Hyperbolic Discounting, Wealth Accumulation, and Consumption

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Abstract. Laboratory and field studies of time preference find that discount rates are much greater in the short-run than in the long-run. Hyperbolic discount functions capture this property. This paper presents simulations of the savings and asset allocation choices of households with hyperbolic preferences. The behavior of the hyperbolic households is compared to the behavior of exponential households. The hyperbolic households hold relatively more illiquid wealth and relatively less liquid wealth. The hyperbolic households borrow much more frequently in the revolving credit market. The hyperbolic households exhibit greater consumption-income comovement and experience a greater drop in consumption around retirement. Moreover, the hyperbolic simulations match observed consumption and balance sheet data much better than the exponential simulations.

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1. Introduction

Our preferences for the long-run tend to conflict with our short-run behavior. In the long-run we intend to clean our offices more regularly, exercise more frequently, and eat more healthfully. But in the short-run we have little interest in filing paperwork, jogging on the stairmaster, and skipping the chocolate soufflé à la mode. Delay of gratification is a nice abstract goal, but instantaneous gratification is disconcertingly tempting.

This gap between long-run intentions and short-run actions is also reflected in our savings decisions. For example, a 1997 survey by Public Agenda found that three quarters of respondents (76%) believe they should be saving more for retirement. Looking only at respondents who believed they were at an age where "You should be seriously saving already," the survey found that 55% reported being "Behind" in their savings and only 6% reported being "Ahead" (Farkas and Johnson, 1997). Such results led the authors of the Public Agenda report to conclude, "The gaps between people's attitudes, intentions, and behavior are troubling and threaten increased insecurity and dissatisfaction for people when they retire. Americans are simply not doing what logic — and their own reasoning — suggests they should be doing." A 1993 Luntz Webber/Merril Lynch survey found similar answers when it asked baby boomers "What percentage of your annual household income do you think you should save for retirement? ('Target saving')" and "What percentage of your annual household income are you now saving for retirement? ('Actual saving')." The median reported short-fall between target and actual saving was 10% and the mean gap was 11.1% (Bernheim 1995).

This survey evidence resonates with popular and professional financial planning advice. Financial planners clearly recognize self-control limits when they advise consumers to "use whatever means possible to remove a set amount of money from your bank account each month before you have a chance to spend it."[1] Advisers believe that consumers often make savings decisions that they are then incapable of carrying out. Consumers heed this advice, sometimes willingly fore-

[1] *New York Times,* 'Your Money' column, Rankin (1993).

going liquidity even in the absence of higher returns. Without invoking some form of self-control problem, it is not easy to explain 10,000,000 Christmas club accounts and intentional income tax overwithholding.[2]

However, it is difficult to document convincingly the impact of self-control problems. Much of the evidence described above is either anecdotal or based on attitudinal survey questions that are hard to interpret. Almost all mainstream economic analysis is based on consumer *choices*, not consumer *attitudes* about ambiguous concepts like "target saving."

In this paper we provide new *behavioral* evidence for the role of self-control problems. To structure our exploration, we adopt a conceptual framework that integrates a standard economic theory of lifecycle planning and a psychological model of self-control. This integrated model achieves two goals. First, it provides a parsimonious formal framework in which to quantitatively evaluate the effects of self-control problems. Second, it makes testable predictions about consumer choices, enabling us to run empirical horse races between our self-control model and the standard consumption model.

We build our framework on three organizing principles. First, our model adopts the technological assumptions of mainstream consumption models like those originally developed by Carroll (1992, 1997), Deaton (1991), and Zeldes (1989b). These authors assume stochastic income and incomplete markets — consumers can't borrow against uncertain future labor income. We extend this literature allowing consumers to borrow on credit cards. Second, we include a partially illiquid asset in the consumers' menu of investment options. Third, we assume that consumers have a short-run preference for instantaneous gratification and a long-run preference to act patiently — "hyperbolic" discounting.

Such hyperbolic time preferences set up a conflict between today's preferences and the preferences which will be held in the future. From today's perspective, the discount rate between two

[2] For evidence on Christmas club use, see Simmons Market Research Bureau (1996). For evidence on widespread overwithholding, even in the absence of underwithholding penalties, see Shapiro and Slemrod (1995).

far off periods, t and $t+1$, is a long-term low discount rate. However, from the time t perspective, the discount rate between t and $t+1$ is a short-term high discount rate. This type of preference 'change' is reflected in many common experiences. For example, today I may desire to cut back on credit card expenditures next month (i.e., act patiently in the future), but when next month actually rolls around my taste at that time will be to further postpone any sacrifices.

Hence, hyperbolic consumers will report a gap between what they feel they should save and what they actually save. Prescriptive saving rates will lie above actual savings rates, since short-run preferences for instantaneous gratification will undermine the consumer's effort to implement long-run optimal plans. However, the hyperbolic consumer is not doomed to face poverty in retirement. Illiquid assets can help the hyperbolic consumer commit herself to the patient, welfare-enhancing course of action. Hence, the availability of illiquid assets becomes a critical determinant of household savings and welfare. However, too much illiquidity can be problematic. Consumers face substantial uninsurable labor income risk, and need to use liquid assets to smooth their consumption. Hyperbolic agents seek an investment portfolio that strikes the right balance between commitment and flexibility.

In the body of this paper we present lifecycle buffer stock simulations of the savings and asset allocation choices of households with hyperbolic preferences. The behavior of the hyperbolic households is compared to the behavior of exponential households. Both sets of households are calibrated to hold levels of pre-retirement wealth that match observed levels of wealth holding in the Survey of Consumer Finances. Despite the fact that our calibration imposes identical levels of total wealth for hyperbolics and exponentials, numerous differences between the predictions of these two types of models nevertheless arise.

First, the hyperbolic households invest comparatively less of their wealth in liquid assets. They hold relatively low levels of liquid wealth measured either as a fraction of labor income or as a share of total wealth. Analogously, hyperbolic households also borrow more aggressively in the revolving credit market (i.e., on credit cards). The low levels of liquid wealth and high rates of credit card borrowing generated by hyperbolic simulations match empirical measures from the SCF much better than the results of exponential simulations.

Because the hyperbolic households have low levels of liquid assets and high levels of credit card debt, they are unable to smooth their consumption paths in the presence of predictable changes in income. We use data from our calibrated models and from the Panel Study of Income Dynamics to regress consumption growth on expected income growth. Our calibrated hyperbolic simulations generate coefficients of approximately 0.25, very close to empirical coefficients estimated from the PSID. By contrast, our calibrated exponential simulations generate comovement coefficients of only .03. Similarly, hyperbolic simulations generate substantial drops in consumption around retirement, matching empirical estimates. The exponential simulations fail to replicate this pattern. All in all, our analysis suggests that the hyperbolic buffer stock model matches observed consumption data better than the exponential buffer stock model.

The rest of the paper substantiates these claims. In section 2, we discuss the hyperbolic discount function. In section 3, we present our benchmark model, which can accomodate either exponential or hyperbolic preferences. In section 4 we provide some analytic results that help us evaluate the model's predictions and provide intuition for the simulations that follow. In section 5 we present our simulation results and empirical comparisons. In section 6 we conclude.

2. Hyperbolic Discounting

Robert Strotz (1956) first suggested that discount rates are higher in the short-run than in the long-run. Almost every experimental study on time preference has supported his conjecture (Ainslie 1992). When two rewards are both far away in time, in general decision-makers act relatively patiently (*e.g.,* I prefer two apples in 101 days, rather than one apple in 100 days). But when both rewards are brought forward in time, preferences exhibit a reversal, becoming much more impatient (I prefer one apple right now, rather than two apples tomorrow).[3] Experiments of this form have

[3]This example is from Thaler (1981).

been done with a wide range of real rewards, including money, durable goods, fruit juice, sweets, video rentals, relief from noxious noise, and access to video games.[4] Such reversals should be well-understood by everyone who willfully sets the alarm clock the night before, only to oversleep — with the help of the snooze button — the morning after.

When researchers use subject choices to estimate the shape of the discount function, the estimates consistently approximate generalized hyperbolas: events τ periods away are discounted with factor $(1+\alpha\tau)^{-\gamma/\alpha}$, with $\alpha, \gamma > 0$ (Loewenstein and Prelec 1992).[5] Figure 1 graphs the generalized hyperbolic discount function with parameters $\alpha = 4$ and $\gamma = 1$. Figure 1 also plots the standard exponential discount function, δ^τ, assuming $\delta = .944$ (the annual discount factor used in our simulations).

In addition, the figure plots a third discount function, which Laibson (1997a) adopted as an analytically tractable approximation of the generalized hyperbolic discount function. This "quasi-hyperbolic function" is a discrete time function with values $\{1, \beta\cdot\delta, \beta\cdot\delta^2, \beta\cdot\delta^3, \ldots\}$. This function approximates the sharp short-run drop in valuation implied by the generalized hyperbolic discount function. The $\beta\delta^\tau$ function was first employed by Phelps and Pollak (1968).[6] However, their use of this structure was motivated in a different way. Their application was one of imperfect intergenerational altruism, and the discount factors apply to non-overlapping generations of a dynasty. Figure 1 plots the particular parameterization, $\beta = .7$ and $\delta = .957$ (the values used in our simulations). Using annual periods, these parameter values roughly match experimentally measured discounting patterns. One year discount *rates* are typically found to exceed $\frac{1}{3} \approx 1-\beta\delta \approx -\ln\beta\delta$. By contrast, long-horizon annualized discount rates are typically found to lie close to $0 \approx 1-\delta \approx -\ln\delta$ (Ainslie 1992).

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[4] See Solnick et al 1980, Thaler 1981, Navarick 1982, Millar and Navarick 1984, King and Logue 1987, Kirby and Herrnstein 1995, Kirby and Marakovic 1995, 1996, Kirby 1997, Read et al 1996. See Ainslie 1992 for a partial review of this literature. See Mulligan 1997 for a critique.

[5] Loewenstein and Prelec (1992) provide an axiomatic derivation of the generalized hyperbolic discount function. See Chung and Herrnstein (1961) for the first use of the hyperbolic discount function. The original psychology literature worked with the special cases $\frac{1}{\tau}$ and $\frac{1}{1+\alpha\tau}$. Ainslie (1992) reviews this literature.

[6] Akerlof (1991) used an intermediate version of this function: $1, \beta, \beta, \beta, \ldots$.

All forms of hyperbolic preferences induce dynamic inconsistency. Consider the discrete-time quasi-hyperbolic function. Note that the discount factor between adjacent periods n and $n+1$ represents the weight placed on utils at time $n+1$ relative to the weight placed on utils at time n. From the perspective of self t, the discount factor between periods t and $t+1$ is $\beta\delta$, but the discount factor that applies between any two later periods is δ. Since we take β to be less than one, this implies a short-term discount factor that is less than the long-term discount factor.[7] From the perspective of self $t+1$, $\beta\delta$ is the relevant discount factor between periods $t+1$ and $t+2$. Hence, self t and self $t+1$ disagree about the desired level of patience which should be used to tradeoff rewards in periods $t+1$ and $t+2$.

Because of this dynamic inconsistency, the hyperbolic consumer is involved in a decision which has intra-personal strategic dimensions. Early selves would like to commit later selves to honor the preferences of those early selves. Later selves do their best to maximize their own interests. Economists have modelled this situation as an intra-personal game played among the consumer's temporally situated selves (Strotz 1956). Recently, hyperbolic discount functions have been used to explain a wide range of anomalous economic choices, including procrastination, contract design, drug addiction, self-deception, retirement timing, and undersaving.[8] We focus here on the implications for lifecycle savings decisions.

In the sections that follow, we analyse two versions of the hyperbolic model – sophisticates and naifs. The sophisticated hyperbolic consumers correctly predict that later selves will not honor the preferences of early selves. By constrast, the naif consumers make current choices under the false belief that later selves will act in the interests of the current self. The naif assumption was first proposed by Strotz (1956) and has been carefully studied by Akerlof (1991) and O'Donoghue and Rabin (1997a, 1997b).

[7]Note that a discount factor, say θ, is inversely related to the discount rate, $-\ln\theta$.

[8]For example, see Akerlof (1991), Barro (1997), Benabou and Tirole (2000), Carillo and Marriotti (2000), Diamond and Koszegi (1998), Laibson (1994,1996,1997a), O'Donoghue and Rabin (1997a, 1997b).

3. MODEL

We analyze a special case of the model developed in Laibson, Repetto and Tobacman (2000), hereafter LRT. This model is based on the simulation literature pioneered by Carroll (1992, 1997), Deaton (1991), and Zeldes (1989b) and extended by Hubbard, Skinner, and Zeldes (1994, 1995), Engen, Gale, and Scholz (1994), Gourinchas and Parker (1999), and Laibson, Repetto and Tobacman (1998).

The LRT (2000) model incorporates most of the features of previous lifecycle simulation models and adds new features, including credit cards, time-varying household size, and illiquid assets. We divide the current presentation of the model into six domains: 1) demographics, 2) income from transfers and wages, 3) illiquid assets, liquid assets, and non-collateralized debt, 4) budget constraints, 5) preferences, and 6) equilibrium. A more general version of the model appears in LRT (2000), which also includes a precise description of the empirical calibration of the model.[9] We summarize those calibration decisions below, but refer interested readers to LRT (2000) for the details. Most of our calibration decisions are standard for the consumption literature except for our calibration of preferences, which is discussed in the second to last subsection below.

3.1. Demographics.

We present results for households whose head has a high school diploma and does not have any more advanced degree. Such households represent roughly 52% of all US households. For comparison, about 26% of household heads do not have a high school diploma, and 22% of household heads have a college degree. We have replicated our analysis for households in these other educational categories, and the conclusions are quantitatively similar (LRT 2000).

Households face a time-varying, exogenous hazard rate of survival s_t, where t indexes age. Survival rates are taken from the life tables of the U.S. National Center for Health Statistics (1993).

Households live for a maximum of $T + N$ periods, where T and N are exogenous variables

[9] This more general model allows consumers to declare bankruptcy and allows the consumer to borrow against illiquid collateral (e.g., mortgages on housing).

that represent respectively the maximum length of pre-retirement life and the maximum length of retirement. We assume that independent economic life begins at age 20, and that consumers live for a maximum of 90 years ($T+N$). If a household is alive at age $20 \leq t \leq T$, then the household is in the workforce. If a household is alive at age $T < t \leq T+N$, then the household is retired. To calculate the typical retirement age we look at households that experienced a transition into retirement in the Panel Study of Income Dynamics (using the Bernheim et al, 1997, definition of retirement). We find that the mean age at which households with only a high school diploma begin retirement is age 63, and hence we set $T = 63$.

We assume that household composition — number of adults and non-adults — varies over the life-cycle. Households always contain a household head and a spouse, but the number of adult and non-adult dependents varies. To calibrate the age-varying number of dependents, we use the PSID and condition on households with a head and a spouse. To construct the predicted life-cycle profile of household size, we smooth the observed profiles of dependent children and dependent adults.

Following Blundell et al (1994), we define effective household size as the number of adults plus 0.4 times the number of children. We assume that the total number of adults is equal to two (head and spouse) plus the number of predicted dependent adults. Our predicted measure of effective household size exhibits a hump shape pattern, with a peak at age 47.

3.2. Income from transfers and wages. Let Y_t represent all after-tax income from transfers and wages. Hence, Y_t includes labor income, inheritances, private defined-benefit pensions, and all government transfers. Since we assume labor is supplied inelastically, Y_t is exogenous. Let $y_t \equiv \ln(Y_t)$. We refer to y_t as "labor income," to simplify exposition. During working life ($20 \leq t \leq T$):

$$y_t = f^W(t) + u_t + \nu_t^W \tag{1}$$

where $f^W(t)$ is a cubic polynomial in age, u_t is an autocorrelated process (AR1), and ν_t^W is iid and normally distributed, $N(0, \sigma^2_{\nu,W})$. During retirement ($T < t \leq T + N$):

$$y_t = f^R(t) + \nu_t^R \tag{2}$$

where $f^R(t)$ is linear in age, and ν_t^R is iid and normally distributed, $N(0, \sigma^2_{\nu,R})$.

These income processes are estimated with data from the Panel Study of Income Dynamics. We include household fixed effects, family size, cohort effects, and state-level unemployment rates as additional control variables. Our estimated parameter values are almost identical to the values reported by Hubbard et al (1994), who estimate an identical after-tax income process.

3.3. Illiquid assets, liquid assets, and non-collateralized debt. Let Z_t represent illiquid asset holdings at age t. The illiquid asset generates two sources of returns: capital gains and consumption flows. We assume that in all periods Z is bounded below by zero, so $Z_t \geq 0$. The liquidation of this asset generates transaction costs, which are described below.

Let X_t represent liquid asset holdings at the beginning of period t, excluding current labor income. So $X_t + Y_t$ represents total liquid asset holdings at the beginning of period t. To model non-collateralized borrowing — i.e., credit card borrowing — we permit X_t to lie below zero, but we introduce a credit limit equal to some fraction of current (average) income

$$X_t \geq -\lambda \cdot \bar{Y}_t$$

where $\bar{Y}_t$ is average income at age t. We calibrate the credit limit $\lambda \cdot \bar{Y}_t$ using actual credit limits reported in the 1995 SCF, setting $\lambda = .30$.

3.4. Dynamic and within-period budget constraints. Let I_t^X represent net investment into the liquid asset X during period t. Let I_t^Z represent net investment into the illiquid asset Z

during period t. Hence the dynamic budget constraints are given by,

$$X_{t+1} = R^X \cdot (X_t + I_t^X) \tag{3}$$

$$Z_{t+1} = R^Z \cdot (Z_t + I_t^Z) \tag{4}$$

where R^X and R^Z are the gross real after-tax interest rates, respectively, on liquid wealth and illiquid wealth. We assume that the interest rate on liquid wealth depends on whether the consumer is borrowing or saving in her liquid accounts. We interpret liquid borrowing as credit card debt.

$$R^X = \begin{cases} R^{CC} & \text{if} \quad X_t + I_t^X < 0 \\ R & \text{if} \quad X_t + I_t^X > 0 \end{cases}.$$

Naturally, R^{CC} is the interest rate on credit card debt, and R represents the interest rate on positive stocks of liquid wealth. The within-period budget constraint is:

$$C_t = Y_t - I_t^X - I_t^Z - \psi(I_t^Z),$$

where $\psi(I_t^Z)$ is the transaction cost on the sale of Z. Hence, the state variables at the beginning of period t are liquid wealth $(X_t + Y_t)$, illiquid wealth (Z_t), and the value of the Markov process (u_t). The choice variables are net investment in liquid wealth (I_t^X) and net investment in illiquid wealth (I_t^Z). Consumption is calculated as a residual.

We set the value of the after-tax real return on liquid savings, $R - 1$, equal to .0375 (3.75 percentage points). This assumes that liquid assets are invested in a diversified portfolio of stocks and bonds ($\frac{2}{3}$ stocks and $\frac{1}{3}$ bonds), and that the effective tax rate on real returns is 25%.

We set the real return on credit card loans, $R^{CC} - 1$, to 0.1075, three percentage points *below* the mean debt-weighted real interest rate measured by the Federal Reserve Board. We do this to implicitly capture the effect of bankruptcy. Actual annual bankruptcy rates of roughly one percent per year imply that the effective interest rate is at least one percentage point below the observed interest rate.

We set the real return on illiquid assets to 0, but assume that illiquid assets generate a consumption flow equal to 5.00 percent of the value of the illiquid asset. Hence, illiquid assets have the same pre-tax gross return as liquid assets, but illiquid assets generate consumption flows that are by-and-large not taxed (e.g., housing). Hence, the "after-tax" return on illiquid assets is considerably higher than the after-tax return on other assets.

We assume an extreme form of transaction costs:

$$\psi(I^Z) = \begin{cases} 0 & \text{if} \quad I^Z > 0 \\ \$10,000 + (.10) \cdot I^Z & \text{if} \quad I^Z \leq 0 \end{cases} .$$

In other words, purchases of the illiquid asset generate no transaction costs, but sales generate a \$10,000 fixed cost and a .10 proportional cost. These costs were chosen to match those that arise in real estate transactions, including brokerage commissions, legal fees, bank fees, moving costs, and search costs.

3.5. Preferences. We assume that the discount function is quasi-hyperbolic ($\beta\delta^t$). To analyze the decisions of an agent with dynamically inconsistent preferences, we must specify the preferences of all of the temporally distinct selves. We index these selves by their lifecycle position, $t \in \{20, 21, ..., T+N-1, T+N\}$. Self t has instantaneous payoff function

$$u(C_t, Z_t, n_t) = n_t \cdot \frac{\left(\frac{C_t + \gamma Z_t}{n_t}\right)^{1-\rho} - 1}{1-\rho}$$

and continuation payoffs given by:

$$\beta \sum_{i=1}^{T+N-t} \delta^i \left(\Pi_{j=1}^{i-1} s_{t+j}\right) \left[s_{t+i} \cdot u(C_{t+i}, Z_{t+i}, n_{t+i}) + (1 - s_{t+i}) \cdot B(X_{t+i}, Z_{t+i})\right]. \tag{5}$$

Note that n_t is the effective household size,

$$n_t = ([\# \text{ adults}_t] + \kappa[\# \text{ of children}_t]),$$

where κ is the adult-equivalence weight on children, ρ is the coefficient of relative risk aversion, γZ_t represents the consumption flow generated by Z_t, s_{t+1} is the probability of surviving to age $t+1$ conditional on being alive at age t, and $B(\cdot)$ represents the payoff in the death state, which incorporates a bequest motive.[10] The first expression in the bracketed term in Equation 5 represents utility flows that arise in period $t+i$ if the household survives to age $t+i$. The second expression in the bracketed term represents termination payoffs in period $t+i$ which arise if the household dies between period $t+i-1$ and $t+i$.

We adopt a utility function with a constant coefficient of relative risk aversion. In our benchmark calibration we set the coefficient of relative risk aversion, ρ, equal to two, a value which lies in the middle of the range of values that are commonly used in the consumption literature (i.e., $\rho \in [.5, 5]$).[11]

In Section 5 we simulate exponential and hyperbolic households. In these simulations we assume that the economy is either populated exclusively by exponential households (i.e., $\beta = 1$) or exclusively by hyperbolic households (either sophisticates or naifs), which we model by setting $\beta = .7$. We set β in accordance with the experimental evidence discussed in Section 2.

Having fixed all of the other parameters, we are left with one free parameter in our exponential simulations, $\delta_{\text{exponential}}$, one free parameter in our hyperbolic-sophisticate simulations, $\delta_{\text{hyperbolic}}$, and one free parameter in our naif simulations, δ_{naif}. In our simulations we pick the various δ values so that our simulations replicate the actual level of pre-retirement wealth holdings. Specifically, we calibrate δ such that the *simulated* median ratio of total wealth to income for individuals between

[10] See LRT (2000) for details on the construction of the bequest payoff function.

[11] See Laibson, Repetto, and Tobacman (1998) for a detailed discussion of calibration of ρ, and an argument that ρ is closer to .5 than to 5.

ages 50 and 59 matches the *actual* median in the data (SCF). When we construct total wealth from the SCF, we include liquid assets (checking accounts, savings accounts, money market accounts, call accounts, CD's, bonds, stocks, mutual funds, cash, less credit card debt), and illiquid assets (IRA's, defined contribution pension plans, life insurance, trusts, annuities, vehicles, home equity, real estate, business equity, jewelry/furniture/antiques, home durables, less education loans). We do not include defined benefit pension wealth, such as claims on the Social Security System. When we measure total wealth in our simulations, we add: $X + Z + \frac{Y}{24}$, where X represents liquid assets (excluding current labor income), Z represents illiquid assets, and Y represents annual after-tax labor income. The last term is included to reflect average cash-inventories used for (continuous) consumption. If labor income is paid in equal monthly installments, $\frac{Y}{12}$, and consumption is smoothly spread over time, then average cash inventories will be $\frac{Y}{24}$.

The SCF data is taken from the 1983, 1989, 1992, and 1995 surveys. We match the mean of the medians across those four years of surveys. The mean median ratio of net wealth to income for individuals between ages 50 and 59 is 3.2 for households whose head's highest educational attainment is a high school education.

The discount factors that replicate this wealth to income ratio are .9437 for the exponential model, .9571 for the sophisticated hyperbolic model, and .9495 for the naif model. Since hyperbolic consumers have two sources of discounting — β and δ — the hyperbolic δ's must be higher than the exponential δ's. Recall that the hyperbolic and exponential discount functions are calibrated to generate the same amount of pre-retirement wealth accumulation. In this manner we "equalize" the underlying willingness to save between the exponential and hyperbolic consumers. The calibrated long-term discount factors are sensible when compared to discount factors that have been used in similar exercises by other authors. Finally, note that these discount factors do not include mortality effects which reduce the respective discount factors by an additional one percent on average per year.

3.6. Equilibrium. When $\beta < 1$ the household has dynamically inconsistent preferences, and hence the consumption problem cannot be treated as a straightforward dynamic optimization problem. If the consumer is *sophisticated*, she will realize that late selves will not implement the policies that are optimal from the perspective of early selves.

Following the work of Strotz (1956), we model a sophisticated consumer as a sequence of rational players in an *intra*-personal game. Selves $\{20, 21, ..., T+N-1, T+N\}$ are the players in this game. Taking the strategies of other selves as given, self t picks a strategy for time t that is optimal from its perspective. This strategy is a mapping from the (Markov) state variables, $\{t,\ X+Y,\ Z,\ u\}$, to the non-redundant choice variables $\{I^X,\ I^Z\}$. An equilibrium is a fixed point in the strategy space, such that all strategies are optimal given the strategies of the other players. We solve for the equilibrium strategies using a numerically implemented backwards induction algorithm.

We also consider the case where early hyperbolic selves mistakenly expect later selves to honor the preferences of the early selves. This is the naif case, discussed above. Such naifs have optimistic forecasts in the sense that they believe that future selves will carry out the wishes of the current self. Under this belief, the current self constructs the sequence of actions that maximizes the preferences of the current self. The current self then implements the first action in that sequence, expecting future selves to implement the remaining actions. Of course, those future selves conduct their own optimization and implement potentially conflicting actions.

In the discussions below we emphasize discussion of the standard exponential case (δ^τ), and the sophisticated hyperbolic case ($\beta\delta^\tau$, with rational beliefs about future selves). We sometimes omit the naif case ($\beta\delta^\tau$, with optimistic beliefs about future selves), since the results of the sophistication case and the naif case are generally quite similar. We refer to sophisticated hyperbolics as "hyperbolics" and naif hyperbolics as "naifs."

4. Analytic Results

In the standard exponential discounting model (i.e., $\beta = 1$ above, with no illiquid assets, and time-varying mortality and household size effects), the equilibrium path satisfies the well-known Exponential Euler Relation

$$u'(C_t) = \mathrm{E}_t R\delta u'(C_{t+1}).$$

Intuitively, the marginal utility of consuming an additional dollar today, $u'(C_t)$, must equal the marginal utility of saving that dollar. A saved dollar grows to R dollars by next year. Utils next period are discounted with factor δ. Hence, the value of today's marginal savings is given by $\mathrm{E}_t R\delta u'(C_{t+1})$. The expectation operator integrates over uncertain future consumption.

Harris and Laibson (2000) show that the Exponential Euler Equation has a natural generalization in the sophisticated hyperbolic economy.[12]

$$u'(C_t) = E_t R\left[\beta\delta\left(\frac{\partial C_{t+1}}{\partial X_{t+1}}\right) + \delta\left(1 - \frac{\partial C_{t+1}}{\partial X_{t+1}}\right)\right]u'(C_{t+1})$$

The difference between the Exponential Euler Relation and the Hyperbolic Euler Relation is that the latter replaces the constant exponential discount factor, δ, by the bracketed term above,

$$\left[\beta\delta\left(\frac{\partial C_{t+1}}{\partial X_{t+1}}\right) + \delta\left(1 - \frac{\partial C_{t+1}}{\partial X_{t+1}}\right)\right].$$

This effective discount factor is a weighted average of the short-run discount factor $\beta\delta$ and the long-run discount factor δ. The respective weights are $\left(\frac{\partial C_{t+1}}{\partial X_{t+1}}\right)$, the marginal propensity to consume out of liquid wealth, and $\left(1 - \frac{\partial C_{t+1}}{\partial X_{t+1}}\right)$. Since $\beta < 1$, the effective discount factor is stochastic and endogenous to the model.

In the sophisticated hyperbolic model, the effective discount factor is negatively related to the

[12]This particular form of the generalization applies when the consumption function is Lipschitz continuous, a property which holds in a neighborhood of $\beta = 1$ (Harris and Laibson 2000).

future marginal propensity to consume (MPC). To gain intuition for this effect, consider a consumer at time 0 who is thinking about saving a marginal dollar for the future. The consumer at time zero — 'self 0' — expects future selves to overconsume relative to the consumption rate that self 0 prefers those future selves to implement. Hence, on the equilibrium path, self 0 values marginal saving more than marginal consumption at any future time period. From self 0's perspective, therefore, it matters how a marginal unit of wealth at time period 1 will be divided between savings and consumption by self 1. Self 1's MPC determines this division. Since self 0 values marginal saving more than marginal consumption at time period 1, self 0 values the future less the higher the expected MPC at time period 1.

The effective discount factor in the Hyperbolic Euler Relation varies significantly over time. Consumers who expect to have low levels of future cash-on-hand will expect $\frac{\partial C_{t+1}}{\partial X_{t+1}}$ to be close to one,[13] implying that the effective discount factor will approximately equal $\beta\delta$. Assuming that periods are annual with a standard calibration of $\beta = .7$ and $\delta = .95$, the effective discount rate would be $-\ln(.7 \times .95) = .41$. By contrast, consumers with high levels of future cash-on-hand will expect $\frac{\partial C_{t+1}}{\partial X_{t+1}}$ to be close to zero,[14] implying that the effective discount factor will approximately equal δ. In this case, the effective discount rate will be $-\ln(.95) = .05$.

Sophisticated hyperbolic consumers have an incentive to keep themselves liquidity constrained (Laibson, 1997a). By storing wealth in illiquid form, hyperbolic consumers prevent themselves from overspending in the future. Early selves intentionally try to constrain the consumption of future selves. This has the effect of raising the future marginal propensity to consume out of the (constrained) stock of liquid wealth. The high marginal propensity to consume generates high effective discount rates ($\approx .41$), explaining why hyperbolics are frequently willing to borrow on credit cards.

[13] Low levels of cash-on-hand imply that the agent is liquidity constrained. Hence, low levels of cash-on-hand imply a high MPC. See Harris and Laibson (2000) for simulated hyperbolic consumption functions in buffer stock models.

[14] When the agent is not liquidity constrained, marginal consumption is approximately equal to the annuity value of marginal increments of wealth. Hence, the local slope of the consumption function is close to the real interest rate.

Sophisticated hyperbolics recognize that illiquid wealth will be spent much more slowly than liquid wealth. Illiquid wealth — e.g., housing — generates marginal utility flows for many periods in the future. The consumer discounts utility flows τ periods away with factor $\beta\delta^{\tau}$. When discounting consumption increments over long-horizons, a hyperbolic consumer uses an effective discount rate of

$$\lim_{\tau\to\infty}\left[-\ln(\beta\delta^{\tau})^{\frac{1}{\tau}}\right] = \lim_{\tau\to\infty}\left[-\frac{1}{\tau}\ln(\beta) - \ln(\delta)\right] = -\ln(\delta).$$

Hence, illiquid wealth accumulation is primarily driven by δ, not β, implying that the consumer accumulates illiquid wealth as if she had a discount rate of $-\ln(\delta) = .05$.

With the potential for effective discount rates of 41% per year, the model predicts widespread borrowing on credit cards at 15% – 20% annual interest rates. However, the hyperbolic model simultaneously predicts that most consumers will accumulate large stocks of illiquid wealth, basing accumulation decisions on a relatively low discount rate of .05. Furthermore, with high levels of credit card debts and low levels of liquid assets, hyperbolic consumers are unable to smooth consumption even if income changes are expected. Thus the hyperbolic model predicts that consumption will track income for most consumers.

5. Simulation Results

We first present simulation results for exponential households ($\beta = 1$, $\delta = .9437$), and then compare these exponential households to simulated hyperbolic households ($\beta = .7$, $\delta = .9571$).[15] Recall that all households — exponential, hyperbolic and naifs — have preferences that are chosen to match the median empirical wealth to income ratios during the pre-retirement years (age 50-59).

Figure 2 plots the mean consumption profile for exponential households. This profile covaries with the mean labor income profile, which is also plotted. This comovement is driven by two

[15] In the figures that follow, we only report the results for sophisticated hyperbolic consumers. The plots for naif consumers are qualitatively the same. We do report the naif results when we compare our simulation results to the empirical data. In this section, we refer to hyperbolic sophisticates as "hyperbolics", and to hyperbolic naifs as "naifs".

factors. First, low income early in life holds down consumption, since consumers cannot borrow against future income.[16] Second, consumption needs peak in mid-life. The number of adult-equivalent dependents reaches a peak of 1.29 at age 47.[17] Figure 3 plots the realized consumption and income paths for a single typical household. This figure demonstrates both high and low frequency consumption-income comovement.

Figure 4 compares the profile of mean consumption of exponential households and the profile of mean consumption of hyperbolic households. These two consumption profiles are almost indistinguishable. The only differences arise at the very beginning of life, around retirement, and at the very end of life. At the beginning of life, hyperbolic consumers go on a credit card financed spending spree,[18] leading to higher consumption than the exponentials. Around retirement, hyperbolic consumption falls more steeply than exponential consumption, since hyperbolic households have most of their wealth in illiquid assets that they cannot cost-effectively sell to smooth consumption. At the end of life, hyperbolic consumers have more illiquid assets to sell, supporting a higher level of late-life consumption.

Figure 5 panel A plots the mean levels of liquid financial assets ($X_t^+ + \frac{Y_t}{24}$), illiquid assets (Z_t), and total assets ($Z_t + X_t + \frac{Y_t}{24}$) for our simulated exponential households. Figure 5 panel B plots the mean level of liquid financial liabilities (X_t^-) for our simulated exponential households. Note that $X_t^+ = \max\{0, X_t\}$ and $X_t^- = \min\{0, X_t\}$. Credit card borrowing, (X_t^-), grows quickly early in life, reaching a temporary steady state around age 30. In the exponential simulations mean balances are always small (compared to empirical benchmarks), with average balances of only \$900. The empirical mean is over \$4,500.[19] Liquid financial assets accumulate until they reach a temporary plateau at age 30. This buffer stock of liquid wealth is used to ride out transitory shocks during

[16] The credit card borrowing limit is .30 of *one* year's income, not enough to smooth consumption over the lifecycle.

[17] The number of children reaches a peak of 2.09 at age 36. The number of dependent adults reaches a peak of .91 at age 51. See LRT (2000) for more details.

[18] See Gourinchas and Parker (1999) for empirical evidence on an early life consumption boom.

[19] This average balance includes households in all education categories. It is calculated on the basis of aggregate information reported by the Federal Reserve. This figure is consistent with values from a proprietary account-level data set assembled by David Gross and Nicholas Souleles (1999a, 1999b, 2000). See LRT (2000).

working life. More liquid wealth is accumulated in the decade before retirement (ages 53-63), to smooth out the drop in labor income at retirement. Illiquid accumulation begins at age 30 and peaks at age 63. Late in life, illiquid wealth is sold, transformed into liquid wealth, and then consumed.

Figure 6 compares the total asset profiles of exponential and hyperbolic consumers. These two profiles are similar, except during the retirement years. Exponential consumers dissave more quickly in retirement, because they have less illiquid wealth in their portfolios. The contrast in illiquid wealth holding is plotted in Figure 7, which shows that relative to exponentials, hyperbolics begin accumulating illiquid wealth earlier, continue accumulating illiquid wealth later, and hence accumulate more. Conversely, hyperbolics accumulate less liquid wealth. Figure 8 (panels A and B) plot the liquid financial assets ($X_t^+ + \frac{Y_t}{24}$) and liquid financial liabilities (X_t^-) of hyperbolics and exponentials. Hyperbolics hold more credit card debt – with average balances of \$3,408 – and less positive financial assets than exponentials. Hyperbolics constrain their own future selves from overconsuming by holding a disproportionately low share of their wealth in liquid form.

In the next sub-section we formally evaluate the differences between the hyperbolic models and the exponential model. In addition, we evaluate the predictions of these models using empirical evidence about wealth accumulation, asset allocation, and consumption over the lifecycle.

5.1. Wealth accumulation and asset allocation. Exponential and hyperbolic consumers make very different asset allocation choices, although exponentials and hyperbolics are calibrated to hold identical amounts of pre-retirement total wealth (recall Figures 6, 7, and 8). Exponentials hold relatively more liquid wealth and hence relatively less illiquid wealth. Although both hyperbolics and exponentials pay "costs" for illiquidity, these costs are at least partially offset for hyperbolics because they value the commitment produced by illiquidity. Hence, on net, illiquidity is more costly for an exponential than for a hyperbolic consumer.

To quantitatively evaluate the differences between hyperbolic and exponential households we calculated the ratio of liquid financial assets to labor income for every household at every age:

$$LWR_{it} = \frac{X_{it}^{+} + \frac{Y_{it}}{24}}{Y_{it}}$$

We then tabulated the percentage of simulated households for whom this liquid wealth ratio, LWR_{it}, is less than $\frac{1}{12}$. In other words, we ask what percentage of households have liquid financial asset holdings that are less than one month of labor income. The results of this tabulation of simulated data are reported in columns 1, 2 and 3 of Table 1. For example, from ages 40 to 49, 28% of simulated exponential households hold liquid financial assets that are less than one month of labor income. The analogous number for hyperbolics is 62%, and 82% for naifs. For comparison, the 1995 SCF implies that between 58% and 74% of actual households hold liquid financial assets that are less than one month of labor income. This range of empirical values arises because we consider a range of definitions of liquid financial assets in Table 1.

In general, the empirical percentages fall slowly from approximately 80% to 60% between ages 20 and 60. The empirical percentages then fall much more rapidly, ending at around 30% after age 70. The hyperbolic model provides a much better fit to the data during the decades between ages 20 and 59. Between ages 60 and 69, the exponential simulations and hyperbolic simulations both do poorly, since the data lie roughly between the predictions of the two models. Only in the period after age 70 does the exponential model beat the hyperbolic model.

For example, consider the intermediate definition of liquid financial assets (column 5) which includes cash, checking accounts, saving accounts and money market accounts. Using this definition the lifecycle profile of the empirical percentages is 81%, 76%, 69%, 59%, 32%, and 29% (ages 20-29, 30-39, 40-49, 50-59, 60-69, and 70 and over, respectively). By contrast, the associated exponential simulation percentages are 48%, 28%, 28%, 24%, 9%, and 23%. Note that this exponential profile lies *everywhere* below the empirical profile, with an average difference of -32%. However, the associated hyperbolic simulation percentages are 66%, 61%, 62%, 57%, 58%, and

54%. This profile crosses the empirical profile, with an average difference of only 2%. Moreover, the exponential simulations would still lose to the hyperbolic simulations even if we chose the most inclusive empirical definition of liquid financial assets (column 5). The naif consumers also come closer to the data than the exponentials, with percentages of 76%, 72%, 82%, 74%, 52%, and 41%, with an average difference of 9%.

We can also evaluate the model by looking at the predicted share of liquid financial assets in household portfolios. In Table 2, we evaluate the liquid wealth share

$$LWS_{it} = \frac{X_{it}^{+} + \frac{Y_{it}}{24}}{X_{it}^{+} + \frac{Y_{it}}{24} + Z_{it}}.$$

The mean hyperbolic liquid wealth share matches the SCF data more closely than the mean exponential liquid wealth share. Indeed, the mean hyperbolic liquid wealth shares always lie *between* the actual data and the exponential share. For example, in column 5 of Table 2 we report the average empirical liquid wealth share, using the intermediate definition of liquid financial assets, which includes cash, checking accounts, saving accounts and money market accounts. Using this definition the lifecycle profile of the empirical shares is 11%, 6%, 5%, 5%, 10%, and 12% (ages 20-29, 30-39, 40-49, 50-59, 60-69, 70+). By contrast, the associated exponential shares are 97%, 65%, 35%, 20%, 27%, and 57%. Note that this exponential profile lies *everywhere* above the empirical profile, with an average difference of 42%. The associated hyperbolic shares are 86%, 46%, 24%, 13%, 12%, and 56%. This profile lies between the exponential profile and the empirical profile. Hence, the hyperbolic simulation outperforms the exponential simulation. But the hyperbolic shares are still far above the empirical shares, with an average difference of 31%. These relative comparisons still arise even when we choose the least inclusive empirical definition of liquid financial assets (column 4). The naifs fall between the hyperbolics and exponentials, with ratios of 93%, 56%, 29%, 15%, 17%, and 56%, for an average difference of 36%.

Another important form of liquidity is revolving credit. Low levels of liquid financial assets are naturally associated with high levels of revolving debt. We next contrast exponential and hyperbolic consumers by comparing their simulated propensities to borrow on credit cards (LRT, 2000). At any point in time only 19% of exponential consumers borrow on their credit cards, compared to 51% of hyperbolics and 60% of naifs. In the 1995 Survey of Consumer Finances, 70% of households with credit cards report that they did not fully pay their credit card bill the last time that they mailed in a payment. Hyperbolics and naifs come much closer to matching these self-reports.

Analogous results arise when we measure the average amount borrowed by exponential and hyperbolic consumers. On average, simulated exponential households owe $900 of credit card debt, *including* the households with no debt. By contrast, simulated hyperbolic households owe $3,408 of credit card debt. Naif consumers owe $4,777 on average. The actual amount of credit card debt owed per household with a credit card is approximately $4,600 (*including* households with no debt, but *excluding* the float). Again, the hyperbolic and naif simulations provide a better approximation than the exponential simulations.

5.2. Comovement of consumption and labor income. To formally measure consumption-income comovement, we ran the following regression using our simulated data:

$$\Delta \ln(C_{it}) = \alpha E_{t-1} \Delta \ln(Y_{it}) + X_{it}\beta + \varepsilon_{it} \tag{6}$$

The control variables, X_{it}, include a full set of age dummies.[20] The first panel of Table 3 reports the results of this regression for simulated exponential consumers. If consumption includes the flow provided by the illiquid asset, the coefficient on $E_{t-1}\Delta \ln(Y_{it})$, α, is around 0.03. If consumption is restricted to the flow of non-durables, then the coefficient rises to 0.058. These estimates are little changed when we include other control variables and drop the age dummies (equation 2 in the Table).

[20]Without age dummies, the estimated coefficient is little changed.

By contrast, estimates of α using *empirical* data lie generally between 0 to .5, with "consensus estimates" around 0.2. For example, Hall and Mishkin (1982) report a statistically significant coefficient of .200, Hayashi (1985) reports a significant coefficient of .158, Altonji and Siow (1987) report an insignificant coefficient of .091, Attanasio and Weber (1993) report an insignificant coefficient of .119, Attanasio and Weber (1995) report an insignificant coefficient of .100, Shea (1995) reports a marginally significant coefficient of .888, Lusardi (1996) reports a significant coefficient of .368, and Souleles (1999) reports a significant coefficient of .344. [21]

We have replicated standard comovement regresssions using the PSID surveys from 1978 to 1992, for different definitions of consumption and for alternative assumptions on the measurement error of income.[22] We report these empirical estimates in Table 4. We use a range of instruments for $E_{t-1}\Delta\ln(Y_{it})$: lagged income, lagged hours worked by the head and spouse, and race and marital status dummies.[23] Almost all of our empirical estimates of α are statistically significant and fall between .19 and .33, in line with the empirical literature, and far above the value implied by the exponential simulation (0.03 to 0.06).

We have also run Equation 6 on the simulated hyperbolic and naif data. The second and third panels of Table 3 report the results of this analysis. The excess sensitivity coefficient, α, is consistently estimated above .16, in line with the available empirical evidence. Since hyperbolic and naif consumers hold low levels of liquid wealth and borrow more on their credit cards, they are much more likely to hit binding liquidity constraints, and hence display greater income-consumption comovement than their exponential counterparts.

We also use our simulations to investigate income-consumption comovement around retirement. Banks et al (1998) and Bernheim et al (1997) argue that consumption anomalously falls during the

[21] See Deaton (1992) and Browning and Lusardi (1996) for discussion of the excess sensitivity literature.

[22] To measure consumption we used the information on food consumption (at home, away from home and food stamps), and on rent and utilities. To interpret the PSID timing of food consumption, we followed Zeldes (1989a) and Shea (1995). Not all PSID surveys collect information on the different consumption categories and on Federal Income Taxes, which we need to estimate after-tax income. Hence, we were forced to use only the surveys with complete information.

[23] Similar sets of instruments were used by Altonji and Siow (1987), Runkle (1991), and Lusardi (1996).

mid-60's, at the same time that workers are retiring and labor income is falling. We estimate the following regression on our simulated data:

$$\Delta \ln(C_{it}) = I_{it}^{\text{RETIRE}}\gamma + X_{it}\beta + \varepsilon_{it}$$

where I_{it}^{RETIRE} is a set of dummy variables that take the value of one in periods $t-1$, t, $t+1$ and $t+2$ if period t is the age of retirement ($t = 63$ in our simulations), and X_{it} is a vector of control variables, including age-contingent mortality rates and changes in effective household size. This regression mimics the analysis of Banks et al (1998), except Banks et al do not include a dummy variable for the age of retirement. Table 5 reports our estimates of the vector γ using simulated exponential and hyperbolic data. For exponential data, we find coefficients of -.0078, -.0107, -.001, and -.0105 for the dummies in years $t-1$, t, $t+1$ and $t+2$, with a total predicted drop in consumption of 3% around retirement. The analogous numbers for sophisticated hyperbolic consumers are -.0225, -.0569, -.0403, and -.0254, with a total predicted drop of 14.5%. The corresponding figures for naif consumers are -.0237, -.0893, -.0323, and -0.0104, with a total fall of 15.6%. Hence, hyperbolic and naif consumers are predicted to experience a much larger consumption drop at retirement, even if retirement is an exogenous, completely predictable event.

These hyperbolic results match analogous empirical estimates. Table 6 reports our empirical estimates of the consumption drops that US households experience around retirement, using data from the PSID. Using our most comprehensive measure of consumption (column 5), the estimated coefficients predict a total drop in consumption of 11.6% around retirement. This drop is little changed if we exclude households who retired at ages 55 or younger; that is, if we exclude households who may have retired unexpectedly early. Again, the hyperbolic and naif consumers come much closer to the observed data than their exponential counterparts.

6. Conclusions

We have compared simulations of exponential, sophisticated hyperbolic, and naif hyperbolic economies. We calibrate our three simulations so that each one matches the median empirical wealth to income ratio during the pre-retirement years.

Relative to exponential households, both types of hyperbolic households hold relatively low levels of liquid wealth measured either as a fraction of labor income or as a share of total wealth. Hyperbolic households also borrow more aggressively in the revolving credit market (i.e., on credit cards). These hyperbolic simulations more successfully match empirical balance sheet data.

Because the hyperbolic households have low levels of liquid assets and high levels of credit card debt, they are unable to smooth their consumption paths in the presence of predictable changes in income. Using simulated data from our calibrated models and empirical data from the Panel Study of Income Dynamics, we regress consumption growth on expected income growth. Our calibrated hyperbolic simulations generate associated coefficients of approximately 0.25, very close to empirical coefficients estimated from the PSID. By contrast, our calibrated exponential simulations generate comovement coefficients of only .03. Similarly, hyperbolic simulations generate substantial drops in consumption around retirement, matching empirical estimates. The exponential simulations fail to replicate this pattern. All in all, our analysis suggests that the hyperbolic buffer stock model matches observed consumption and wealth data better than the exponential buffer stock model.

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Table 1. Percentage of Households with Liquid Assets Less than One Month of Income

	Simulated data			Actual data		
Age Group	Exponential	Hyperbolic Soph.	Hyperbolic Naif	(1)	(2)	(3)
20-29	0,48	0,66	0,76	0,82	0,81	0,74
30-39	0,28	0,61	0,72	0,79	0,76	0,64
40-49	0,28	0,62	0,82	0,74	0,69	0,58
50-59	0,24	0,57	0,74	0,65	0,59	0,50
60-69	0,09	0,58	0,52	0,42	0,32	0,24
70+	0,23	0,54	0,41	0,38	0,29	0,22

Sources: 1995 SCF and authors' simulations.

The table reports the fraction of households who hold less than a month's income in liquid wealth.

(1) Cash, checking and savings accounts.

(2) Definition (1) plus money market accounts.

(3) Definition (2) plus call accounts, CDs, bonds, stocks and mutual funds.

Table 2. Share of Assets in Liquid Form

	Simulations			Survey of Consumer Finances		
Age Group	Exponential	Hyperbolic	Hyperbolic Naif	Definition 1	Definition 2	Definition 3
20-29	0,97	0,86	0,93	0,10	0,11	0,16
30-39	0,65	0,46	0,56	0,05	0,06	0,11
40-49	0,35	0,24	0,29	0,04	0,05	0,08
50-59	0,20	0,13	0,15	0,04	0,05	0,09
60-69	0,27	0,12	0,17	0,09	0,10	0,20
70+	0,57	0,56	0,56	0,09	0,12	0,24

Sources: 1995 SCF and authors' simulations.
Definition 1 includes cash, checking accounts, and savings accounts.
Definition 2 includes definition 1 plus money market accounts.
Definition 3 includes definition 2 plus call accounts, CDs, bonds, stocks and mutual funds.
Illiquid wealth includes all assets not included in the corresponding liquid wealth definition, plus IRAs, DC plans, life insurance, trusts, annuities, vehicles, home equity, real estate, business equity, jewelry, furniture, antiques, and home durables.

Table 3. Consumption-Income Comovement
Simulated Data

		Growth in Consumption[a]	
		Total	Non-Durable
Exponential Simulation			
Equation 1[b]	$ED \ln Y$	0,032	0,058
		(53.2)	(75.7)
Equation 2[c]	$ED \ln Y$	0,035	0,062
		(56.7)	(79.1)
Hyperbolic-Sophisticate Simulation			
Equation 1[b]	$ED \ln Y$	0,166	0,244
		(160.0)	(180.8)
Equation 2[c]	$ED \ln Y$	0,179	0,268
		(170.5)	(194.1)
Hyperbolic-Naif Simulation			
Equation 1[b]	$ED \ln Y$	0,325	0,437
		(232.6)	(255.8)
Equation 2[c]	$ED \ln Y$	0,349	0,476
		(246.4)	(275.0)

Source: Authors' simulations. Standard errors are in parentheses. The results are based on 5000 simulated households over a 70 year life-cycle.

[a]The dependent variable is the change in the natural logarithm of consumption. Total consumption refers to non-durable consumption plus the consumption flow generated by the illiquid asset.

[b] The independent variables are the expected change in the natural logarithm of income, and a full set of age dummies.

[c] The independent variables are the expected change in the natural logarithm of income, the change in effective family size, mortality effects, and a constant.

Table 4. Consumption and Income Comovements
High School Graduates

	Food consumption						Food + Rent[c]			Food + Rent+Utilities[c]		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
ED lnY	0,187	0,186	0,271	0,229	0,230	0,308	0,314	0,297	0,325	0,285	0,293	0,227
	(0.122)	(0.120)	(0.164)	(0.129)	(0.127)	(0.160)	(0.147)	(0.141)	(0.204)	(0.133)	(0.135)	(0.128)
Measurement error	White noise	MA(1)	MA(2)	White noise	MA(1)	MA(2)	White noise	MA(1)	MA(2)	White noise	MA(1)	MA(2)
Instruments												
$\ln Y_{t-2}$	yes			yes			yes			yes		
$\ln Y_{t-3}$	yes	yes		yes	yes		yes	yes		yes	yes	
$\ln Y_{t-4}$	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Other inst.[a]				yes	yes	yes						
Overid test	23,56	14,90		44,63	38,49	29,85	2,36	0,53		7,26	1,04	
p-value	0,00	0,00		0,00	0,00	0,00	0,31	0,47		0,03	0,31	
F test[b]	3,71	3,93	6,23	2,75	2,96	3,37						
p-value	0,01	0,02	0,01	0,00	0,00	0,00						
Number of obs.	12398	12416	12436	12398	12416	12436	11791	11807	11825	10374	10390	10406

Source: 1978-1992 PSID. Standard errors are in parentheses.

The table reports the 2SLS coefficient α on the following regression model:

$\ln(C_t)-\ln(C_{t-1})=\alpha(\ln(Y_t)-\ln(Y_{t-1}))+(\text{age dummies})\beta+(\text{time dummies})\gamma+\delta(\text{family size}_t-\text{family size}_{t-1})+\varepsilon_t$

where C_t is household consumption, and Y_t is household after tax non-asset income.

[a] Other instruments: race and marital status dummies, and lagged once and twice annual head's and spouse's work hours.

[b] F test of the joint significance of the instruments in the first stage regression.

[c] If the household is a homeowner, rent is 0.05 times the market value of the primary residence.

If the household neither owns nor rents, rent is the self-reported rental value of the home if it were rented.

Table 5. Consumption Drops at Retirement[a]
Simulated Data

Consumption growth in period	Total	Non-Durable
Exponential Simulation		
t-1	-.0078	-.0151
	(-6.99)	(-10.51)
t: Retirement year	-.0107	-.0186
	(-9.69)	(-13.06)
t+1	-.0010	-.0182
	(-9.06)	(-12.82)
t+2	-.0105	-.0180
	(-9.89)	(-12.70)
Hyperbolic-Sophisticate Simulation		
t-1	-.0225	-.0443
	(-11.38)	(-16.92)
t: Retirement year	-.0569	-.1078
	(-28.88)	(-41.23)
t+1	-.0403	-.0692
	(-20.57)	(-26.57)
t+2	-.0254	-.0433
	(-13.02)	(-16.68)
Hyperbolic-Naif Simulation		
t-1	-.0237	-.0421
	(-8.57)	(-12.22)
t: Retirement year	-.0893	-.1597
	(-32.37)	(-46.56)
t+1	-.0323	-.0548
	(-11.75)	(-16.04)
t+2	-.0104	-.0159
	(-3.80)	(-4.67)

Source: Authors' simulations.

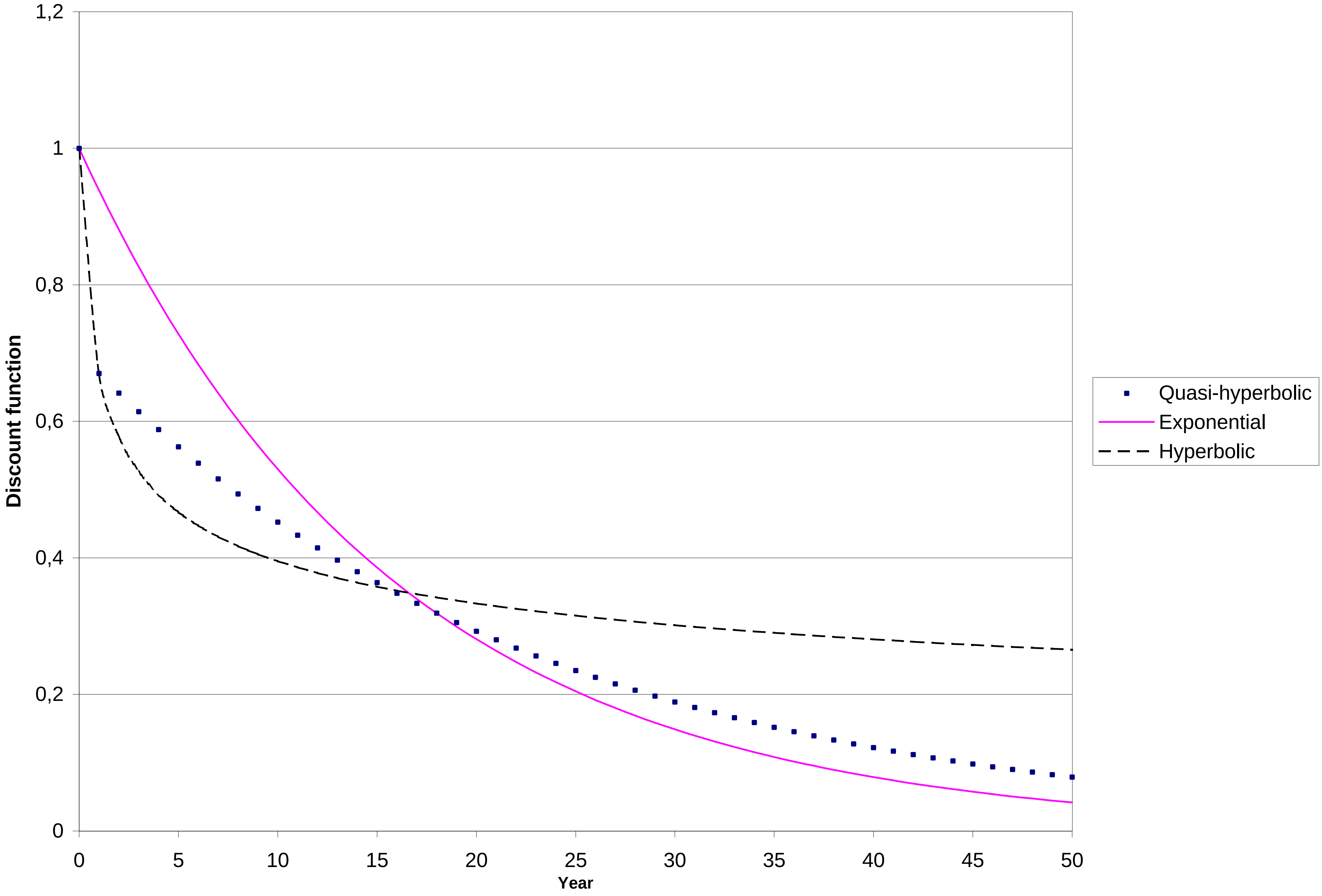
[a] The table reports the coefficients on dummies that are equal to one in years t-1, t, t+1, and t+2, respectively, if the household retired in year t, and zero otherwise. The dependent variable is the change in the natural logarithm of consumption. Total consumption includes non durable consumption and the flow generated by the illiquid asset. Standard errors are in parentheses. All regressions include a constant, the change the effective household size, and mortality effects.

Table 6. Consumption Drops at Retirement[a]
High School Graduates

	Food		Food+Rent[b]		Food+Rent+Utilities[b]	
Cons. growth in year	All	Over 55	All	Over 55	All	Over 55
t-1	-0,027	-0,018	-0,043	-0,038	-0,040	-0,033
	0,038	0,040	0,030	0,032	0,031	0,033
t: Retirement year	-0,001	-0,006	-0,020	-0,017	-0,016	-0,011
	0,027	0,029	0,021	0,022	0,025	0,028
t+1	-0,095	-0,087	-0,051	-0,051	-0,064	-0,065
	0,035	0,037	0,023	0,024	0,026	0,027
t+2	0,031	0,016	0,013	0,011	0,004	0,001
	0,052	0,053	0,033	0,035	0,042	0,045

Source: PSID 1978-1992.
[a]The dependent variable is the change in the natural log of consumption.
The table reports the coefficients on dummies that are equal to 1 in years t-1, t, t+1 and t+2, respectively, if the household retired in year t, and zero otherwise.
All regressions include time dummies, the change in the effective family size, and age-varying mortality effects. The Over 55 sample excludes households who retired at ages 55 or younger.
The retirement dummies are constructed on the basis of the head's self-reported employment status.
[b] If the household is a homeowner, rent is 0.05 times the market value of the primary residence.
If the household neither owns nor rents, rent is the self-reported rental value of the home if it were rented.

Figure 1. Discount functions



Source: Authors' calculations. Exponential: δ^t, with δ=0.939; hyperbolic: $(1+\alpha t)^{-\gamma/\alpha}$, with α=4 and γ=1; and quasi-hyperbolic: $\{1,\beta\delta,\beta\delta^2,\beta\delta^3,...\}$, with β=0.7 and δ=0.957.

Figure 2: Simulated Mean Income and Consumption of Exponential Households

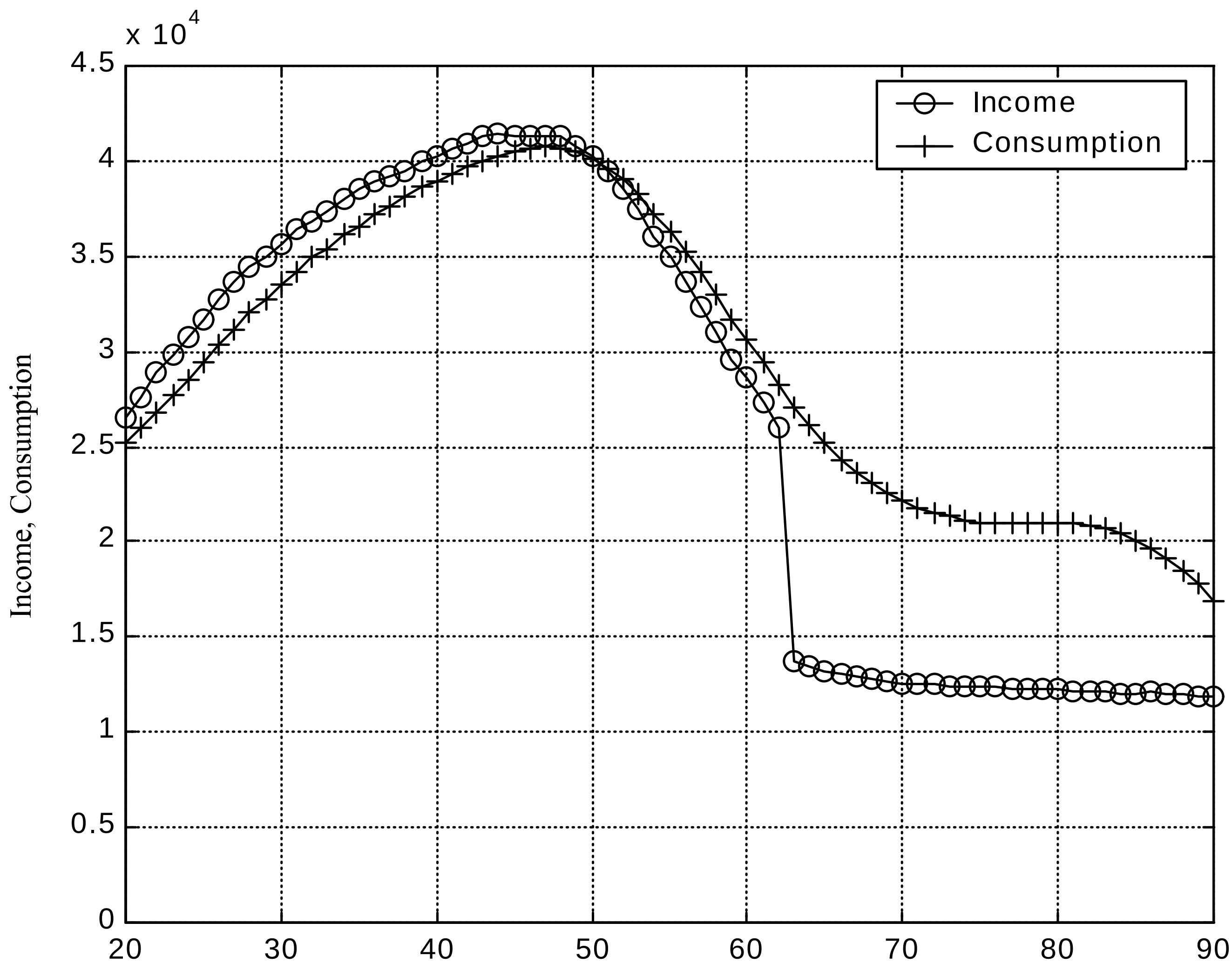


Figure 3: Simulated Income and Consumption of a Typical Exponential Household

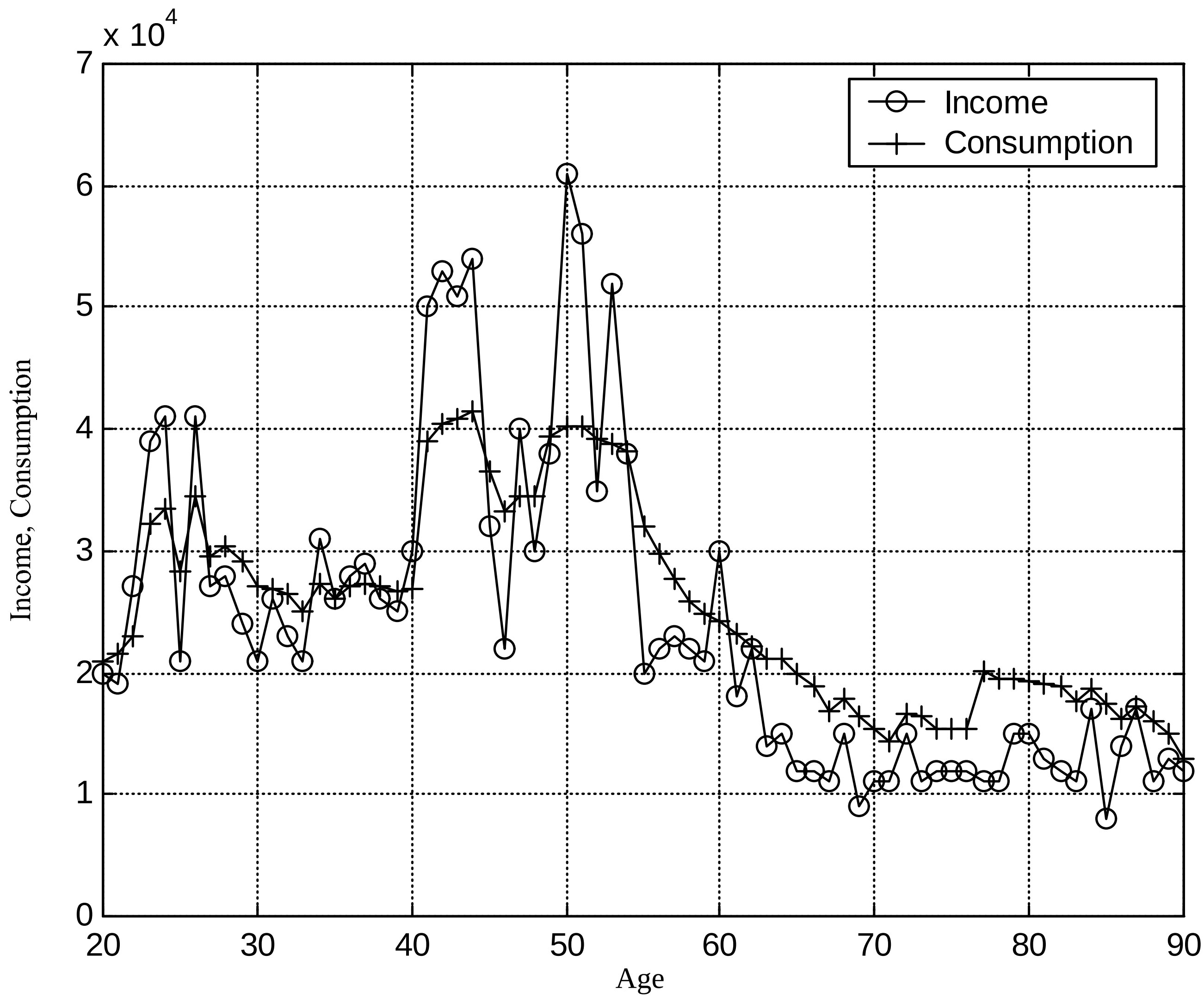


Figure 4: Mean Consumption of Exponential and Hyperbolic Households

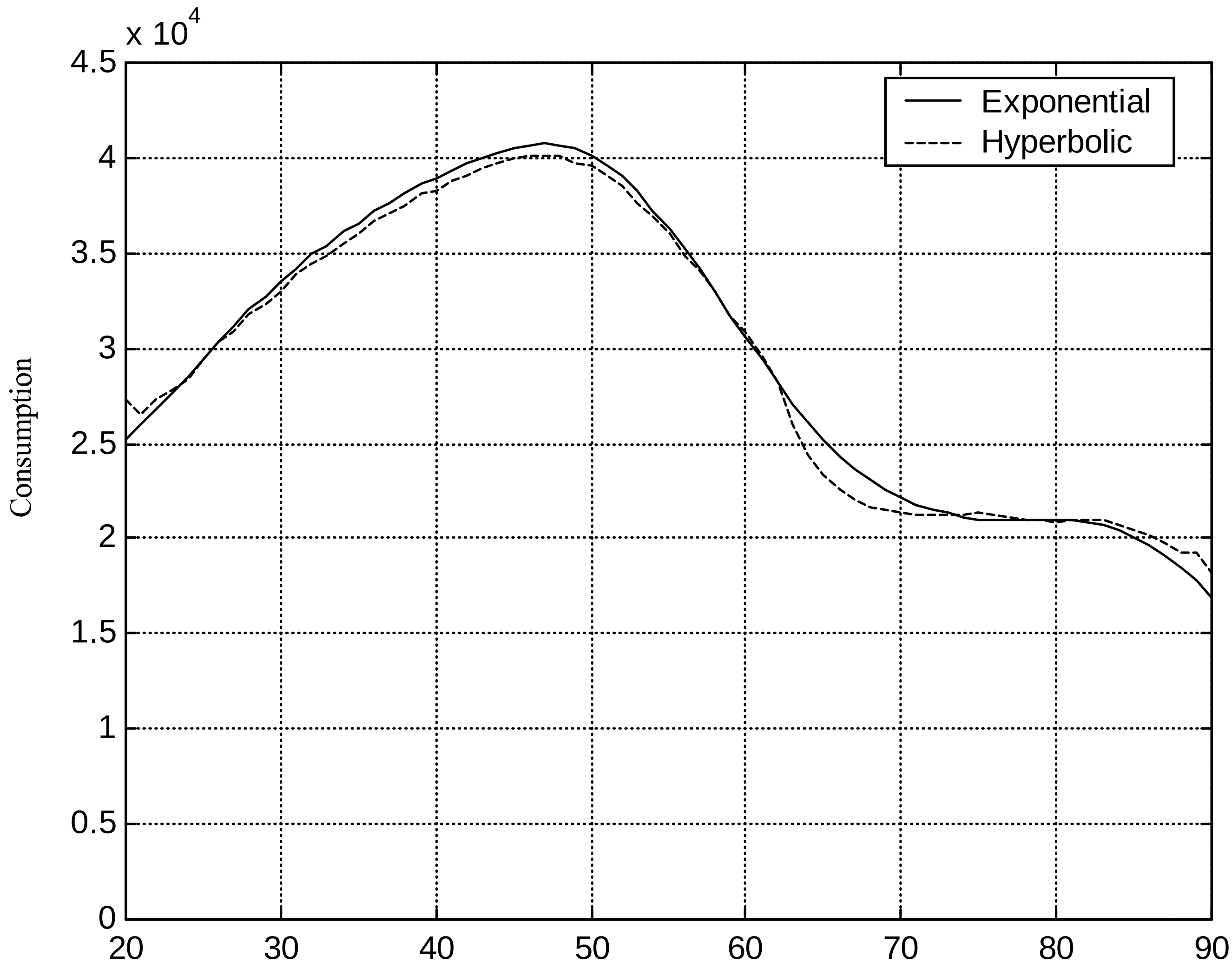
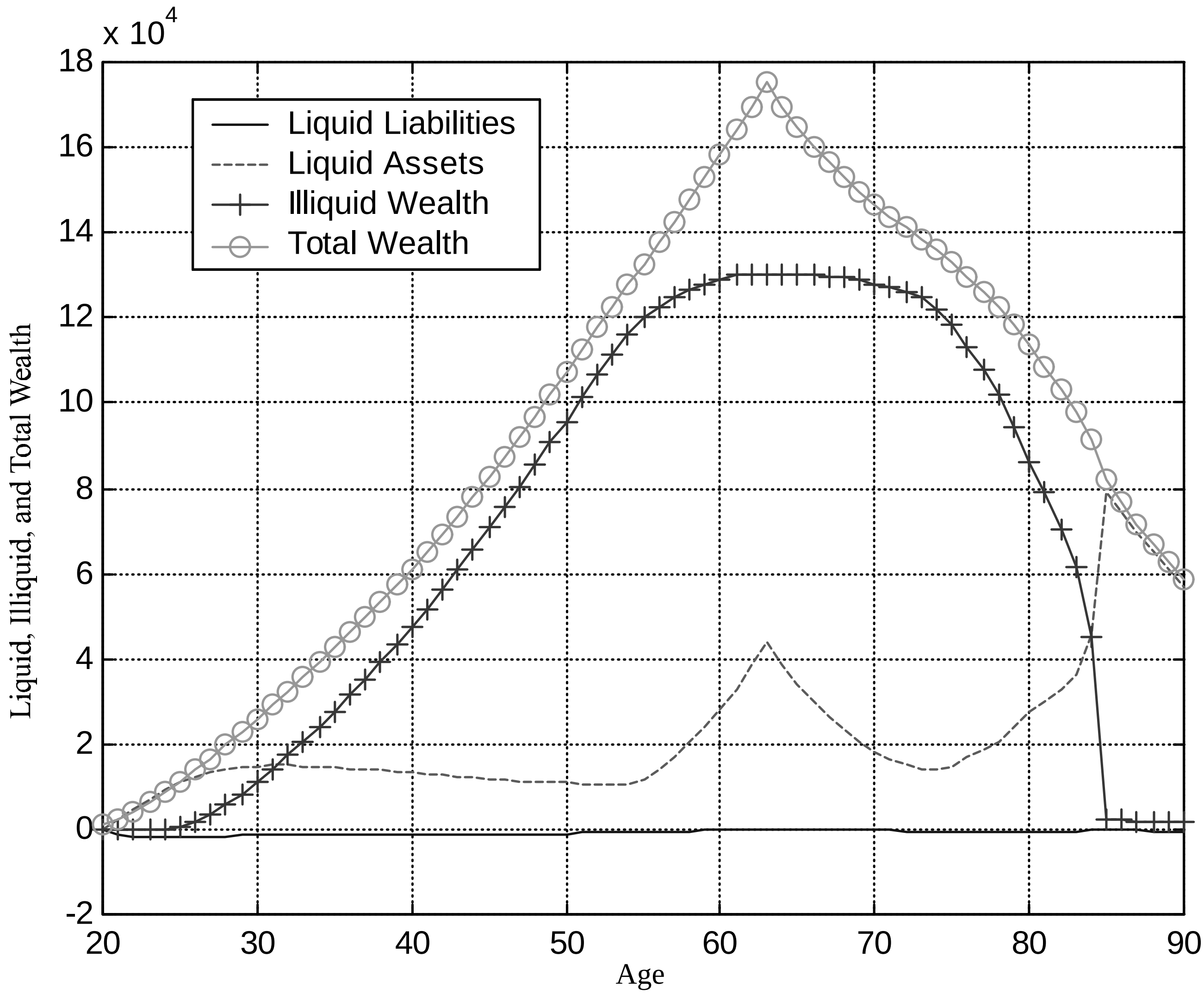
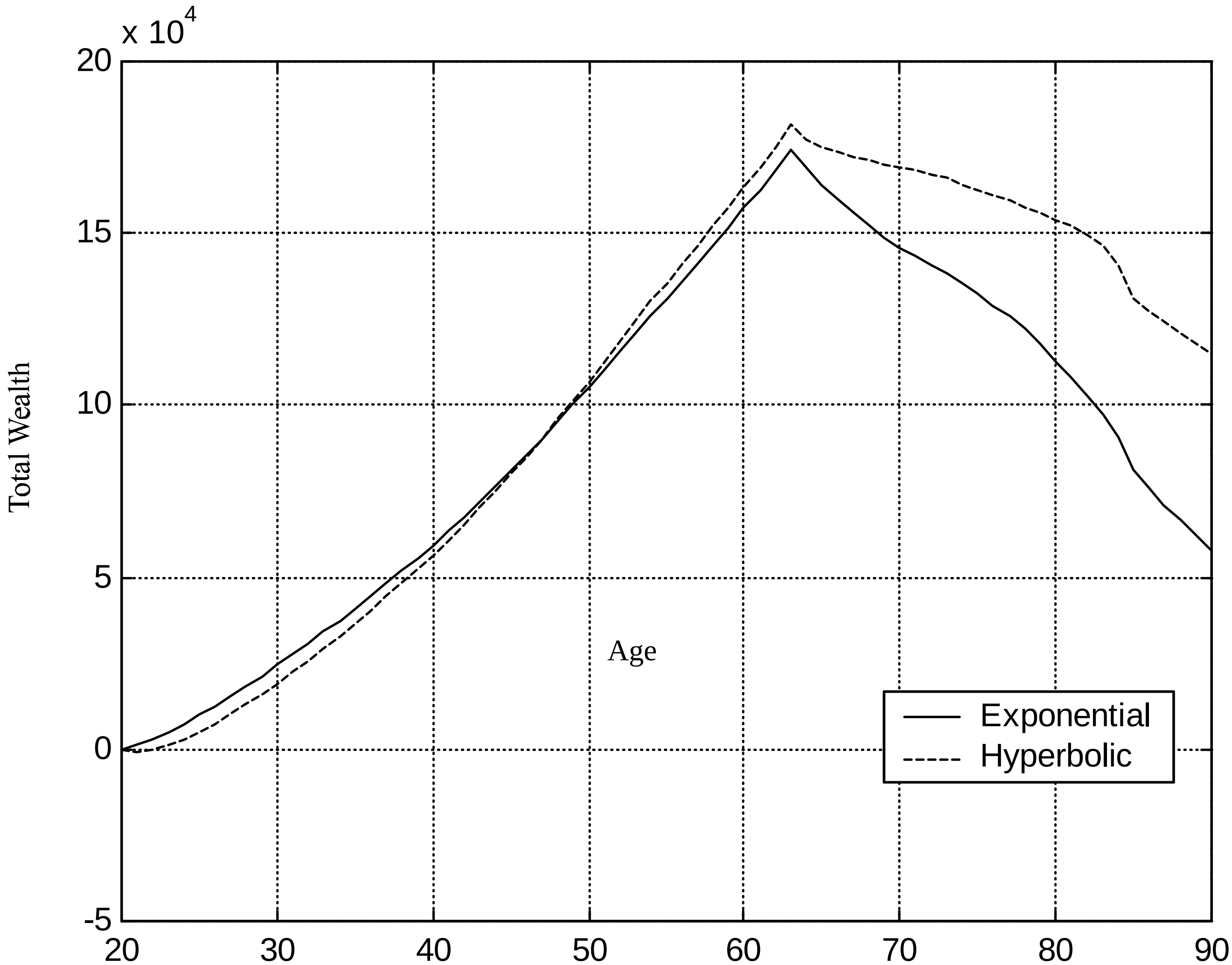


Figure 5: Simulated Mean Liquid Liabilities, Liquid Assets, Illiquid Wealth, and Total Wealth for Exponential Households



Source: Authors' simulations.
The figure plots the simulated mean level of liquid liabilities, liquid assets, illiquid wealth, and total wealth for households with high school graduate heads.

Figure 6: Mean Total Wealth of Exponential and Hyperbolic Households



Source: Authors' simulations.
The figure plots average wealth over the life-cycle for simulated exponential and hyperbolic households with high school graduate heads.

Figure 7: Mean Illiquid Wealth of Exponential and Hyperbolic Households

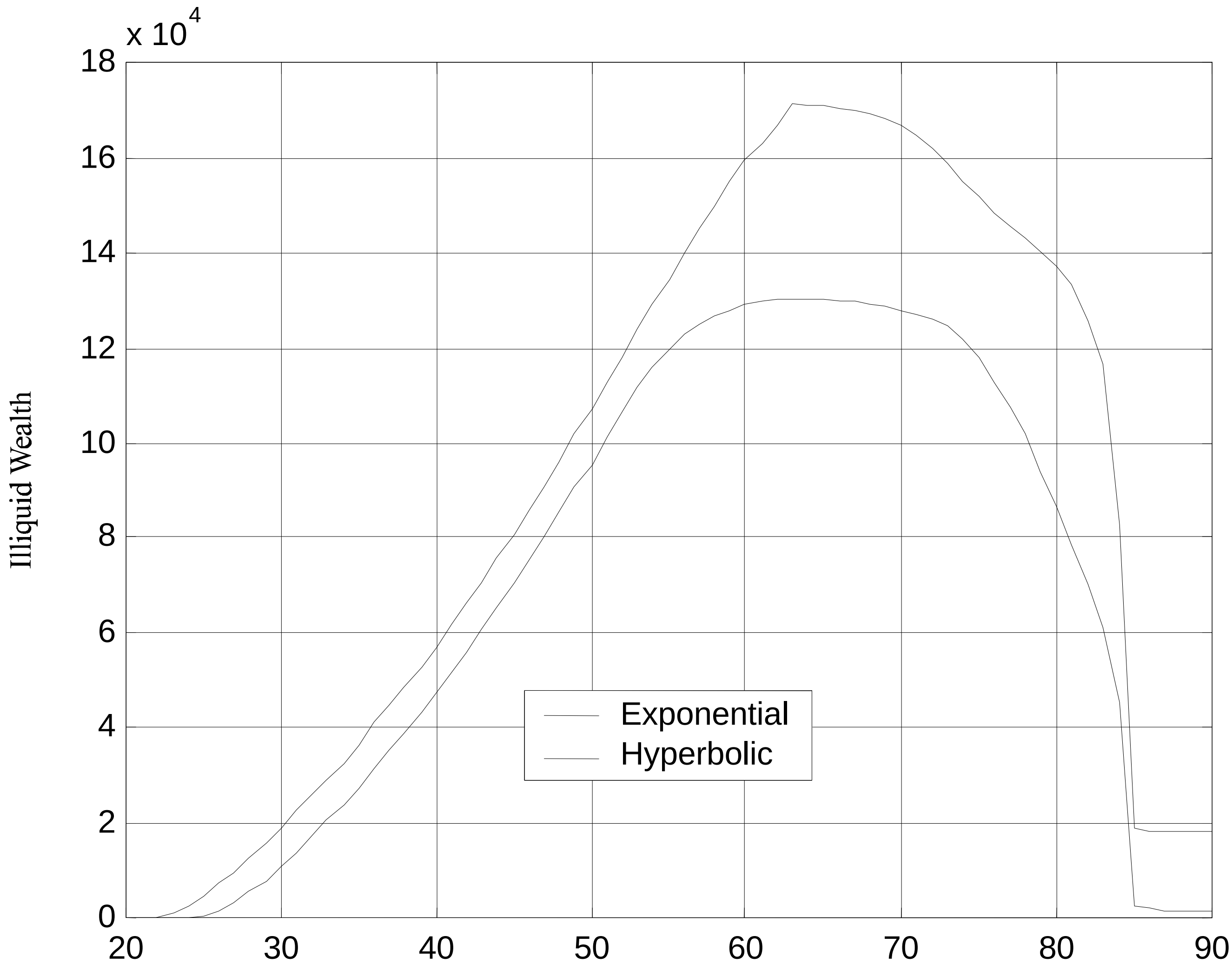


Figure 8: Mean Liquid Liabilities and Assets of Exponential and Hyperbolic Households