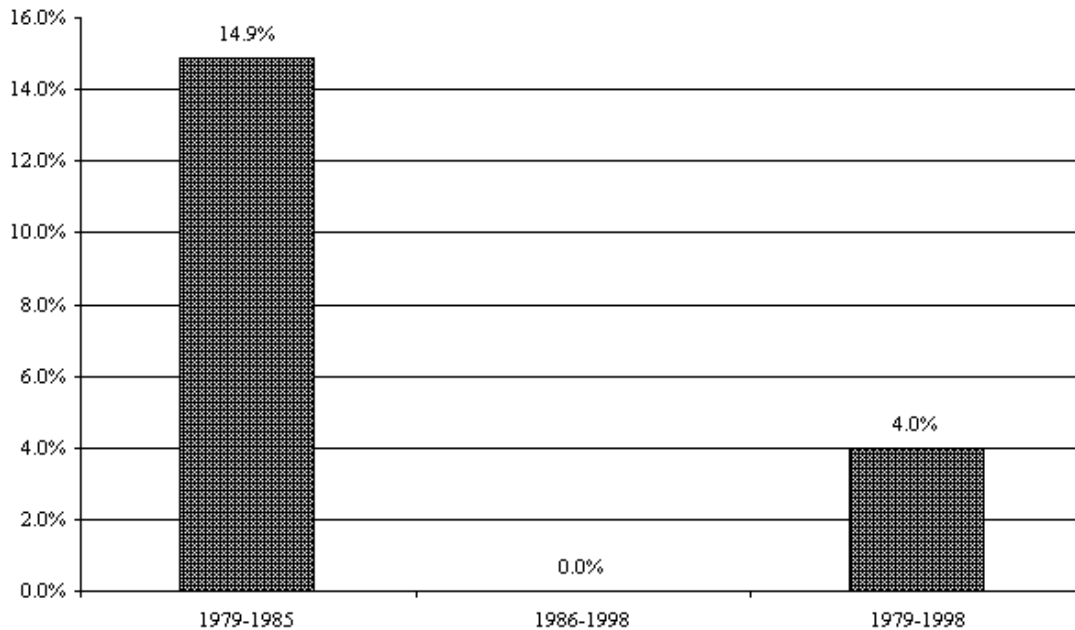
**Assumptions Underlying the Computation of the Index**

	cov q,y	covq,w	cov q, i	var q	$\phi$	var w	var y	cov w y	cov i, y	cov w,i	var i
<b>1979-1985</b>	-0.04738	-0.05233	0.00038	0.56114	0.03000	0.01057	0.00924	0.00476	0.00059	-0.00104	0.00114
<b>1986-1998</b>	-0.00059	-0.00136	0.00008	0.01029	0.01500	0.00213	0.00099	0.00016	0.00001	-0.00003	4E-05
<b>1979-1998</b>	-0.01593	-0.01870	0.00023	0.19440	0.02250	0.00519	0.00387	0.00151	0.00023	-0.00040	0.00041

	<i>g</i>	<i>b</i>	<i>w</i>	<i>y</i>	<i>q</i>	<i>i</i>
<b>1979-1985</b>	35.0%	38.6%	-3.5%	-0.4%	7.3%	12.3%
<b>1986-1998</b>	35.4%	41.4%	1.0%	3.9%	-9.6%	6.3%
<b>1979-1998</b>	35.3%	40.4%	-0.6%	2.4%	-3.7%	8.4%

**Exceso de pérdidas en relación al óptimo (%)**



**Excess Losses as Compared to the Optimal Portfolio (%)**

Graph 1 shows the evolution of index “I”. We can see that the whole loss is concentrated in 1979-1985. Over the period, the poor performance of the fully Dollarized portfolio is associated to the poor behavior of the stochastic processes governing the economy at the time. In the first place, the extremely high variability of

<sup>16</sup> For purposes of obtaining innovations for the first period, we were forced to first extend our

the real exchange rate, driven by the collapse of the *Tablita*, resulted in an extremely high variability of the public budget. In the second place, the joint occurrence of a strong activity level contraction and a dramatic deflation in US Dollar terms resulted in a strong negative covariance between these two variables, which caused the debt cost to rise when the government's revenues were falling. In addition, the covariance between deflation in US Dollar terms and the LIBO rate was positive for the period, which implies that the cost of the US Dollar-denominated debt was rising when the general debt cost was also rising. The simultaneous fall in wages, a determining factor for a negative covariance over the period between this variable and deflation in US Dollars, resulted in a fall in endogenous social security expenditures and public servants' wages, which was not adequate to offset the above-mentioned negative effects of the US Dollar debt.

Conversely to the situation in the previous period, the full Dollarization of the portfolio was the optimal alternative in 1986-1998. In fact, in this period several factors were witnessed which favored incurring in US Dollar-denominated debt. In the first place, the variability of the real exchange rate was significantly lower. The fall in the covariance between US Dollar deflation and the economic activity level, together with a fall in the covariance between the economic activity level and the LIBO rate, shows that, even if  $f$  is assumed to be lower, the US Dollar-denominated debt proved to be more attractive.

For the whole period, given the high variance of  $q$ , the effective debt portfolio contributed an excessive amount of volatility to the budget. In addition, it is worth noting the counter-cyclic behavior of deflation in US Dollar terms and the pro-cyclic

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sample to include data since 1974.

behavior of the international interest rate.. Thus, the fully Dollarized portfolio was suboptimal in 1979-1998.

In this sense, it is also worth noting that the loss for the whole of the sample results from the loss suffered in the Period 1. Between 1979 and 1985, the excess cost of the fully Dollarized portfolio, as measured by index “I”, was more than three times its level in 1979-1998.

As a summary of this analysis broken down per period two issues should be emphasized. In the first place, the stochastic behavior of the economy is essential to determine the optimal portfolio. In the second place, in spite of the moderately advantageous behavior of the Dollarized portfolio during an expansive period, full Dollarization is suboptimal. In fact, one of the most important features that may be deducted from the analysis broken down per period is the extremely poor performance of the debt portfolio in crisis years.

The current debt portfolio is bad because it is associated to a pro-cyclic service cost; but it is even worse because its worst behavior coincides with the worst times for the economy.

#### **IV. Findings and Closing Comments**

This paper clearly shows the need to create new public debt instruments, indexed bonds, as a means to improve the Uruguayan government’s debt management and to reduce the global volatility of the fiscal risk.

The diversification of the debt portfolio should be given priority among public debt management policies in the coming years. The existing extremely high concentration of securities denominated in foreign currency may only be explained by a government willing to minimize *ex ante* debt costs; but this solution has proven not to

be optimal if a wider objective implying the minimization of global budget risks is taken into consideration.

At present, the Uruguayan government as a whole is highly exposed to the exchange risk.. Indexed debt would allow transferring a portion of such risk to the holders of public securities. In addition, issuing indexed papers would allow stabilizing the real debt service. Under the model handled in this paper, using indexed securities would allow achieving a more predictable real expenditure pattern, and this would in turn help keeping stable tax aliquots over time.

Although some might argue that indexed papers would cause inflation to last longer, this allegation does not seem to be appropriate in today's world, in particular given the economic performance of the countries presently issuing this kind of papers. The countries that have recently issued indexed debt are precisely those that have historically had a low inflation level, like Australia, New Zealand, England, Sweden, and the USA. Issuing indexed bonds is even more meaningful in stable macroeconomic environments.

This paper has particularly emphasized tax smoothing allegations to promote issuing indexed debt. It is worth however noting, as we get to the end of the paper, that other allegations also favor issuing indexed securities. In the first place, the introduction of indexed papers may be considered as a means to complete financial markets. In theory, instruments may be said to help completing the markets if they create return patterns that could not be created by a mix of the existing papers. Thus, investors may be insured against certain nature states in a manner that was not available before. Indexed bonds do, doubtless, meet this definition, and may thus contribute to improving the general wellbeing of society.

Only the government is in a position to create a market for indexed instruments, given their associated short-term costs. Once the minimum conditions have been set, the private sector will be likely to keep developing such market. In particular, the creation of indexed instruments would benefit borrowers in the non-tradables sector of the economy, since this new financial technology would offer them a hedge against the exchange risk.

Indexed bonds may be particularly attractive for the Managers of Private Retirement Funds. These institutional investors might match these indexed assets against their long-term liabilities (i.e. the retirement savings accumulated in individual accounts). These liabilities are directly linked to the evolution of wages and indirectly to the evolution of the purchasing power of money.

In addition, indexed bonds would eliminate the lack of currency matching that has existed up to this date between the deficits accrued by the social security system and their funding. Up to this date, the government has borrowed in US Dollars to fund social security expenditures denominated in Uruguayan Pesos. The Managers of Private Retirement Funds purchase US Dollars in the market, and then lend such currency to the government. And the government sells these US Dollars to BCU for purposes of funding the Social Security Office, and releases once again Pesos to the market. This mechanism is exercising an additional pressure on the exchange rate. Indexed securities would solve this problem, as the government would no longer need to operate in the exchange market and the social security system would be funded in the same currency.

Finally, it is worth noting that for indexed bonds to be really attractive, a generally accepted indexed account unit is required, following the variation of the CPI with a minimum lag, daily adjusted with a simple and clear formula, a unit that may not

be the object of any sort of manipulation. The existence of such a unit is a necessary condition for the existence of indexed securities, but it is not the only condition required.

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## Exhibit 1 – The Role of Risk Premiums

Deriving function  $\theta^*(\phi)$  for  $\phi$ , we obtain:

$$(*) \frac{\delta\theta^*}{\delta\phi} = \frac{-\Delta\phi^2 + 2\phi(b_1\sigma_q^2 - \Psi) + \Delta\sigma_q^2}{b_1^2(\sigma_q^2 + \phi^2)^2}$$

The sign of this derivative is given by the sign of the numerator. It should be noted that this sign suffers two changes in the two zeroes of the numerator's polynomial. Using second-degree root rules offers us major information in this connection. In the first place, we know that the product of the roots of a second-degree polynomial of the form  $ax^2 + bx + c = F(x)$  is equal to  $c/a$ . In this case the quotient is negative, which implies one root is positive and the other negative. The sum of the roots is equal to  $-b/a$ , in this case a positive number, since  $a$  is negative and  $b$  is positive (for reasonable values of the parameters). We thus know that the roots are not centered around zero, and that they are biased towards positive numbers.

We have established that the share of foreign currency debt grows in the interval between the two roots of the polynomial we are examining. We should however ask whether this is enough to state that growth will occur in the corresponding stretch of  $\theta^*$  (i.e. when  $0 \leq \theta^* \leq 1$ ). To find an answer to the above question, we shall compare the value of the premium that fully Dollarizes the debt portfolio,

$$\phi_{\theta^*=1} = \frac{b_1\sigma_q^2 - \Psi}{\Delta},$$

where the value of  $\phi$  corresponds to the positive root of the polynomial in the numerator of (\*):

$$\bar{\phi}_2 = \frac{(b_1\sigma_q^2 - \Psi) + \sqrt{(b_1\sigma_q^2 - \Psi)^2 + 4\Delta^2\sigma_q^2}}{2\Delta}.$$

The difference between these two values is given by

$$\bar{\phi}_2 - \phi_{\theta^*=1} = \frac{\sqrt{(b_1\sigma_q^2 - \Psi)^2 + 4\Delta^2\sigma_q^2}}{2\Delta} - \frac{b_1\sigma_q^2 - \Psi}{2\Delta} > 0.$$

$\bar{\phi}_2$

This function clearly grows; exceeds the 1 mark; at  $\bar{\phi}_2$  reaches a maximum; and then starts to fall. In the boundary, when the premium extends towards infinite,  $\theta^*$  extends towards 1. This curious situation derives from the form chosen for the government's loss function. Given its quadratic form, it is indifferent for the authority to have a tax and a subsidy of the same amount. If the original problem were not subject to restraint (iv), in the interval  $1 < \phi < \bar{\phi}_2$  it would be possible to cut taxes by making an additional placement of foreign currency debt and using the resources thus obtained to grant loans in indexed instruments. This would be an advantageous mechanism if the financial profit derived from such intermediation allowed reducing taxes as necessary to close the government's budget.

Once taxes are zero (at  $\phi = \bar{\phi}_2$ ,  $V(\phi)=0$ ), any additional rise in the risk premium will reduce the share of foreign currency debt in the optimal debt portfolio. In fact, if the premium continues to rise from  $\bar{\phi}_2$ , the optimal strategy to preserve fiscal equilibrium (i.e. to keep  $V(\phi) = 0$ ) would be to gradually cut loans in indexed currency, thus preventing the excess gains created by the mismatching of currencies from resulting in a subsidy to the private sector ( $\tau < 0$ ). In the boundary, placing a very very small amount of credit in indexed currency would be enough to fund all fiscal needs.

## Exhibit 2 - The Anticipated Sign of the Model's Parameters

It is worth asking which are, pursuant to economic theory, the anticipated signs of covariances  $\sigma_{qy}$ ,  $\sigma_{qi^*}$ ,  $\sigma_{qw}$ ,  $\sigma_{q\pi}$ ,  $\sigma_{\pi y}$ ,  $\sigma_{\pi i^*}$ , and  $\sigma_{\pi w}$ . Although in some cases the signs of these parameters seem to be clearly established, in other cases they are uncertain, as they depend on the kind of shock affecting the economy.

To the effects of clarifying the analysis, we are considering four different shocks, and we shall examine the response of each of variables  $q$ ,  $p$ ,  $y$ ,  $i^*$  and  $w$  to each of such shocks. The shocks to be considered are as follows: capital inflows, productivity gains, an improvement in the terms of trade, and a positive demand shock (expansive fiscal or monetary policy).

### Covariances whose anticipated signs are clearly established:

1)  $\sigma_{qy} < 0$  – In the case of capital inflows, the GDP growth rate should rise and the real exchange rate –approximated through the US Dollar/CPI ratio- should appreciate. Domestic productivity gains would have similar results. The GDP rises under the twofold effect of a rise in productivity and a rise in the labor used, and the real exchange rate appreciates due to the productivity gain as regards our trade partners. An improvement in the terms of trade increases the available income and, thus, the demand for goods and the GDP. The effect on the real exchange rate is a little harder to identify, but insofar as there is a rise in the demand for non-tradables, the real exchange rate should be expected to fall. Demand shocks also result, in the short run, in an expansion of the GDP and an appreciation of the real exchange rate.

2)  $\sigma_{qi^*} > 0$  – A fall in the nominal international interest rate failing to be encompassed by a fall in the anticipated inflation rate would result in an outflow of capitals from developed countries towards emerging markets. This fall should be, thus, associated to a higher inflow of capitals to Uruguay. An appreciation of the exchange

rate would result from the higher supply of foreign exchange. Thus, the covariance between  $q$  and  $i^*$  should be positive.

3)  $\sigma_{\pi i^*} < 0$  – From the above, it is evident that capital inflows exercise pressures on the price of domestic goods, and thus the inflation rate would tend to rise. A fall in  $i^*$  should be encompassed by a rise in  $\pi$ .

Covariances with uncertain signs:

4)  $\sigma_{\pi y}$  ? – Capital inflows or demand shocks should result in a positive association between the inflation rate and the GDP growth rate. Yet, supply shocks, as for instance productivity gains, would yield the opposite sign. We are also uncertain as to the effect of the variations in the terms of trade on domestic inflation. We conclude, thus, that the sign of this covariance is uncertain, depending on the relative importance of the shocks affecting the economy.

5)  $\sigma_{\pi w}$  ? – The same applies to the relationship between inflation and the real wage. On the one hand, if we take capital inflows into consideration, the covariance between  $\pi$  and  $w$  should be expected to have a positive sign. On the other hand, if we assume that the economy is mostly affected by productivity shocks, the opposite sign should be anticipated.

6)  $\sigma_{q w}$  ? – A real appreciation is usually encompassed by a rise in the real wage. Yet, in theory, this should not always be the case. If, for instance, the government implements an unexpected expansion of the amount of money, assuming that nominal wages are predetermined, this monetary expansion would lead to a rise in inflation, a fall in the real wage, and a higher production level in the short term. In this case, the covariance between  $q$  and  $w$  would have a positive sign.

7)  $\sigma_{\pi q}$  ? – The covariance between inflation and the real exchange rate is also uncertain under economic theory considerations. In the event of major supply shocks we

would find a positive covariance, but if demand shocks prevail or if the economy faces strong foreign capital inflows, the sign of this covariance should be positive [sic].

The above comments may be summarized in the following double-entry chart:

**Chart 6**

<i><b>Shock/Covar.</b></i>	$\sigma_{q,y}$	$\sigma_{q,w}$	$\sigma_{q,\pi}$	$\sigma_{y,\pi}$	$\sigma_{w,\pi}$	$\sigma_{q,i^*}$	$\sigma_{i^*,\pi}$
<i>Capital Inflows</i>	-	-	-	+	+	+	-
<i>Supply Shock (Productivity Gain)</i>	-	-	+	-	-	0	0
<i>Improvement in Terms of Trade</i>	-	-	?	?	?	0	0
<i>Demand Shock (Fiscal/Monetary Expansion)</i>	-	+	-	+	-	0	0
<b>Average</b>	-	?	?	?	?	+	-

It is worth noting that theoretical correlations are likely to be diluted in actual data, due to the lags existing in the responses of the different variables in the real world.

### Innovation Series – Sliding VARs

